

# Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.1-Quadratic/1.2.1.5-  
 $a+b-x+c-x^2-p-d+e-x+f-x^2-q$

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# Chapter 1

## Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [ 123 ]. This is test number [ 36 ].

### 1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 ( 123 )	% 0.00 ( 0 )
Mathematica	% 100.00 ( 123 )	% 0.00 ( 0 )
Maple	% 98.37 ( 121 )	% 1.63 ( 2 )
Maxima	% 54.47 ( 67 )	% 45.53 ( 56 )
Fricas	% 90.24 ( 111 )	% 9.76 ( 12 )
Sympy	% 34.96 ( 43 )	% 65.04 ( 80 )
Giac	% 73.17 ( 90 )	% 26.83 ( 33 )
Mupad	% 43.09 ( 53 )	% 56.91 ( 70 )

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> <li>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.</li> <li>2. antiderivative contains a special function and the optimal antiderivative does not.</li> <li>3. antiderivative contains the imaginary unit and the optimal antiderivative does not.</li> </ol>
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

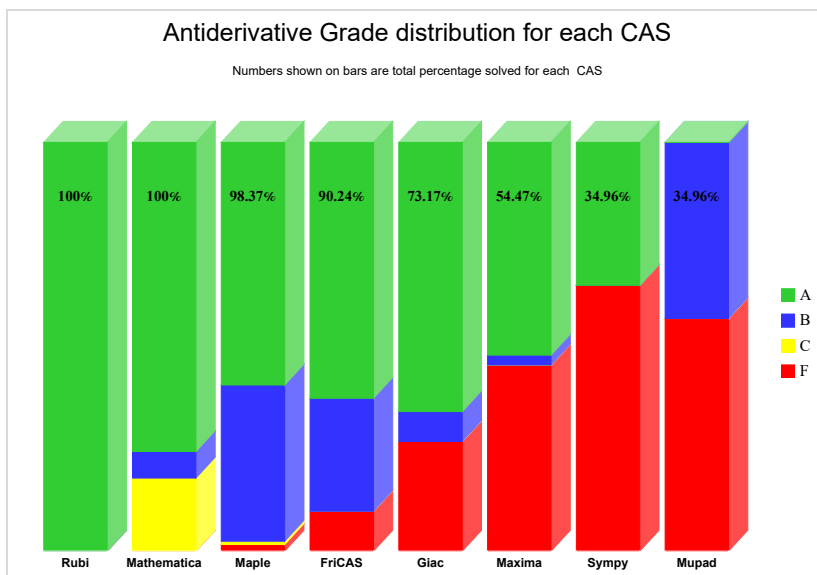
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

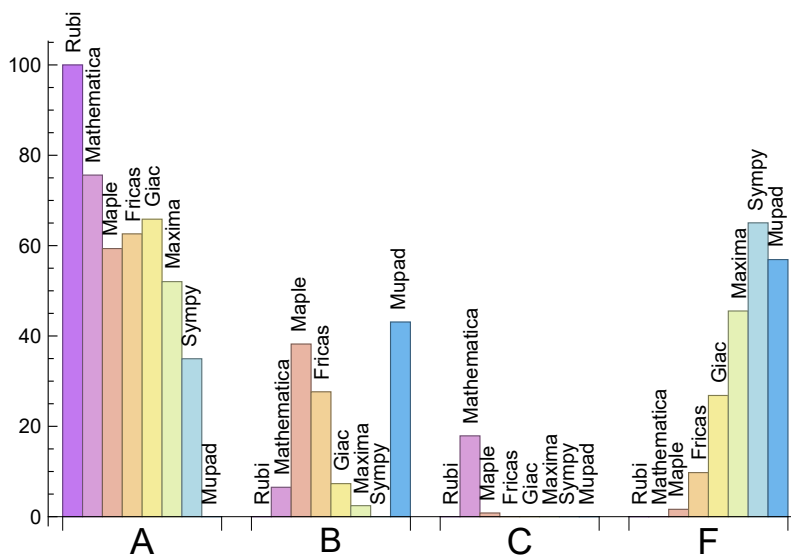
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	75.61	6.50	17.89	0.00
Maple	59.35	38.21	0.81	1.63
Maxima	52.03	2.44	0.00	45.53
Fricas	62.60	27.64	0.00	9.76
Sympy	34.96	0.00	0.00	65.04
Giac	65.85	7.32	0.00	26.83
Mupad	0.00	43.09	0.00	56.91

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This is the typical normal failure **F**.

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	2	100.00 %	0.00 %	0.00 %
Maxima	56	64.29 %	0.00 %	35.71 %
Fricas	12	41.67 %	58.33 %	0.00 %
Sympy	80	82.50 %	17.50 %	0.00 %
Giac	33	18.18 %	15.15 %	66.67 %
Mupad	70	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

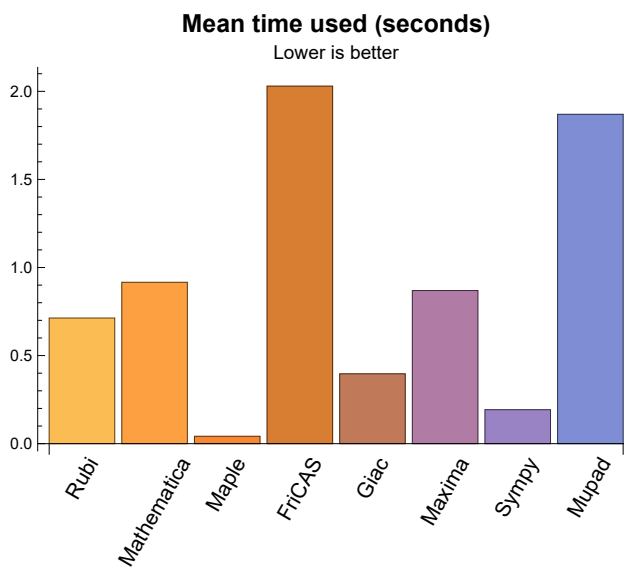
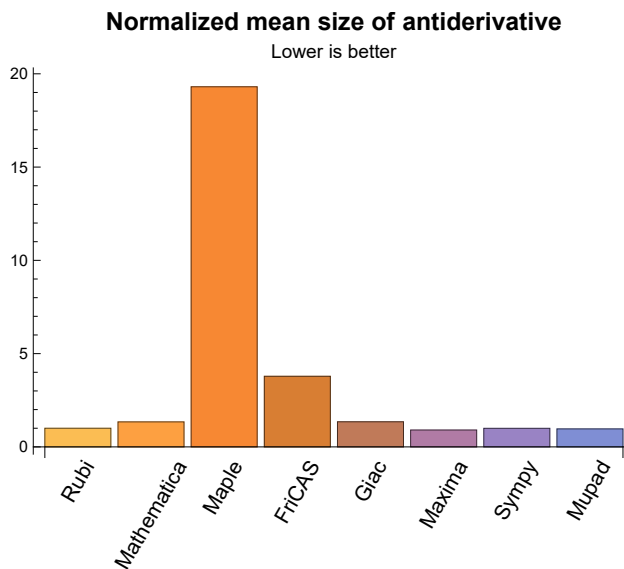
## 1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.71	205.62	1.00	128.00	1.00
Mathematica	0.92	322.87	1.34	95.00	1.00
Maple	0.04	6251.50	19.31	136.00	0.82
Maxima	0.87	90.12	0.90	72.00	0.83
Fricas	2.03	827.68	3.79	112.00	1.24
Sympy	0.19	76.86	0.99	73.00	0.98
Giac	0.40	203.17	1.35	72.50	0.79
Mupad	1.87	105.40	0.96	64.00	0.83

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.



## 1.4 list of integrals that has no closed form antiderivative

{

## 1.5 list of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

**Mupad** {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {}

**Mathematica** {6, 106, 107, 108, 113, 122, 123}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

**Mupad** Verification phase not implemented yet.



## 1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima abs\_integrate was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

## 1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error Exception raised: NotImplementedError

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

## 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function LeafSize is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user slelievre at <https://>

[ask.sagemath.org/question/57123/could-we-have-a-leaf\\_count-function-in-base-sagemath/](http://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/)

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount = 1
```

## 1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

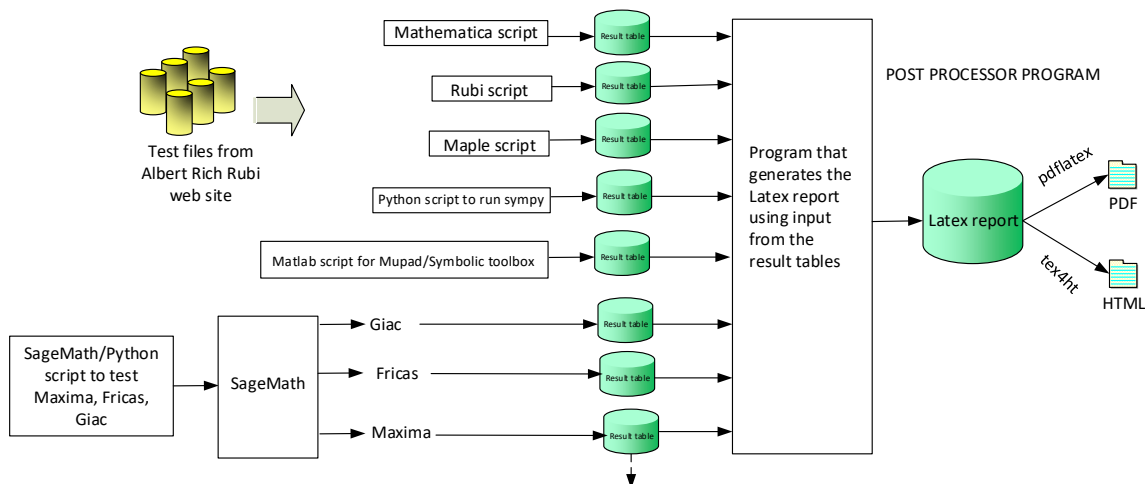
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine, 'cos(x)*sin(x)')
the_variable = evalin(symengine, 'x')
anti = int(integrand, the_variable)
```

Which gives  $\sin(x)^2/2$

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



**One record (line) per one integral result. The line is CSV comma separated. This is description of each record**

1. integer. the problem number.
  2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
  3. integer. Leaf size of result.
  4. integer. Leaf size of the optimal antiderivative.
  5. number. CPU time used to solve this integral. 0 if failed.
  6. string. The integral in Latex format
  7. string. The input used in CAS own syntax.
  8. string. The result (antiderivative) produced by CAS in Latex format
  9. string. The optimal antiderivative in Latex format.
  10. integer. 0 or 1. Indicates if problem has known antiderivative or not
  11. String. The result (antiderivative) in CAS own syntax.
  12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
- The following field present only in Rubi and Mathematica Tables*
13. integer. 1 if result was verified or 0 if not verified.
- The following fields present only in Rubi Tables*
14. integer. Number of rules used.
  15. integer. Integrand leaf size.
  16. real number. Ratio of field 14 over field 15
  17. integer. 1 if result was verified or 0 if not verified.
  18. String of form "{n,n,...}" which is list of the rules used by Rubi

**High level overview of the CAS independent integration test build system**

# Chapter 2

## detailed summary tables of results

### 2.1 List of integrals sorted by grade for each CAS

#### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123 }

B grade: { }

C grade: { }

F grade: { }

#### 2.1.2 Mathematica

A grade: { 1, 9, 10, 11, 12, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 65, 66, 67, 68, 72, 73, 74, 75, 79, 80, 81, 82, 86, 87, 88, 89, 93, 94, 95, 96, 100, 101, 102, 103, 104, 105, 106, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 122, 123 }

B grade: { 2, 3, 4, 5, 6, 7, 107, 108 }

C grade: { 8, 13, 14, 62, 63, 64, 69, 70, 71, 76, 77, 78, 83, 84, 85, 90, 91, 92, 97, 98, 99, 121 }

F grade: { }

### 2.1.3 Maple

A grade: { 1, 8, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 65, 66, 67, 68, 72, 73, 74, 75, 79, 80, 81, 82, 86, 87, 88, 89, 93, 96, 111, 120, 122, 123 }

B grade: { 2, 3, 4, 5, 6, 7, 14, 62, 63, 64, 69, 70, 71, 76, 77, 78, 83, 84, 85, 90, 91, 92, 94, 95, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 112, 113, 114, 115, 116, 117, 118, 119, 121 }

C grade: { 11 }

F grade: { 9, 10 }

### 2.1.4 Maxima

A grade: { 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 65, 66, 67, 68, 72, 73, 74, 75, 79, 80, 81, 82, 86, 87, 88, 89, 96 }

B grade: { 93, 94, 95 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 62, 63, 64, 69, 70, 71, 76, 77, 78, 83, 84, 85, 90, 91, 92, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123 }

### 2.1.5 FriCAS

A grade: { 1, 8, 11, 12, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 65, 66, 67, 68, 72, 73, 74, 75, 79, 80, 81, 82, 86, 87, 88, 93, 94, 96, 100, 101, 105, 109, 110, 111, 119 }

B grade: { 2, 3, 4, 5, 6, 7, 62, 63, 64, 69, 70, 71, 76, 77, 78, 83, 84, 85, 89, 90, 91, 92, 95, 97, 98, 99, 104, 112, 114, 115, 116, 118, 120, 121 }

C grade: { }

F grade: { 9, 10, 13, 102, 103, 106, 107, 108, 113, 117, 122, 123 }

### 2.1.6 Sympy

A grade: { 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123 }

## 2.1.7 Giac

A grade: { 1, 11, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 65, 66, 67, 68, 72, 73, 74, 75, 79, 80, 81, 82, 86, 87, 88, 89, 93, 94, 95, 96, 100, 101, 105, 109, 110, 111, 114, 115, 116, 118, 119, 120 }

B grade: { 2, 3, 4, 5, 8, 12, 14, 104, 121 }

C grade: { }

F grade: { 6, 7, 9, 10, 13, 62, 63, 64, 69, 70, 71, 76, 77, 78, 83, 84, 85, 90, 91, 92, 97, 98, 99, 102, 103, 106, 107, 108, 112, 113, 117, 122, 123 }

## 2.1.8 Mupad

A grade: { }

B grade: { 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 89, 96, 100, 101, 116, 120 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 97, 98, 99, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 117, 118, 119, 121, 122, 123 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as  $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	87	136	0	205	0	84	-1
normalized size	1	1.00	0.85	1.33	0.00	2.01	0.00	0.82	-0.01
time (sec)	N/A	0.094	0.195	0.028	0.000	1.207	0.000	0.256	0.000
Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	178	491	0	1079	0	847	-1
normalized size	1	1.00	2.17	5.99	0.00	13.16	0.00	10.33	-0.01
time (sec)	N/A	0.111	0.394	0.060	0.000	1.713	0.000	3.037	0.000
Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	161	307	0	813	0	703	-1
normalized size	1	1.00	2.44	4.65	0.00	12.32	0.00	10.65	-0.02
time (sec)	N/A	0.079	0.228	0.029	0.000	1.428	0.000	1.898	0.000



Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	296	829	0	1544	0	1166	-1
normalized size	1	1.00	2.29	6.43	0.00	11.97	0.00	9.04	-0.01
time (sec)	N/A	0.169	0.923	0.033	0.000	1.865	0.000	2.265	0.000

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	1746	1884	0	3818	0	2986	-1
normalized size	1	1.00	7.79	8.41	0.00	17.04	0.00	13.33	-0.00
time (sec)	N/A	0.427	6.382	0.032	0.000	9.944	0.000	4.870	0.000

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	3382	3695	0	8134	0	0	-1
normalized size	1	1.00	10.31	11.27	0.00	24.80	0.00	0.00	-0.00
time (sec)	N/A	0.970	6.607	0.038	0.000	40.657	0.000	0.000	0.000

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	490	1377	0	2005	0	0	-1
normalized size	1	1.00	3.02	8.50	0.00	12.38	0.00	0.00	-0.01
time (sec)	N/A	0.306	1.965	0.033	0.000	3.066	0.000	0.000	0.000

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	84	27	0	38	0	52	-1
normalized size	1	1.00	3.00	0.96	0.00	1.36	0.00	1.86	-0.04
time (sec)	N/A	0.017	0.074	0.022	0.000	0.844	0.000	0.219	0.000

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	142	0	0	0	0	0	-1
normalized size	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.143	0.253	0.000	0.908	0.000	0.000	0.000

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	172	0	0	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.134	0.243	0.302	0.000	0.905	0.000	0.000	0.000

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	27	16	0	22	0	49	-1
normalized size	1	1.00	0.56	0.33	0.00	0.46	0.00	1.02	-0.02
time (sec)	N/A	0.015	0.020	0.046	0.000	0.862	0.000	0.229	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	66	84	0	82	0	143	-1
normalized size	1	1.00	0.94	1.20	0.00	1.17	0.00	2.04	-0.01
time (sec)	N/A	0.051	0.088	0.021	0.000	0.912	0.000	0.260	0.000

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1077	1077	600	661	0	0	0	0	-1
normalized size	1	1.00	0.56	0.61	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.099	1.639	0.248	0.000	0.861	0.000	0.000	0.000

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	159	341	0	161	0	171	-1
normalized size	1	1.00	1.62	3.48	0.00	1.64	0.00	1.74	-0.01
time (sec)	N/A	0.210	0.395	0.030	0.000	0.955	0.000	0.215	0.000

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	55	54	54	65	54	54
normalized size	1	1.00	1.00	0.81	0.79	0.79	0.96	0.79	0.79
time (sec)	N/A	0.049	0.003	0.001	0.460	0.722	0.086	0.206	0.062

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	45	44	44	53	44	44
normalized size	1	1.00	1.00	0.80	0.79	0.79	0.95	0.79	0.79
time (sec)	N/A	0.038	0.002	0.001	0.447	0.720	0.081	0.181	0.034

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	35	34	34	41	34	34
normalized size	1	1.00	1.00	0.80	0.77	0.77	0.93	0.77	0.77
time (sec)	N/A	0.029	0.001	0.001	0.434	0.691	0.075	0.209	0.025

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
normalized size	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.016	0.001	0.001	0.432	0.692	0.062	0.182	0.023

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	34	33	33	49	33	35
normalized size	1	1.00	1.00	0.81	0.79	0.79	1.17	0.79	0.83
time (sec)	N/A	0.040	0.016	0.005	0.975	0.647	0.141	0.176	3.393

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	34	36	45	42	36	35
normalized size	1	1.00	1.00	0.79	0.84	1.05	0.98	0.84	0.81
time (sec)	N/A	0.027	0.015	0.005	0.969	0.734	0.157	0.209	0.038

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	53	47	56	75	63	46	55
normalized size	1	1.00	0.83	0.73	0.88	1.17	0.98	0.72	0.86
time (sec)	N/A	0.037	0.026	0.005	0.958	0.796	0.182	0.175	0.049

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	80	65	64	64	76	64	64
normalized size	1	1.00	1.00	0.81	0.80	0.80	0.95	0.80	0.80
time (sec)	N/A	0.060	0.003	0.002	0.452	0.772	0.102	0.201	0.085

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	66	55	54	54	63	54	54
normalized size	1	1.00	1.00	0.83	0.82	0.82	0.95	0.82	0.82
time (sec)	N/A	0.048	0.002	0.001	0.438	0.507	0.086	0.215	0.054

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	44	44	51	44	44
normalized size	1	1.00	1.00	0.83	0.81	0.81	0.94	0.81	0.81
time (sec)	N/A	0.042	0.002	0.001	0.437	0.980	0.082	0.202	0.034

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	35	34	34	41	34	34
normalized size	1	1.00	1.00	0.76	0.74	0.74	0.89	0.74	0.74
time (sec)	N/A	0.030	0.001	0.001	0.435	1.108	0.072	0.192	0.025

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	53	44	43	43	63	43	45
normalized size	1	1.00	0.95	0.79	0.77	0.77	1.12	0.77	0.80
time (sec)	N/A	0.051	0.020	0.004	0.968	0.715	0.150	0.210	3.449

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	59	51	52	78	65	52	51
normalized size	1	1.00	0.94	0.81	0.83	1.24	1.03	0.83	0.81
time (sec)	N/A	0.061	0.032	0.006	0.961	0.871	0.198	0.172	0.050

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	53	47	56	75	63	46	55
normalized size	1	1.00	0.83	0.73	0.88	1.17	0.98	0.72	0.86
time (sec)	N/A	0.052	0.024	0.006	0.968	0.844	0.199	0.185	3.442

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	63	57	76	105	83	56	75
normalized size	1	1.00	0.74	0.67	0.89	1.24	0.98	0.66	0.88
time (sec)	N/A	0.063	0.044	0.007	0.962	0.841	0.229	0.183	3.468

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	96	75	74	74	92	74	74
normalized size	1	1.00	1.00	0.78	0.77	0.77	0.96	0.77	0.77
time (sec)	N/A	0.071	0.004	0.000	0.432	0.779	0.097	0.176	0.121

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	65	64	64	78	64	64
normalized size	1	1.00	1.00	0.79	0.78	0.78	0.95	0.78	0.78
time (sec)	N/A	0.056	0.002	0.001	0.426	0.803	0.090	0.205	0.081

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	55	54	54	65	54	54
normalized size	1	1.00	1.00	0.81	0.79	0.79	0.96	0.79	0.79
time (sec)	N/A	0.051	0.002	0.002	0.429	0.769	0.084	0.202	0.054

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	45	44	44	53	44	44
normalized size	1	1.00	1.00	0.80	0.79	0.79	0.95	0.79	0.79
time (sec)	N/A	0.032	0.001	0.001	0.434	0.826	0.077	0.201	0.033

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	63	54	53	53	76	53	55
normalized size	1	1.00	0.90	0.77	0.76	0.76	1.09	0.76	0.79
time (sec)	N/A	0.053	0.023	0.004	0.963	0.875	0.159	0.211	0.043

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	61	62	88	78	62	61
normalized size	1	1.00	1.00	0.79	0.81	1.14	1.01	0.81	0.79
time (sec)	N/A	0.072	0.027	0.008	0.961	0.845	0.188	0.206	3.431

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	78	63	72	118	85	62	71
normalized size	1	1.00	0.93	0.75	0.86	1.40	1.01	0.74	0.85
time (sec)	N/A	0.087	0.037	0.009	0.970	0.882	0.235	0.189	3.426

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	72	64	63	63	87	63	65
normalized size	1	1.00	0.86	0.76	0.75	0.75	1.04	0.75	0.77
time (sec)	N/A	0.057	0.028	0.006	0.976	0.802	0.167	0.211	3.441

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	63	54	53	53	73	53	55
normalized size	1	1.00	0.90	0.77	0.76	0.76	1.04	0.76	0.79
time (sec)	N/A	0.056	0.021	0.004	0.968	1.024	0.152	0.213	0.042

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	52	44	43	43	60	43	45
normalized size	1	1.00	0.93	0.79	0.77	0.77	1.07	0.77	0.80
time (sec)	N/A	0.050	0.017	0.004	0.958	0.852	0.155	0.182	3.445

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	34	33	33	46	33	35
normalized size	1	1.00	1.00	0.81	0.79	0.79	1.10	0.79	0.83
time (sec)	N/A	0.035	0.010	0.004	0.950	0.863	0.141	0.207	0.039

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	60	59	59	83	59	79
normalized size	1	1.00	1.00	0.82	0.81	0.81	1.14	0.81	1.08
time (sec)	N/A	0.053	0.032	0.006	0.963	0.807	0.244	0.191	0.187

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	94	77	78	117	102	78	95
normalized size	1	1.00	1.00	0.82	0.83	1.24	1.09	0.83	1.01
time (sec)	N/A	0.089	0.083	0.007	0.960	0.962	0.322	0.198	3.567

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	104	89	98	177	119	88	115
normalized size	1	1.00	0.90	0.77	0.85	1.54	1.03	0.77	1.00
time (sec)	N/A	0.124	0.155	0.008	0.959	1.019	0.363	0.180	0.179



Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	91	71	72	98	90	72	72
normalized size	1	1.00	1.00	0.78	0.79	1.08	0.99	0.79	0.79
time (sec)	N/A	0.086	0.052	0.010	0.964	1.168	0.198	0.183	3.462

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	61	62	88	75	62	61
normalized size	1	1.00	1.00	0.79	0.81	1.14	0.97	0.81	0.79
time (sec)	N/A	0.073	0.028	0.008	0.960	0.732	0.191	0.195	3.424

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	51	52	78	61	52	52
normalized size	1	1.00	1.00	0.81	0.83	1.24	0.97	0.83	0.83
time (sec)	N/A	0.063	0.031	0.007	0.966	0.970	0.189	0.173	3.404

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	34	36	45	42	36	36
normalized size	1	1.00	1.00	0.79	0.84	1.05	0.98	0.84	0.84
time (sec)	N/A	0.026	0.015	0.004	0.964	0.949	0.154	0.189	0.040

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	94	77	78	117	102	78	96
normalized size	1	1.00	1.00	0.82	0.83	1.24	1.09	0.83	1.02
time (sec)	N/A	0.088	0.061	0.007	0.963	0.926	0.319	0.192	3.578

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	106	94	96	167	122	96	115
normalized size	1	1.00	0.83	0.74	0.76	1.31	0.96	0.76	0.91
time (sec)	N/A	0.123	0.057	0.010	0.961	0.862	0.359	0.193	0.177

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	136	106	118	237	143	110	135
normalized size	1	1.00	0.92	0.72	0.80	1.60	0.97	0.74	0.91
time (sec)	N/A	0.161	0.076	0.011	0.968	1.070	0.400	0.202	3.586

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	98	73	82	128	95	72	81
normalized size	1	1.00	1.00	0.74	0.84	1.31	0.97	0.73	0.83
time (sec)	N/A	0.113	0.038	0.008	0.951	0.818	0.241	0.207	0.045

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	84	63	72	118	82	62	72
normalized size	1	1.00	1.00	0.75	0.86	1.40	0.98	0.74	0.86
time (sec)	N/A	0.087	0.037	0.007	0.958	0.820	0.229	0.183	0.049

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	51	47	56	75	63	46	56
normalized size	1	1.00	0.80	0.73	0.88	1.17	0.98	0.72	0.88
time (sec)	N/A	0.053	0.029	0.006	0.961	0.873	0.201	0.186	3.474

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	51	47	56	75	61	46	55
normalized size	1	1.00	0.80	0.73	0.88	1.17	0.95	0.72	0.86
time (sec)	N/A	0.033	0.028	0.007	0.960	0.917	0.183	0.205	0.044

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	99	89	98	177	122	88	116
normalized size	1	1.00	0.86	0.77	0.85	1.54	1.06	0.77	1.01
time (sec)	N/A	0.124	0.161	0.008	0.973	0.833	0.357	0.211	3.580

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	136	106	116	227	143	110	136
normalized size	1	1.00	0.85	0.66	0.72	1.42	0.89	0.69	0.85
time (sec)	N/A	0.160	0.124	0.011	0.974	0.868	0.427	0.217	3.597

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	151	118	138	297	163	116	155
normalized size	1	1.00	0.83	0.65	0.76	1.64	0.90	0.64	0.86
time (sec)	N/A	0.204	0.095	0.013	0.972	0.734	0.470	0.219	3.588

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	85	166	177	98	0	93	221
normalized size	1	1.00	0.41	0.80	0.85	0.47	0.00	0.45	1.06
time (sec)	N/A	0.309	0.296	0.031	1.006	0.660	0.000	0.249	5.028

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	75	132	143	88	0	83	187
normalized size	1	1.00	0.45	0.80	0.86	0.53	0.00	0.50	1.13
time (sec)	N/A	0.181	0.171	0.008	0.982	0.899	0.000	0.254	4.692

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	65	98	109	78	0	73	153
normalized size	1	1.00	0.52	0.79	0.88	0.63	0.00	0.59	1.23
time (sec)	N/A	0.099	0.105	0.007	0.976	0.847	0.000	0.217	4.185

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	55	64	75	68	0	63	119
normalized size	1	1.00	0.67	0.78	0.91	0.83	0.00	0.77	1.45
time (sec)	N/A	0.040	0.052	0.006	0.961	1.012	0.000	0.238	3.835

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	185	2065	0	2016	0	0	-1
normalized size	1	1.00	1.06	11.87	0.00	11.59	0.00	0.00	-0.01
time (sec)	N/A	0.441	0.457	0.149	0.000	2.105	0.000	0.000	0.000

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	214	16357	0	2102	0	0	-1
normalized size	1	1.00	1.14	87.01	0.00	11.18	0.00	0.00	-0.01
time (sec)	N/A	0.393	1.036	0.232	0.000	2.365	0.000	0.000	0.000

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	299	44343	0	2182	0	0	-1
normalized size	1	1.00	1.34	198.85	0.00	9.78	0.00	0.00	-0.00
time (sec)	N/A	0.459	2.075	0.372	0.000	2.816	0.000	0.000	0.000

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	95	185	206	108	0	103	-1
normalized size	1	1.00	0.41	0.80	0.89	0.47	0.00	0.45	-0.00
time (sec)	N/A	0.342	0.367	0.035	1.022	0.668	0.000	0.255	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	85	151	172	98	0	93	-1
normalized size	1	1.00	0.45	0.80	0.91	0.52	0.00	0.49	-0.01
time (sec)	N/A	0.190	0.231	0.007	0.991	0.988	0.000	0.256	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	75	117	138	88	0	83	-1
normalized size	1	1.00	0.51	0.80	0.94	0.60	0.00	0.56	-0.01
time (sec)	N/A	0.122	0.141	0.009	0.983	0.805	0.000	0.257	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	65	83	104	78	0	73	-1
normalized size	1	1.00	0.62	0.79	0.99	0.74	0.00	0.70	-0.01
time (sec)	N/A	0.050	0.080	0.006	0.963	0.739	0.000	0.230	0.000

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	310	3460	0	2027	0	0	-1
normalized size	1	1.00	1.57	17.56	0.00	10.29	0.00	0.00	-0.01
time (sec)	N/A	0.488	0.693	0.053	0.000	1.975	0.000	0.000	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	530	28185	0	2150	0	0	-1
normalized size	1	1.00	2.28	121.49	0.00	9.27	0.00	0.00	-0.00
time (sec)	N/A	0.575	2.562	0.158	0.000	2.643	0.000	0.000	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	1262	81552	0	2183	0	0	-1
normalized size	1	1.00	5.66	365.70	0.00	9.79	0.00	0.00	-0.00
time (sec)	N/A	0.433	5.342	0.350	0.000	2.609	0.000	0.000	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	105	204	235	118	0	113	-1
normalized size	1	1.00	0.41	0.80	0.93	0.46	0.00	0.44	-0.00
time (sec)	N/A	0.373	0.452	0.040	1.021	0.811	0.000	0.536	0.000

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	95	170	201	108	0	103	-1
normalized size	1	1.00	0.45	0.80	0.95	0.51	0.00	0.49	-0.00
time (sec)	N/A	0.220	0.296	0.009	0.996	1.005	0.000	0.549	0.000

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	85	136	167	98	0	93	-1
normalized size	1	1.00	0.50	0.80	0.98	0.58	0.00	0.55	-0.01
time (sec)	N/A	0.130	0.183	0.007	1.028	0.795	0.000	0.516	0.000

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	75	102	133	88	0	83	-1
normalized size	1	1.00	0.59	0.80	1.04	0.69	0.00	0.65	-0.01
time (sec)	N/A	0.062	0.112	0.006	0.974	0.696	0.000	0.391	0.000

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	229	4860	0	2010	0	0	-1
normalized size	1	1.00	1.03	21.89	0.00	9.05	0.00	0.00	-0.00
time (sec)	N/A	0.539	1.047	0.053	0.000	2.250	0.000	0.000	0.000

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	685	40028	0	2161	0	0	-1
normalized size	1	1.00	2.69	156.97	0.00	8.47	0.00	0.00	-0.00
time (sec)	N/A	0.660	1.655	0.157	0.000	2.739	0.000	0.000	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	1009	119458	0	2240	0	0	-1
normalized size	1	1.00	3.59	425.12	0.00	7.97	0.00	0.00	-0.00
time (sec)	N/A	0.655	1.996	0.384	0.000	2.978	0.000	0.000	0.000

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	75	147	148	88	0	83	-1
normalized size	1	1.00	0.41	0.79	0.80	0.48	0.00	0.45	-0.01
time (sec)	N/A	0.312	0.241	0.019	0.999	0.769	0.000	0.503	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	65	113	114	78	0	73	-1
normalized size	1	1.00	0.45	0.79	0.80	0.55	0.00	0.51	-0.01
time (sec)	N/A	0.168	0.134	0.008	0.977	0.682	0.000	0.524	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	55	79	80	68	0	63	-1
normalized size	1	1.00	0.54	0.78	0.79	0.67	0.00	0.62	-0.01
time (sec)	N/A	0.088	0.075	0.008	0.965	0.731	0.000	0.487	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	45	45	46	58	0	53	-1
normalized size	1	1.00	0.76	0.76	0.78	0.98	0.00	0.90	-0.02
time (sec)	N/A	0.033	0.039	0.006	0.956	0.812	0.000	0.530	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	176	684	0	2002	0	0	-1
normalized size	1	1.00	1.19	4.62	0.00	13.53	0.00	0.00	-0.01
time (sec)	N/A	0.314	0.300	0.004	0.000	2.250	0.000	0.000	0.000



Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	287	5225	0	2102	0	0	-1
normalized size	1	1.00	1.53	27.79	0.00	11.18	0.00	0.00	-0.01
time (sec)	N/A	0.429	0.996	0.006	0.000	2.601	0.000	0.000	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	1277	13040	0	2183	0	0	-1
normalized size	1	1.00	5.73	58.48	0.00	9.79	0.00	0.00	-0.00
time (sec)	N/A	0.469	6.230	0.007	0.000	2.372	0.000	0.000	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	95	166	148	112	0	82	-1
normalized size	1	1.00	0.57	1.00	0.89	0.67	0.00	0.49	-0.01
time (sec)	N/A	0.204	0.373	0.029	0.985	0.914	0.000	0.251	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	65	132	114	102	0	72	-1
normalized size	1	1.00	0.52	1.06	0.92	0.82	0.00	0.58	-0.01
time (sec)	N/A	0.127	0.232	0.008	0.973	0.945	0.000	0.281	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	55	98	80	92	0	62	-1
normalized size	1	1.00	0.67	1.20	0.98	1.12	0.00	0.76	-0.01
time (sec)	N/A	0.071	0.143	0.008	0.959	0.884	0.000	0.237	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	64	46	82	0	53	87
normalized size	1	1.00	1.00	1.42	1.02	1.82	0.00	1.18	1.93
time (sec)	N/A	0.029	0.076	0.006	0.955	0.804	0.000	0.234	0.227

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	202	718	0	2083	0	0	-1
normalized size	1	1.00	1.15	4.08	0.00	11.84	0.00	0.00	-0.01
time (sec)	N/A	0.408	1.287	0.034	0.000	2.341	0.000	0.000	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	740	5942	0	2173	0	0	-1
normalized size	1	1.00	3.51	28.16	0.00	10.30	0.00	0.00	-0.00
time (sec)	N/A	0.473	1.525	0.099	0.000	2.526	0.000	0.000	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	231	18981	0	2263	0	0	-1
normalized size	1	1.00	0.94	77.16	0.00	9.20	0.00	0.00	-0.00
time (sec)	N/A	0.525	2.206	0.208	0.000	3.041	0.000	0.000	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	75	214	253	132	0	81	-1
normalized size	1	1.00	0.51	1.46	1.72	0.90	0.00	0.55	-0.01
time (sec)	N/A	0.167	0.513	0.031	1.002	0.647	0.000	0.252	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	65	180	219	122	0	72	-1
normalized size	1	1.00	0.62	1.71	2.09	1.16	0.00	0.69	-0.01
time (sec)	N/A	0.105	0.338	0.008	1.020	0.602	0.000	0.268	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	55	146	185	112	0	61	-1
normalized size	1	1.00	0.81	2.15	2.72	1.65	0.00	0.90	-0.01
time (sec)	N/A	0.061	0.236	0.007	0.978	0.684	0.000	0.267	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	33	30	59	51	0	29	29
normalized size	1	1.00	0.70	0.64	1.26	1.09	0.00	0.62	0.62
time (sec)	N/A	0.022	0.103	0.005	0.432	0.569	0.000	0.232	0.090

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	218	751	0	2133	0	0	-1
normalized size	1	1.00	1.10	3.77	0.00	10.72	0.00	0.00	-0.01
time (sec)	N/A	0.456	0.868	0.039	0.000	2.042	0.000	0.000	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	296	5975	0	2253	0	0	-1
normalized size	1	1.00	1.26	25.53	0.00	9.63	0.00	0.00	-0.00
time (sec)	N/A	0.543	1.153	0.104	0.000	1.992	0.000	0.000	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	242	19014	0	2343	0	0	-1
normalized size	1	1.00	0.90	70.68	0.00	8.71	0.00	0.00	-0.00
time (sec)	N/A	0.589	1.958	0.209	0.000	1.994	0.000	0.000	0.000

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	436	436	657	1429	0	1269	0	638	1299
normalized size	1	1.00	1.51	3.28	0.00	2.91	0.00	1.46	2.98
time (sec)	N/A	0.789	0.935	0.023	0.000	1.322	0.000	0.365	5.312

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	173	453	0	465	0	212	320
normalized size	1	1.00	0.99	2.59	0.00	2.66	0.00	1.21	1.83
time (sec)	N/A	0.164	0.273	0.007	0.000	0.770	0.000	0.299	3.906

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	431	431	417	6019	0	0	0	0	-1
normalized size	1	1.00	0.97	13.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.051	1.219	0.052	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	488	488	555	22287	0	0	0	0	-1
normalized size	1	1.00	1.14	45.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.929	5.093	0.037	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	564	564	829	2458	0	2179	0	1150	-1
normalized size	1	1.00	1.47	4.36	0.00	3.86	0.00	2.04	-0.00
time (sec)	N/A	0.935	1.762	0.025	0.000	3.790	0.000	0.598	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	392	862	0	839	0	417	-1
normalized size	1	1.00	1.66	3.65	0.00	3.56	0.00	1.77	-0.00
time (sec)	N/A	0.230	0.606	0.009	0.000	1.268	0.000	0.612	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	679	678	1232	22523	0	0	0	0	-1
normalized size	1	1.00	1.81	33.17	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	11.033	4.576	0.029	0.000	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	704	704	2843	72576	0	0	0	0	-1
normalized size	1	1.00	4.04	103.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	11.950	6.831	0.039	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	671	669	4727	178044	0	0	0	0	-1
normalized size	1	1.00	7.04	265.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	11.597	7.229	0.066	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	717	717	615	1930	0	1583	0	824	-1
normalized size	1	1.00	0.86	2.69	0.00	2.21	0.00	1.15	-0.00
time (sec)	N/A	2.710	1.346	0.028	0.000	3.195	0.000	0.528	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	251	706	0	637	0	304	-1
normalized size	1	1.00	0.79	2.23	0.00	2.02	0.00	0.96	-0.00
time (sec)	N/A	0.626	0.503	0.014	0.000	1.435	0.000	0.632	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	96	185	0	227	0	98	-1
normalized size	1	1.00	0.83	1.59	0.00	1.96	0.00	0.84	-0.01
time (sec)	N/A	0.111	0.146	0.008	0.000	1.455	0.000	0.499	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	374	374	376	761	0	11287	0	0	-1
normalized size	1	1.00	1.01	2.03	0.00	30.18	0.00	0.00	-0.00
time (sec)	N/A	0.579	1.454	0.025	0.000	14.925	0.000	0.000	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	789	787	1377	3858	0	0	0	0	-1
normalized size	1	1.00	1.75	4.89	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	8.210	6.689	0.031	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	649	649	745	2827	0	3143	0	1099	-1
normalized size	1	1.00	1.15	4.36	0.00	4.84	0.00	1.69	-0.00
time (sec)	N/A	2.106	1.664	0.030	0.000	5.599	0.000	0.423	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	288	1011	0	1305	0	407	-1
normalized size	1	1.00	0.93	3.27	0.00	4.22	0.00	1.32	-0.00
time (sec)	N/A	0.447	0.733	0.015	0.000	3.263	0.000	0.352	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	113	249	0	429	0	122	143
normalized size	1	1.00	1.02	2.24	0.00	3.86	0.00	1.10	1.29
time (sec)	N/A	0.080	0.311	0.006	0.000	1.744	0.000	0.312	3.734

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	666	666	700	4099	0	0	0	0	-1
normalized size	1	1.00	1.05	6.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.829	5.343	0.028	0.000	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	891	891	872	4635	0	3995	0	1401	-1
normalized size	1	1.00	0.98	5.20	0.00	4.48	0.00	1.57	-0.00
time (sec)	N/A	1.768	2.191	0.031	0.000	12.868	0.000	0.455	0.000

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	444	444	387	1786	0	1581	0	587	-1
normalized size	1	1.00	0.87	4.02	0.00	3.56	0.00	1.32	-0.00
time (sec)	N/A	0.451	1.257	0.014	0.000	6.519	0.000	0.338	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	147	185	0	286	0	240	175
normalized size	1	1.00	1.12	1.41	0.00	2.18	0.00	1.83	1.34
time (sec)	N/A	0.085	0.378	0.007	0.000	4.021	0.000	0.322	3.708

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	81	144	0	154	0	205	-1
normalized size	1	1.00	1.59	2.82	0.00	3.02	0.00	4.02	-0.02
time (sec)	N/A	0.066	0.041	0.024	0.000	0.755	0.000	0.276	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1432	1432	670	928	0	0	0	0	-1
normalized size	1	1.00	0.47	0.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	6.219	2.341	0.272	0.000	1.039	0.000	0.000	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	652	652	390	418	0	0	0	0	-1
normalized size	1	1.00	0.60	0.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.677	0.611	0.468	0.000	0.930	0.000	0.000	0.000



## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio  $\frac{\text{number of rules}}{\text{integrand size}}$  is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [14] had the largest ratio of [.4444]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	4	1.00	29	0.138
2	A	2	2	1.00	31	0.065
3	A	2	2	1.00	27	0.074
4	A	4	4	1.00	27	0.148
5	A	5	5	1.00	27	0.185
6	A	6	5	1.00	27	0.185
7	A	4	4	1.00	31	0.129
8	A	2	2	1.00	23	0.087
9	A	2	2	1.00	31	0.065
10	A	2	2	1.00	34	0.059
11	A	3	3	1.00	22	0.136
12	A	6	5	1.00	18	0.278
13	A	3	3	1.00	26	0.115
14	A	16	12	1.00	27	0.444
15	A	2	1	1.00	23	0.043

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
16	A	2	1	1.00	23	0.043
17	A	2	1	1.00	23	0.043
18	A	2	1	1.00	21	0.048
19	A	6	5	1.00	23	0.217
20	A	4	4	1.00	23	0.174
21	A	5	5	1.00	23	0.217
22	A	2	1	1.00	25	0.040
23	A	2	1	1.00	25	0.040
24	A	2	1	1.00	25	0.040
25	A	2	1	1.00	23	0.043
26	A	6	5	1.00	25	0.200
27	A	7	6	1.00	25	0.240
28	A	5	4	1.00	25	0.160
29	A	6	5	1.00	25	0.200
30	A	2	1	1.00	25	0.040
31	A	2	1	1.00	25	0.040
32	A	2	1	1.00	25	0.040
33	A	2	1	1.00	23	0.043
34	A	6	5	1.00	25	0.200
35	A	7	6	1.00	25	0.240
36	A	8	6	1.00	25	0.240
37	A	6	5	1.00	25	0.200
38	A	6	5	1.00	25	0.200
39	A	6	5	1.00	25	0.200
40	A	6	5	1.00	23	0.217
41	A	9	5	1.00	25	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
42	A	10	6	1.00	25	0.240
43	A	11	7	1.00	25	0.280
44	A	7	6	1.00	25	0.240
45	A	7	6	1.00	25	0.240
46	A	7	6	1.00	25	0.240
47	A	4	4	1.00	23	0.174
48	A	10	6	1.00	25	0.240
49	A	11	7	1.00	25	0.280
50	A	12	7	1.00	25	0.280
51	A	8	6	1.00	25	0.240
52	A	8	6	1.00	25	0.240
53	A	5	4	1.00	25	0.160
54	A	5	5	1.00	23	0.217
55	A	11	7	1.00	25	0.280
56	A	12	7	1.00	25	0.280
57	A	13	7	1.00	25	0.280
58	A	11	5	1.00	27	0.185
59	A	9	5	1.00	27	0.185
60	A	7	5	1.00	27	0.185
61	A	5	5	1.00	25	0.200
62	A	8	7	1.00	27	0.259
63	A	6	5	1.00	27	0.185
64	A	7	6	1.00	27	0.222
65	A	12	5	1.00	27	0.185
66	A	10	5	1.00	27	0.185
67	A	8	5	1.00	27	0.185

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
68	A	6	5	1.00	25	0.200
69	A	9	8	1.00	27	0.296
70	A	10	9	1.00	27	0.333
71	A	7	6	1.00	27	0.222
72	A	13	5	1.00	27	0.185
73	A	11	5	1.00	27	0.185
74	A	9	5	1.00	27	0.185
75	A	7	5	1.00	25	0.200
76	A	10	9	1.00	27	0.333
77	A	11	9	1.00	27	0.333
78	A	11	10	1.00	27	0.370
79	A	10	4	1.00	27	0.148
80	A	8	4	1.00	27	0.148
81	A	6	4	1.00	27	0.148
82	A	4	4	1.00	25	0.160
83	A	5	4	1.00	27	0.148
84	A	6	5	1.00	27	0.185
85	A	7	6	1.00	27	0.222
86	A	9	5	1.00	27	0.185
87	A	7	5	1.00	27	0.185
88	A	5	5	1.00	27	0.185
89	A	4	4	1.00	25	0.160
90	A	6	5	1.00	27	0.185
91	A	7	6	1.00	27	0.222
92	A	8	6	1.00	27	0.222
93	A	8	5	1.00	27	0.185

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
94	A	6	5	1.00	27	0.185
95	A	5	4	1.00	27	0.148
96	A	3	3	1.00	25	0.120
97	A	7	6	1.00	27	0.222
98	A	8	6	1.00	27	0.222
99	A	9	6	1.00	27	0.222
100	A	7	5	1.00	27	0.185
101	A	5	5	1.00	25	0.200
102	A	8	5	1.00	27	0.185
103	A	6	4	1.00	27	0.148
104	A	8	5	1.00	27	0.185
105	A	6	5	1.00	25	0.200
106	A	9	6	1.00	27	0.222
107	A	10	7	1.00	27	0.259
108	A	7	5	1.00	27	0.185
109	A	8	4	1.00	27	0.148
110	A	6	4	1.00	27	0.148
111	A	4	4	1.00	25	0.160
112	A	5	3	1.00	27	0.111
113	A	6	4	1.00	27	0.148
114	A	7	5	1.00	27	0.185
115	A	5	5	1.00	27	0.185
116	A	4	4	1.00	25	0.160
117	A	6	4	1.00	27	0.148
118	A	6	5	1.00	27	0.185
119	A	5	4	1.00	27	0.148

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
120	A	3	3	1.00	25	0.120
121	A	5	4	1.00	27	0.148
122	A	3	3	1.00	29	0.103
123	A	3	3	1.00	29	0.103

# Chapter 3

## Listing of integrals

3.1 
$$\int \frac{a+bx+\frac{bf x^2}{e}}{\sqrt{d+ex+fx^2}} dx$$

Optimal. Leaf size=102

$$\frac{\left(8af - b\left(\frac{4df}{e} + e\right)\right) \tanh^{-1}\left(\frac{e+2fx}{2\sqrt{f}\sqrt{d+ex+fx^2}}\right)}{8f^{3/2}} + \frac{bx\sqrt{d+ex+fx^2}}{2e} + \frac{b\sqrt{d+ex+fx^2}}{4f}$$

[Out]  $1/8*(8*a*f-b*(e+4*d*f/e))*\operatorname{arctanh}(1/2*(2*f*x+e)/f^{(1/2)})/(f*x^2+e*x+d)^{(1/2)}/f^{(3/2)}+1/4*b*(f*x^2+e*x+d)^{(1/2)}/f+1/2*b*x*(f*x^2+e*x+d)^{(1/2)}/e$

**Rubi** [A] time = 0.09, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$ , Rules used = {1661, 640, 621, 206}

$$\frac{\left(8af - b\left(\frac{4df}{e} + e\right)\right) \tanh^{-1}\left(\frac{e+2fx}{2\sqrt{f}\sqrt{d+ex+fx^2}}\right)}{8f^{3/2}} + \frac{bx\sqrt{d+ex+fx^2}}{2e} + \frac{b\sqrt{d+ex+fx^2}}{4f}$$

Antiderivative was successfully verified.

[In] `Int[(a + b*x + (b*f*x^2)/e)/Sqrt[d + e*x + f*x^2], x]`

[Out]  $(b*\operatorname{Sqrt}[d + e*x + f*x^2])/(4*f) + (b*x*\operatorname{Sqrt}[d + e*x + f*x^2])/(2*e) + ((8*a*f - b*(e + (4*d*f)/e))*\operatorname{ArcTanh}[(e + 2*f*x)/(2*\operatorname{Sqrt}[f]*\operatorname{Sqrt}[d + e*x + f*x^2])])/(8*f^{(3/2)})$

Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 621

Int[1/Sqrt[(a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 640

Int[((d\_) + (e\_)\*(x\_))\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

### Rule 1661

Int[(Pq\_)\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(q + 2\*p + 1)), x] + Dist[1/(c\*(q + 2\*p + 1)), Int[(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*(q + 2\*p + 1)\*Pq - a\*e\*(q - 1)\*x^(q - 2) - b\*e\*(q + p)\*x^(q - 1) - c\*e\*(q + 2\*p + 1)\*x^q, x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

### Rubi steps

$$\begin{aligned} \int \frac{a + bx + \frac{bfx^2}{e}}{\sqrt{d + ex + fx^2}} dx &= \frac{bx\sqrt{d + ex + fx^2}}{2e} + \frac{\int \frac{(2a - \frac{bd}{e})f + \frac{bfx}{2}}{\sqrt{d + ex + fx^2}} dx}{2f} \\ &= \frac{b\sqrt{d + ex + fx^2}}{4f} + \frac{bx\sqrt{d + ex + fx^2}}{2e} + \frac{(-be + 8af - \frac{4bdf}{e}) \int \frac{1}{\sqrt{d + ex + fx^2}} dx}{8f} \\ &= \frac{b\sqrt{d + ex + fx^2}}{4f} + \frac{bx\sqrt{d + ex + fx^2}}{2e} + \frac{(-be + 8af - \frac{4bdf}{e}) \text{Subst}\left(\int \frac{1}{4f - x^2} dx, x, \frac{e + 2fx}{\sqrt{d + ex + fx^2}}\right)}{4f} \\ &= \frac{b\sqrt{d + ex + fx^2}}{4f} + \frac{bx\sqrt{d + ex + fx^2}}{2e} - \frac{(be - 8af + \frac{4bdf}{e}) \tanh^{-1}\left(\frac{e + 2fx}{2\sqrt{f}\sqrt{d + ex + fx^2}}\right)}{8f^{3/2}} \end{aligned}$$



**Mathematica [A]** time = 0.19, size = 87, normalized size = 0.85

$$\frac{2b\sqrt{f}(e+2fx)\sqrt{d+x(e+fx)} - (b(4df+e^2) - 8aef) \tanh^{-1}\left(\frac{e+2fx}{2\sqrt{f}\sqrt{d+x(e+fx)}}\right)}{8ef^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x + (b\*f\*x^2)/e)/Sqrt[d + e\*x + f\*x^2], x]

[Out] (2\*b\*Sqrt[f]\*(e + 2\*f\*x)\*Sqrt[d + x\*(e + f\*x)] - (-8\*a\*e\*f + b\*(e^2 + 4\*d\*f))\*ArcTanh[(e + 2\*f\*x)/(2\*Sqrt[f]\*Sqrt[d + x\*(e + f\*x)])]/(8\*e\*f^(3/2))

**fricas [A]** time = 1.21, size = 205, normalized size = 2.01

$$\left[ \frac{(be^2 + 4(bd - 2ae)f)\sqrt{f} \log(-8f^2x^2 - 8efx - e^2 - 4\sqrt{fx^2 + ex + d}(2fx + e)\sqrt{f} - 4df) - 4(2bf^2x + be^2)}{16ef^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x+b\*f\*x^2/e)/(f\*x^2+e\*x+d)^(1/2), x, algorithm="fricas")

[Out] [-1/16\*((b\*e^2 + 4\*(b\*d - 2\*a\*e)\*f)\*sqrt(f)\*log(-8\*f^2\*x^2 - 8\*e\*f\*x - e^2 - 4\*sqrt(f\*x^2 + e\*x + d)\*(2\*f\*x + e)\*sqrt(f) - 4\*d\*f) - 4\*(2\*b\*f^2\*x + b\*e\*f)\*sqrt(f\*x^2 + e\*x + d))/(e\*f^2), 1/8\*((b\*e^2 + 4\*(b\*d - 2\*a\*e)\*f)\*sqrt(-f)\*arctan(1/2\*sqrt(f\*x^2 + e\*x + d)\*(2\*f\*x + e)\*sqrt(-f)/(f^2\*x^2 + e\*f\*x + d\*f)) + 2\*(2\*b\*f^2\*x + b\*e\*f)\*sqrt(f\*x^2 + e\*x + d))/(e\*f^2)]

**giac [A]** time = 0.26, size = 84, normalized size = 0.82

$$\frac{1}{4}\sqrt{fx^2 + xe + d}\left(2bx e^{(-1)} + \frac{b}{f}\right) + \frac{(4bdf - 8afe + be^2)e^{(-1)} \log\left(\left|-2\left(\sqrt{f}x - \sqrt{fx^2 + xe + d}\right)\sqrt{f} - e\right|\right)}{8f^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x+b\*f\*x^2/e)/(f\*x^2+e\*x+d)^(1/2), x, algorithm="giac")

[Out] 1/4\*sqrt(f\*x^2 + x\*e + d)\*(2\*b\*x\*e^(-1) + b/f) + 1/8\*(4\*b\*d\*f - 8\*a\*f\*e + b\*e^2)\*e^(-1)\*log(abs(-2\*(sqrt(f)\*x - sqrt(f\*x^2 + x\*e + d))\*sqrt(f) - e))/f^(3/2)

**maple [A]** time = 0.03, size = 136, normalized size = 1.33

$$\frac{a \ln\left(\frac{fx + \frac{e}{2}}{\sqrt{f}} + \sqrt{fx^2 + ex + d}\right)}{\sqrt{f}} - \frac{bd \ln\left(\frac{fx + \frac{e}{2}}{\sqrt{f}} + \sqrt{fx^2 + ex + d}\right)}{2e\sqrt{f}} - \frac{be \ln\left(\frac{fx + \frac{e}{2}}{\sqrt{f}} + \sqrt{fx^2 + ex + d}\right)}{8f^{\frac{3}{2}}} + \frac{\sqrt{fx^2 + ex + d}}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x)`

[Out]  $\frac{1}{2}bx\sqrt{fx^2+ex+d}/e + \frac{1}{4}b\sqrt{fx^2+ex+d}/f - \frac{1}{8}eb/f^{3/2} \ln\left(\frac{(1/2*e+fx)/f^{1/2}+(fx^2+ex+d)^{1/2}}{(1/2*e+fx)/f^{1/2}+(fx^2+ex+d)^{1/2}}\right) - \frac{1}{2}eb/f^{1/2}d \ln\left(\frac{(1/2*e+fx)/f^{1/2}+(fx^2+ex+d)^{1/2}}{(1/2*e+fx)/f^{1/2}+(fx^2+ex+d)^{1/2}}\right) + a \ln\left(\frac{(1/2*e+fx)/f^{1/2}+(fx^2+ex+d)^{1/2}}{(1/2*e+fx)/f^{1/2}+(fx^2+ex+d)^{1/2}}\right)$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*d\*f-e^2>0)', see `assume?` for more details)Is 4\*d\*f-e^2 zero or nonzero?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + bx + \frac{bfx^2}{e}}{\sqrt{fx^2 + ex + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x + (b*f*x^2)/e)/(d + e*x + f*x^2)^(1/2),x)`

[Out] `int((a + b*x + (b*f*x^2)/e)/(d + e*x + f*x^2)^(1/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ae}{\sqrt{d+ex+fx^2}} dx + \int \frac{bex}{\sqrt{d+ex+fx^2}} dx + \int \frac{bfx^2}{\sqrt{d+ex+fx^2}} dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*x+b*f*x**2/e)/(f*x**2+e*x+d)**(1/2),x)`

[Out] `(Integral(a*e/sqrt(d + e*x + f*x**2), x) + Integral(b*e*x/sqrt(d + e*x + f*x**2), x) + Integral(b*f*x**2/sqrt(d + e*x + f*x**2), x))/e`

$$3.2 \quad \int \frac{1}{\sqrt{d+ex+fx^2} \left( a+bx+\frac{bf x^2}{e} \right)} dx$$

Optimal. Leaf size=82

$$-\frac{2\sqrt{e} \tanh^{-1} \left( \frac{(e+2fx)\sqrt{bd-ae}}{\sqrt{e} \sqrt{be-4af} \sqrt{d+ex+fx^2}} \right)}{\sqrt{bd-ae} \sqrt{be-4af}}$$

[Out]  $-2*\operatorname{arctanh}((2*f*x+e)*(-a*e+b*d)^{(1/2)}/e^{(1/2)}/(-4*a*f+b*e)^{(1/2)}/(f*x^2+e*x+d)^{(1/2}))*e^{(1/2)}/(-a*e+b*d)^{(1/2)}/(-4*a*f+b*e)^{(1/2)}$

**Rubi [A]** time = 0.11, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$ , Rules used = {982, 208}

$$-\frac{2\sqrt{e} \tanh^{-1} \left( \frac{(e+2fx)\sqrt{bd-ae}}{\sqrt{e} \sqrt{be-4af} \sqrt{d+ex+fx^2}} \right)}{\sqrt{bd-ae} \sqrt{be-4af}}$$

Antiderivative was successfully verified.

[In] `Int[1/(Sqrt[d + e*x + f*x^2]*(a + b*x + (b*f*x^2)/e)),x]`

[Out]  $(-2*\operatorname{Sqrt}[e]*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b*d - a*e]*(e + 2*f*x))/(\operatorname{Sqrt}[e]*\operatorname{Sqrt}[b*e - 4*a*f]*\operatorname{Sqrt}[d + e*x + f*x^2])])/(\operatorname{Sqrt}[b*d - a*e]*\operatorname{Sqrt}[b*e - 4*a*f])$

Rule 208

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

Rule 982

`Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0]`

Rubi steps

$$\int \frac{1}{\sqrt{d+ex+fx^2} \left(a+bx+\frac{bfx^2}{e}\right)} dx = -\left((2e) \text{Subst}\left(\int \frac{1}{e(be-4af)-(bd-ae)x^2} dx, x, \frac{e+2fx}{\sqrt{d+ex+fx^2}}\right)\right)$$

$$= -\frac{2\sqrt{e} \tanh^{-1}\left(\frac{\sqrt{bd-ae}(e+2fx)}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{\sqrt{bd-ae}\sqrt{be-4af}}$$

**Mathematica [B]** time = 0.39, size = 178, normalized size = 2.17

$$\frac{\sqrt{e} \left( \tanh^{-1}\left(\frac{-\sqrt{e}(e+2fx)\sqrt{be-4af}-\sqrt{b}(e^2-4df)}{4f\sqrt{bd-ae}\sqrt{d+x(e+fx)}}\right) + \tanh^{-1}\left(\frac{\sqrt{b}(e^2-4df)-\sqrt{e}(e+2fx)\sqrt{be-4af}}{4f\sqrt{bd-ae}\sqrt{d+x(e+fx)}}\right) \right)}{\sqrt{bd-ae}\sqrt{be-4af}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[d + e\*x + f\*x^2]\*(a + b\*x + (b\*f\*x^2)/e)),x]

[Out] (Sqrt[e]\*(ArcTanh[(-(Sqrt[b]\*(e^2 - 4\*d\*f)) - Sqrt[e]\*Sqrt[b\*e - 4\*a\*f]\*(e + 2\*f\*x))/(4\*Sqrt[b\*d - a\*e]\*f\*Sqrt[d + x\*(e + f\*x)])] + ArcTanh[(Sqrt[b]\*(e^2 - 4\*d\*f) - Sqrt[e]\*Sqrt[b\*e - 4\*a\*f]\*(e + 2\*f\*x))/(4\*Sqrt[b\*d - a\*e]\*f\*Sqrt[d + x\*(e + f\*x)])]))/(Sqrt[b\*d - a\*e]\*Sqrt[b\*e - 4\*a\*f])

**fricas [B]** time = 1.71, size = 1079, normalized size = 13.16

$$\left[ \frac{1}{2} \sqrt{\frac{e}{b^2de - abe^2 - 4(abd - a^2e)f}} \log \left( \frac{8b^2d^2e^4 - 8abde^5 + a^2e^6 + 16a^2d^2e^2f^2 + (b^2e^4f^2 + 16(b^2d^2 - 8abde + \dots))}{\dots} \right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*x+b\*f\*x^2/e)/(f\*x^2+e\*x+d)^(1/2),x, algorithm="fricas")

[Out] [1/2\*sqrt(e/(b^2\*d\*e - a\*b\*e^2 - 4\*(a\*b\*d - a^2\*e)\*f))\*log((8\*b^2\*d^2\*e^4 - 8\*a\*b\*d\*e^5 + a^2\*e^6 + 16\*a^2\*d^2\*e^2\*f^2 + (b^2\*e^4\*f^2 + 16\*(b^2\*d^2 - 8\*a\*b\*d\*e + 8\*a^2\*e^2)\*f^4 + 8\*(3\*b^2\*d\*e^2 - 4\*a\*b\*e^3)\*f^3)\*x^4 + 2\*(b^2\*e^5\*f + 16\*(b^2\*d^2\*e - 8\*a\*b\*d\*e^2 + 8\*a^2\*e^3)\*f^3 + 8\*(3\*b^2\*d\*e^3 - 4\*a\*b\*e^4)\*f^2)\*x^3 + (b^2\*e^6 - 32\*(3\*a\*b\*d^2\*e - 4\*a^2\*d\*e^2)\*f^3 + 16\*(3\*b^2\*d^2\*e^2 - 13\*a\*b\*d\*e^3 + 10\*a^2\*e^4)\*f^2 + 2\*(16\*b^2\*d\*e^4 - 19\*a\*b\*e^5)\*f)\*x^2 - 8\*(4\*a\*b\*d^2\*e^3 - 3\*a^2\*d\*e^4)\*f + 2\*(4\*b^2\*d\*e^5 - 3\*a\*b\*e^6 - 16\*(3\*a\*b\*d^2\*e^2 - 4\*a^2\*d\*e^3)\*f^2 + 8\*(2\*b^2\*d^2\*e^3 - 5\*a\*b\*d\*e^4 + 2\*a^2

```

2*e^5)*f)*x - 4*(2*b^3*d^2*e^4 - 3*a*b^2*d*e^5 + a^2*b*e^6 - 2*(16*(a*b^2*d
^2 - 3*a^2*b*d*e + 2*a^3*e^2)*f^4 - 4*(b^3*d^2*e - 4*a*b^2*d*e^2 + 3*a^2*b*
e^3)*f^3 - (b^3*d*e^3 - a*b^2*e^4)*f^2)*x^3 + 16*(a^2*b*d^2*e^2 - a^3*d*e^3
)*f^2 - 3*(16*(a*b^2*d^2*e - 3*a^2*b*d*e^2 + 2*a^3*e^3)*f^3 - 4*(b^3*d^2*e^
2 - 4*a*b^2*d*e^3 + 3*a^2*b*e^4)*f^2 - (b^3*d*e^4 - a*b^2*e^5)*f)*x^2 - 4*(
3*a*b^2*d^2*e^3 - 4*a^2*b*d*e^4 + a^3*e^5)*f + (b^3*d*e^5 - a*b^2*e^6 + 32*
(a^2*b*d^2*e - a^3*d*e^2)*f^3 - 40*(a*b^2*d^2*e^2 - 2*a^2*b*d*e^3 + a^3*e^4
)*f^2 + 2*(4*b^3*d^2*e^3 - 11*a*b^2*d*e^4 + 7*a^2*b*e^5)*f)*x)*sqrt(f*x^2 +
e*x + d)*sqrt(e/(b^2*d*e - a*b*e^2 - 4*(a*b*d - a^2*e)*f))/(b^2*f^2*x^4 +
2*b^2*e*f*x^3 + 2*a*b*e^2*x + a^2*e^2 + (b^2*e^2 + 2*a*b*e*f)*x^2)), -sqrt
(-e/(b^2*d*e - a*b*e^2 - 4*(a*b*d - a^2*e)*f))*arctan(-1/2*(2*b*d*e^2 - a*e
^3 - 4*a*d*e*f + (b*e^2*f + 4*(b*d - 2*a*e)*f^2)*x^2 + (b*e^3 + 4*(b*d*e -
2*a*e^2)*f)*x)*sqrt(f*x^2 + e*x + d)*sqrt(-e/(b^2*d*e - a*b*e^2 - 4*(a*b*d
- a^2*e)*f))/(2*e*f^2*x^3 + 3*e^2*f*x^2 + d*e^2 + (e^3 + 2*d*e*f)*x))]

```

**giac** [B] time = 3.04, size = 847, normalized size = 10.33

$$\sqrt{-4abdf e + b^2de^2 + 4a^2fe^2 - abe^3} \log\left(\left|-4\left(\sqrt{f}x - \sqrt{fx^2 + xe + d}\right)^2 bdf^2 + 8\left(\sqrt{f}x - \sqrt{fx^2 + xe + d}\right)^2 af^2e\right.\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*x+b\*f\*x^2/e)/(f\*x^2+e\*x+d)^(1/2),x, algorithm="giac")

```

[Out] sqrt(-4*a*b*d*f*e + b^2*d*e^2 + 4*a^2*f*e^2 - a*b*e^3)*log(abs(-4*(sqrt(f)*
x - sqrt(f*x^2 + x*e + d))^2*b*d*f^2 + 8*(sqrt(f)*x - sqrt(f*x^2 + x*e + d)
)^2*a*f^2*e - 4*(sqrt(f)*x - sqrt(f*x^2 + x*e + d))*b*d*f^(3/2)*e + 4*b*d^2
*f^2 - (sqrt(f)*x - sqrt(f*x^2 + x*e + d))^2*b*f*e^2 + 8*(sqrt(f)*x - sqrt(
f*x^2 + x*e + d))*a*f^(3/2)*e^2 + 4*sqrt(-4*a*b*d*f*e + b^2*d*e^2 + 4*a^2*f
*e^2 - a*b*e^3)*(sqrt(f)*x - sqrt(f*x^2 + x*e + d))^2*f^(3/2) - 3*b*d*f*e^2
- (sqrt(f)*x - sqrt(f*x^2 + x*e + d))*b*sqrt(f)*e^3 + 4*sqrt(-4*a*b*d*f*e
+ b^2*d*e^2 + 4*a^2*f*e^2 - a*b*e^3)*(sqrt(f)*x - sqrt(f*x^2 + x*e + d))*f*
e + 2*a*f*e^3 + sqrt(-4*a*b*d*f*e + b^2*d*e^2 + 4*a^2*f*e^2 - a*b*e^3)*sqrt
(f)*e^2))/(4*a*b*d*f - b^2*d*e - 4*a^2*f*e + a*b*e^2) - sqrt(-4*a*b*d*f*e +
b^2*d*e^2 + 4*a^2*f*e^2 - a*b*e^3)*log(abs(-4*(sqrt(f)*x - sqrt(f*x^2 + x*
e + d))^2*b*d*f^2 + 8*(sqrt(f)*x - sqrt(f*x^2 + x*e + d))^2*a*f^2*e - 4*(sq
rt(f)*x - sqrt(f*x^2 + x*e + d))*b*d*f^(3/2)*e + 4*b*d^2*f^2 - (sqrt(f)*x -
sqrt(f*x^2 + x*e + d))^2*b*f*e^2 + 8*(sqrt(f)*x - sqrt(f*x^2 + x*e + d))*a
*f^(3/2)*e^2 - 4*sqrt(-4*a*b*d*f*e + b^2*d*e^2 + 4*a^2*f*e^2 - a*b*e^3)*(sq
rt(f)*x - sqrt(f*x^2 + x*e + d))^2*f^(3/2) - 3*b*d*f*e^2 - (sqrt(f)*x - sqr
t(f*x^2 + x*e + d))*b*sqrt(f)*e^3 - 4*sqrt(-4*a*b*d*f*e + b^2*d*e^2 + 4*a^2
*f*e^2 - a*b*e^3)*(sqrt(f)*x - sqrt(f*x^2 + x*e + d))*f*e + 2*a*f*e^3 - sqr
t(-4*a*b*d*f*e + b^2*d*e^2 + 4*a^2*f*e^2 - a*b*e^3)*sqrt(f)*e^2))/(4*a*b*d*
f - b^2*d*e - 4*a^2*f*e + a*b*e^2)

```

**maple [B]** time = 0.06, size = 491, normalized size = 5.99

$$e \ln \left( \frac{-\frac{2(ae-bd)}{b} + \frac{\sqrt{-(4af-be)be} \left( x - \frac{-be + \sqrt{-(4af-be)be}}{2bf} \right)}{b} + 2\sqrt{-\frac{ae-bd}{b}} \sqrt{\left( x - \frac{-be + \sqrt{-(4af-be)be}}{2bf} \right)^2} + \frac{\sqrt{-(4af-be)be} \left( x - \frac{-be + \sqrt{-(4af-be)be}}{2bf} \right)}{b} - \frac{ae-bd}{b}}{x - \frac{-be + \sqrt{-(4af-be)be}}{2bf}} \right) + \frac{\sqrt{-(4af-be)be} \sqrt{-\frac{ae-bd}{b}}}{\sqrt{-(4af-be)be} \sqrt{-\frac{ae-bd}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b\*x+b\*f\*x^2/e)/(f\*x^2+e\*x+d)^(1/2),x)

[Out] 
$$-e/(-b*e*(4*a*f-b*e))^{(1/2)}/(-1/b*(a*e-b*d))^{(1/2)}*\ln\left(\frac{-2/b*(a*e-b*d)+(-b*e*(4*a*f-b*e))^{(1/2)}/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)+2*(-1/b*(a*e-b*d))^{(1/2)}*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)^2*f+(-b*e*(4*a*f-b*e))^{(1/2)}/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)-1/b*(a*e-b*d))^{(1/2)}}{(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)+e/(-b*e*(4*a*f-b*e))^{(1/2)}/(-1/b*(a*e-b*d))^{(1/2)}*\ln\left(\frac{-2/b*(a*e-b*d)-(-b*e*(4*a*f-b*e))^{(1/2)}/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)+2*(-1/b*(a*e-b*d))^{(1/2)}*((x+1/2*(b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)^2*f-(-b*e*(4*a*f-b*e))^{(1/2)}/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)-1/b*(a*e-b*d))^{(1/2)}}{(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)}\right)$$

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*x+b\*f\*x^2/e)/(f\*x^2+e\*x+d)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*f-b\*e>0)', see `assume?` for more details)Is 4\*a\*f-b\*e positive, negative or zero?

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{f x^2 + e x + d} \left( a + b x + \frac{b f x^2}{e} \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((d + e*x + f*x^2)^(1/2)*(a + b*x + (b*f*x^2)/e)),x)`

[Out] `int(1/((d + e*x + f*x^2)^(1/2)*(a + b*x + (b*f*x^2)/e)), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$e \int \frac{1}{ae\sqrt{d+ex+fx^2} + bex\sqrt{d+ex+fx^2} + bfx^2\sqrt{d+ex+fx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*x+b*f*x**2/e)/(f*x**2+e*x+d)**(1/2),x)`

[Out] `e*Integral(1/(a*e*sqrt(d + e*x + f*x**2) + b*e*x*sqrt(d + e*x + f*x**2) + b*f*x**2*sqrt(d + e*x + f*x**2)), x)`

$$3.3 \quad \int \frac{1}{\sqrt{a+bx+cx^2} (d+bx+cx^2)} dx$$

Optimal. Leaf size=66

$$-\frac{2 \tanh^{-1} \left( \frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd} \sqrt{a+bx+cx^2}} \right)}{\sqrt{a-d} \sqrt{b^2-4cd}}$$

[Out]  $-2*\operatorname{arctanh}((2*c*x+b)*(a-d)^{(1/2)}/(b^2-4*c*d)^{(1/2)}/(c*x^2+b*x+a)^{(1/2)})/(a-d)^{(1/2)}/(b^2-4*c*d)^{(1/2)}$

**Rubi [A]** time = 0.08, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$ , Rules used = {982, 208}

$$-\frac{2 \tanh^{-1} \left( \frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd} \sqrt{a+bx+cx^2}} \right)}{\sqrt{a-d} \sqrt{b^2-4cd}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + b\*x + c\*x^2]\*(d + b\*x + c\*x^2)), x]

[Out]  $(-2*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a-d]*(b+2*c*x))/(\operatorname{Sqrt}[b^2-4*c*d]*\operatorname{Sqrt}[a+b*x+c*x^2])])/(\operatorname{Sqrt}[a-d]*\operatorname{Sqrt}[b^2-4*c*d])$

Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 982

Int[1/(((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*Sqrt[(d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2]), x\_Symbol] :> Dist[-2\*e, Subst[Int[1/(e\*(b\*e - 4\*a\*f) - (b\*d - a\*e)\*x^2), x], x, (e + 2\*f\*x)/Sqrt[d + e\*x + f\*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && EqQ[c\*e - b\*f, 0]

Rubi steps



$$\int \frac{1}{\sqrt{a+bx+cx^2} (d+bx+cx^2)} dx = - \left( (2b) \text{Subst} \left( \int \frac{1}{b(b^2-4cd) - (ab-bd)x^2} dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}} \right) \right)$$

$$= - \frac{2 \tanh^{-1} \left( \frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd} \sqrt{a+bx+cx^2}} \right)}{\sqrt{a-d} \sqrt{b^2-4cd}}$$

**Mathematica [B]** time = 0.23, size = 161, normalized size = 2.44

$$\frac{\tanh^{-1} \left( \frac{4ac-2cx\sqrt{b^2-4cd}-b(\sqrt{b^2-4cd}+b)}{4c\sqrt{a-d}\sqrt{a+x(b+cx)}} \right) + \tanh^{-1} \left( \frac{-2c(2a+x\sqrt{b^2-4cd})-b\sqrt{b^2-4cd}+b^2}{4c\sqrt{a-d}\sqrt{a+x(b+cx)}} \right)}{\sqrt{a-d}\sqrt{b^2-4cd}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[a + b\*x + c\*x^2]\*(d + b\*x + c\*x^2)), x]

[Out] (ArcTanh[(4\*a\*c - b\*(b + Sqrt[b^2 - 4\*c\*d]) - 2\*c\*Sqrt[b^2 - 4\*c\*d]\*x)/(4\*c\*Sqrt[a - d]\*Sqrt[a + x\*(b + c\*x)])] + ArcTanh[(b^2 - b\*Sqrt[b^2 - 4\*c\*d] - 2\*c\*(2\*a + Sqrt[b^2 - 4\*c\*d]\*x))/(4\*c\*Sqrt[a - d]\*Sqrt[a + x\*(b + c\*x)])])/(Sqrt[a - d]\*Sqrt[b^2 - 4\*c\*d])

**fricas [B]** time = 1.43, size = 813, normalized size = 12.32

$$\left[ \log \left( \frac{8a^2b^4 + (b^4c^2 + 24ab^2c^3 + 16a^2c^4 + 128c^4d^2 - 32(b^2c^3 + 4ac^4)d)x^4 + 2(b^5c + 24ab^3c^2 + 16a^2bc^3 + 128bc^3d^2 - 32(b^3c^2 + 4abc^3)d)x^3 + (b^4 + 24ab^2c + 16a^2c^2)d^2 + (b^6 + 32ab^4c + 48a^2b^2c^2 + 32(5b^2c^2 + 4ac^3)d^2 - 2(19b^4c + 104ab^2c^2 + 48a^2c^3)d)x^2 - 4(2ab^3 + 2(b^2c^2 + 4ac^3 - 8c^3d)x^3 + 3(b^3c + 4ab^2c^2 - 8b^2cd)x^2 - (b^3 + 4ab^2c)d + (b^4 + 8ab^2c - 2(5b^2c + 4ac^2)d)x)*\sqrt{ab^2 + 4cd} - (b^2 + 4ac)d)*\sqrt{c^2x^2 + bx + a} - 8(ab^4 + 4a^2b^2c)}{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x^2+b\*x+d)/(c\*x^2+b\*x+a)^(1/2), x, algorithm="fricas")

[Out] [1/2\*log((8\*a^2\*b^4 + (b^4\*c^2 + 24\*a\*b^2\*c^3 + 16\*a^2\*c^4 + 128\*c^4\*d^2 - 32\*(b^2\*c^3 + 4\*a\*c^4)\*d)\*x^4 + 2\*(b^5\*c + 24\*a\*b^3\*c^2 + 16\*a^2\*b\*c^3 + 128\*b\*c^3\*d^2 - 32\*(b^3\*c^2 + 4\*a\*b\*c^3)\*d)\*x^3 + (b^4 + 24\*a\*b^2\*c + 16\*a^2\*c^2)\*d^2 + (b^6 + 32\*a\*b^4\*c + 48\*a^2\*b^2\*c^2 + 32\*(5\*b^2\*c^2 + 4\*a\*c^3)\*d^2 - 2\*(19\*b^4\*c + 104\*a\*b^2\*c^2 + 48\*a^2\*c^3)\*d)\*x^2 - 4\*(2\*a\*b^3 + 2\*(b^2\*c^2 + 4\*a\*c^3 - 8\*c^3\*d)\*x^3 + 3\*(b^3\*c + 4\*a\*b^2\*c^2 - 8\*b^2\*c\*d)\*x^2 - (b^3 + 4\*a\*b^2\*c)\*d + (b^4 + 8\*a\*b^2\*c - 2\*(5\*b^2\*c + 4\*a\*c^2)\*d)\*x)\*sqrt(a\*b^2 + 4\*c\*d) - (b^2 + 4\*a\*c)\*d)\*sqrt(c\*x^2 + b\*x + a) - 8\*(a\*b^4 + 4\*a^2\*b^2\*c)

```
*d + 2*(4*a*b^5 + 16*a^2*b^3*c + 16*(b^3*c + 4*a*b*c^2)*d^2 - (3*b^5 + 40*a
*b^3*c + 48*a^2*b*c^2)*d)*x)/(c^2*x^4 + 2*b*c*x^3 + 2*b*d*x + (b^2 + 2*c*d)
*x^2 + d^2))/sqrt(a*b^2 + 4*c*d^2 - (b^2 + 4*a*c)*d), -sqrt(-a*b^2 - 4*c*d^
2 + (b^2 + 4*a*c)*d)*arctan(-1/2*(2*a*b^2 + (b^2*c + 4*a*c^2 - 8*c^2*d)*x^2
- (b^2 + 4*a*c)*d + (b^3 + 4*a*b*c - 8*b*c*d)*x)*sqrt(-a*b^2 - 4*c*d^2 + (
b^2 + 4*a*c)*d)*sqrt(c*x^2 + b*x + a)/(a^2*b^3 + 4*a*b*c*d^2 + 2*(a*b^2*c^2
+ 4*c^3*d^2 - (b^2*c^2 + 4*a*c^3)*d)*x^3 + 3*(a*b^3*c + 4*b*c^2*d^2 - (b^3
*c + 4*a*b*c^2)*d)*x^2 - (a*b^3 + 4*a^2*b*c)*d + (a*b^4 + 2*a^2*b^2*c + 4*(
b^2*c + 2*a*c^2)*d^2 - (b^4 + 6*a*b^2*c + 8*a^2*c^2)*d)*x))/(a*b^2 + 4*c*d^
2 - (b^2 + 4*a*c)*d)]
```

**giac [B]** time = 1.90, size = 703, normalized size = 10.65

$$\log\left(\frac{\left(-\left(\sqrt{c}x - \sqrt{cx^2 + bx + a}\right)^2 b^2 c - 4\left(\sqrt{c}x - \sqrt{cx^2 + bx + a}\right)^2 ac^2 + 8\left(\sqrt{c}x - \sqrt{cx^2 + bx + a}\right)^2 c^2 d - \left(\sqrt{c}x - \sqrt{cx^2 + bx + a}\right)^2 b^3 \sqrt{c} - 4\left(\sqrt{c}x - \sqrt{cx^2 + bx + a}\right)^2 a^2 c^2 d + 8\left(\sqrt{c}x - \sqrt{cx^2 + bx + a}\right)^2 b^2 c^2 d - 3a^2 b^2 c + 4\sqrt{a^2 b^2 - b^2 d - 4a^2 c d + 4c^2 d^2} \left(\sqrt{c}x - \sqrt{cx^2 + bx + a}\right)^2 c^{3/2} + 4a^2 c^2 d + 2b^2 c d + 4\sqrt{a^2 b^2 - b^2 d - 4a^2 c d + 4c^2 d^2} \left(\sqrt{c}x - \sqrt{cx^2 + bx + a}\right)^2 b^2 c^{3/2} + \sqrt{a^2 b^2 - b^2 d - 4a^2 c d + 4c^2 d^2} \left(\sqrt{c}x - \sqrt{cx^2 + bx + a}\right)^2 b^2 \sqrt{c}}{\sqrt{a^2 b^2 - b^2 d - 4a^2 c d + 4c^2 d^2}} + \log\left(\frac{\left(-\left(\sqrt{c}x - \sqrt{cx^2 + bx + a}\right)^2 b^2 c - 4\left(\sqrt{c}x - \sqrt{cx^2 + bx + a}\right)^2 a^2 c^2 d + 8\left(\sqrt{c}x - \sqrt{cx^2 + bx + a}\right)^2 b^2 c^2 d - \left(\sqrt{c}x - \sqrt{cx^2 + bx + a}\right)^2 b^3 \sqrt{c} - 4\left(\sqrt{c}x - \sqrt{cx^2 + bx + a}\right)^2 a^2 c^2 d + 8\left(\sqrt{c}x - \sqrt{cx^2 + bx + a}\right)^2 b^2 c^2 d - 3a^2 b^2 c - 4\sqrt{a^2 b^2 - b^2 d - 4a^2 c d + 4c^2 d^2} \left(\sqrt{c}x - \sqrt{cx^2 + bx + a}\right)^2 c^{3/2} + 4a^2 c^2 d + 2b^2 c d - 4\sqrt{a^2 b^2 - b^2 d - 4a^2 c d + 4c^2 d^2} \left(\sqrt{c}x - \sqrt{cx^2 + bx + a}\right)^2 b^2 c^{3/2} + \sqrt{a^2 b^2 - b^2 d - 4a^2 c d + 4c^2 d^2} \left(\sqrt{c}x - \sqrt{cx^2 + bx + a}\right)^2 b^2 \sqrt{c}}{\sqrt{a^2 b^2 - b^2 d - 4a^2 c d + 4c^2 d^2}}\right)}{\sqrt{b^2 - 4cd} \sqrt{a - d}} + \log\left(\frac{2a - 2d - \sqrt{b^2 - 4cd} \left(x + \frac{b + \sqrt{b^2 - 4cd}}{2c}\right)}{2a - 2d + \sqrt{b^2 - 4cd} \left(x - \frac{-b + \sqrt{b^2 - 4cd}}{2c}\right) + 2\sqrt{a - d} \sqrt{a + \left(x - \frac{-b + \sqrt{b^2 - 4cd}}{2c}\right)^2 c - d + \sqrt{b^2 - 4cd} \left(x - \frac{-b + \sqrt{b^2 - 4cd}}{2c}\right)}}{x - \frac{-b + \sqrt{b^2 - 4cd}}{2c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x^2+b\*x+d)/(c\*x^2+b\*x+a)^(1/2),x, algorithm="giac")

```
[Out] -log(abs(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*c - 4*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^2*a*c^2 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^2*d
- (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*sqrt(c) - 4*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))*a*b*c^(3/2) + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^(3/2
)*d - 3*a*b^2*c + 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^2*c^(3/2) + 4*a^2*c^2 + 2*b^2*c*d + 4*sqrt(a*b^2 - b^2
*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c + sqrt(a*b^
2 - b^2*d - 4*a*c*d + 4*c*d^2)*b^2*sqrt(c))/sqrt(a*b^2 - b^2*d - 4*a*c*d +
4*c*d^2) + log(abs(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*c - 4*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^2 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^2*c^2*d - (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*sqrt(c) - 4*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))*a*b*c^(3/2) + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))*b*c^(3/2)*d - 3*a*b^2*c - 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(3/2) + 4*a^2*c^2 + 2*b^2*c*d - 4*sqrt(
a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c
- sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*b^2*sqrt(c))/sqrt(a*b^2 - b^2*d
- 4*a*c*d + 4*c*d^2)
```

**maple [B]** time = 0.03, size = 307, normalized size = 4.65

$$\frac{\ln\left(\frac{2a - 2d + \sqrt{b^2 - 4cd} \left(x - \frac{-b + \sqrt{b^2 - 4cd}}{2c}\right) + 2\sqrt{a - d} \sqrt{a + \left(x - \frac{-b + \sqrt{b^2 - 4cd}}{2c}\right)^2 c - d + \sqrt{b^2 - 4cd} \left(x - \frac{-b + \sqrt{b^2 - 4cd}}{2c}\right)}}{x - \frac{-b + \sqrt{b^2 - 4cd}}{2c}}\right)}{\sqrt{b^2 - 4cd} \sqrt{a - d}} + \ln\left(\frac{2a - 2d - \sqrt{b^2 - 4cd} \left(x + \frac{b + \sqrt{b^2 - 4cd}}{2c}\right)}{2a - 2d + \sqrt{b^2 - 4cd} \left(x - \frac{-b + \sqrt{b^2 - 4cd}}{2c}\right) + 2\sqrt{a - d} \sqrt{a + \left(x - \frac{-b + \sqrt{b^2 - 4cd}}{2c}\right)^2 c - d + \sqrt{b^2 - 4cd} \left(x - \frac{-b + \sqrt{b^2 - 4cd}}{2c}\right)}}{x - \frac{-b + \sqrt{b^2 - 4cd}}{2c}}\right)}{\sqrt{b^2 - 4cd} \sqrt{a - d}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x^2+b*x+d)/(c*x^2+b*x+a)^(1/2), x)`

[Out]  $\frac{1}{(b^2-4cd)^{1/2}(a-d)^{1/2}} \ln\left(\frac{(2a-2d-(b^2-4cd)^{1/2})(x+1/2((b^2-4cd)^{1/2}+b)/c)+2(a-d)^{1/2}((x+1/2((b^2-4cd)^{1/2}+b)/c)^2c-(b^2-4cd)^{1/2}(x+1/2((b^2-4cd)^{1/2}+b)/c)+a-d)^{1/2}}{(x+1/2((b^2-4cd)^{1/2}+b)/c)}\right) - \frac{1}{(b^2-4cd)^{1/2}(a-d)^{1/2}} \ln\left(\frac{(2a-2d+(b^2-4cd)^{1/2})(x-1/2(-b+(b^2-4cd)^{1/2})/c)+2(a-d)^{1/2}((x-1/2(-b+(b^2-4cd)^{1/2})/c)^2c+(b^2-4cd)^{1/2}(x-1/2(-b+(b^2-4cd)^{1/2})/c)+a-d)^{1/2}}{(x-1/2(-b+(b^2-4cd)^{1/2})/c)}\right)$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x^2+b*x+d)/(c*x^2+b*x+a)^(1/2), x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*c\*d-b^2>0)', see 'assume?' for more details) Is 4\*c\*d-b^2 positive, negative or zero?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{cx^2 + bx + a} (cx^2 + bx + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((a + b*x + c*x^2)^(1/2)*(d + b*x + c*x^2)), x)`

[Out] `int(1/((a + b*x + c*x^2)^(1/2)*(d + b*x + c*x^2)), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + bx + cx^2} (bx + cx^2 + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x**2+b*x+d)/(c*x**2+b*x+a)**(1/2), x)`

[Out] `Integral(1/(sqrt(a + b*x + c*x**2)*(b*x + c*x**2 + d)), x)`

$$3.4 \quad \int \frac{1}{\sqrt{a+bx+cx^2} (d+bx+cx^2)^2} dx$$

**Optimal.** Leaf size=129

$$\frac{(4c(a-2d)+b^2) \tanh^{-1}\left(\frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd}\sqrt{a+bx+cx^2}}\right)}{(a-d)^{3/2}(b^2-4cd)^{3/2}} - \frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)(b^2-4cd)(bx+cx^2+d)}$$

[Out] (b^2+4\*c\*(a-2\*d))\*arctanh((2\*c\*x+b)\*(a-d)^(1/2)/(b^2-4\*c\*d)^(1/2)/(c\*x^2+b\*x+a)^(1/2))/(a-d)^(3/2)/(b^2-4\*c\*d)^(3/2)-(2\*c\*x+b)\*(c\*x^2+b\*x+a)^(1/2)/(a-d)/(b^2-4\*c\*d)/(c\*x^2+b\*x+d)

**Rubi [A]** time = 0.17, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$ , Rules used = {974, 12, 982, 208}

$$\frac{(4c(a-2d)+b^2) \tanh^{-1}\left(\frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd}\sqrt{a+bx+cx^2}}\right)}{(a-d)^{3/2}(b^2-4cd)^{3/2}} - \frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)(b^2-4cd)(bx+cx^2+d)}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + b\*x + c\*x^2]\*(d + b\*x + c\*x^2)^2), x]

[Out] -(((b + 2\*c\*x)\*Sqrt[a + b\*x + c\*x^2])/((a - d)\*(b^2 - 4\*c\*d)\*(d + b\*x + c\*x^2))) + ((b^2 + 4\*c\*(a - 2\*d))\*ArcTanh[(Sqrt[a - d]\*(b + 2\*c\*x))/(Sqrt[b^2 - 4\*c\*d]\*Sqrt[a + b\*x + c\*x^2]])/((a - d)^(3/2)\*(b^2 - 4\*c\*d)^(3/2))

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 974

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_)\*((d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2)^(q\_), x\_Symbol] := Simp[((2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f) + c\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*x)\*(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^(q + 1))/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e

```

- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q,
0]

```

### Rule 982

```

Int[1/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]

```

### Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a+bx+cx^2} (d+bx+cx^2)^2} dx &= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)(b^2-4cd)(d+bx+cx^2)} + \frac{\int -\frac{c^2(b^2+4c(a-2d))(a-d)}{2\sqrt{a+bx+cx^2}(d+bx+cx^2)} dx}{c^2(a-d)^2(b^2-4cd)} \\
&= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)(b^2-4cd)(d+bx+cx^2)} - \frac{(b^2+4c(a-2d)) \int \frac{1}{\sqrt{a+bx+cx^2}(d+bx+cx^2)}}{2(a-d)(b^2-4cd)} \\
&= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)(b^2-4cd)(d+bx+cx^2)} + \frac{(b(b^2+4c(a-2d))) \operatorname{Subst}\left(\int \frac{1}{b(b^2+4c(a-2d)+bx^2)} dx\right)}{(a-d)(b^2-4cd)} \\
&= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)(b^2-4cd)(d+bx+cx^2)} + \frac{(b^2+4c(a-2d)) \tanh^{-1}\left(\frac{\sqrt{a-d}}{\sqrt{b^2-4cd}}\right)}{(a-d)^{3/2}(b^2-4cd)^{3/2}}
\end{aligned}$$



$$\begin{aligned} &^2)*d^2 - 2*(a*b^5 + 4*a^2*b^3*c)*d)*x), -1/2*(\sqrt{-a*b^2 - 4*c*d^2 + (b^2 \\ &+ 4*a*c)*d)*(8*c*d^2 - (b^2*c + 4*a*c^2 - 8*c^2*d)*x^2 - (b^2 + 4*a*c)*d - \\ &(b^3 + 4*a*b*c - 8*b*c*d)*x)*\arctan(-1/2*(2*a*b^2 + (b^2*c + 4*a*c^2 - 8*c \\ &^2*d)*x^2 - (b^2 + 4*a*c)*d + (b^3 + 4*a*b*c - 8*b*c*d)*x)*\sqrt{-a*b^2 - 4* \\ &c*d^2 + (b^2 + 4*a*c)*d)*\sqrt{c*x^2 + b*x + a}/(a^2*b^3 + 4*a*b*c*d^2 + 2*( \\ &a*b^2*c^2 + 4*c^3*d^2 - (b^2*c^2 + 4*a*c^3)*d)*x^3 + 3*(a*b^3*c + 4*b*c^2*d \\ &^2 - (b^3*c + 4*a*b*c^2)*d)*x^2 - (a*b^3 + 4*a^2*b*c)*d + (a*b^4 + 2*a^2*b^ \\ &2*c + 4*(b^2*c + 2*a*c^2)*d^2 - (b^4 + 6*a*b^2*c + 8*a^2*c^2)*d)*x)) + 2*(a \\ &*b^3 + 4*b*c*d^2 - (b^3 + 4*a*b*c)*d + 2*(a*b^2*c + 4*c^2*d^2 - (b^2*c + 4* \\ &a*c^2)*d)*x)*\sqrt{c*x^2 + b*x + a}/(a^2*b^4*d + 16*c^2*d^5 - 8*(b^2*c + 4* \\ &a*c^2)*d^4 + (b^4 + 16*a*b^2*c + 16*a^2*c^2)*d^3 - 2*(a*b^4 + 4*a^2*b^2*c)* \\ &d^2 + (a^2*b^4*c + 16*c^3*d^4 - 8*(b^2*c^2 + 4*a*c^3)*d^3 + (b^4*c + 16*a*b \\ &^2*c^2 + 16*a^2*c^3)*d^2 - 2*(a*b^4*c + 4*a^2*b^2*c^2)*d)*x^2 + (a^2*b^5 + \\ &16*b*c^2*d^4 - 8*(b^3*c + 4*a*b*c^2)*d^3 + (b^5 + 16*a*b^3*c + 16*a^2*b*c^2 \\ &)*d^2 - 2*(a*b^5 + 4*a^2*b^3*c)*d)*x)] \end{aligned}$$

**giac [B]** time = 2.27, size = 1166, normalized size = 9.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x^2+b\*x+d)^2/(c\*x^2+b\*x+a)^(1/2),x, algorithm="giac")

[Out] 
$$\begin{aligned} &-1/2*((b^2 + 4*a*c - 8*c*d)*\log(\text{abs}((\sqrt{c})x - \sqrt{c*x^2 + b*x + a}))^2*b \\ &^2*c + 4*(\sqrt{c})x - \sqrt{c*x^2 + b*x + a})^2*a*c^2 - 8*(\sqrt{c})x - \sqrt{ \\ &c*x^2 + b*x + a})^2*c^2*d + (\sqrt{c})x - \sqrt{c*x^2 + b*x + a})*b^3*\sqrt{c} \\ &+ 4*(\sqrt{c})x - \sqrt{c*x^2 + b*x + a})*a*b*c^{(3/2)} - 8*(\sqrt{c})x - \sqrt{ \\ &c*x^2 + b*x + a})*b*c^{(3/2)*d} + 3*a*b^2*c + 4*\sqrt{a*b^2 - b^2*d - 4*a*c*d \\ &+ 4*c*d^2}*(\sqrt{c})x - \sqrt{c*x^2 + b*x + a})^2*c^{(3/2)} - 4*a^2*c^2 - 2*b^ \\ &2*c*d + 4*\sqrt{a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2}*(\sqrt{c})x - \sqrt{c*x^2 + \\ &b*x + a})*b*c + \sqrt{a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2}*b^2*\sqrt{c}))/\sqrt{ \\ &(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2) - (b^2 + 4*a*c - 8*c*d)*\log(\text{abs}((\sqrt{c} \\ &)*x - \sqrt{c*x^2 + b*x + a}))^2*b^2*c + 4*(\sqrt{c})x - \sqrt{c*x^2 + b*x + a} \\ &)^2*a*c^2 - 8*(\sqrt{c})x - \sqrt{c*x^2 + b*x + a})^2*c^2*d + (\sqrt{c})x - \sqrt{ \\ &c*x^2 + b*x + a})*b^3*\sqrt{c} + 4*(\sqrt{c})x - \sqrt{c*x^2 + b*x + a})*a* \\ &b*c^{(3/2)} - 8*(\sqrt{c})x - \sqrt{c*x^2 + b*x + a})*b*c^{(3/2)*d} + 3*a*b^2*c - \\ &4*\sqrt{a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2}*(\sqrt{c})x - \sqrt{c*x^2 + b*x + \\ &a})^2*c^{(3/2)} - 4*a^2*c^2 - 2*b^2*c*d - 4*\sqrt{a*b^2 - b^2*d - 4*a*c*d + 4* \\ &c*d^2}*(\sqrt{c})x - \sqrt{c*x^2 + b*x + a})*b*c - \sqrt{a*b^2 - b^2*d - 4*a*c \\ &*d + 4*c*d^2}*b^2*\sqrt{c}))/\sqrt{a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2}))/ \\ &(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2) + ((\sqrt{c})x - \sqrt{c*x^2 + b*x + a})^2*b^2* \\ &\sqrt{c} + 4*(\sqrt{c})x - \sqrt{c*x^2 + b*x + a})^2*a*c^{(3/2)} - 8*(\sqrt{c})x \\ &- \sqrt{c*x^2 + b*x + a})^2*c^{(3/2)*d} + (\sqrt{c})x - \sqrt{c*x^2 + b*x + a})* \\ &b^3 + 4*(\sqrt{c})x - \sqrt{c*x^2 + b*x + a})*a*b*c - 8*(\sqrt{c})x - \sqrt{c*x \\ &^2 + b*x + a})*b*c*d + 3*a*b^2*\sqrt{c} - 4*a^2*c^{(3/2)} - 2*b^2*\sqrt{c}*d)/ \end{aligned}$$

$$\begin{aligned} & ((\sqrt{c}x - \sqrt{cx^2 + bx + a})^4c + 2(\sqrt{c}x - \sqrt{cx^2 + bx + a})^3b\sqrt{c} + (\sqrt{c}x - \sqrt{cx^2 + bx + a})^2b^2 - 2(\sqrt{c}x - \sqrt{cx^2 + bx + a})^2ac + 4(\sqrt{c}x - \sqrt{cx^2 + bx + a})^2cd - 2(\sqrt{c}x - \sqrt{cx^2 + bx + a})ab\sqrt{c} + 4(\sqrt{c}x - \sqrt{cx^2 + bx + a})b\sqrt{c}d - ab^2 + a^2c + b^2d)(ab^2 - b^2d - 4ac*d + 4cd^2) \end{aligned}$$

**maple [B]** time = 0.03, size = 829, normalized size = 6.43

$$2c \ln \left( \frac{2a-2d+\sqrt{b^2-4cd} \left( x-\frac{-b+\sqrt{b^2-4cd}}{2c} \right) + 2\sqrt{a-d} \sqrt{a+\left( x-\frac{-b+\sqrt{b^2-4cd}}{2c} \right)^2 c-d+\sqrt{b^2-4cd} \left( x-\frac{-b+\sqrt{b^2-4cd}}{2c} \right)}}{x-\frac{-b+\sqrt{b^2-4cd}}{2c}} \right) - 2c \ln \left( \frac{2a-2d-\sqrt{b^2-4cd} \left( x+\frac{b+\sqrt{b^2-4cd}}{2c} \right)}{x+\frac{b+\sqrt{b^2-4cd}}{2c}} \right)$$


---


$$(b^2 - 4cd)^{\frac{3}{2}} \sqrt{a-d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c\*x^2+b\*x+d)^2/(c\*x^2+b\*x+a)^(1/2), x)

[Out] 
$$\begin{aligned} & -2/(b^2-4cd)^{3/2}c/(a-d)^{1/2}*\ln((2a-2d-(b^2-4cd)^{1/2}*(x+1/2*(b+(b^2-4cd)^{1/2}))/c)+2*(a-d)^{1/2}*(a+(x+1/2*(b+(b^2-4cd)^{1/2}))/c))^2*c-d-(b^2-4cd)^{1/2}*(x+1/2*(b+(b^2-4cd)^{1/2}))/c)^{1/2})/(x+1/2*(b+(b^2-4cd)^{1/2}))/c)+2/(b^2-4cd)^{3/2}c/(a-d)^{1/2}*\ln((2a-2d+(b^2-4cd)^{1/2}*(x-1/2*(-b+(b^2-4cd)^{1/2}))/c)+2*(a-d)^{1/2}*(a+(x-1/2*(-b+(b^2-4cd)^{1/2}))/c))^2*c-d+(b^2-4cd)^{1/2}*(x-1/2*(-b+(b^2-4cd)^{1/2}))/c)^{1/2})/(x-1/2*(-b+(b^2-4cd)^{1/2}))/c)-1/(b^2-4cd)/(a-d)/(x-1/2/c*(b^2-4cd)^{1/2}+1/2/c*b)*(a+(x-1/2*(-b+(b^2-4cd)^{1/2}))/c))^2*c-d+(b^2-4cd)^{1/2}*(x-1/2*(-b+(b^2-4cd)^{1/2}))/c)^{1/2}+1/2/(b^2-4cd)^{1/2}/(a-d)^{3/2}*\ln((2a-2d+(b^2-4cd)^{1/2}*(x-1/2*(-b+(b^2-4cd)^{1/2}))/c)+2*(a-d)^{1/2}*(a+(x-1/2*(-b+(b^2-4cd)^{1/2}))/c))^2*c-d+(b^2-4cd)^{1/2}*(x-1/2*(-b+(b^2-4cd)^{1/2}))/c)^{1/2})/(x-1/2*(-b+(b^2-4cd)^{1/2}))/c)-1/(b^2-4cd)/(a-d)/(x+1/2/c*(b^2-4cd)^{1/2}+1/2/c*b)*(a+(x+1/2*(b+(b^2-4cd)^{1/2}))/c))^2*c-d-(b^2-4cd)^{1/2}*(x+1/2*(b+(b^2-4cd)^{1/2}))/c)^{1/2}-1/2/(b^2-4cd)^{1/2}/(a-d)^{3/2}*\ln((2a-2d-(b^2-4cd)^{1/2}*(x+1/2*(b+(b^2-4cd)^{1/2}))/c)+2*(a-d)^{1/2}*(a+(x+1/2*(b+(b^2-4cd)^{1/2}))/c))^2*c-d-(b^2-4cd)^{1/2}*(x+1/2*(b+(b^2-4cd)^{1/2}))/c)^{1/2})/(x+1/2*(b+(b^2-4cd)^{1/2}))/c) \end{aligned}$$

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + bx + a} (cx^2 + bx + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(1/(c\*x^2+b\*x+d)^2/(c\*x^2+b\*x+a)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c\*x^2 + b\*x + a)\*(c\*x^2 + b\*x + d)^2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{cx^2 + bx + a} (cx^2 + bx + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x + c\*x^2)^(1/2)\*(d + b\*x + c\*x^2)^2),x)

[Out] int(1/((a + b\*x + c\*x^2)^(1/2)\*(d + b\*x + c\*x^2)^2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + bx + cx^2} (bx + cx^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x\*\*2+b\*x+d)\*\*2/(c\*x\*\*2+b\*x+a)\*\*(1/2),x)

[Out] Integral(1/(sqrt(a + b\*x + c\*x\*\*2)\*(b\*x + c\*x\*\*2 + d)\*\*2), x)

$$3.5 \quad \int \frac{1}{\sqrt{a+bx+cx^2} (d+bx+cx^2)^3} dx$$

**Optimal.** Leaf size=224

$$\frac{(16c^2(3a^2 - 8ad + 8d^2) + 8b^2c(a - 4d) + 3b^4) \tanh^{-1}\left(\frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd}\sqrt{a+bx+cx^2}}\right)}{4(a-d)^{5/2}(b^2-4cd)^{5/2}} + \frac{3(b+2cx)(4c(a-2d)+b^2)\sqrt{a+bx+cx^2}}{4(a-d)^2(b^2-4cd)^2(bx+cx^2+d)}$$

[Out]  $-1/4*(3*b^4+8*b^2*c*(a-4*d)+16*c^2*(3*a^2-8*a*d+8*d^2))*\operatorname{arctanh}((2*c*x+b)*(a-d)^{(1/2)}/(b^2-4*c*d)^{(1/2)}/(c*x^2+b*x+a)^{(1/2)})/(a-d)^{(5/2)}/(b^2-4*c*d)^{(5/2)}-1/2*(2*c*x+b)*(c*x^2+b*x+a)^{(1/2)}/(a-d)/(b^2-4*c*d)/(c*x^2+b*x+d)^2+3/4*(b^2+4*c*(a-2*d))*(2*c*x+b)*(c*x^2+b*x+a)^{(1/2)}/(a-d)^2/(b^2-4*c*d)^2/(c*x^2+b*x+d)$

**Rubi [A]** time = 0.43, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {974, 1060, 12, 982, 208}

$$\frac{(16c^2(3a^2 - 8ad + 8d^2) + 8b^2c(a - 4d) + 3b^4) \tanh^{-1}\left(\frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd}\sqrt{a+bx+cx^2}}\right)}{4(a-d)^{5/2}(b^2-4cd)^{5/2}} + \frac{3(b+2cx)(4c(a-2d)+b^2)\sqrt{a+bx+cx^2}}{4(a-d)^2(b^2-4cd)^2(bx+cx^2+d)}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + b\*x + c\*x^2]\*(d + b\*x + c\*x^2)^3), x]

[Out]  $-((b + 2*c*x)*\operatorname{Sqrt}[a + b*x + c*x^2])/(2*(a - d)*(b^2 - 4*c*d)*(d + b*x + c*x^2)^2) + (3*(b^2 + 4*c*(a - 2*d))*(b + 2*c*x)*\operatorname{Sqrt}[a + b*x + c*x^2])/(4*(a - d)^2*(b^2 - 4*c*d)^2*(d + b*x + c*x^2)) - ((3*b^4 + 8*b^2*c*(a - 4*d) + 16*c^2*(3*a^2 - 8*a*d + 8*d^2))*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a - d]*(b + 2*c*x))/(\operatorname{Sqrt}[b^2 - 4*c*d]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(4*(a - d)^{(5/2)}*(b^2 - 4*c*d)^{(5/2)})$

**Rule 12**

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

**Rule 208**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

**Rule 974**

```

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]

```

### Rule 982

```

Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0]

```

### Rule 1060

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Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)

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)^2 - (b\*d - a\*e)\*(c\*e - b\*f), 0] && !( !IntegerQ[p] && !LtQ[q, -1]) && !  
IGtQ[q, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{1}{\sqrt{a+bx+cx^2} (d+bx+cx^2)^3} dx &= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)(b^2-4cd)(d+bx+cx^2)^2} + \frac{\int \frac{-\frac{1}{2}c^2(a-d)(3b^2+12ac-16cd)-4bc^3(a-d)x}{\sqrt{a+bx+cx^2}(d+bx+cx^2)^2} dx}{2c^2(a-d)^2(b^2-4cd)} \\
 &= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)(b^2-4cd)(d+bx+cx^2)^2} + \frac{3(b^2+4c(a-2d))(b+2cx)\sqrt{a+bx+cx^2}}{4(a-d)^2(b^2-4cd)^2(d+bx+cx^2)} \\
 &= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)(b^2-4cd)(d+bx+cx^2)^2} + \frac{3(b^2+4c(a-2d))(b+2cx)\sqrt{a+bx+cx^2}}{4(a-d)^2(b^2-4cd)^2(d+bx+cx^2)} \\
 &= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)(b^2-4cd)(d+bx+cx^2)^2} + \frac{3(b^2+4c(a-2d))(b+2cx)\sqrt{a+bx+cx^2}}{4(a-d)^2(b^2-4cd)^2(d+bx+cx^2)} \\
 &= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)(b^2-4cd)(d+bx+cx^2)^2} + \frac{3(b^2+4c(a-2d))(b+2cx)\sqrt{a+bx+cx^2}}{4(a-d)^2(b^2-4cd)^2(d+bx+cx^2)}
 \end{aligned}$$

**Mathematica [B]** time = 6.38, size = 1746, normalized size = 7.79

result too large to display

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[a + b\*x + c\*x^2]\*(d + b\*x + c\*x^2)^3),x]

[Out] 
$$\begin{aligned}
 &(-2*c^2*(a + b*x + c*x^2))/((a - d)*(b^2 - 4*c*d)^(3/2)*(b - Sqrt[b^2 - 4*c*d] \\
 &+ 2*c*x)^2*Sqrt[a + x*(b + c*x)]) + (6*c^2*(a + b*x + c*x^2))/((a - d)* \\
 &(b^2 - 4*c*d)^2*(b - Sqrt[b^2 - 4*c*d] + 2*c*x)*Sqrt[a + x*(b + c*x)]) + (2 \\
 &*c^2*(a + b*x + c*x^2))/((a - d)*(b^2 - 4*c*d)^(3/2)*(b + Sqrt[b^2 - 4*c*d] \\
 &+ 2*c*x)^2*Sqrt[a + x*(b + c*x)]) + (6*c^2*(a + b*x + c*x^2))/((a - d)*(b^2 \\
 &- 4*c*d)^2*(b + Sqrt[b^2 - 4*c*d] + 2*c*x)*Sqrt[a + x*(b + c*x)]) + (6*c^2 \\
 &*Sqrt[a + b*x + c*x^2]*ArcTanh[(b^2 - 4*a*c - b*Sqrt[b^2 - 4*c*d] - 2*c*Sqrt[b^2 - 4*c*d]*x)/(4*c*Sqrt[a - d]*Sqrt[a + b*x + c*x^2]))/(Sqrt[a - d]*(b^2 - 4*c*d)^(5/2)*Sqrt[a + x*(b + c*x)]) + (3*c*Sqrt[a + b*x + c*x^2]*ArcT
 \end{aligned}$$

$$\operatorname{anh}\left[\frac{(b^2 - 4ac - b\sqrt{b^2 - 4cd} - 2c\sqrt{b^2 - 4cd})x}{(4c\sqrt{a-d}\sqrt{a+bx+cx^2})}\right] / (2(a-d)^{3/2}(b^2 - 4cd)^{3/2}\sqrt{a+x(b+cx)}) + (6c^2\sqrt{a+bx+cx^2}\operatorname{ArcTanh}[(4ac - b(b+\sqrt{b^2 - 4cd}) - 2c\sqrt{b^2 - 4cd})x] / (4c\sqrt{a-d}\sqrt{a+bx+cx^2})) / (\sqrt{a-d}(b^2 - 4cd)^{5/2}\sqrt{a+x(b+cx)}) + (3c\sqrt{a+bx+cx^2}\operatorname{ArcTanh}[(4ac - b(b+\sqrt{b^2 - 4cd}) - 2c\sqrt{b^2 - 4cd})x] / (4c\sqrt{a-d}\sqrt{a+bx+cx^2})) / (2(a-d)^{3/2}(b^2 - 4cd)^{3/2}\sqrt{a+x(b+cx)}) + (4c^3\sqrt{a+bx+cx^2} * ((-2c^2(-b+\sqrt{b^2 - 4cd}) - 2c^2(b+2\sqrt{b^2 - 4cd}))\sqrt{a+bx+cx^2}) / ((4ac^2 + 2b*c*(-b+\sqrt{b^2 - 4cd}) + c*(-b+\sqrt{b^2 - 4cd})^2)*(-b+\sqrt{b^2 - 4cd} - 2cx) + (4c\sqrt{a-d}*(b(-2c^2(-b+\sqrt{b^2 - 4cd}) + 2c^2(b+2\sqrt{b^2 - 4cd})) - 2(4ac^3 - c^2(-b+\sqrt{b^2 - 4cd}))(b+2\sqrt{b^2 - 4cd}))))\operatorname{ArcTanh}[(4ac - b(-b+\sqrt{b^2 - 4cd}) - (2bc + 2c*(-b+\sqrt{b^2 - 4cd}))x] / (4c\sqrt{a-d}\sqrt{a+bx+cx^2})) / ((4ac^2 + 2b*c*(-b+\sqrt{b^2 - 4cd}) + c*(-b+\sqrt{b^2 - 4cd})^2)*(16ac^2 + 8b*c*(-b+\sqrt{b^2 - 4cd}) + 4c*(-b+\sqrt{b^2 - 4cd})^2))) / ((b^2 - 4cd)^{3/2}(4ac^2 + 2b*c*(-b+\sqrt{b^2 - 4cd}) + c*(-b+\sqrt{b^2 - 4cd})^2)\sqrt{a+x(b+cx)}) + (4c^3\sqrt{a+bx+cx^2} * (((-2c^2(b-2\sqrt{b^2 - 4cd}) + 2c^2(b+\sqrt{b^2 - 4cd}))\sqrt{a+bx+cx^2}) / ((4ac^2 - 2b*c*(b+\sqrt{b^2 - 4cd}) + c*(b+\sqrt{b^2 - 4cd})^2)*(b+\sqrt{b^2 - 4cd} + 2cx) + (4c\sqrt{a-d}*(b(2c^2(b-2\sqrt{b^2 - 4cd}) + 2c^2(b+\sqrt{b^2 - 4cd})) - 2(4ac^3 + c^2(b-2\sqrt{b^2 - 4cd}))(b+\sqrt{b^2 - 4cd}))))\operatorname{ArcTanh}[(4ac - b(b+\sqrt{b^2 - 4cd}) - (-2bc + 2c*(b+\sqrt{b^2 - 4cd}))x] / (4c\sqrt{a-d}\sqrt{a+bx+cx^2})) / ((4ac^2 - 2b*c*(b+\sqrt{b^2 - 4cd}) + c*(b+\sqrt{b^2 - 4cd})^2)*(16ac^2 - 8b*c*(b+\sqrt{b^2 - 4cd}) + 4c*(b+\sqrt{b^2 - 4cd})^2))) / ((b^2 - 4cd)^{3/2}(4ac^2 - 2b*c*(b+\sqrt{b^2 - 4cd}) + c*(b+\sqrt{b^2 - 4cd})^2)\sqrt{a+x(b+cx)})$$

**fricas** [B] time = 9.94, size = 3818, normalized size = 17.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cx^2+bx+d)^3/(cx^2+bx+a)^(1/2),x, algorithm="fricas")

[Out] [1/16\*((128\*c^2\*d^4 + (3\*b^4\*c^2 + 8\*a\*b^2\*c^3 + 48\*a^2\*c^4 + 128\*c^4\*d^2 - 32\*(b^2\*c^3 + 4\*a\*c^4)\*d)\*x^4 - 32\*(b^2\*c + 4\*a\*c^2)\*d^3 + 2\*(3\*b^5\*c + 8\*a\*b^3\*c^2 + 48\*a^2\*b\*c^3 + 128\*b\*c^3\*d^2 - 32\*(b^3\*c^2 + 4\*a\*b\*c^3)\*d)\*x^3 + (3\*b^4 + 8\*a\*b^2\*c + 48\*a^2\*c^2)\*d^2 + (3\*b^6 + 8\*a\*b^4\*c + 48\*a^2\*b^2\*c^2 + 256\*c^3\*d^3 + 64\*(b^2\*c^2 - 4\*a\*c^3)\*d^2 - 2\*(13\*b^4\*c + 56\*a\*b^2\*c^2 - 48\*a^2\*c^3)\*d)\*x^2 + 2\*(128\*b\*c^2\*d^3 - 32\*(b^3\*c + 4\*a\*b\*c^2)\*d^2 + (3\*b^5 + 8\*a\*b^3\*c + 48\*a^2\*b\*c^2)\*d)\*x)\*sqrt(a\*b^2 + 4\*c\*d^2 - (b^2 + 4\*a\*c)\*d) \*log((8\*a^2\*b^4 + (b^4\*c^2 + 24\*a\*b^2\*c^3 + 16\*a^2\*c^4 + 128\*c^4\*d^2 - 32\*(

$$\begin{aligned}
& b^2*c^3 + 4*a*c^4)*d)*x^4 + 2*(b^5*c + 24*a*b^3*c^2 + 16*a^2*b*c^3 + 128*b*c^3*d^2 - 32*(b^3*c^2 + 4*a*b*c^3)*d)*x^3 + (b^4 + 24*a*b^2*c + 16*a^2*c^2)*d^2 + (b^6 + 32*a*b^4*c + 48*a^2*b^2*c^2 + 32*(5*b^2*c^2 + 4*a*c^3)*d^2 - 2*(19*b^4*c + 104*a*b^2*c^2 + 48*a^2*c^3)*d)*x^2 - 4*(2*a*b^3 + 2*(b^2*c^2 + 4*a*c^3 - 8*c^3*d)*x^3 + 3*(b^3*c + 4*a*b*c^2 - 8*b*c^2*d)*x^2 - (b^3 + 4*a*b*c)*d + (b^4 + 8*a*b^2*c - 2*(5*b^2*c + 4*a*c^2)*d)*x)*sqrt(a*b^2 + 4*c*d^2 - (b^2 + 4*a*c)*d)*sqrt(c*x^2 + b*x + a) - 8*(a*b^4 + 4*a^2*b^2*c)*d + 2*(4*a*b^5 + 16*a^2*b^3*c + 16*(b^3*c + 4*a*b*c^2)*d^2 - (3*b^5 + 40*a*b^3*c + 48*a^2*b*c^2)*d)*x)/(c^2*x^4 + 2*b*c*x^3 + 2*b*d*x + (b^2 + 2*c*d)*x^2 + d^2)) - 4*(2*a^2*b^5 + 128*b*c^2*d^4 - 52*(b^3*c + 4*a*b*c^2)*d^3 - 6*(a*b^4*c^2 + 4*a^2*b^2*c^3 - 32*c^4*d^3 + 12*(b^2*c^3 + 4*a*c^4)*d^2 - (b^4*c^2 + 16*a*b^2*c^3 + 16*a^2*c^4)*d)*x^3 + 5*(b^5 + 16*a*b^3*c + 16*a^2*b*c^2)*d^2 - 9*(a*b^5*c + 4*a^2*b^3*c^2 - 32*b*c^3*d^3 + 12*(b^3*c^2 + 4*a*b*c^3)*d^2 - (b^5*c + 16*a*b^3*c^2 + 16*a^2*b*c^3)*d)*x^2 - 7*(a*b^5 + 4*a^2*b^3*c)*d - (3*a*b^6 + 8*a^2*b^4*c - 256*c^3*d^4 + 8*(b^2*c^2 + 52*a*c^3)*d^3 + 2*(13*b^4*c - 8*a*b^2*c^2 - 80*a^2*c^3)*d^2 - (3*b^6 + 34*a*b^4*c - 8*a^2*b^2*c^2)*d)*x)*sqrt(c*x^2 + b*x + a))/(a^3*b^6*d^2 + 64*c^3*d^8 - 48*(b^2*c^2 + 4*a*c^3)*d^7 + 12*(b^4*c + 12*a*b^2*c^2 + 16*a^2*c^3)*d^6 - (b^6 + 36*a*b^4*c + 144*a^2*b^2*c^2 + 64*a^3*c^3)*d^5 + 3*(a*b^6 + 12*a^2*b^4*c + 16*a^3*b^2*c^2)*d^4 + (a^3*b^6*c^2 + 64*c^5*d^6 - 48*(b^2*c^4 + 4*a*c^5)*d^5 + 12*(b^4*c^3 + 12*a*b^2*c^4 + 16*a^2*c^5)*d^4 - (b^6*c^2 + 36*a*b^4*c^3 + 144*a^2*b^2*c^4 + 64*a^3*c^5)*d^3 + 3*(a*b^6*c^2 + 12*a^2*b^4*c^3 + 16*a^3*b^2*c^4)*d^2 - 3*(a^2*b^6*c^2 + 4*a^3*b^4*c^3)*d)*x^4 - 3*(a^2*b^6 + 4*a^3*b^4*c)*d^3 + 2*(a^3*b^7*c + 64*b*c^4*d^6 - 48*(b^3*c^3 + 4*a*b*c^4)*d^5 + 12*(b^5*c^2 + 12*a*b^3*c^3 + 16*a^2*b*c^4)*d^4 - (b^7*c + 36*a*b^5*c^2 + 144*a^2*b^3*c^3 + 64*a^3*b*c^4)*d^3 + 3*(a*b^7*c + 12*a^2*b^5*c^2 + 16*a^3*b^3*c^3)*d^2 - 3*(a^2*b^7*c + 4*a^3*b^5*c^2)*d)*x^3 + (a^3*b^8 + 128*c^4*d^7 - 32*(b^2*c^3 + 12*a*c^4)*d^6 - 24*(b^4*c^2 - 4*a*b^2*c^3 - 16*a^2*c^4)*d^5 + 2*(5*b^6*c + 36*a*b^4*c^2 - 48*a^2*b^2*c^3 - 64*a^3*c^4)*d^4 - (b^8 + 30*a*b^6*c + 72*a^2*b^4*c^2 - 32*a^3*b^2*c^3)*d^3 + 3*(a*b^8 + 10*a^2*b^6*c + 8*a^3*b^4*c^2)*d^2 - (3*a^2*b^8 + 10*a^3*b^6*c)*d)*x^2 + 2*(a^3*b^7*d + 64*b*c^3*d^7 - 48*(b^3*c^2 + 4*a*b*c^3)*d^6 + 12*(b^5*c + 12*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 - (b^7 + 36*a*b^5*c + 144*a^2*b^3*c^2 + 64*a^3*b*c^3)*d^4 + 3*(a*b^7 + 12*a^2*b^5*c + 16*a^3*b^3*c^2)*d^3 - 3*(a^2*b^7 + 4*a^3*b^5*c)*d^2)*x), -1/8*((128*c^2*d^4 + (3*b^4*c^2 + 8*a*b^2*c^3 + 48*a^2*c^4 + 128*c^4*d^2 - 32*(b^2*c^3 + 4*a*c^4)*d)*x^4 - 32*(b^2*c + 4*a*c^2)*d^3 + 2*(3*b^5*c + 8*a*b^3*c^2 + 48*a^2*b*c^3 + 128*b*c^3*d^2 - 32*(b^3*c^2 + 4*a*b*c^3)*d)*x^3 + (3*b^4 + 8*a*b^2*c + 48*a^2*c^2)*d^2 + (3*b^6 + 8*a*b^4*c + 48*a^2*b^2*c^2 + 256*c^3*d^3 + 64*(b^2*c^2 - 4*a*c^3)*d^2 - 2*(13*b^4*c + 56*a*b^2*c^2 - 48*a^2*c^3)*d)*x^2 + 2*(128*b*c^2*d^3 - 32*(b^3*c + 4*a*b*c^2)*d^2 + (3*b^5 + 8*a*b^3*c + 48*a^2*b*c^2)*d)*x)*sqrt(-a*b^2 - 4*c*d^2 + (b^2 + 4*a*c)*d)*arctan(-1/2*(2*a*b^2 + (b^2*c + 4*a*c^2 - 8*c^2*d)*x^2 - (b^2 + 4*a*c)*d + (b^3 + 4*a*b*c - 8*b*c*d)*x)*sqrt(-a*b^2 - 4*c*d^2 + (b^2 + 4*a*c)*d)*sqrt(c*x^2 + b*x + a)/(a^2*b^3 + 4*a*b*c*d^2 + 2*(a*b^2*c^2 + 4*c^3*d^2 - (b^2*c^2 + 4*a*c^3)*d)*x^3 + 3*(a*b^3*c + 4*b*c^2*d^2 - (b^3*c + 4*a*b*c^2)
\end{aligned}$$

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*d)*x^2 - (a*b^3 + 4*a^2*b*c)*d + (a*b^4 + 2*a^2*b^2*c + 4*(b^2*c + 2*a*c^2
)*d^2 - (b^4 + 6*a*b^2*c + 8*a^2*c^2)*d)*x)) + 2*(2*a^2*b^5 + 128*b*c^2*d^4
- 52*(b^3*c + 4*a*b*c^2)*d^3 - 6*(a*b^4*c^2 + 4*a^2*b^2*c^3 - 32*c^4*d^3 +
12*(b^2*c^3 + 4*a*c^4)*d^2 - (b^4*c^2 + 16*a*b^2*c^3 + 16*a^2*c^4)*d)*x^3
+ 5*(b^5 + 16*a*b^3*c + 16*a^2*b*c^2)*d^2 - 9*(a*b^5*c + 4*a^2*b^3*c^2 - 32
*b*c^3*d^3 + 12*(b^3*c^2 + 4*a*b*c^3)*d^2 - (b^5*c + 16*a*b^3*c^2 + 16*a^2*
b*c^3)*d)*x^2 - 7*(a*b^5 + 4*a^2*b^3*c)*d - (3*a*b^6 + 8*a^2*b^4*c - 256*c^
3*d^4 + 8*(b^2*c^2 + 52*a*c^3)*d^3 + 2*(13*b^4*c - 8*a*b^2*c^2 - 80*a^2*c^3
)*d^2 - (3*b^6 + 34*a*b^4*c - 8*a^2*b^2*c^2)*d)*x)*sqrt(c*x^2 + b*x + a))/(
a^3*b^6*d^2 + 64*c^3*d^8 - 48*(b^2*c^2 + 4*a*c^3)*d^7 + 12*(b^4*c + 12*a*b^
2*c^2 + 16*a^2*c^3)*d^6 - (b^6 + 36*a*b^4*c + 144*a^2*b^2*c^2 + 64*a^3*c^3)
*d^5 + 3*(a*b^6 + 12*a^2*b^4*c + 16*a^3*b^2*c^2)*d^4 + (a^3*b^6*c^2 + 64*c^
5*d^6 - 48*(b^2*c^4 + 4*a*c^5)*d^5 + 12*(b^4*c^3 + 12*a*b^2*c^4 + 16*a^2*c^
5)*d^4 - (b^6*c^2 + 36*a*b^4*c^3 + 144*a^2*b^2*c^4 + 64*a^3*c^5)*d^3 + 3*(a
*b^6*c^2 + 12*a^2*b^4*c^3 + 16*a^3*b^2*c^4)*d^2 - 3*(a^2*b^6*c^2 + 4*a^3*b^
4*c^3)*d)*x^4 - 3*(a^2*b^6 + 4*a^3*b^4*c)*d^3 + 2*(a^3*b^7*c + 64*b*c^4*d^6
- 48*(b^3*c^3 + 4*a*b*c^4)*d^5 + 12*(b^5*c^2 + 12*a*b^3*c^3 + 16*a^2*b*c^4
)*d^4 - (b^7*c + 36*a*b^5*c^2 + 144*a^2*b^3*c^3 + 64*a^3*b*c^4)*d^3 + 3*(a*
b^7*c + 12*a^2*b^5*c^2 + 16*a^3*b^3*c^3)*d^2 - 3*(a^2*b^7*c + 4*a^3*b^5*c^2
)*d)*x^3 + (a^3*b^8 + 128*c^4*d^7 - 32*(b^2*c^3 + 12*a*c^4)*d^6 - 24*(b^4*c
^2 - 4*a*b^2*c^3 - 16*a^2*c^4)*d^5 + 2*(5*b^6*c + 36*a*b^4*c^2 - 48*a^2*b^2
*c^3 - 64*a^3*c^4)*d^4 - (b^8 + 30*a*b^6*c + 72*a^2*b^4*c^2 - 32*a^3*b^2*c^
3)*d^3 + 3*(a*b^8 + 10*a^2*b^6*c + 8*a^3*b^4*c^2)*d^2 - (3*a^2*b^8 + 10*a^3
*b^6*c)*d)*x^2 + 2*(a^3*b^7*d + 64*b*c^3*d^7 - 48*(b^3*c^2 + 4*a*b*c^3)*d^6
+ 12*(b^5*c + 12*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 - (b^7 + 36*a*b^5*c + 144*a
^2*b^3*c^2 + 64*a^3*b*c^3)*d^4 + 3*(a*b^7 + 12*a^2*b^5*c + 16*a^3*b^3*c^2)*
d^3 - 3*(a^2*b^7 + 4*a^3*b^5*c)*d^2)*x)]

```

**giac [B]** time = 4.87, size = 2986, normalized size = 13.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```

[Out] -1/8*((3*b^4 + 8*a*b^2*c + 48*a^2*c^2 - 32*b^2*c*d - 128*a*c^2*d + 128*c^2*
d^2)*log(abs(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*c - 4*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^2*a*c^2 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^
2*d - (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*sqrt(c) - 4*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))*a*b*c^(3/2) + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^
(3/2)*d - 3*a*b^2*c + 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^2*c^(3/2) + 4*a^2*c^2 + 2*b^2*c*d + 4*sqrt(a*b^2 -
b^2*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c + sqrt(
a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*b^2*sqrt(c))/sqrt(a*b^2 - b^2*d - 4*a*c
*d + 4*c*d^2) - (3*b^4 + 8*a*b^2*c + 48*a^2*c^2 - 32*b^2*c*d - 128*a*c^2*d

```

$$\begin{aligned}
& + 128*c^2*d^2)*\log(\text{abs}(-(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^2*b^2*c - 4*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^2*a*c^2 + 8*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^2*c^2*d - (\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))*b^3*\text{sqrt}(c) - 4*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))*a*b*c^(3/2) + 8*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))*b*c^(3/2)*d - 3*a*b^2*c - 4*\text{sqrt}(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*( \text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^2*c^(3/2) + 4*a^2*c^2 + 2*b^2*c*d - 4*\text{sqrt}(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*( \text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))*b*c - \text{sqrt}(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*b^2*\text{sqrt}(c)))/\text{sqrt}(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2))/(a^2*b^4 - 2*a*b^4*d - 8*a^2*b^2*c*d + b^4*d^2 + 16*a*b^2*c*d^2 + 16*a^2*c^2*d^2 - 8*b^2*c*d^3 - 32*a*c^2*d^3 + 16*c^2*d^4) - \\
& 1/4*(3*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^6*b^4*c^(3/2) + 8*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^6*a*b^2*c^(5/2) + 48*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^6*a^2*c^(7/2) - 32*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^6*b^2*c^(5/2)*d - 128*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^6*a*c^(7/2)*d + 128*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^6*c^(7/2)*d^2 + 9*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^5*b^5*c + 24*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^5*a*b^3*c^2 + 144*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^5*a^2*b*c^3 - 96*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^5*b^3*c^2*d - 384*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^5*a*b*c^3*d + 384*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^5*b*c^3*d^2 + 9*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^4*b^6*\text{sqrt}(c) + 15*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^4*a*b^4*c^(3/2) + 120*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^4*a^2*b^2*c^(5/2) - 144*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^4*a^3*c^(7/2) - 78*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^4*b^4*c^(3/2)*d - 240*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^4*a*b^2*c^(5/2)*d + 672*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^4*a^2*c^(7/2)*d + 192*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^4*b^2*c^(5/2)*d^2 - 1152*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^4*a*c^(7/2)*d^2 + 768*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^4*c^(7/2)*d^3 + 3*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^3*b^7 - 10*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^3*a*b^5*c - 288*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^3*a^3*b*c^3 + 4*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^3*b^5*c*d + 160*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^3*a*b^3*c^2*d + 1344*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^3*a^2*b*c^3*d - 256*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^3*b^3*c^2*d^2 - 2304*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^3*a*b*c^3*d^2 + 1536*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^3*b*c^3*d^3 - 14*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^2*a*b^6*\text{sqrt}(c) - 71*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^2*a^2*b^4*c^(3/2) - 200*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^2*a^3*b^2*c^(5/2) + 144*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^2*a^4*c^(7/2) + 23*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^2*b^6*\text{sqrt}(c)*d + 280*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^2*a*b^4*c^(3/2)*d + 1168*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^2*a^2*b^2*c^(5/2)*d - 640*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^2*a^3*c^(7/2)*d - 272*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^2*b^4*c^(3/2)*d^2 - 2048*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^2*a*b^2*c^(5/2)*d^2 + 640*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^2*a^2*c^(7/2)*d^2 + 1152*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^2*b^2*c^(5/2)*d^3 - 5*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))*a*b^7 - 47*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))*a^2*b^5*c - 56*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))*a^3*b^3*c^2 + 144*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))*a^4*b*
\end{aligned}$$





$$\begin{aligned}
& (b^2-4cd)^{1/2}/c)^{1/2}/(x+1/2*(b+(b^2-4cd)^{1/2})/c))*b^2-3/2/(b^2-4cd)^{3/2}/(a-d)^{5/2}*\ln((2a-2d-(b^2-4cd)^{1/2}*(x+1/2*(b+(b^2-4cd)^{1/2})/c)+2*(a-d)^{1/2}*(a+(x+1/2*(b+(b^2-4cd)^{1/2})/c)^2*c-d-(b^2-4cd)^{1/2}*(x+1/2*(b+(b^2-4cd)^{1/2})/c))^{1/2})/(x+1/2*(b+(b^2-4cd)^{1/2})/c))*c*d+1/(b^2-4cd)^{3/2}*c/(a-d)^{3/2}*\ln((2a-2d-(b^2-4cd)^{1/2}*(x+1/2*(b+(b^2-4cd)^{1/2})/c)+2*(a-d)^{1/2}*(a+(x+1/2*(b+(b^2-4cd)^{1/2})/c)^2*c-d-(b^2-4cd)^{1/2}*(x+1/2*(b+(b^2-4cd)^{1/2})/c))^{1/2})/(x+1/2*(b+(b^2-4cd)^{1/2})/c))+6*c^2/(b^2-4cd)^{5/2}/(a-d)^{1/2}*\ln((2a-2d-(b^2-4cd)^{1/2}*(x+1/2*(b+(b^2-4cd)^{1/2})/c)+2*(a-d)^{1/2}*(a+(x+1/2*(b+(b^2-4cd)^{1/2})/c)^2*c-d-(b^2-4cd)^{1/2}*(x+1/2*(b+(b^2-4cd)^{1/2})/c))^{1/2})/(x+1/2*(b+(b^2-4cd)^{1/2})/c))-6*c^2/(b^2-4cd)^{5/2}/(a-d)^{1/2}*\ln((2a-2d+(b^2-4cd)^{1/2}*(x-1/2*(-b+(b^2-4cd)^{1/2})/c)+2*(a-d)^{1/2}*(a+(x-1/2*(-b+(b^2-4cd)^{1/2})/c))^2*c-d+(b^2-4cd)^{1/2}*(x-1/2*(-b+(b^2-4cd)^{1/2})/c))^{1/2})/(x-1/2*(-b+(b^2-4cd)^{1/2})/c))+3/(b^2-4cd)^2*c/(a-d)/(x+1/2*b/c-1/2*(b^2-4cd)^{1/2}/c)*(a+(x-1/2*(-b+(b^2-4cd)^{1/2})/c))^2*c-d+(b^2-4cd)^{1/2}*(x-1/2*(-b+(b^2-4cd)^{1/2})/c))^{1/2}+3/(b^2-4cd)^2*c/(a-d)/(x+1/2*b/c+1/2*(b^2-4cd)^{1/2}/c)*(a+(x+1/2*(b+(b^2-4cd)^{1/2})/c))^2*c-d-(b^2-4cd)^{1/2}*(x+1/2*(b+(b^2-4cd)^{1/2})/c))^{1/2}
\end{aligned}$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + bx + a} (cx^2 + bx + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x^2+b\*x+d)^3/(c\*x^2+b\*x+a)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c\*x^2 + b\*x + a)\*(c\*x^2 + b\*x + d)^3), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{cx^2 + bx + a} (cx^2 + bx + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x + c\*x^2)^(1/2)\*(d + b\*x + c\*x^2)^3),x)

[Out] int(1/((a + b\*x + c\*x^2)^(1/2)\*(d + b\*x + c\*x^2)^3), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+d)**3/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

$$3.6 \quad \int \frac{1}{\sqrt{a+bx+cx^2} (d+bx+cx^2)^4} dx$$

**Optimal.** Leaf size=328

$$\frac{(b+2cx)(16c^2(15a^2-44ad+44d^2)+8b^2c(7a-22d)+15b^4)\sqrt{a+bx+cx^2}}{24(a-d)^3(b^2-4cd)^3(bx+cx^2+d)} + \frac{(4c(a-2d)+b^2)(16c^2(5a^2-8ad+4d^2)+8b^2c(7a-22d)+15b^4)\sqrt{a+bx+cx^2}}{24(a-d)^3(b^2-4cd)^3(bx+cx^2+d)}$$

[Out]  $\frac{1}{8}(b^2+4c(a-2d))(5b^4-8b^2c(a+4d)+16c^2(5a^2-8ad+4d^2))\operatorname{arctanh}\left(\frac{(2cx+b)(a-d)^{1/2}}{(b^2-4cd)^{1/2}(cx^2+bx+a)^{1/2}}\right)/(a-d)^{7/2}/(b^2-4cd)^{7/2}-\frac{1}{3}(2cx+b)(cx^2+bx+a)^{1/2}/(a-d)/(b^2-4cd)/(cx^2+bx+d)^3+\frac{5}{12}(b^2+4c(a-2d))(2cx+b)(cx^2+bx+a)^{1/2}/(a-d)^2/(b^2-4cd)^2/(cx^2+bx+d)^2-\frac{1}{24}(15b^4+8b^2c(7a-22d)+16c^2(15a^2-44ad+44d^2))(2cx+b)(cx^2+bx+a)^{1/2}/(a-d)^3/(b^2-4cd)^3/(cx^2+bx+d)$

**Rubi [A]** time = 0.97, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {974, 1060, 12, 982, 208}

$$\frac{(b+2cx)(16c^2(15a^2-44ad+44d^2)+8b^2c(7a-22d)+15b^4)\sqrt{a+bx+cx^2}}{24(a-d)^3(b^2-4cd)^3(bx+cx^2+d)} + \frac{(4c(a-2d)+b^2)(16c^2(5a^2-8ad+4d^2)+8b^2c(7a-22d)+15b^4)\sqrt{a+bx+cx^2}}{24(a-d)^3(b^2-4cd)^3(bx+cx^2+d)}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + b\*x + c\*x^2]\*(d + b\*x + c\*x^2)^4), x]

[Out]  $-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(3(a-d)(b^2-4cd)(d+bx+cx^2)^3)} + \frac{5(b^2+4c(a-2d))(b+2cx)\sqrt{a+bx+cx^2}}{(12(a-d)^2(b^2-4cd)^2(d+bx+cx^2)^2)} - \frac{((15b^4+8b^2c(7a-22d)+16c^2(15a^2-44ad+44d^2))(b+2cx)\sqrt{a+bx+cx^2})}{(24(a-d)^3(b^2-4cd)^3(d+bx+cx^2))} + \frac{((b^2+4c(a-2d))(5b^4-8b^2c(a+4d)+16c^2(5a^2-8ad+4d^2))\operatorname{ArcTanh}\left[\frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd}\sqrt{a+bx+cx^2}}\right])}{(8(a-d)^{7/2}(b^2-4cd)^{7/2})}$

**Rule 12**

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

**Rule 208**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 974

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_)\*((d\_) + (e\_)\*(x\_) + (f\_)\*(x\_)^2)^(q\_), x\_Symbol] := Simp[((2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f) + c\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*x)\*(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^(q + 1))/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1)), x] - Dist[1/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1)), Int[(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^q\*Simp[2\*c\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1) - (2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(a\*f\*(p + 1) - c\*d\*(p + 2)) - e\*(b^2\*c\*e - 2\*a\*c^2\*e - b^3\*f - b\*c\*(c\*d - 3\*a\*f))\*(p + q + 2) + (2\*f\*(2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f))\*(p + q + 2) - (2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(b\*f\*(p + 1) - c\*e\*(2\*p + q + 4)))\*x + c\*f\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(2\*p + 2\*q + 5)\*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && LtQ[p, -1] && NeQ[(c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f), 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]

### Rule 982

Int[1/(((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)\*Sqrt[(d\_) + (e\_)\*(x\_) + (f\_)\*(x\_)^2]), x\_Symbol] := Dist[-2\*e, Subst[Int[1/(e\*(b\*e - 4\*a\*f) - (b\*d - a\*e)\*x^2), x], x, (e + 2\*f\*x)/Sqrt[d + e\*x + f\*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && EqQ[c\*e - b\*f, 0]

### Rule 1060

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_)\*((A\_) + (B\_)\*(x\_) + (C\_)\*(x\_)^2)\*((d\_) + (e\_)\*(x\_) + (f\_)\*(x\_)^2)^(q\_), x\_Symbol] := Simp[((a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^(q + 1)\*((A\*c - a\*C)\*(2\*a\*c\*e - b\*(c\*d + a\*f)) + (A\*b - a\*B)\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f)) + c\*(A\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f)) - B\*(b\*c\*d - 2\*a\*c\*e + a\*b\*f) + C\*(b^2\*d - a\*b\*e - 2\*a\*(c\*d - a\*f)))\*x)/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1)), x] + Dist[1/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1)), Int[(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^q\*Simp[(b\*B - 2\*A\*c - 2\*a\*C)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1) + (b^2\*(C\*d + A\*f) - b\*(B\*c\*d + A\*c\*e + a\*C\*e + a\*B\*f) + 2\*(A\*c\*(c\*d - a\*f) - a\*(c\*C\*d - B\*c\*e - a\*C\*f)))\*(a\*f\*(p + 1) - c\*d\*(p + 2)) - e\*((A\*c - a\*C)\*(2\*a\*c\*e - b\*(c\*d + a\*f)) + (A\*b - a\*B)\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f)))\*(p + q + 2) - (2\*f\*((A\*c - a\*C)\*(2\*a\*c\*e - b\*(c\*d + a\*f)) + (A\*b - a\*B)\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f)))\*(p + q + 2) - (b^2\*(C\*d + A\*f) - b\*(B\*c\*d +

```

A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !
IGtQ[q, 0]

```

### Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a+bx+cx^2} (d+bx+cx^2)^4} dx &= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)(b^2-4cd)(d+bx+cx^2)^3} + \frac{\int \frac{-\frac{1}{2}c^2(a-d)(5b^2+20ac-24cd)-8bc^3(a-d)x}{\sqrt{a+bx+cx^2}(d+bx+cx^2)^3} dx}{3c^2(a-d)^2(b^2-4cd)} \\
&= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)(b^2-4cd)(d+bx+cx^2)^3} + \frac{5(b^2+4c(a-2d))(b+2cx)\sqrt{a+bx+cx^2}}{12(a-d)^2(b^2-4cd)^2(d+bx+cx^2)^2} \\
&= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)(b^2-4cd)(d+bx+cx^2)^3} + \frac{5(b^2+4c(a-2d))(b+2cx)\sqrt{a+bx+cx^2}}{12(a-d)^2(b^2-4cd)^2(d+bx+cx^2)} \\
&= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)(b^2-4cd)(d+bx+cx^2)^3} + \frac{5(b^2+4c(a-2d))(b+2cx)\sqrt{a+bx+cx^2}}{12(a-d)^2(b^2-4cd)^2(d+bx+cx^2)} \\
&= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)(b^2-4cd)(d+bx+cx^2)^3} + \frac{5(b^2+4c(a-2d))(b+2cx)\sqrt{a+bx+cx^2}}{12(a-d)^2(b^2-4cd)^2(d+bx+cx^2)} \\
&= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)(b^2-4cd)(d+bx+cx^2)^3} + \frac{5(b^2+4c(a-2d))(b+2cx)\sqrt{a+bx+cx^2}}{12(a-d)^2(b^2-4cd)^2(d+bx+cx^2)}
\end{aligned}$$

**Mathematica [B]** time = 6.61, size = 3382, normalized size = 10.31

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[1/(Sqrt[a + b\*x + c\*x^2]\*(d + b\*x + c\*x^2)^4), x]

```
[Out] (-8*c^3*(a + b*x + c*x^2))/(3*(a - d)*(b^2 - 4*c*d)^2*(b - Sqrt[b^2 - 4*c*d]
+ 2*c*x)^3*Sqrt[a + x*(b + c*x)]) + (8*c^3*(a + b*x + c*x^2))/((a - d)*(b
^2 - 4*c*d)^(5/2)*(b - Sqrt[b^2 - 4*c*d] + 2*c*x)^2*Sqrt[a + x*(b + c*x)])
- (20*c^3*(a + b*x + c*x^2))/((a - d)*(b^2 - 4*c*d)^3*(b - Sqrt[b^2 - 4*c*d]
+ 2*c*x)*Sqrt[a + x*(b + c*x)]) - (8*c^3*(a + b*x + c*x^2))/(3*(a - d)*(b
^2 - 4*c*d)^2*(b + Sqrt[b^2 - 4*c*d] + 2*c*x)^3*Sqrt[a + x*(b + c*x)]) - (8
*c^3*(a + b*x + c*x^2))/((a - d)*(b^2 - 4*c*d)^(5/2)*(b + Sqrt[b^2 - 4*c*d]
+ 2*c*x)^2*Sqrt[a + x*(b + c*x)]) - (20*c^3*(a + b*x + c*x^2))/((a - d)*(b
^2 - 4*c*d)^3*(b + Sqrt[b^2 - 4*c*d] + 2*c*x)*Sqrt[a + x*(b + c*x)]) - (20*
c^3*Sqrt[a + b*x + c*x^2]*ArcTanh[(b^2 - 4*a*c - b*Sqrt[b^2 - 4*c*d] - 2*c*
Sqrt[b^2 - 4*c*d]*x)/(4*c*Sqrt[a - d]*Sqrt[a + b*x + c*x^2]))/(Sqrt[a - d]
*(b^2 - 4*c*d)^(7/2)*Sqrt[a + x*(b + c*x)]) - (5*c^2*Sqrt[a + b*x + c*x^2]*
ArcTanh[(b^2 - 4*a*c - b*Sqrt[b^2 - 4*c*d] - 2*c*Sqrt[b^2 - 4*c*d]*x)/(4*c*
Sqrt[a - d]*Sqrt[a + b*x + c*x^2]))/((a - d)^(3/2)*(b^2 - 4*c*d)^(5/2)*Sqr
t[a + x*(b + c*x)]) - (20*c^3*Sqrt[a + b*x + c*x^2]*ArcTanh[(4*a*c - b*(b +
Sqrt[b^2 - 4*c*d]) - 2*c*Sqrt[b^2 - 4*c*d]*x)/(4*c*Sqrt[a - d]*Sqrt[a + b*
x + c*x^2]))/(Sqrt[a - d]*(b^2 - 4*c*d)^(7/2)*Sqrt[a + x*(b + c*x)]) - (5*
c^2*Sqrt[a + b*x + c*x^2]*ArcTanh[(4*a*c - b*(b + Sqrt[b^2 - 4*c*d]) - 2*c*
Sqrt[b^2 - 4*c*d]*x)/(4*c*Sqrt[a - d]*Sqrt[a + b*x + c*x^2]))/((a - d)^(3/
2)*(b^2 - 4*c*d)^(5/2)*Sqrt[a + x*(b + c*x)]) - (16*c^4*Sqrt[a + b*x + c*x^
2]*((( -2*c^2*(-b + Sqrt[b^2 - 4*c*d]) - 2*c^2*(b + 2*Sqrt[b^2 - 4*c*d]))*Sq
rt[a + b*x + c*x^2])/((4*a*c^2 + 2*b*c*(-b + Sqrt[b^2 - 4*c*d]) + c*(-b + S
qrt[b^2 - 4*c*d])^2)*(-b + Sqrt[b^2 - 4*c*d] - 2*c*x)) + (4*c*Sqrt[a - d]*(
b*(-2*c^2*(-b + Sqrt[b^2 - 4*c*d]) + 2*c^2*(b + 2*Sqrt[b^2 - 4*c*d])) - 2*(
4*a*c^3 - c^2*(-b + Sqrt[b^2 - 4*c*d]))*(b + 2*Sqrt[b^2 - 4*c*d])))*ArcTanh[
(-4*a*c - b*(-b + Sqrt[b^2 - 4*c*d]) - (2*b*c + 2*c*(-b + Sqrt[b^2 - 4*c*d]
))*x)/(4*c*Sqrt[a - d]*Sqrt[a + b*x + c*x^2]))/((4*a*c^2 + 2*b*c*(-b + Sqr
t[b^2 - 4*c*d]) + c*(-b + Sqrt[b^2 - 4*c*d])^2)*(16*a*c^2 + 8*b*c*(-b + Sqr
t[b^2 - 4*c*d]) + 4*c*(-b + Sqrt[b^2 - 4*c*d])^2))))/(b^2 - 4*c*d)^(5/2)*(
4*a*c^2 + 2*b*c*(-b + Sqrt[b^2 - 4*c*d]) + c*(-b + Sqrt[b^2 - 4*c*d])^2)*Sq
rt[a + x*(b + c*x)]) - (16*c^4*Sqrt[a + b*x + c*x^2]*(-1/2*((4*c^2*(-b + Sq
rt[b^2 - 4*c*d]) + 2*c^2*(2*b + 3*Sqrt[b^2 - 4*c*d]))*Sqrt[a + b*x + c*x^2]
)/((4*a*c^2 + 2*b*c*(-b + Sqrt[b^2 - 4*c*d]) + c*(-b + Sqrt[b^2 - 4*c*d])^2
)*(-b + Sqrt[b^2 - 4*c*d] - 2*c*x)^2) - (((10*c^3*Sqrt[b^2 - 4*c*d]*(-b + S
qrt[b^2 - 4*c*d]) + 2*c^3*(10*b^2 - 16*a*c - 24*c*d + 5*b*Sqrt[b^2 - 4*c*d]
))*Sqrt[a + b*x + c*x^2])/((4*a*c^2 + 2*b*c*(-b + Sqrt[b^2 - 4*c*d]) + c*(-
b + Sqrt[b^2 - 4*c*d])^2)*(-b + Sqrt[b^2 - 4*c*d] - 2*c*x)) + (4*c*Sqrt[a -
d]*(b*(10*c^3*Sqrt[b^2 - 4*c*d]*(-b + Sqrt[b^2 - 4*c*d]) - 2*c^3*(10*b^2 -
16*a*c - 24*c*d + 5*b*Sqrt[b^2 - 4*c*d])) - 2*(-20*a*c^4*Sqrt[b^2 - 4*c*d]
+ c^3*(-b + Sqrt[b^2 - 4*c*d]))*(10*b^2 - 16*a*c - 24*c*d + 5*b*Sqrt[b^2 -
4*c*d])))*ArcTanh[(-4*a*c - b*(-b + Sqrt[b^2 - 4*c*d]) - (2*b*c + 2*c*(-b +
Sqrt[b^2 - 4*c*d]))*x)/(4*c*Sqrt[a - d]*Sqrt[a + b*x + c*x^2]))/((4*a*c^2
+ 2*b*c*(-b + Sqrt[b^2 - 4*c*d]) + c*(-b + Sqrt[b^2 - 4*c*d])^2)*(16*a*c^2
+ 8*b*c*(-b + Sqrt[b^2 - 4*c*d]) + 4*c*(-b + Sqrt[b^2 - 4*c*d])^2))))/(2*(4
*a*c^2 + 2*b*c*(-b + Sqrt[b^2 - 4*c*d]) + c*(-b + Sqrt[b^2 - 4*c*d])^2))))/
```

$$\begin{aligned}
& (3*(b^2 - 4*c*d)^2*(4*a*c^2 + 2*b*c*(-b + \text{Sqrt}[b^2 - 4*c*d]) + c*(-b + \text{Sqrt}[b^2 - 4*c*d]))^2*\text{Sqrt}[a + x*(b + c*x)]) - (16*c^4*\text{Sqrt}[a + b*x + c*x^2]*(( -2*c^2*(b - 2*\text{Sqrt}[b^2 - 4*c*d]) + 2*c^2*(b + \text{Sqrt}[b^2 - 4*c*d]))*\text{Sqrt}[a + b*x + c*x^2])/((4*a*c^2 - 2*b*c*(b + \text{Sqrt}[b^2 - 4*c*d]) + c*(b + \text{Sqrt}[b^2 - 4*c*d]))^2)*(b + \text{Sqrt}[b^2 - 4*c*d] + 2*c*x)) + (4*c*\text{Sqrt}[a - d]*(b*(2*c^2*(b - 2*\text{Sqrt}[b^2 - 4*c*d]) + 2*c^2*(b + \text{Sqrt}[b^2 - 4*c*d])) - 2*(4*a*c^3 + c^2*(b - 2*\text{Sqrt}[b^2 - 4*c*d])*(b + \text{Sqrt}[b^2 - 4*c*d])))*\text{ArcTanh}[(4*a*c - b*(b + \text{Sqrt}[b^2 - 4*c*d]) - (-2*b*c + 2*c*(b + \text{Sqrt}[b^2 - 4*c*d]))*x)/(4*c*\text{Sqrt}[a - d]*\text{Sqrt}[a + b*x + c*x^2])))/((4*a*c^2 - 2*b*c*(b + \text{Sqrt}[b^2 - 4*c*d]) + c*(b + \text{Sqrt}[b^2 - 4*c*d]))^2)*(16*a*c^2 - 8*b*c*(b + \text{Sqrt}[b^2 - 4*c*d]) + 4*c*(b + \text{Sqrt}[b^2 - 4*c*d])^2)))/((b^2 - 4*c*d)^(5/2)*(4*a*c^2 - 2*b*c*(b + \text{Sqrt}[b^2 - 4*c*d]) + c*(b + \text{Sqrt}[b^2 - 4*c*d])^2)*\text{Sqrt}[a + x*(b + c*x)]) - (16*c^4*\text{Sqrt}[a + b*x + c*x^2]*(-1/2*((2*c^2*(2*b - 3*\text{Sqrt}[b^2 - 4*c*d]) - 4*c^2*(b + \text{Sqrt}[b^2 - 4*c*d]))*\text{Sqrt}[a + b*x + c*x^2])/((4*a*c^2 - 2*b*c*(b + \text{Sqrt}[b^2 - 4*c*d]) + c*(b + \text{Sqrt}[b^2 - 4*c*d])^2)*(b + \text{Sqrt}[b^2 - 4*c*d] + 2*c*x)^2) - (((-10*c^3*\text{Sqrt}[b^2 - 4*c*d]*(b + \text{Sqrt}[b^2 - 4*c*d]) - 2*c^3*(10*b^2 - 16*a*c - 24*c*d - 5*b*\text{Sqrt}[b^2 - 4*c*d]))*\text{Sqrt}[a + b*x + c*x^2])/((4*a*c^2 - 2*b*c*(b + \text{Sqrt}[b^2 - 4*c*d]) + c*(b + \text{Sqrt}[b^2 - 4*c*d])^2)*(b + \text{Sqrt}[b^2 - 4*c*d] + 2*c*x)) + (4*c*\text{Sqrt}[a - d]*(b*(-10*c^3*\text{Sqrt}[b^2 - 4*c*d]*(b + \text{Sqrt}[b^2 - 4*c*d]) + 2*c^3*(10*b^2 - 16*a*c - 24*c*d - 5*b*\text{Sqrt}[b^2 - 4*c*d])) - 2*(-20*a*c^4*\text{Sqrt}[b^2 - 4*c*d] + c^3*(b + \text{Sqrt}[b^2 - 4*c*d]))*(10*b^2 - 16*a*c - 24*c*d - 5*b*\text{Sqrt}[b^2 - 4*c*d])))*\text{ArcTanh}[(4*a*c - b*(b + \text{Sqrt}[b^2 - 4*c*d]) - (-2*b*c + 2*c*(b + \text{Sqrt}[b^2 - 4*c*d]))*x)/(4*c*\text{Sqrt}[a - d]*\text{Sqrt}[a + b*x + c*x^2])))/((4*a*c^2 - 2*b*c*(b + \text{Sqrt}[b^2 - 4*c*d]) + c*(b + \text{Sqrt}[b^2 - 4*c*d])^2)*(16*a*c^2 - 8*b*c*(b + \text{Sqrt}[b^2 - 4*c*d]) + 4*c*(b + \text{Sqrt}[b^2 - 4*c*d])^2)))/((2*(4*a*c^2 - 2*b*c*(b + \text{Sqrt}[b^2 - 4*c*d]) + c*(b + \text{Sqrt}[b^2 - 4*c*d])^2)))/((3*(b^2 - 4*c*d)^2*(4*a*c^2 - 2*b*c*(b + \text{Sqrt}[b^2 - 4*c*d]) + c*(b + \text{Sqrt}[b^2 - 4*c*d])^2)*\text{Sqrt}[a + x*(b + c*x)]))
\end{aligned}$$

**fricas** [B] time = 40.66, size = 8134, normalized size = 24.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x^2+b\*x+d)^4/(c\*x^2+b\*x+a)^(1/2),x, algorithm="fricas")

[Out] [1/96\*(3\*(1024\*c^3\*d^6 - (5\*b^6\*c^3 + 12\*a\*b^4\*c^4 + 48\*a^2\*b^2\*c^5 + 320\*a^3\*c^6 - 1024\*c^6\*d^3 + 384\*(b^2\*c^5 + 4\*a\*c^6))\*d^2 - 24\*(3\*b^4\*c^4 + 8\*a\*b^2\*c^5 + 48\*a^2\*c^6)\*d)\*x^6 - 384\*(b^2\*c^2 + 4\*a\*c^3)\*d^5 - 3\*(5\*b^7\*c^2 + 12\*a\*b^5\*c^3 + 48\*a^2\*b^3\*c^4 + 320\*a^3\*b\*c^5 - 1024\*b\*c^5\*d^3 + 384\*(b^3\*c^4 + 4\*a\*b\*c^5))\*d^2 - 24\*(3\*b^5\*c^3 + 8\*a\*b^3\*c^4 + 48\*a^2\*b\*c^5)\*d)\*x^5 + 24\*(3\*b^4\*c + 8\*a\*b^2\*c^2 + 48\*a^2\*c^3)\*d^4 - 3\*(5\*b^8\*c + 12\*a\*b^6\*c^2 + 48\*a^2\*b^4\*c^3 + 320\*a^3\*b^2\*c^4 - 1024\*c^5\*d^4 - 128\*(5\*b^2\*c^4 - 12\*a\*c^5))\*d^3 + 24\*(13\*b^4\*c^3 + 56\*a\*b^2\*c^4 - 48\*a^2\*c^5)\*d^2 - (67\*b^6\*c^2 + 180\*



$$\begin{aligned}
& a*b^4*c^3 + 1104*a^2*b^2*c^4 - 320*a^3*c^5)*d)*x^4 - (5*b^6 + 12*a*b^4*c + \\
& 48*a^2*b^2*c^2 + 320*a^3*c^3)*d^3 - (5*b^9 + 12*a*b^7*c + 48*a^2*b^5*c^2 + \\
& 320*a^3*b^3*c^3 - 6144*b*c^4*d^4 + 256*(5*b^3*c^3 + 36*a*b*c^4)*d^3 - 48*(b \\
& ^5*c^2 - 8*a*b^3*c^3 + 144*a^2*b*c^4)*d^2 - 6*(7*b^7*c + 20*a*b^5*c^2 + 144 \\
& *a^2*b^3*c^3 - 320*a^3*b*c^4)*d)*x^3 + 3*(1024*c^4*d^5 + 128*(5*b^2*c^3 - 1 \\
& 2*a*c^4)*d^4 - 24*(13*b^4*c^2 + 56*a*b^2*c^3 - 48*a^2*c^4)*d^3 + (67*b^6*c \\
& + 180*a*b^4*c^2 + 1104*a^2*b^2*c^3 - 320*a^3*c^4)*d^2 - (5*b^8 + 12*a*b^6*c \\
& + 48*a^2*b^4*c^2 + 320*a^3*b^2*c^3)*d)*x^2 + 3*(1024*b*c^3*d^5 - 384*(b^3*c \\
& ^2 + 4*a*b*c^3)*d^4 + 24*(3*b^5*c + 8*a*b^3*c^2 + 48*a^2*b*c^3)*d^3 - (5*b \\
& ^7 + 12*a*b^5*c + 48*a^2*b^3*c^2 + 320*a^3*b*c^3)*d^2)*x)*sqrt(a*b^2 + 4*c*d \\
& ^2 - (b^2 + 4*a*c)*d)*log((8*a^2*b^4 + (b^4*c^2 + 24*a*b^2*c^3 + 16*a^2*c^4 \\
& + 128*c^4*d^2 - 32*(b^2*c^3 + 4*a*c^4)*d)*x^4 + 2*(b^5*c + 24*a*b^3*c^2 + \\
& 16*a^2*b*c^3 + 128*b*c^3*d^2 - 32*(b^3*c^2 + 4*a*b*c^3)*d)*x^3 + (b^4 + 24 \\
& *a*b^2*c + 16*a^2*c^2)*d^2 + (b^6 + 32*a*b^4*c + 48*a^2*b^2*c^2 + 32*(5*b^2 \\
& *c^2 + 4*a*c^3)*d^2 - 2*(19*b^4*c + 104*a*b^2*c^2 + 48*a^2*c^3)*d)*x^2 - 4* \\
& (2*a*b^3 + 2*(b^2*c^2 + 4*a*c^3 - 8*c^3*d)*x^3 + 3*(b^3*c + 4*a*b*c^2 - 8*b \\
& *c^2*d)*x^2 - (b^3 + 4*a*b*c)*d + (b^4 + 8*a*b^2*c - 2*(5*b^2*c + 4*a*c^2)* \\
& d)*x)*sqrt(a*b^2 + 4*c*d^2 - (b^2 + 4*a*c)*d)*sqrt(c*x^2 + b*x + a) - 8*(a* \\
& b^4 + 4*a^2*b^2*c)*d + 2*(4*a*b^5 + 16*a^2*b^3*c + 16*(b^3*c + 4*a*b*c^2)*d \\
& ^2 - (3*b^5 + 40*a*b^3*c + 48*a^2*b*c^2)*d)*x)/(c^2*x^4 + 2*b*c*x^3 + 2*b*d \\
& *x + (b^2 + 2*c*d)*x^2 + d^2)) - 4*(8*a^3*b^7 + 4608*b*c^3*d^6 - 2592*(b^3*c \\
& ^2 + 4*a*b*c^3)*d^5 + 2*(15*a*b^6*c^3 + 56*a^2*b^4*c^4 + 240*a^3*b^2*c^5 + \\
& 2816*c^6*d^4 - 1408*(b^2*c^5 + 4*a*c^6)*d^3 + 4*(59*b^4*c^4 + 584*a*b^2*c^ \\
& 5 + 944*a^2*c^6)*d^2 - (15*b^6*c^3 + 292*a*b^4*c^4 + 1168*a^2*b^2*c^5 + 960 \\
& *a^3*c^6)*d)*x^5 + 4*(123*b^5*c + 1352*a*b^3*c^2 + 1968*a^2*b*c^3)*d^4 + 5* \\
& (15*a*b^7*c^2 + 56*a^2*b^5*c^3 + 240*a^3*b^3*c^4 + 2816*b*c^5*d^4 - 1408*(b \\
& ^3*c^4 + 4*a*b*c^5)*d^3 + 4*(59*b^5*c^3 + 584*a*b^3*c^4 + 944*a^2*b*c^5)*d^ \\
& 2 - (15*b^7*c^2 + 292*a*b^5*c^3 + 1168*a^2*b^3*c^4 + 960*a^3*b*c^5)*d)*x^4 \\
& - (33*b^7 + 940*a*b^5*c + 3760*a^2*b^3*c^2 + 2112*a^3*b*c^3)*d^3 + 4*(15*a* \\
& b^8*c + 51*a^2*b^6*c^2 + 220*a^3*b^4*c^3 + 3456*c^5*d^5 + 16*(63*b^2*c^4 - \\
& 452*a*c^5)*d^4 - 4*(273*b^4*c^3 + 584*a*b^2*c^4 - 1264*a^2*c^5)*d^3 + 8*(27 \\
& *b^6*c^2 + 233*a*b^4*c^3 + 236*a^2*b^2*c^4 - 160*a^3*c^5)*d^2 - (15*b^8*c + \\
& 267*a*b^6*c^2 + 992*a^2*b^4*c^3 + 560*a^3*b^2*c^4)*d)*x^3 + (59*a*b^7 + 58 \\
& 4*a^2*b^5*c + 944*a^3*b^3*c^2)*d^2 + (15*a*b^9 + 26*a^2*b^7*c + 120*a^3*b^5 \\
& *c^2 + 20736*b*c^4*d^5 - 32*(251*b^3*c^3 + 1356*a*b*c^4)*d^4 + 8*(61*b^5*c^ \\
& 2 + 1768*a*b^3*c^3 + 3792*a^2*b*c^4)*d^3 + 4*(29*b^7*c - 124*a*b^5*c^2 - 18 \\
& 88*a^2*b^3*c^3 - 1920*a^3*b*c^4)*d^2 - (15*b^9 + 142*a*b^7*c + 112*a^2*b^5*c \\
& ^2 - 1440*a^3*b^3*c^3)*d)*x^2 - 34*(a^2*b^7 + 4*a^3*b^5*c)*d - 2*(5*a^2*b^ \\
& 8 + 12*a^3*b^6*c - 4608*c^4*d^6 - 864*(b^2*c^3 - 12*a*c^4)*d^5 + 4*(329*b^4 \\
& *c^2 + 456*a*b^2*c^3 - 1968*a^2*c^4)*d^4 - (283*b^6*c + 2356*a*b^4*c^2 + 12 \\
& 96*a^2*b^2*c^3 - 2112*a^3*c^4)*d^3 + (20*b^8 + 413*a*b^6*c + 1304*a^2*b^4*c \\
& ^2 + 336*a^3*b^2*c^3)*d^2 - (25*a*b^8 + 142*a^2*b^6*c + 264*a^3*b^4*c^2)*d) \\
& *x)*sqrt(c*x^2 + b*x + a))/(a^4*b^8*d^3 + 256*c^4*d^11 - 256*(b^2*c^3 + 4*a \\
& *c^4)*d^10 + 32*(3*b^4*c^2 + 32*a*b^2*c^3 + 48*a^2*c^4)*d^9 - 16*(b^6*c + 2 \\
& 4*a*b^4*c^2 + 96*a^2*b^2*c^3 + 64*a^3*c^4)*d^8 + (b^8 + 64*a*b^6*c + 576*a^
\end{aligned}$$

$$\begin{aligned}
& 2*b^4*c^2 + 1024*a^3*b^2*c^3 + 256*a^4*c^4)*d^7 - 4*(a*b^8 + 24*a^2*b^6*c + \\
& 96*a^3*b^4*c^2 + 64*a^4*b^2*c^3)*d^6 + (a^4*b^8*c^3 + 256*c^7*d^8 - 256*(b \\
& ^2*c^6 + 4*a*c^7)*d^7 + 32*(3*b^4*c^5 + 32*a*b^2*c^6 + 48*a^2*c^7)*d^6 - 16 \\
& *(b^6*c^4 + 24*a*b^4*c^5 + 96*a^2*b^2*c^6 + 64*a^3*c^7)*d^5 + (b^8*c^3 + 64 \\
& *a*b^6*c^4 + 576*a^2*b^4*c^5 + 1024*a^3*b^2*c^6 + 256*a^4*c^7)*d^4 - 4*(a*b \\
& ^8*c^3 + 24*a^2*b^6*c^4 + 96*a^3*b^4*c^5 + 64*a^4*b^2*c^6)*d^3 + 2*(3*a^2*b \\
& ^8*c^3 + 32*a^3*b^6*c^4 + 48*a^4*b^4*c^5)*d^2 - 4*(a^3*b^8*c^3 + 4*a^4*b^6*c \\
& ^4)*d)*x^6 + 2*(3*a^2*b^8 + 32*a^3*b^6*c + 48*a^4*b^4*c^2)*d^5 + 3*(a^4*b^ \\
& 9*c^2 + 256*b*c^6*d^8 - 256*(b^3*c^5 + 4*a*b*c^6)*d^7 + 32*(3*b^5*c^4 + 32* \\
& a*b^3*c^5 + 48*a^2*b*c^6)*d^6 - 16*(b^7*c^3 + 24*a*b^5*c^4 + 96*a^2*b^3*c^5 \\
& + 64*a^3*b*c^6)*d^5 + (b^9*c^2 + 64*a*b^7*c^3 + 576*a^2*b^5*c^4 + 1024*a^3 \\
& *b^3*c^5 + 256*a^4*b*c^6)*d^4 - 4*(a*b^9*c^2 + 24*a^2*b^7*c^3 + 96*a^3*b^5*c \\
& ^4 + 64*a^4*b^3*c^5)*d^3 + 2*(3*a^2*b^9*c^2 + 32*a^3*b^7*c^3 + 48*a^4*b^5*c \\
& ^4)*d^2 - 4*(a^3*b^9*c^2 + 4*a^4*b^7*c^3)*d)*x^5 - 4*(a^3*b^8 + 4*a^4*b^6*c \\
& ^4 + 3*(a^4*b^10*c - 1024*a*c^6*d^8 + 256*c^6*d^9 - 32*(5*b^4*c^4 - 48* \\
& a^2*c^6)*d^7 + 16*(5*b^6*c^3 + 40*a*b^4*c^4 - 64*a^3*c^6)*d^6 - (15*b^8*c^2 \\
& + 320*a*b^6*c^3 + 960*a^2*b^4*c^4 - 256*a^4*c^6)*d^5 + (b^10*c + 60*a*b^8*c \\
& ^2 + 480*a^2*b^6*c^3 + 640*a^3*b^4*c^4)*d^4 - 2*(2*a*b^10*c + 45*a^2*b^8*c \\
& ^2 + 160*a^3*b^6*c^3 + 80*a^4*b^4*c^4)*d^3 + 2*(3*a^2*b^10*c + 30*a^3*b^8*c \\
& ^2 + 40*a^4*b^6*c^3)*d^2 - (4*a^3*b^10*c + 15*a^4*b^8*c^2)*d)*x^4 + (a^4*b^ \\
& 11 + 1536*b*c^5*d^9 - 256*(5*b^3*c^4 + 24*a*b*c^5)*d^8 + 64*(5*b^5*c^3 + 80 \\
& *a*b^3*c^4 + 144*a^2*b*c^5)*d^7 - 256*(5*a*b^5*c^3 + 30*a^2*b^3*c^4 + 24*a^ \\
& 3*b*c^5)*d^6 - 2*(5*b^9*c - 960*a^2*b^5*c^3 - 2560*a^3*b^3*c^4 - 768*a^4*b \\
& ^c^5)*d^5 + (b^11 + 40*a*b^9*c - 1280*a^3*b^5*c^3 - 1280*a^4*b^3*c^4)*d^4 - \\
& 4*(a*b^11 + 15*a^2*b^9*c - 80*a^4*b^5*c^3)*d^3 + 2*(3*a^2*b^11 + 20*a^3*b^9 \\
& *c)*d^2 - 2*(2*a^3*b^11 + 5*a^4*b^9*c)*d)*x^3 + 3*(a^4*b^10*d - 1024*a*c^5* \\
& d^9 + 256*c^5*d^10 - 32*(5*b^4*c^3 - 48*a^2*c^5)*d^8 + 16*(5*b^6*c^2 + 40*a \\
& *b^4*c^3 - 64*a^3*c^5)*d^7 - (15*b^8*c + 320*a*b^6*c^2 + 960*a^2*b^4*c^3 - \\
& 256*a^4*c^5)*d^6 + (b^10 + 60*a*b^8*c + 480*a^2*b^6*c^2 + 640*a^3*b^4*c^3)* \\
& d^5 - 2*(2*a*b^10 + 45*a^2*b^8*c + 160*a^3*b^6*c^2 + 80*a^4*b^4*c^3)*d^4 + \\
& 2*(3*a^2*b^10 + 30*a^3*b^8*c + 40*a^4*b^6*c^2)*d^3 - (4*a^3*b^10 + 15*a^4*b \\
& ^8*c)*d^2)*x^2 + 3*(a^4*b^9*d^2 + 256*b*c^4*d^10 - 256*(b^3*c^3 + 4*a*b*c^4 \\
& ))*d^9 + 32*(3*b^5*c^2 + 32*a*b^3*c^3 + 48*a^2*b*c^4)*d^8 - 16*(b^7*c + 24*a \\
& *b^5*c^2 + 96*a^2*b^3*c^3 + 64*a^3*b*c^4)*d^7 + (b^9 + 64*a*b^7*c + 576*a^2 \\
& *b^5*c^2 + 1024*a^3*b^3*c^3 + 256*a^4*b*c^4)*d^6 - 4*(a*b^9 + 24*a^2*b^7*c \\
& + 96*a^3*b^5*c^2 + 64*a^4*b^3*c^3)*d^5 + 2*(3*a^2*b^9 + 32*a^3*b^7*c + 48*a \\
& ^4*b^5*c^2)*d^4 - 4*(a^3*b^9 + 4*a^4*b^7*c)*d^3)*x), -1/48*(3*(1024*c^3*d^6 \\
& - (5*b^6*c^3 + 12*a*b^4*c^4 + 48*a^2*b^2*c^5 + 320*a^3*c^6 - 1024*c^6*d^3 \\
& + 384*(b^2*c^5 + 4*a*c^6)*d^2 - 24*(3*b^4*c^4 + 8*a*b^2*c^5 + 48*a^2*c^6)*d \\
& ))*x^6 - 384*(b^2*c^2 + 4*a*c^3)*d^5 - 3*(5*b^7*c^2 + 12*a*b^5*c^3 + 48*a^2* \\
& b^3*c^4 + 320*a^3*b*c^5 - 1024*b*c^5*d^3 + 384*(b^3*c^4 + 4*a*b*c^5)*d^2 - \\
& 24*(3*b^5*c^3 + 8*a*b^3*c^4 + 48*a^2*b*c^5)*d)*x^5 + 24*(3*b^4*c + 8*a*b^2* \\
& c^2 + 48*a^2*c^3)*d^4 - 3*(5*b^8*c + 12*a*b^6*c^2 + 48*a^2*b^4*c^3 + 320*a^ \\
& 3*b^2*c^4 - 1024*c^5*d^4 - 128*(5*b^2*c^4 - 12*a*c^5)*d^3 + 24*(13*b^4*c^3 \\
& + 56*a*b^2*c^4 - 48*a^2*c^5)*d^2 - (67*b^6*c^2 + 180*a*b^4*c^3 + 1104*a^2*b
\end{aligned}$$

$$\begin{aligned}
&^2*c^4 - 320*a^3*c^5)*d)*x^4 - (5*b^6 + 12*a*b^4*c + 48*a^2*b^2*c^2 + 320*a^3*c^3)*d^3 - (5*b^9 + 12*a*b^7*c + 48*a^2*b^5*c^2 + 320*a^3*b^3*c^3 - 6144*b*c^4*d^4 + 256*(5*b^3*c^3 + 36*a*b*c^4)*d^3 - 48*(b^5*c^2 - 8*a*b^3*c^3 + 144*a^2*b*c^4)*d^2 - 6*(7*b^7*c + 20*a*b^5*c^2 + 144*a^2*b^3*c^3 - 320*a^3*b*c^4)*d)*x^3 + 3*(1024*c^4*d^5 + 128*(5*b^2*c^3 - 12*a*c^4)*d^4 - 24*(13*b^4*c^2 + 56*a*b^2*c^3 - 48*a^2*c^4)*d^3 + (67*b^6*c + 180*a*b^4*c^2 + 1104*a^2*b^2*c^3 - 320*a^3*c^4)*d^2 - (5*b^8 + 12*a*b^6*c + 48*a^2*b^4*c^2 + 320*a^3*b^2*c^3)*d)*x^2 + 3*(1024*b*c^3*d^5 - 384*(b^3*c^2 + 4*a*b*c^3)*d^4 + 24*(3*b^5*c + 8*a*b^3*c^2 + 48*a^2*b*c^3)*d^3 - (5*b^7 + 12*a*b^5*c + 48*a^2*b^3*c^2 + 320*a^3*b*c^3)*d^2)*x)*sqrt(-a*b^2 - 4*c*d^2 + (b^2 + 4*a*c)*d)*arctan(-1/2*(2*a*b^2 + (b^2*c + 4*a*c^2 - 8*c^2*d)*x^2 - (b^2 + 4*a*c)*d + (b^3 + 4*a*b*c - 8*b*c*d)*x)*sqrt(-a*b^2 - 4*c*d^2 + (b^2 + 4*a*c)*d)*sqrt(c*x^2 + b*x + a)/(a^2*b^3 + 4*a*b*c*d^2 + 2*(a*b^2*c^2 + 4*c^3*d^2 - (b^2*c^2 + 4*a*c^3)*d)*x^3 + 3*(a*b^3*c + 4*b*c^2*d^2 - (b^3*c + 4*a*b*c^2)*d)*x^2 - (a*b^3 + 4*a^2*b*c)*d + (a*b^4 + 2*a^2*b^2*c + 4*(b^2*c + 2*a*c^2)*d^2 - (b^4 + 6*a*b^2*c + 8*a^2*c^2)*d)*x)) + 2*(8*a^3*b^7 + 4608*b*c^3*d^6 - 2592*(b^3*c^2 + 4*a*b*c^3)*d^5 + 2*(15*a*b^6*c^3 + 56*a^2*b^4*c^4 + 240*a^3*b^2*c^5 + 2816*c^6*d^4 - 1408*(b^2*c^5 + 4*a*c^6)*d^3 + 4*(59*b^4*c^4 + 584*a*b^2*c^5 + 944*a^2*c^6)*d^2 - (15*b^6*c^3 + 292*a*b^4*c^4 + 1168*a^2*b^2*c^5 + 960*a^3*c^6)*d)*x^5 + 4*(123*b^5*c + 1352*a*b^3*c^2 + 1968*a^2*b*c^3)*d^4 + 5*(15*a*b^7*c^2 + 56*a^2*b^5*c^3 + 240*a^3*b^3*c^4 + 2816*b*c^5*d^4 - 1408*(b^3*c^4 + 4*a*b*c^5)*d^3 + 4*(59*b^5*c^3 + 584*a*b^3*c^4 + 944*a^2*b*c^5)*d)*x^4 - (33*b^7 + 940*a*b^5*c + 3760*a^2*b^3*c^2 + 2112*a^3*b*c^3)*d^3 + 4*(15*a*b^8*c + 51*a^2*b^6*c^2 + 220*a^3*b^4*c^3 + 3456*c^5*d^5 + 16*(63*b^2*c^4 - 452*a*c^5)*d^4 - 4*(273*b^4*c^3 + 584*a*b^2*c^4 - 1264*a^2*c^5)*d^3 + 8*(27*b^6*c^2 + 233*a*b^4*c^3 + 236*a^2*b^2*c^4 - 160*a^3*c^5)*d^2 - (15*b^8*c + 267*a*b^6*c^2 + 992*a^2*b^4*c^3 + 560*a^3*b^2*c^4)*d)*x^3 + (59*a*b^7 + 584*a^2*b^5*c + 944*a^3*b^3*c^2)*d^2 + (15*a*b^9 + 26*a^2*b^7*c + 120*a^3*b^5*c^2 + 20736*b*c^4*d^5 - 32*(251*b^3*c^3 + 1356*a*b*c^4)*d^4 + 8*(61*b^5*c^2 + 1768*a*b^3*c^3 + 3792*a^2*b*c^4)*d^3 + 4*(29*b^7*c - 124*a*b^5*c^2 - 1888*a^2*b^3*c^3 - 1920*a^3*b*c^4)*d^2 - (15*b^9 + 142*a*b^7*c + 112*a^2*b^5*c^2 - 1440*a^3*b^3*c^3)*d)*x^2 - 34*(a^2*b^7 + 4*a^3*b^5*c)*d - 2*(5*a^2*b^8 + 12*a^3*b^6*c - 4608*c^4*d^6 - 864*(b^2*c^3 - 12*a*c^4)*d^5 + 4*(329*b^4*c^2 + 456*a*b^2*c^3 - 1968*a^2*c^4)*d^4 - (283*b^6*c + 2356*a*b^4*c^2 + 1296*a^2*b^2*c^3 - 2112*a^3*c^4)*d^3 + (20*b^8 + 413*a*b^6*c + 1304*a^2*b^4*c^2 + 336*a^3*b^2*c^3)*d^2 - (25*a*b^8 + 142*a^2*b^6*c + 264*a^3*b^4*c^2)*d)*x)*sqrt(c*x^2 + b*x + a))/(a^4*b^8*d^3 + 256*c^4*d^11 - 256*(b^2*c^3 + 4*a*c^4)*d^10 + 32*(3*b^4*c^2 + 32*a*b^2*c^3 + 48*a^2*c^4)*d^9 - 16*(b^6*c + 24*a*b^4*c^2 + 96*a^2*b^2*c^3 + 64*a^3*c^4)*d^8 + (b^8 + 64*a*b^6*c + 576*a^2*b^4*c^2 + 1024*a^3*b^2*c^3 + 256*a^4*c^4)*d^7 - 4*(a*b^8 + 24*a^2*b^6*c + 96*a^3*b^4*c^2 + 64*a^4*b^2*c^3)*d^6 + (a^4*b^8*c^3 + 256*c^7*d^8 - 256*(b^2*c^6 + 4*a*c^7)*d^7 + 32*(3*b^4*c^5 + 32*a*b^2*c^6 + 48*a^2*c^7)*d^6 - 16*(b^6*c^4 + 24*a*b^4*c^5 + 96*a^2*b^2*c^6 + 64*a^3*c^7)*d^5 + (b^8*c^3 + 64*a*b^6*c^4 + 576*a^2*b^4*c^5 + 1024*a^3*b^2*c^6 + 256*a^4*c^7)*d^
\end{aligned}$$

$$\begin{aligned}
& 4 - 4*(a*b^8*c^3 + 24*a^2*b^6*c^4 + 96*a^3*b^4*c^5 + 64*a^4*b^2*c^6)*d^3 + \\
& 2*(3*a^2*b^8*c^3 + 32*a^3*b^6*c^4 + 48*a^4*b^4*c^5)*d^2 - 4*(a^3*b^8*c^3 + \\
& 4*a^4*b^6*c^4)*d)*x^6 + 2*(3*a^2*b^8 + 32*a^3*b^6*c + 48*a^4*b^4*c^2)*d^5 + \\
& 3*(a^4*b^9*c^2 + 256*b*c^6*d^8 - 256*(b^3*c^5 + 4*a*b*c^6)*d^7 + 32*(3*b^5 \\
& *c^4 + 32*a*b^3*c^5 + 48*a^2*b*c^6)*d^6 - 16*(b^7*c^3 + 24*a*b^5*c^4 + 96*a \\
& ^2*b^3*c^5 + 64*a^3*b*c^6)*d^5 + (b^9*c^2 + 64*a*b^7*c^3 + 576*a^2*b^5*c^4 \\
& + 1024*a^3*b^3*c^5 + 256*a^4*b*c^6)*d^4 - 4*(a*b^9*c^2 + 24*a^2*b^7*c^3 + 9 \\
& 6*a^3*b^5*c^4 + 64*a^4*b^3*c^5)*d^3 + 2*(3*a^2*b^9*c^2 + 32*a^3*b^7*c^3 + 4 \\
& 8*a^4*b^5*c^4)*d^2 - 4*(a^3*b^9*c^2 + 4*a^4*b^7*c^3)*d)*x^5 - 4*(a^3*b^8 + \\
& 4*a^4*b^6*c)*d^4 + 3*(a^4*b^10*c - 1024*a*c^6*d^8 + 256*c^6*d^9 - 32*(5*b^4 \\
& *c^4 - 48*a^2*c^6)*d^7 + 16*(5*b^6*c^3 + 40*a*b^4*c^4 - 64*a^3*c^6)*d^6 - ( \\
& 15*b^8*c^2 + 320*a*b^6*c^3 + 960*a^2*b^4*c^4 - 256*a^4*c^6)*d^5 + (b^10*c + \\
& 60*a*b^8*c^2 + 480*a^2*b^6*c^3 + 640*a^3*b^4*c^4)*d^4 - 2*(2*a*b^10*c + 45 \\
& *a^2*b^8*c^2 + 160*a^3*b^6*c^3 + 80*a^4*b^4*c^4)*d^3 + 2*(3*a^2*b^10*c + 30 \\
& *a^3*b^8*c^2 + 40*a^4*b^6*c^3)*d^2 - (4*a^3*b^10*c + 15*a^4*b^8*c^2)*d)*x^4 \\
& + (a^4*b^11 + 1536*b*c^5*d^9 - 256*(5*b^3*c^4 + 24*a*b*c^5)*d^8 + 64*(5*b^ \\
& 5*c^3 + 80*a*b^3*c^4 + 144*a^2*b*c^5)*d^7 - 256*(5*a*b^5*c^3 + 30*a^2*b^3*c \\
& ^4 + 24*a^3*b*c^5)*d^6 - 2*(5*b^9*c - 960*a^2*b^5*c^3 - 2560*a^3*b^3*c^4 - \\
& 768*a^4*b*c^5)*d^5 + (b^11 + 40*a*b^9*c - 1280*a^3*b^5*c^3 - 1280*a^4*b^3*c \\
& ^4)*d^4 - 4*(a*b^11 + 15*a^2*b^9*c - 80*a^4*b^5*c^3)*d^3 + 2*(3*a^2*b^11 + \\
& 20*a^3*b^9*c)*d^2 - 2*(2*a^3*b^11 + 5*a^4*b^9*c)*d)*x^3 + 3*(a^4*b^10*d - 1 \\
& 024*a*c^5*d^9 + 256*c^5*d^10 - 32*(5*b^4*c^3 - 48*a^2*c^5)*d^8 + 16*(5*b^6* \\
& c^2 + 40*a*b^4*c^3 - 64*a^3*c^5)*d^7 - (15*b^8*c + 320*a*b^6*c^2 + 960*a^2* \\
& b^4*c^3 - 256*a^4*c^5)*d^6 + (b^10 + 60*a*b^8*c + 480*a^2*b^6*c^2 + 640*a^3 \\
& *b^4*c^3)*d^5 - 2*(2*a*b^10 + 45*a^2*b^8*c + 160*a^3*b^6*c^2 + 80*a^4*b^4*c \\
& ^3)*d^4 + 2*(3*a^2*b^10 + 30*a^3*b^8*c + 40*a^4*b^6*c^2)*d^3 - (4*a^3*b^10 \\
& + 15*a^4*b^8*c)*d^2)*x^2 + 3*(a^4*b^9*d^2 + 256*b*c^4*d^10 - 256*(b^3*c^3 + \\
& 4*a*b*c^4)*d^9 + 32*(3*b^5*c^2 + 32*a*b^3*c^3 + 48*a^2*b*c^4)*d^8 - 16*(b^ \\
& 7*c + 24*a*b^5*c^2 + 96*a^2*b^3*c^3 + 64*a^3*b*c^4)*d^7 + (b^9 + 64*a*b^7*c \\
& + 576*a^2*b^5*c^2 + 1024*a^3*b^3*c^3 + 256*a^4*b*c^4)*d^6 - 4*(a*b^9 + 24* \\
& a^2*b^7*c + 96*a^3*b^5*c^2 + 64*a^4*b^3*c^3)*d^5 + 2*(3*a^2*b^9 + 32*a^3*b^ \\
& 7*c + 48*a^4*b^5*c^2)*d^4 - 4*(a^3*b^9 + 4*a^4*b^7*c)*d^3)*x]
\end{aligned}$$

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x^2+b\*x+d)^4/(c\*x^2+b\*x+a)^(1/2),x, algorithm="giac")

[Out] Timed out

**maple** [B] time = 0.04, size = 3695, normalized size = 11.27

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(1/(c*x^2+b*x+d)^4/(c*x^2+b*x+a)^{(1/2)}, x)$

[Out] 
$$\frac{5}{2} \frac{(b^2-4cd)^2}{(a-d)^3} \frac{(x+1/2b/c+1/2(b^2-4cd)^{1/2}/c)^{(1/2)}}{(a+(x+1/2(b+(b^2-4cd)^{1/2}))/c)^{2cd-(b^2-4cd)^{1/2}} (x+1/2(b+(b^2-4cd)^{1/2}))/c)^{(1/2)}} * c^2 d - (b^2-4cd)^{1/2} (x+1/2(b+(b^2-4cd)^{1/2}))/c)^{(1/2)} * c^2 d + 5/2 (b^2-4cd)^2 / (a-d)^3 / (x+1/2b/c-1/2(b^2-4cd)^{1/2}/c)^{(1/2)} * (a+(x-1/2(-b+(b^2-4cd)^{1/2}))/c)^{2cd+(b^2-4cd)^{1/2}} (x-1/2(-b+(b^2-4cd)^{1/2}))/c)^{(1/2)} * c^2 d - 3/4 (b^2-4cd)^{3/2} / (a-d)^{5/2} * c \ln((2a-2d+(b^2-4cd)^{1/2}) (x-1/2(-b+(b^2-4cd)^{1/2}))/c) + 2(a-d)^{1/2} (a+(x-1/2(-b+(b^2-4cd)^{1/2}))/c)^{2cd+(b^2-4cd)^{1/2}} (x-1/2(-b+(b^2-4cd)^{1/2}))/c)^{(1/2)} / (x-1/2(-b+(b^2-4cd)^{1/2}))/c) - 1/3 (b^2-4cd)^2 / (a-d) / (x+1/2b/c+1/2(b^2-4cd)^{1/2}/c)^3 (a+(x+1/2(b+(b^2-4cd)^{1/2}))/c)^{2cd-(b^2-4cd)^{1/2}} (x+1/2(b+(b^2-4cd)^{1/2}))/c)^{(1/2)} - 3 (b^2-4cd)^{5/2} * c^2 / (a-d)^{3/2} * \ln((2a-2d-(b^2-4cd)^{1/2}) (x+1/2(b+(b^2-4cd)^{1/2}))/c) + 2(a-d)^{1/2} (a+(x+1/2(b+(b^2-4cd)^{1/2}))/c)^{2cd-(b^2-4cd)^{1/2}} (x+1/2(b+(b^2-4cd)^{1/2}))/c)^{(1/2)} / (x+1/2(b+(b^2-4cd)^{1/2}))/c) - 1/3 (b^2-4cd)^2 / (a-d) / (x+1/2b/c-1/2(b^2-4cd)^{1/2}/c)^3 (a+(x-1/2(-b+(b^2-4cd)^{1/2}))/c)^{2cd+(b^2-4cd)^{1/2}} (x-1/2(-b+(b^2-4cd)^{1/2}))/c)^{(1/2)} + 3 (b^2-4cd)^{5/2} * c^2 / (a-d)^{3/2} * \ln((2a-2d+(b^2-4cd)^{1/2}) (x-1/2(-b+(b^2-4cd)^{1/2}))/c) + 2(a-d)^{1/2} (a+(x-1/2(-b+(b^2-4cd)^{1/2}))/c)^{2cd+(b^2-4cd)^{1/2}} (x-1/2(-b+(b^2-4cd)^{1/2}))/c)^{(1/2)} / (x-1/2(-b+(b^2-4cd)^{1/2}))/c) + 20c^3 / (b^2-4cd)^{7/2} / (a-d)^{1/2} * \ln((2a-2d+(b^2-4cd)^{1/2}) (x-1/2(-b+(b^2-4cd)^{1/2}))/c) + 2(a-d)^{1/2} (a+(x-1/2(-b+(b^2-4cd)^{1/2}))/c)^{2cd+(b^2-4cd)^{1/2}} (x-1/2(-b+(b^2-4cd)^{1/2}))/c)^{(1/2)} / (x-1/2(-b+(b^2-4cd)^{1/2}))/c) - 20c^3 / (b^2-4cd)^{7/2} / (a-d)^{1/2} * \ln((2a-2d-(b^2-4cd)^{1/2}) (x+1/2(b+(b^2-4cd)^{1/2}))/c) + 2(a-d)^{1/2} (a+(x+1/2(b+(b^2-4cd)^{1/2}))/c)^{2cd-(b^2-4cd)^{1/2}} (x+1/2(b+(b^2-4cd)^{1/2}))/c)^{(1/2)} / (x+1/2(b+(b^2-4cd)^{1/2}))/c) - 5/12 (b^2-4cd)^{3/2} / (a-d)^2 / (x+1/2b/c+1/2(b^2-4cd)^{1/2}/c)^2 (a+(x+1/2(b+(b^2-4cd)^{1/2}))/c)^{2cd-(b^2-4cd)^{1/2}} (x+1/2(b+(b^2-4cd)^{1/2}))/c)^{(1/2)} - 5/16 (b^2-4cd)^{3/2} / (a-d)^{7/2} * \ln((2a-2d-(b^2-4cd)^{1/2}) (x+1/2(b+(b^2-4cd)^{1/2}))/c) + 2(a-d)^{1/2} (a+(x+1/2(b+(b^2-4cd)^{1/2}))/c)^{2cd-(b^2-4cd)^{1/2}} (x+1/2(b+(b^2-4cd)^{1/2}))/c)^{(1/2)} / (x+1/2(b+(b^2-4cd)^{1/2}))/c) * b^2 + 3/4 (b^2-4cd)^{3/2} / (a-d)^{5/2} * c \ln((2a-2d-(b^2-4cd)^{1/2}) (x+1/2(b+(b^2-4cd)^{1/2}))/c) + 2(a-d)^{1/2} (a+(x+1/2(b+(b^2-4cd)^{1/2}))/c)^{2cd-(b^2-4cd)^{1/2}} (x+1/2(b+(b^2-4cd)^{1/2}))/c)^{(1/2)} / (x+1/2(b+(b^2-4cd)^{1/2}))/c) + 5/12 (b^2-4cd)^{3/2} / (a-d)^2 / (x+1/2b/c-1/2(b^2-4cd)^{1/2}/c)^2 (a+(x-1/2(-b+(b^2-4cd)^{1/2}))/c)^{2cd+(b^2-4cd)^{1/2}} (x-1/2(-b+(b^2-4cd)^{1/2}))/c)^{(1/2)} + 5/16 (b^2-4cd)^{3/2} / (a-d)^{7/2} * \ln((2a-2d+(b^2-4cd)^{1/2}) (x-1/2(-b+(b^2-4cd)^{1/2}))/c) + 2(a-d)^{1/2} (a+(x-1/2(-b+(b^2-4cd)^{1/2}))/c)^{2cd+(b^2-4cd)^{1/2}} (x-1/2(-b+(b^2-4cd)^{1/2}))/c)^{(1/2)} / (x-1/2(-b+(b^2-4cd)^{1/2}))/c) * b^2 - 5/8 (b^2-4cd)^2 / (a-d)^3 / (x+1/2b/c+1/2(b^2-4cd)^{1/2}/c) (a+(x+1/2(b+(b^2-4cd)^{1/2}))/c)^{2cd-(b^2-4cd)^{1/2}}$$

$$\begin{aligned}
& *d)^{(1/2)} * (x + 1/2 * (b + (b^2 - 4 * c * d)^{(1/2)}) / c)^{(1/2)} * b^2 - 10 / (b^2 - 4 * c * d)^3 * c^2 / \\
& (a - d) / (x + 1/2 * b / c + 1/2 * (b^2 - 4 * c * d)^{(1/2)} / c) * (a + (x + 1/2 * (b + (b^2 - 4 * c * d)^{(1/2)}) / c) \\
& ^2 * c - d - (b^2 - 4 * c * d)^{(1/2)} * (x + 1/2 * (b + (b^2 - 4 * c * d)^{(1/2)}) / c)^{(1/2)} + 2 / (b^2 - 4 * c * \\
& d)^{(5/2)} * c / (a - d) / (x + 1/2 * b / c - 1/2 * (b^2 - 4 * c * d)^{(1/2)} / c)^2 * (a + (x - 1/2 * (-b + (b^2 - 4 * c * \\
& * c * d)^{(1/2)}) / c)^2 * c - d + (b^2 - 4 * c * d)^{(1/2)} * (x - 1/2 * (-b + (b^2 - 4 * c * d)^{(1/2)}) / c)^{(1/2)} + 3/2 / (b^2 - 4 * c * d)^{(5/2)} * c / (a - d)^{(5/2)} * \ln((2 * a - 2 * d + (b^2 - 4 * c * d)^{(1/2)} * (x - 1 \\
& / 2 * (-b + (b^2 - 4 * c * d)^{(1/2)}) / c) + 2 * (a - d)^{(1/2)} * (a + (x - 1/2 * (-b + (b^2 - 4 * c * d)^{(1/2)}) / c) \\
& ^2 * c - d + (b^2 - 4 * c * d)^{(1/2)} * (x - 1/2 * (-b + (b^2 - 4 * c * d)^{(1/2)}) / c)^{(1/2)}) / (x - 1/2 \\
& * (-b + (b^2 - 4 * c * d)^{(1/2)}) / c) * b^2 - 6 / (b^2 - 4 * c * d)^{(5/2)} * c^2 / (a - d)^{(5/2)} * \ln((2 * a \\
& - 2 * d + (b^2 - 4 * c * d)^{(1/2)} * (x - 1/2 * (-b + (b^2 - 4 * c * d)^{(1/2)}) / c) + 2 * (a - d)^{(1/2)} * (a + (x \\
& - 1/2 * (-b + (b^2 - 4 * c * d)^{(1/2)}) / c)^2 * c - d + (b^2 - 4 * c * d)^{(1/2)} * (x - 1/2 * (-b + (b^2 - 4 * c * \\
& d)^{(1/2)}) / c)^{(1/2)}) / (x - 1/2 * (-b + (b^2 - 4 * c * d)^{(1/2)}) / c) * d - 2 / (b^2 - 4 * c * d)^{(5/2)} \\
& ) * c / (a - d) / (x + 1/2 * b / c + 1/2 * (b^2 - 4 * c * d)^{(1/2)} / c)^2 * (a + (x + 1/2 * (b + (b^2 - 4 * c * d)^{(1 \\
& / 2)) / c)^2 * c - d - (b^2 - 4 * c * d)^{(1/2)} * (x + 1/2 * (b + (b^2 - 4 * c * d)^{(1/2)}) / c)^{(1/2)} - 7/3 / \\
& (b^2 - 4 * c * d)^2 * c / (a - d)^2 / (x + 1/2 * b / c + 1/2 * (b^2 - 4 * c * d)^{(1/2)} / c) * (a + (x + 1/2 * (b + (b \\
& ^2 - 4 * c * d)^{(1/2)}) / c)^2 * c - d - (b^2 - 4 * c * d)^{(1/2)} * (x + 1/2 * (b + (b^2 - 4 * c * d)^{(1/2)}) / c) \\
& )^{(1/2)} - 3/2 / (b^2 - 4 * c * d)^{(5/2)} * c / (a - d)^{(5/2)} * \ln((2 * a - 2 * d - (b^2 - 4 * c * d)^{(1/2)} * ( \\
& x + 1/2 * (b + (b^2 - 4 * c * d)^{(1/2)}) / c) + 2 * (a - d)^{(1/2)} * (a + (x + 1/2 * (b + (b^2 - 4 * c * d)^{(1/2)} \\
& ) / c)^2 * c - d - (b^2 - 4 * c * d)^{(1/2)} * (x + 1/2 * (b + (b^2 - 4 * c * d)^{(1/2)}) / c)^{(1/2)}) / (x + 1/2 \\
& * (b + (b^2 - 4 * c * d)^{(1/2)}) / c) * b^2 + 6 / (b^2 - 4 * c * d)^{(5/2)} * c^2 / (a - d)^{(5/2)} * \ln((2 * a - \\
& 2 * d - (b^2 - 4 * c * d)^{(1/2)} * (x + 1/2 * (b + (b^2 - 4 * c * d)^{(1/2)}) / c) + 2 * (a - d)^{(1/2)} * (a + (x + 1 \\
& / 2 * (b + (b^2 - 4 * c * d)^{(1/2)}) / c)^2 * c - d - (b^2 - 4 * c * d)^{(1/2)} * (x + 1/2 * (b + (b^2 - 4 * c * d)^{( \\
& 1/2)) / c)^{(1/2)}) / (x + 1/2 * (b + (b^2 - 4 * c * d)^{(1/2)}) / c) * d - 10 / (b^2 - 4 * c * d)^3 * c^2 / (a \\
& - d) / (x + 1/2 * b / c - 1/2 * (b^2 - 4 * c * d)^{(1/2)} / c) * (a + (x - 1/2 * (-b + (b^2 - 4 * c * d)^{(1/2)}) / c) \\
& ^2 * c - d + (b^2 - 4 * c * d)^{(1/2)} * (x - 1/2 * (-b + (b^2 - 4 * c * d)^{(1/2)}) / c)^{(1/2)} - 5/8 / (b^2 - 4 \\
& * c * d)^2 / (a - d)^3 / (x + 1/2 * b / c - 1/2 * (b^2 - 4 * c * d)^{(1/2)} / c) * (a + (x - 1/2 * (-b + (b^2 - 4 * c * \\
& d)^{(1/2)}) / c)^2 * c - d + (b^2 - 4 * c * d)^{(1/2)} * (x - 1/2 * (-b + (b^2 - 4 * c * d)^{(1/2)}) / c)^{(1/2)} \\
& ) * b^2 - 7/3 / (b^2 - 4 * c * d)^2 * c / (a - d)^2 / (x + 1/2 * b / c - 1/2 * (b^2 - 4 * c * d)^{(1/2)} / c) * (a + (x \\
& - 1/2 * (-b + (b^2 - 4 * c * d)^{(1/2)}) / c)^2 * c - d + (b^2 - 4 * c * d)^{(1/2)} * (x - 1/2 * (-b + (b^2 - 4 * c * \\
& d)^{(1/2)}) / c)^{(1/2)} + 5/4 / (b^2 - 4 * c * d)^{(3/2)} / (a - d)^{(7/2)} * \ln((2 * a - 2 * d - (b^2 - 4 * c * \\
& d)^{(1/2)} * (x + 1/2 * (b + (b^2 - 4 * c * d)^{(1/2)}) / c) + 2 * (a - d)^{(1/2)} * (a + (x + 1/2 * (b + (b^2 - 4 * \\
& c * d)^{(1/2)}) / c)^2 * c - d - (b^2 - 4 * c * d)^{(1/2)} * (x + 1/2 * (b + (b^2 - 4 * c * d)^{(1/2)}) / c)^{(1/2)} \\
& ) / (x + 1/2 * (b + (b^2 - 4 * c * d)^{(1/2)}) / c) * c * d - 5/4 / (b^2 - 4 * c * d)^{(3/2)} / (a - d)^{(7/2)} * \\
& \ln((2 * a - 2 * d + (b^2 - 4 * c * d)^{(1/2)} * (x - 1/2 * (-b + (b^2 - 4 * c * d)^{(1/2)}) / c) + 2 * (a - d)^{(1/2)} \\
& ) * (a + (x - 1/2 * (-b + (b^2 - 4 * c * d)^{(1/2)}) / c)^2 * c - d + (b^2 - 4 * c * d)^{(1/2)} * (x - 1/2 * (-b + (b \\
& ^2 - 4 * c * d)^{(1/2)}) / c)^{(1/2)}) / (x - 1/2 * (-b + (b^2 - 4 * c * d)^{(1/2)}) / c) * c * d
\end{aligned}$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + bx + a} (cx^2 + bx + d)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x^2+b\*x+d)^4/(c\*x^2+b\*x+a)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c\*x^2 + b\*x + a)\*(c\*x^2 + b\*x + d)^4), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{cx^2 + bx + a} (cx^2 + bx + d)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x + c\*x^2)^(1/2)\*(d + b\*x + c\*x^2)^4), x)

[Out] int(1/((a + b\*x + c\*x^2)^(1/2)\*(d + b\*x + c\*x^2)^4), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x\*\*2+b\*x+d)\*\*4/(c\*x\*\*2+b\*x+a)\*\*(1/2), x)

[Out] Timed out

$$3.7 \quad \int \frac{1}{\sqrt{d+ex+fx^2} (ae+bex+bf x^2)^2} dx$$

**Optimal.** Leaf size=162

$$\frac{(8aef - b(4df + e^2)) \tanh^{-1}\left(\frac{(e+2fx)\sqrt{bd-ae}}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{e^{3/2}(bd-ae)^{3/2}(be-4af)^{3/2}} - \frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)(ae+bex+bf x^2)}$$

[Out]  $-(8*a*e*f-b*(4*d*f+e^2))*\operatorname{arctanh}\left(\frac{(e+2*f*x)*(-a*e+b*d)^{(1/2)}/e^{(1/2)}}{(f*x^2+e*x+d)^{(1/2)}/e^{(3/2)}/(-a*e+b*d)^{(3/2)}/(-4*a*f+b*e)^{(3/2)}-b*(2*f*x+e)*(f*x^2+e*x+d)^{(1/2)}/e/(-a*e+b*d)/(-4*a*f+b*e)/(b*f*x^2+b*e*x+a*e)}\right)-\frac{b*(e+2*f*x)*\sqrt{d+e*x+f*x^2}}{e*(b*d-a*e)*(b*e-4*a*f)*(a*e+b*x+e*f*x^2)}$

**Rubi [A]** time = 0.31, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$ , Rules used = {974, 12, 982, 208}

$$\frac{(8aef - b(4df + e^2)) \tanh^{-1}\left(\frac{(e+2fx)\sqrt{bd-ae}}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{e^{3/2}(bd-ae)^{3/2}(be-4af)^{3/2}} - \frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)(ae+bex+bf x^2)}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[d + e\*x + f\*x^2]\*(a\*e + b\*e\*x + b\*f\*x^2)^2), x]

[Out]  $-\left(\frac{b*(e+2*f*x)*\sqrt{d+e*x+f*x^2}}{e*(b*d-a*e)*(b*e-4*a*f)*(a*e+b*e*x+b*f*x^2)}\right) - \left(\frac{(8*a*e*f-b*(e^2+4*d*f))*\operatorname{ArcTanh}\left[\frac{\sqrt{b*d-a*e}*(e+2*f*x)}{\sqrt{e}\sqrt{b*e-4*a*f}\sqrt{d+e*x+f*x^2}}\right]}{e^{(3/2)}*(b*d-a*e)^{(3/2)}*(b*e-4*a*f)^{(3/2)}}\right)$

**Rule 12**

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

**Rule 208**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

**Rule 974**

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_)\*((d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2)^(q\_), x\_Symbol] := Simp[((2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*



```

a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q,
0]

```

### Rule 982

```

Int[1/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]

```

### Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{d+ex+fx^2} (ae+bex+bf x^2)^2} dx &= -\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)(ae+bex+bf x^2)} + \frac{\int \frac{b(bd-ae)f^2(8aef-b(e^2+4df))}{2\sqrt{d+ex+fx^2}(ae+bex+bf x^2)}}{be(bd-ae)^2 f^2 (be-4af)} \\
&= -\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)(ae+bex+bf x^2)} + \frac{(8aef-b(e^2+4df)) \int}{2e(bd-ae)} \\
&= -\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)(ae+bex+bf x^2)} - \frac{(8aef-b(e^2+4df)) \operatorname{Su}}{e^3/2(bd-ae)} \\
&= -\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)(ae+bex+bf x^2)} - \frac{(8aef-b(e^2+4df)) \operatorname{ta}}{e^3/2(bd-ae)}
\end{aligned}$$

**Mathematica [B]** time = 1.96, size = 490, normalized size = 3.02

$$2f \left( \frac{(b(4df+e^2)-8aef) \tanh^{-1}\left(\frac{-\sqrt{e}(e+2fx)\sqrt{be-4af}-\sqrt{b}(e^2-4df)}{4f\sqrt{bd-ae}\sqrt{d+x(e+fx)}}\right)}{4f(bd-ae)^{3/2}(be-4af)^{3/2}} - \frac{e \tanh^{-1}\left(\frac{\sqrt{b}(e^2-4df)-\sqrt{e}(e+2fx)\sqrt{be-4af}}{4f\sqrt{bd-ae}\sqrt{d+x(e+fx)}}\right)}{4f(bd-ae)^{3/2}\sqrt{be-4af}} + \frac{\tanh^{-1}\left(\frac{\sqrt{e}(e+2fx)\sqrt{be-4af}-\sqrt{b}(e^2-4df)}{4f\sqrt{bd-ae}\sqrt{d+x(e+fx)}}\right)}{\sqrt{bd-ae}(be-4af)^{3/2}} \right) e^{3/2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[d + e\*x + f\*x^2]\*(a\*e + b\*e\*x + b\*f\*x^2)^2), x]

[Out] (2\*f\*(-((Sqrt[b]\*Sqrt[e]\*Sqrt[d + x\*(e + f\*x)])/((b\*d - a\*e)\*(b\*e - 4\*a\*f)\*(- (Sqrt[e]\*Sqrt[b\*e - 4\*a\*f]) + Sqrt[b]\*(e + 2\*f\*x)))) - (Sqrt[b]\*Sqrt[e]\*Sqrt[d + x\*(e + f\*x)])/((b\*d - a\*e)\*(b\*e - 4\*a\*f)\*(Sqrt[e]\*Sqrt[b\*e - 4\*a\*f] + Sqrt[b]\*(e + 2\*f\*x))) - ((-8\*a\*e\*f + b\*(e^2 + 4\*d\*f))\*ArcTanh[(- (Sqrt[b]\*(e^2 - 4\*d\*f)) - Sqrt[e]\*Sqrt[b\*e - 4\*a\*f]\*(e + 2\*f\*x))/(4\*Sqrt[b\*d - a\*e]\*f\*Sqrt[d + x\*(e + f\*x)])])/(4\*(b\*d - a\*e)^(3/2)\*f\*(b\*e - 4\*a\*f)^(3/2)) - (e\*ArcTanh[(Sqrt[b]\*(e^2 - 4\*d\*f) - Sqrt[e]\*Sqrt[b\*e - 4\*a\*f]\*(e + 2\*f\*x))/(4\*Sqrt[b\*d - a\*e]\*f\*Sqrt[d + x\*(e + f\*x)])])/(4\*(b\*d - a\*e)^(3/2)\*f\*Sqrt[b\*e - 4\*a\*f]) + ArcTanh[(- (Sqrt[b]\*(e^2 - 4\*d\*f)) + Sqrt[e]\*Sqrt[b\*e - 4\*a\*f]\*(e + 2\*f\*x))/(4\*Sqrt[b\*d - a\*e]\*f\*Sqrt[d + x\*(e + f\*x)])])/(Sqrt[b\*d - a\*e]\*(b\*e - 4\*a\*f)^(3/2)))/e^(3/2)

**fricas [B]** time = 3.07, size = 2005, normalized size = 12.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*f\*x^2+b\*e\*x+a\*e)^2/(f\*x^2+e\*x+d)^(1/2), x, algorithm="fricas")

[Out] [-1/4\*(sqrt(b^2\*d\*e^2 - a\*b\*e^3 - 4\*(a\*b\*d\*e - a^2\*e^2)\*f)\*(a\*b\*e^3 + (b^2\*e^2\*f + 4\*(b^2\*d - 2\*a\*b\*e)\*f^2)\*x^2 + 4\*(a\*b\*d\*e - 2\*a^2\*e^2)\*f + (b^2\*e^3 + 4\*(b^2\*d\*e - 2\*a\*b\*e^2)\*f)\*x)\*log((8\*b^2\*d^2\*e^4 - 8\*a\*b\*d\*e^5 + a^2\*e^6 + 16\*a^2\*d^2\*e^2\*f^2 + (b^2\*e^4\*f^2 + 16\*(b^2\*d^2 - 8\*a\*b\*d\*e + 8\*a^2\*e^2)\*f^4 + 8\*(3\*b^2\*d\*e^2 - 4\*a\*b\*e^3)\*f^3)\*x^4 + 2\*(b^2\*e^5\*f + 16\*(b^2\*d^2\*e - 8\*a\*b\*d\*e^2 + 8\*a^2\*e^3)\*f^3 + 8\*(3\*b^2\*d\*e^3 - 4\*a\*b\*e^4)\*f^2)\*x^3 + (b^2\*e^6 - 32\*(3\*a\*b\*d^2\*e - 4\*a^2\*d\*e^2)\*f^3 + 16\*(3\*b^2\*d^2\*e^2 - 13\*a\*b\*d\*e^3 + 10\*a^2\*e^4)\*f^2 + 2\*(16\*b^2\*d\*e^4 - 19\*a\*b\*e^5)\*f)\*x^2 - 4\*sqrt(b^2\*d\*e^2 - a\*b\*e^3 - 4\*(a\*b\*d\*e - a^2\*e^2)\*f)\*(2\*b\*d\*e^3 - a\*e^4 - 4\*a\*d\*e^2\*f + 2\*(b\*e^2\*f^2 + 4\*(b\*d - 2\*a\*e)\*f^3)\*x^3 + 3\*(b\*e^3\*f + 4\*(b\*d\*e - 2\*a\*e^2)\*f^2)\*x^2 + (b\*e^4 - 8\*a\*d\*e\*f^2 + 2\*(4\*b\*d\*e^2 - 5\*a\*e^3)\*f)\*x)\*sqrt(f\*x^2 + e\*x + d) - 8\*(4\*a\*b\*d^2\*e^3 - 3\*a^2\*d\*e^4)\*f + 2\*(4\*b^2\*d\*e^5 - 3\*a\*b\*e^6 - 16\*(3\*a\*b\*d^2\*e^2 - 4\*a^2\*d\*e^3)\*f^2 + 8\*(2\*b^2\*d^2\*e^3 - 5\*a\*b\*d\*e^4 + 2\*a^2\*e^5)\*f)\*x)/(b^2\*f^2\*x^4 + 2\*b^2\*e\*f\*x^3 + 2\*a\*b\*e^2\*x + a^2\*e^2 + (b

$$\begin{aligned} & \left( \frac{2e^2 + 2abef}{x^2} \right) + 4(b^3d^2e^3 - ab^2e^4 - 4(ab^2d^2e^2 - a^2b^2e^3)ef - 2(4(ab^2d^2e - a^2b^2e^2)ef^2 - (b^3d^2e^2 - ab^2e^3)ef)) \sqrt{fx^2 + ex + d} \\ & \left( \frac{1}{(ab^4d^2e^5 - 2a^2b^3d^2e^6 + a^3b^2e^7 + 16(a^3b^2d^2e^3 - 2a^4b^2d^2e^4 + a^5e^5)ef^2 + (16(a^2b^3d^2e^2 - 2a^3b^2d^2e^3 + a^4b^2e^4)ef^3 - 8(ab^4d^2e^3 - 2a^2b^3d^2e^4 + a^3b^2e^5)ef^2 + (b^5d^2e^4 - 2ab^4d^2e^5 + a^2b^3e^6)ef) x^2 - 8(a^2b^3d^2e^4 - 2a^3b^2d^2e^5 + a^4b^2e^6)ef + (b^5d^2e^5 - 2ab^4d^2e^6 + a^2b^3e^7 + 16(a^2b^3d^2e^3 - 2a^3b^2d^2e^4 + a^4b^2e^5)ef^2 - 8(ab^4d^2e^4 - 2a^2b^3d^2e^5 + a^3b^2e^6)ef) x} \right), \\ & \frac{1}{2} \sqrt{-b^2d^2e^2 + ab^2e^3 + 4(ab^2d^2e - a^2e^2)ef} \arctan\left(\frac{-1}{2\sqrt{-b^2d^2e^2 + ab^2e^3 + 4(ab^2d^2e - a^2e^2)ef}}\right) \\ & \left( \frac{1}{(b^2d^2e^3 - ab^2d^2e^4 - 2(4(ab^2d^2e - a^2e^2)ef^3 - (b^2d^2e^2 - ab^2e^3)ef^2) x^3 - 3(4(ab^2d^2e^2 - a^2e^3)ef^2 - (b^2d^2e^3 - ab^2e^4)ef) x^2 - 4(ab^2d^2e^2 - a^2d^2e^3)ef + (b^2d^2e^4 - ab^2e^5 - 8(ab^2d^2e - a^2d^2e^2)ef^2 + 2(b^2d^2e^2 - 3ab^2d^2e^3 + 2a^2e^4)ef) x} \right) - 2(b^3d^2e^3 - ab^2e^4 - 4(ab^2d^2e^2 - a^2b^2e^3)ef - 2(4(ab^2d^2e - a^2b^2e^2)ef^2 - (b^3d^2e^2 - ab^2e^3)ef) x) \sqrt{fx^2 + ex + d} \\ & \left( \frac{1}{(ab^4d^2e^5 - 2a^2b^3d^2e^6 + a^3b^2e^7 + 16(a^3b^2d^2e^3 - 2a^4b^2d^2e^4 + a^5e^5)ef^2 + (16(a^2b^3d^2e^2 - 2a^3b^2d^2e^3 + a^4b^2e^4)ef^3 - 8(ab^4d^2e^3 - 2a^2b^3d^2e^4 + a^3b^2e^5)ef^2 + (b^5d^2e^4 - 2ab^4d^2e^5 + a^2b^3e^6)ef) x^2 - 8(a^2b^3d^2e^4 - 2a^3b^2d^2e^5 + a^4b^2e^6)ef + (b^5d^2e^5 - 2ab^4d^2e^6 + a^2b^3e^7 + 16(a^2b^3d^2e^3 - 2a^3b^2d^2e^4 + a^4b^2e^5)ef^2 - 8(ab^4d^2e^4 - 2a^2b^3d^2e^5 + a^3b^2e^6)ef) x} \right) \end{aligned}$$

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*f\*x^2+b\*e\*x+a\*e)^2/(f\*x^2+e\*x+d)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP  
 UT:sage2:=int(sage0,x);OUTPUT:Unable to divide, perhaps due to rounding error  
 error%{1, [2]%%}, [8, 2, 0, 0, 0]%%}+%{1, [1]%%}, [6, 2, 2, 0, 0]%%}+%{1, [1]%%}, [7, 2, 1, 0, 0]%%}+%{1, [1]%%}, [6, 2, 2, 0, 0]%%}+%{1, [2]%%}, [6, 2, 0, 0, 1]%%}+%{1, [2]%%}, [6, 1, 1, 1, 0]%%}+%{1, [4, 0] : [1, 0, %%{-1, [1]%%}]}%}, [5, 2, 3, 0, 0]%%}+%{1, [12, [1]%%}, 0] : [1, 0, %%{-1, [1]%%}]}%}, [5, 2, 1, 0, 1]%%}+%{1, [24, [1]%%}, 0] : [1, 0, %%{-1, [1]%%}]}%}, [5, 1, 2, 1, 0]%%}+%{1, [4, 2, 4, 0, 0]%%}+%{1, [14, [1]%%}, [4, 2, 2, 0, 1]%%}+%{1, [6, [2]%%}, [4, 2, 0, 0, 2]%%}+%{1, [26, [1]%%}, [4, 1, 3, 1, 0]%%}+%{1, [16, [2]%%}, [4, 1, 1, 1, 1]%%}+%{1, [16, [2]%%}, [4, 0, 2, 2, 0]%%}+%{1, [8

```

,0]: [1,0,%%{-1, [1]%%}]%}, [3,2,3,0,1]%%}+%%{%%{-12, [1]%%},0]: [1,0
,%%{-1, [1]%%}]%}, [3,2,1,0,2]%%}+%%{%%{-12,0]: [1,0,%%{-1, [1]%%}]%},
[3,1,4,1,0]%%}+%%{%%{32, [1]%%},0]: [1,0,%%{-1, [1]%%}]%}, [3,1,2,1,
1]%%}+%%{%%{-32, [1]%%},0]: [1,0,%%{-1, [1]%%}]%}, [3,0,3,2,0]%%}+
%%{-2, [2,2,4,0,1]%%}+%%{%%{10, [1]%%}, [2,2,2,0,2]%%}+%%{%%{-4, [2]%%}
, [2,2,0,0,3]%%}+%%{2, [2,1,5,1,0]%%}+%%{%%{-28, [1]%%}, [2,1,3,1,1]%%}+
%%{%%{8, [2]%%}, [2,1,1,1,2]%%}+%%{%%{24, [1]%%}, [2,0,4,2,0]%%}+%%{%%
{-4,0]: [1,0,%%{-1, [1]%%}]%}, [1,2,3,0,2]%%}+%%{%%{4, [1]%%},0]: [1
,0,%%{-1, [1]%%}]%}, [1,2,1,0,3]%%}+%%{%%{12,0]: [1,0,%%{-1, [1]%%}]%}
, [1,1,4,1,1]%%}+%%{%%{-8, [1]%%},0]: [1,0,%%{-1, [1]%%}]%}, [1,1,2,1
,2]%%}+%%{%%{-8,0]: [1,0,%%{-1, [1]%%}]%}, [1,0,5,2,0]%%}+%%{1, [0,2,4,
0,2]%%}+%%{%%{-2, [1]%%}, [0,2,2,0,3]%%}+%%{%%{1, [2]%%}, [0,2,0,0,4]%%
}+%%{-2, [0,1,5,1,1]%%}+%%{%%{2, [1]%%}, [0,1,3,1,2]%%}+%%{1, [0,0,6,2,
0]%%} / %%{%%{1, [3]%%}, [8,2,0,0,0]%%}+%%{%%{poly1 [%%{-4, [2]%%},0]: [
1,0,%%{-1, [1]%%}]%}, [7,2,1,0,0]%%}+%%{%%{6, [2]%%}, [6,2,2,0,0]%%}+%%
{%%{-4, [3]%%}, [6,2,0,0,1]%%}+%%{%%{8, [3]%%}, [6,1,1,1,0]%%}+%%{%%{p
oly1 [%%{-4, [1]%%},0]: [1,0,%%{-1, [1]%%}]%}, [5,2,3,0,0]%%}+%%{%%{poly1
[%%{12, [2]%%},0]: [1,0,%%{-1, [1]%%}]%}, [5,2,1,0,1]%%}+%%{%%{poly1 [%%
{-24, [2]%%},0]: [1,0,%%{-1, [1]%%}]%}, [5,1,2,1,0]%%}+%%{%%{1, [1]%%}, [
4,2,4,0,0]%%}+%%{%%{-14, [2]%%}, [4,2,2,0,1]%%}+%%{%%{6, [3]%%}, [4,2,0
,0,2]%%}+%%{%%{26, [2]%%}, [4,1,3,1,0]%%}+%%{%%{-16, [3]%%}, [4,1,1,1,1
]%%}+%%{%%{-16, [3]%%}, [4,0,2,2,0]%%}+%%{%%{poly1 [%%{8, [1]%%},0]: [1,0
,%%{-1, [1]%%}]%}, [3,2,3,0,1]%%}+%%{%%{poly1 [%%{-12, [2]%%},0]: [1,0,%%
{-1, [1]%%}]%}, [3,2,1,0,2]%%}+%%{%%{poly1 [%%{-12, [1]%%},0]: [1,0,%%{-
1, [1]%%}]%}, [3,1,4,1,0]%%}+%%{%%{poly1 [%%{32, [2]%%},0]: [1,0,%%{-1, [1
]%%}]%}, [3,1,2,1,1]%%}+%%{%%{poly1 [%%{-32, [2]%%},0]: [1,0,%%{-1, [1]%%
}]%}, [3,0,3,2,0]%%}+%%{%%{-2, [1]%%}, [2,2,4,0,1]%%}+%%{%%{10, [2]%%}
}, [2,2,2,0,2]%%}+%%{%%{-4, [3]%%}, [2,2,0,0,3]%%}+%%{%%{2, [1]%%}, [2,1
,5,1,0]%%}+%%{%%{-28, [2]%%}, [2,1,3,1,1]%%}+%%{%%{8, [3]%%}, [2,1,1,1,
2]%%}+%%{%%{24, [2]%%}, [2,0,4,2,0]%%}+%%{%%{poly1 [%%{-4, [1]%%},0]: [1
,0,%%{-1, [1]%%}]%}, [1,2,3,0,2]%%}+%%{%%{poly1 [%%{4, [2]%%},0]: [1,0,%%
{-1, [1]%%}]%}, [1,2,1,0,3]%%}+%%{%%{poly1 [%%{12, [1]%%},0]: [1,0,%%{-1
, [1]%%}]%}, [1,1,4,1,1]%%}+%%{%%{poly1 [%%{-8, [2]%%},0]: [1,0,%%{-1, [1]
%%}]%}, [1,1,2,1,2]%%}+%%{%%{poly1 [%%{-8, [1]%%},0]: [1,0,%%{-1, [1]%%}
]%%}, [1,0,5,2,0]%%}+%%{%%{1, [1]%%}, [0,2,4,0,2]%%}+%%{%%{-2, [2]%%}, [
0,2,2,0,3]%%}+%%{%%{1, [3]%%}, [0,2,0,0,4]%%}+%%{%%{-2, [1]%%}, [0,1,5,
1,1]%%}+%%{%%{2, [2]%%}, [0,1,3,1,2]%%}+%%{%%{1, [1]%%}, [0,0,6,2,0]%%}
} Error: Bad Argument Value

```

**maple [B]** time = 0.03, size = 1377, normalized size = 8.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(1/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^{(1/2)}, x)$

```
[Out] -2/(4*a*f-b*e)/e*f/(-4*a*f-b*e)*b*e)^(1/2)/(-a*e-b*d)/b)^(1/2)*ln((-2*(a*
e-b*d)/b+(-4*a*f-b*e)*b*e)^(1/2)*(x-1/2*(-b*e+(-4*a*f-b*e)*b*e)^(1/2))/b/
f)/b+2*(-(a*e-b*d)/b)^(1/2)*((x-1/2*(-b*e+(-4*a*f-b*e)*b*e)^(1/2))/b/f)^2*
f+(-4*a*f-b*e)*b*e)^(1/2)*(x-1/2*(-b*e+(-4*a*f-b*e)*b*e)^(1/2))/b/f)/b-(a
*e-b*d)/b)^(1/2))/(x-1/2*(-b*e+(-4*a*f-b*e)*b*e)^(1/2))/b/f))-1/(4*a*f-b*e
)/e/(a*e-b*d)/(x+1/2/f*e-1/2/b/f*(-4*a*f-b*e)*b*e)^(1/2))*((x-1/2*(-b*e+(-
4*a*f-b*e)*b*e)^(1/2))/b/f)^2*f+(-4*a*f-b*e)*b*e)^(1/2)*(x-1/2*(-b*e+(-4
*a*f-b*e)*b*e)^(1/2))/b/f)/b-(a*e-b*d)/b)^(1/2)+1/2/(4*a*f-b*e)/b/e*(-(4*a*
f-b*e)*b*e)^(1/2)/(a*e-b*d)/(-a*e-b*d)/b)^(1/2)*ln((-2*(a*e-b*d)/b+(-4*a*
f-b*e)*b*e)^(1/2)*(x-1/2*(-b*e+(-4*a*f-b*e)*b*e)^(1/2))/b/f)/b+2*(-(a*e-b*
d)/b)^(1/2)*((x-1/2*(-b*e+(-4*a*f-b*e)*b*e)^(1/2))/b/f)^2*f+(-4*a*f-b*e)*
b*e)^(1/2)*(x-1/2*(-b*e+(-4*a*f-b*e)*b*e)^(1/2))/b/f)/b-(a*e-b*d)/b)^(1/2)
)/(x-1/2*(-b*e+(-4*a*f-b*e)*b*e)^(1/2))/b/f))-1/(4*a*f-b*e)/e/(a*e-b*d)/(x
+1/2/f*e+1/2/b/f*(-4*a*f-b*e)*b*e)^(1/2))*((x+1/2*(b*e+(-4*a*f-b*e)*b*e)^(
1/2))/b/f)^2*f-(-4*a*f-b*e)*b*e)^(1/2)*(x+1/2*(b*e+(-4*a*f-b*e)*b*e)^(1/
2))/b/f)/b-(a*e-b*d)/b)^(1/2)-1/2/(4*a*f-b*e)/b/e*(-(4*a*f-b*e)*b*e)^(1/2)/
(a*e-b*d)/(-a*e-b*d)/b)^(1/2)*ln((-2*(a*e-b*d)/b-(-4*a*f-b*e)*b*e)^(1/2)*
(x+1/2*(b*e+(-4*a*f-b*e)*b*e)^(1/2))/b/f)/b+2*(-(a*e-b*d)/b)^(1/2)*((x+1/2
*(b*e+(-4*a*f-b*e)*b*e)^(1/2))/b/f)^2*f-(-4*a*f-b*e)*b*e)^(1/2)*(x+1/2*(b
*e+(-4*a*f-b*e)*b*e)^(1/2))/b/f)/b-(a*e-b*d)/b)^(1/2))/(x+1/2*(b*e+(-4*a*
f-b*e)*b*e)^(1/2))/b/f))+2/(4*a*f-b*e)/e*f/(-4*a*f-b*e)*b*e)^(1/2)/(-a*e-
b*d)/b)^(1/2)*ln((-2*(a*e-b*d)/b-(-4*a*f-b*e)*b*e)^(1/2)*(x+1/2*(b*e+(-4*
a*f-b*e)*b*e)^(1/2))/b/f)/b+2*(-(a*e-b*d)/b)^(1/2)*((x+1/2*(b*e+(-4*a*f-b*
e)*b*e)^(1/2))/b/f)^2*f-(-4*a*f-b*e)*b*e)^(1/2)*(x+1/2*(b*e+(-4*a*f-b*e)*
b*e)^(1/2))/b/f)/b-(a*e-b*d)/b)^(1/2))/(x+1/2*(b*e+(-4*a*f-b*e)*b*e)^(1/2)
)/b/f))
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bf x^2 + bex + ae)^2 \sqrt{fx^2 + ex + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*f*x^2 + b*e*x + a*e)^2*sqrt(f*x^2 + e*x + d)), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(bf x^2 + bex + ae)^2 \sqrt{fx^2 + ex + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*e + b*e*x + b*f*x^2)^2*(d + e*x + f*x^2)^(1/2)),x)
```

```
[Out] int(1/((a*e + b*e*x + b*f*x^2)^2*(d + e*x + f*x^2)^(1/2)), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*f*x**2+b*e*x+a*e)**2/(f*x**2+e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

$$3.8 \quad \int \frac{1}{(4+2x+x^2)\sqrt{5+2x+x^2}} dx$$

**Optimal.** Leaf size=28

$$\frac{\tan^{-1}\left(\frac{x+1}{\sqrt{3}\sqrt{x^2+2x+5}}\right)}{\sqrt{3}}$$

[Out] 1/3\*arctan(1/3\*(1+x)\*3^(1/2)/(x^2+2\*x+5)^(1/2))\*3^(1/2)

**Rubi [A]** time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {982, 204}

$$\frac{\tan^{-1}\left(\frac{x+1}{\sqrt{3}\sqrt{x^2+2x+5}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/((4 + 2\*x + x^2)\*Sqrt[5 + 2\*x + x^2]),x]

[Out] ArcTan[(1 + x)/(Sqrt[3]\*Sqrt[5 + 2\*x + x^2])]/Sqrt[3]

Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :-> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 982

Int[1/(((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*Sqrt[(d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2]), x\_Symbol] :-> Dist[-2\*e, Subst[Int[1/(e\*(b\*e - 4\*a\*f) - (b\*d - a\*e)\*x^2), x], x, (e + 2\*f\*x)/Sqrt[d + e\*x + f\*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && EqQ[c\*e - b\*f, 0]

Rubi steps

$$\int \frac{1}{(4 + 2x + x^2)\sqrt{5 + 2x + x^2}} dx = -\left(4 \operatorname{Subst}\left(\int \frac{1}{-24 - 2x^2} dx, x, \frac{2 + 2x}{\sqrt{5 + 2x + x^2}}\right)\right)$$

$$= \frac{\tan^{-1}\left(\frac{1+x}{\sqrt{3}\sqrt{5+2x+x^2}}\right)}{\sqrt{3}}$$

**Mathematica [C]** time = 0.07, size = 84, normalized size = 3.00

$$\frac{i\left(\tanh^{-1}\left(\frac{-i\sqrt{3}x-i\sqrt{3}+4}{\sqrt{x^2+2x+5}}\right) - \tanh^{-1}\left(\frac{i\sqrt{3}x+i\sqrt{3}+4}{\sqrt{x^2+2x+5}}\right)\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((4 + 2\*x + x^2)\*Sqrt[5 + 2\*x + x^2]), x]

[Out] ((-1/2\*I)\*(ArcTanh[(4 - I\*Sqrt[3] - I\*Sqrt[3]\*x)/Sqrt[5 + 2\*x + x^2]] - ArcTanh[(4 + I\*Sqrt[3] + I\*Sqrt[3]\*x)/Sqrt[5 + 2\*x + x^2]]))/Sqrt[3]

**fricas [A]** time = 0.84, size = 38, normalized size = 1.36

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\sqrt{x^2+2x+5}(x+1) - \frac{1}{3}\sqrt{3}(x^2+2x+4)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+2\*x+4)/(x^2+2\*x+5)^(1/2), x, algorithm="fricas")

[Out] 1/3\*sqrt(3)\*arctan(1/3\*sqrt(3)\*sqrt(x^2 + 2\*x + 5)\*(x + 1) - 1/3\*sqrt(3)\*(x^2 + 2\*x + 4))

**giac [B]** time = 0.22, size = 52, normalized size = 1.86

$$-\frac{1}{3}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x - \sqrt{x^2 + 2x + 5} + 2\right)\right) + \frac{1}{3}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x - \sqrt{x^2 + 2x + 5}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+2\*x+4)/(x^2+2\*x+5)^(1/2), x, algorithm="giac")

[Out] -1/3\*sqrt(3)\*arctan(-1/3\*sqrt(3)\*(x - sqrt(x^2 + 2\*x + 5) + 2)) + 1/3\*sqrt(3)\*arctan(-1/3\*sqrt(3)\*(x - sqrt(x^2 + 2\*x + 5)))



**maple** [A] time = 0.02, size = 27, normalized size = 0.96

$$\frac{\sqrt{3} \arctan\left(\frac{\sqrt{3}(2x+2)}{6\sqrt{x^2+2x+5}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2+2*x+4)/(x^2+2*x+5)^(1/2), x)`

[Out] `1/3*3^(1/2)*arctan(1/6*3^(1/2)/(x^2+2*x+5)^(1/2)*(2*x+2))`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2 + 2x + 5}(x^2 + 2x + 4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^2+2*x+4)/(x^2+2*x+5)^(1/2), x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(x^2 + 2*x + 5)*(x^2 + 2*x + 4)), x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(x^2 + 2x + 4)\sqrt{x^2 + 2x + 5}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((2*x + x^2 + 4)*(2*x + x^2 + 5)^(1/2)), x)`

[Out] `int(1/((2*x + x^2 + 4)*(2*x + x^2 + 5)^(1/2)), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^2 + 2x + 4)\sqrt{x^2 + 2x + 5}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x**2+2*x+4)/(x**2+2*x+5)**(1/2), x)`

[Out] `Integral(1/((x**2 + 2*x + 4)*sqrt(x**2 + 2*x + 5)), x)`

$$3.9 \quad \int \left( a + \frac{ex}{2} + cx^2 \right)^p \left( 2a + ex + 2cx^2 \right)^q dx$$

**Optimal.** Leaf size=136

$$\frac{2^{q+1} \left( -\frac{\sqrt{e^2-16ac}+4cx+e}{\sqrt{e^2-16ac}} \right)^{-p-q-1} (2a+2cx^2+ex)^{p+q+1} {}_2F_1 \left( -p-q, p+q+1; p+q+2; \frac{e+4cx+\sqrt{e^2-16ac}}{2\sqrt{e^2-16ac}} \right)}{(p+q+1)\sqrt{e^2-16ac}}$$

[Out]  $-2^{(1+q)}*(2*c*x^2+e*x+2*a)^{(1+p+q)}*\text{hypergeom}([-p-q, 1+p+q], [2+p+q], 1/2*(e+4*c*x+(-16*a*c+e^2)^{(1/2)})/(-16*a*c+e^2)^{(1/2)})*((-e-4*c*x+(-16*a*c+e^2)^{(1/2)})/(-16*a*c+e^2)^{(1/2)})^{(-1-p-q)}/(1+p+q)/(-16*a*c+e^2)^{(1/2)}$

**Rubi [A]** time = 0.10, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$ , Rules used = {967, 624}

$$\frac{2^{q+1} \left( -\frac{\sqrt{e^2-16ac}+4cx+e}{\sqrt{e^2-16ac}} \right)^{-p-q-1} (2a+2cx^2+ex)^{p+q+1} {}_2F_1 \left( -p-q, p+q+1; p+q+2; \frac{e+4cx+\sqrt{e^2-16ac}}{2\sqrt{e^2-16ac}} \right)}{(p+q+1)\sqrt{e^2-16ac}}$$

Antiderivative was successfully verified.

[In] Int[(a + (e\*x)/2 + c\*x^2)^p\*(2\*a + e\*x + 2\*c\*x^2)^q,x]

[Out]  $-((2^{(1+q)}*((e - \text{Sqrt}[-16*a*c + e^2] + 4*c*x)/\text{Sqrt}[-16*a*c + e^2]))^{(-1-p-q)}*(2*a + e*x + 2*c*x^2)^{(1+p+q)}*\text{Hypergeometric2F1}[-p-q, 1+p+q, 2+p+q, (e + \text{Sqrt}[-16*a*c + e^2] + 4*c*x)/(2*\text{Sqrt}[-16*a*c + e^2])])/(\text{Sqrt}[-16*a*c + e^2]*(1+p+q))$

**Rule 624**

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{q = Rt[b^2 - 4\*a\*c, 2]}, -Simp[((a + b\*x + c\*x^2)^(p+1)\*Hypergeometric2F1[-p, p+1, p+2, (b+q+2\*c\*x)/(2\*q)])/(q\*(p+1)\*((q-b-2\*c\*x)/(2\*q))^(p+1)), x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && !IntegerQ[4\*p]

**Rule 967**

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.)\*((d\_) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2)^(q\_.), x\_Symbol] :> Dist[(c/f)^p, Int[(d + e\*x + f\*x^2)^(p+q), x], x] /; FreeQ[{a, b, c, d, e, f, p, q}, x] && EqQ[c\*d - a\*f, 0] && EqQ[b\*d - a\*e, 0] && (IntegerQ[p] || GtQ[c/f, 0]) && (!IntegerQ[q] || LeafCount[d + e\*x + f\*x^2] <= LeafCount[a + b\*x + c\*x^2])

Rubi steps

$$\int \left(a + \frac{ex}{2} + cx^2\right)^p (2a + ex + 2cx^2)^q dx = 2^{-p} \int (2a + ex + 2cx^2)^{p+q} dx$$

$$= -\frac{2^{1+q} \left(-\frac{e-\sqrt{-16ac+e^2}+4cx}{\sqrt{-16ac+e^2}}\right)^{-1-p-q} (2a + ex + 2cx^2)^{1+p+q} {}_2F_1\left(-p-q, 1\right)}{\sqrt{-16ac+e^2} (1+p+q)}$$

**Mathematica [A]** time = 0.14, size = 142, normalized size = 1.04

$$\frac{2^{q-2} \left(-\sqrt{e^2-16ac} + 4cx + e\right) \left(\frac{\sqrt{e^2-16ac}+4cx+e}{\sqrt{e^2-16ac}}\right)^{-p-q} (2a + x(2cx + e))^{p+q} {}_2F_1\left(-p-q, p+q+1; p+q+2; \frac{-e-4cx}{2\sqrt{e^2-16ac}}\right)}{c(p+q+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + (e\*x)/2 + c\*x^2)^p\*(2\*a + e\*x + 2\*c\*x^2)^q,x]

[Out] (2^(-2 + q)\*(e - Sqrt[-16\*a\*c + e^2] + 4\*c\*x)\*(e + Sqrt[-16\*a\*c + e^2] + 4\*c\*x)/Sqrt[-16\*a\*c + e^2])^(-p - q)\*(2\*a + x\*(e + 2\*c\*x))^(p + q)\*Hypergeometric2F1[-p - q, 1 + p + q, 2 + p + q, (-e + Sqrt[-16\*a\*c + e^2] - 4\*c\*x)/(2\*Sqrt[-16\*a\*c + e^2])]/(c\*(1 + p + q))

**fricas [F]** time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(2cx^2 + ex + 2a\right)^q \left(cx^2 + \frac{1}{2}ex + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+1/2\*e\*x+c\*x^2)^p\*(2\*c\*x^2+e\*x+2\*a)^q,x, algorithm="fricas")

[Out] integral((2\*c\*x^2 + e\*x + 2\*a)^q\*(c\*x^2 + 1/2\*e\*x + a)^p, x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (2cx^2 + ex + 2a)^q \left(cx^2 + \frac{1}{2}ex + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+1/2\*e\*x+c\*x^2)^p\*(2\*c\*x^2+e\*x+2\*a)^q,x, algorithm="giac")

[Out] integrate((2\*c\*x^2 + e\*x + 2\*a)^q\*(c\*x^2 + 1/2\*e\*x + a)^p, x)

**maple** [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \left( c x^2 + \frac{1}{2} e x + a \right)^p (2 c x^2 + e x + 2 a)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+1/2\*e\*x+c\*x^2)^p\*(2\*c\*x^2+e\*x+2\*a)^q,x)

[Out] int((a+1/2\*e\*x+c\*x^2)^p\*(2\*c\*x^2+e\*x+2\*a)^q,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2 c x^2 + e x + 2 a)^q \left( c x^2 + \frac{1}{2} e x + a \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+1/2\*e\*x+c\*x^2)^p\*(2\*c\*x^2+e\*x+2\*a)^q,x, algorithm="maxima")

[Out] integrate((2\*c\*x^2 + e\*x + 2\*a)^q\*(c\*x^2 + 1/2\*e\*x + a)^p, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left( c x^2 + \frac{e x}{2} + a \right)^p (2 c x^2 + e x + 2 a)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + (e\*x)/2 + c\*x^2)^p\*(2\*a + e\*x + 2\*c\*x^2)^q,x)

[Out] int((a + (e\*x)/2 + c\*x^2)^p\*(2\*a + e\*x + 2\*c\*x^2)^q, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+1/2\*e\*x+c\*x\*\*2)\*\*p\*(2\*c\*x\*\*2+e\*x+2\*a)\*\*q,x)

[Out] Timed out

$$3.10 \quad \int \left( a + \frac{cex}{f} + cx^2 \right)^p \left( \frac{af}{c} + ex + fx^2 \right)^q dx$$

Optimal. Leaf size=200

$$\frac{\sqrt{c} 2^{p+q+1} \left( a + \frac{cex}{f} + cx^2 \right)^p \left( \frac{af}{c} + ex + fx^2 \right)^{q+1} \left( \frac{\sqrt{c} \left( -\frac{\sqrt{ce^2 - 4af^2}}{\sqrt{c}} + e + 2fx \right)}{\sqrt{ce^2 - 4af^2}} \right)^{-p-q-1}}{(p+q+1)\sqrt{ce^2 - 4af^2}} {}_2F_1 \left( -p-q, p+q+1; p+q+2; \right)$$

[Out]  $-2^{(1+p+q)} \cdot (a + c \cdot e \cdot x / f + c \cdot x^2)^p \cdot (a \cdot f / c + e \cdot x + f \cdot x^2)^{(1+q)} \cdot \text{hypergeom}([-p-q, 1+p+q], [2+p+q], 1/2 \cdot c^{(1/2)} \cdot (e + 2 \cdot f \cdot x + (-4 \cdot a \cdot f^2 + c \cdot e^2)^{(1/2)} / c^{(1/2)}) / (-4 \cdot a \cdot f^2 + c \cdot e^2)^{(1/2)}) \cdot c^{(1/2)} \cdot (-c^{(1/2)} \cdot (e + 2 \cdot f \cdot x - (-4 \cdot a \cdot f^2 + c \cdot e^2)^{(1/2)} / c^{(1/2)}) / (-4 \cdot a \cdot f^2 + c \cdot e^2)^{(1/2)})^{(-1-p-q)} / (1+p+q) / (-4 \cdot a \cdot f^2 + c \cdot e^2)^{(1/2)}$

**Rubi [A]** time = 0.13, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$ , Rules used = {968, 624}

$$\frac{\sqrt{c} 2^{p+q+1} \left( a + \frac{cex}{f} + cx^2 \right)^p \left( \frac{af}{c} + ex + fx^2 \right)^{q+1} \left( \frac{\sqrt{c} \left( -\frac{\sqrt{ce^2 - 4af^2}}{\sqrt{c}} + e + 2fx \right)}{\sqrt{ce^2 - 4af^2}} \right)^{-p-q-1}}{(p+q+1)\sqrt{ce^2 - 4af^2}} {}_2F_1 \left( -p-q, p+q+1; p+q+2; \right)$$

Antiderivative was successfully verified.

[In] Int[(a + (c\*e\*x)/f + c\*x^2)^p\*((a\*f)/c + e\*x + f\*x^2)^q,x]

[Out]  $-((2^{(1+p+q)} \cdot \text{Sqrt}[c] \cdot (-((\text{Sqrt}[c] \cdot (e - \text{Sqrt}[c \cdot e^2 - 4 \cdot a \cdot f^2]) / \text{Sqrt}[c] + 2 \cdot f \cdot x)) / \text{Sqrt}[c \cdot e^2 - 4 \cdot a \cdot f^2]))^{(-1-p-q)} \cdot (a + (c \cdot e \cdot x) / f + c \cdot x^2)^p \cdot ((a \cdot f) / c + e \cdot x + f \cdot x^2)^{(1+q)} \cdot \text{Hypergeometric2F1}[-p-q, 1+p+q, 2+p+q, (\text{Sqrt}[c] \cdot (e + \text{Sqrt}[c \cdot e^2 - 4 \cdot a \cdot f^2]) / \text{Sqrt}[c] + 2 \cdot f \cdot x)) / (2 \cdot \text{Sqrt}[c \cdot e^2 - 4 \cdot a \cdot f^2])]) / (\text{Sqrt}[c \cdot e^2 - 4 \cdot a \cdot f^2] \cdot (1+p+q)))$

Rule 624

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, -Simp[((a + b\*x + c\*x^2)^(p+1)\*Hypergeometric2F1[-p, p+1, p+2, (b+q+2\*c\*x)/(2\*q)]) / (q\*(p+1)\*((q-b-2\*c\*x)/(2\*q))^(p+1)), x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && !IntegerQ[4\*p]

Rule 968

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((d_) + (e_)*(x_) + (f_)*(x_)
^2)^(q_), x_Symbol] := Dist[(a^IntPart[p]*(a + b*x + c*x^2)^FracPart[p])/
(d^IntPart[p]*(d + e*x + f*x^2)^FracPart[p]), Int[(d + e*x + f*x^2)^(p + q),
x], x] /; FreeQ[{a, b, c, d, e, f, p, q}, x] && EqQ[c*d - a*f, 0] && EqQ[b
*d - a*e, 0] && !IntegerQ[p] && !IntegerQ[q] && !GtQ[c/f, 0]
```

### Rubi steps

$$\int \left(a + \frac{cex}{f} + cx^2\right)^p \left(\frac{af}{c} + ex + fx^2\right)^q dx = \left(\left(a + \frac{cex}{f} + cx^2\right)^p \left(\frac{af}{c} + ex + fx^2\right)^{-p}\right) \int \left(\frac{af}{c} + ex + fx^2\right)^{p+q} dx$$

$$= \frac{2^{1+p+q} \sqrt{c} \left( \frac{\sqrt{c} \left( e - \frac{\sqrt{ce^2 - 4af^2}}{\sqrt{c}} + 2fx \right)}{\sqrt{ce^2 - 4af^2}} \right)^{-1-p-q} \left(a + \frac{cex}{f} + cx^2\right)^p \left(\frac{af}{c} + ex + \dots\right)}{\sqrt{ce^2 - 4af^2}}$$

**Mathematica** [A] time = 0.24, size = 172, normalized size = 0.86

$$\frac{2^{p+q-1} \left(\sqrt{c}(e + 2fx) - \sqrt{ce^2 - 4af^2}\right) \left(a + \frac{cx(e+fx)}{f}\right)^p \left(\frac{af}{c} + x(e + fx)\right)^q \left(\frac{\sqrt{c}(e+2fx)}{\sqrt{ce^2-4af^2}} + 1\right)^{-p-q} {}_2F_1\left(-p-q, p+q+1; \dots\right)}{\sqrt{c} f(p+q+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + (c*e*x)/f + c*x^2)^p*((a*f)/c + e*x + f*x^2)^q,x]
```

```
[Out] (2^(-1 + p + q))*((a*f)/c + x*(e + f*x))^q*(a + (c*x*(e + f*x))/f)^p*(-Sqrt[
c*e^2 - 4*a*f^2] + Sqrt[c]*(e + 2*f*x))*(1 + (Sqrt[c]*(e + 2*f*x))/Sqrt[c*e
^2 - 4*a*f^2])^(-p - q)*Hypergeometric2F1[-p - q, 1 + p + q, 2 + p + q, 1/2
- (Sqrt[c]*(e + 2*f*x))/(2*Sqrt[c*e^2 - 4*a*f^2])]/(Sqrt[c]*f*(1 + p + q)
)
```

**fricas** [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\frac{cfx^2 + cex + af}{c}\right)^q \left(\frac{cfx^2 + cex + af}{f}\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*e*x/f+c*x^2)^p*(a*f/c+e*x+f*x^2)^q,x, algorithm="fricas")
```

[Out] integral(((c\*f\*x^2 + c\*e\*x + a\*f)/c)^q\*((c\*f\*x^2 + c\*e\*x + a\*f)/f)^p, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left( cx^2 + \frac{cex}{f} + a \right)^p \left( fx^2 + ex + \frac{af}{c} \right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c\*e\*x/f+c\*x^2)^p\*(a\*f/c+e\*x+f\*x^2)^q,x, algorithm="giac")

[Out] integrate((c\*x^2 + c\*e\*x/f + a)^p\*(f\*x^2 + e\*x + a\*f/c)^q, x)

**maple** [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \left( cx^2 + \frac{cex}{f} + a \right)^p \left( fx^2 + ex + \frac{af}{c} \right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+c\*e\*x/f+c\*x^2)^p\*(a\*f/c+e\*x+f\*x^2)^q,x)

[Out] int((a+c\*e\*x/f+c\*x^2)^p\*(a\*f/c+e\*x+f\*x^2)^q,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left( cx^2 + \frac{cex}{f} + a \right)^p \left( fx^2 + ex + \frac{af}{c} \right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c\*e\*x/f+c\*x^2)^p\*(a\*f/c+e\*x+f\*x^2)^q,x, algorithm="maxima")

[Out] integrate((c\*x^2 + c\*e\*x/f + a)^p\*(f\*x^2 + e\*x + a\*f/c)^q, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left( ex + fx^2 + \frac{af}{c} \right)^q \left( a + cx^2 + \frac{cex}{f} \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x + f\*x^2 + (a\*f)/c)^q\*(a + c\*x^2 + (c\*e\*x)/f)^p,x)

[Out] int((e\*x + f\*x^2 + (a\*f)/c)^q\*(a + c\*x^2 + (c\*e\*x)/f)^p, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*e*x/f+c*x**2)**p*(a*f/c+e*x+f*x**2)**q,x)
```

```
[Out] Timed out
```



$$3.11 \quad \int \frac{\sqrt{1+2x+x^2}}{\sqrt{1+x^2}} dx$$

Optimal. Leaf size=48

$$\frac{\sqrt{x^2+1}\sqrt{x^2+2x+1}}{x+1} + \frac{\sqrt{x^2+2x+1}\sinh^{-1}(x)}{x+1}$$

[Out] arcsinh(x)\*((1+x)^2)^(1/2)/(1+x)+(x^2+1)^(1/2)\*((1+x)^2)^(1/2)/(1+x)

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {970, 641, 215}

$$\frac{\sqrt{x^2+1}\sqrt{x^2+2x+1}}{x+1} + \frac{\sqrt{x^2+2x+1}\sinh^{-1}(x)}{x+1}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + 2\*x + x^2]/Sqrt[1 + x^2], x]

[Out] (Sqrt[1 + x^2]\*Sqrt[1 + 2\*x + x^2])/(1 + x) + (Sqrt[1 + 2\*x + x^2]\*ArcSinh[x])/(1 + x)

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] :> Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 641

Int[((d\_) + (e\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Simp[(e\*(a + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[d, Int[(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]

#### Rule 970

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_)\*((d\_) + (f\_.)\*(x\_)^2)^(q\_.), x\_Symbol] :> Dist[(a + b\*x + c\*x^2)^FracPart[p]/((4\*c)^IntPart[p]\*(b + 2\*c\*x)^(2\*FracPart[p])), Int[(b + 2\*c\*x)^(2\*p)\*(d + f\*x^2)^q, x], x] /; FreeQ[{a, b, c, d, f, p, q}, x] && EqQ[b^2 - 4\*a\*c, 0] && !IntegerQ[p]

#### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+2x+x^2}}{\sqrt{1+x^2}} dx &= \frac{\sqrt{1+2x+x^2} \int \frac{2+2x}{\sqrt{1+x^2}} dx}{2+2x} \\
&= \frac{\sqrt{1+x^2} \sqrt{1+2x+x^2}}{1+x} + \frac{\left(2\sqrt{1+2x+x^2}\right) \int \frac{1}{\sqrt{1+x^2}} dx}{2+2x} \\
&= \frac{\sqrt{1+x^2} \sqrt{1+2x+x^2}}{1+x} + \frac{\sqrt{1+2x+x^2} \sinh^{-1}(x)}{1+x}
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 27, normalized size = 0.56

$$\frac{\sqrt{(x+1)^2} \left( \sqrt{x^2+1} + \sinh^{-1}(x) \right)}{x+1}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 + 2\*x + x^2]/Sqrt[1 + x^2], x]

[Out] (Sqrt[(1 + x)^2]\*(Sqrt[1 + x^2] + ArcSinh[x]))/(1 + x)

**fricas [A]** time = 0.86, size = 22, normalized size = 0.46

$$\sqrt{x^2+1} - \log\left(-x + \sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1+x)^2)^(1/2)/(x^2+1)^(1/2), x, algorithm="fricas")

[Out] sqrt(x^2 + 1) - log(-x + sqrt(x^2 + 1))

**giac [A]** time = 0.23, size = 49, normalized size = 1.02

$$-\left(\sqrt{2} - \log\left(\sqrt{2} + 1\right)\right) \operatorname{sgn}(x+1) - \log\left(-x + \sqrt{x^2+1}\right) \operatorname{sgn}(x+1) + \sqrt{x^2+1} \operatorname{sgn}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1+x)^2)^(1/2)/(x^2+1)^(1/2), x, algorithm="giac")

[Out] -(sqrt(2) - log(sqrt(2) + 1))\*sgn(x + 1) - log(-x + sqrt(x^2 + 1))\*sgn(x + 1) + sqrt(x^2 + 1)\*sgn(x + 1)

**maple** [C] time = 0.05, size = 16, normalized size = 0.33

$$\left(\operatorname{arcsinh}(x) + \sqrt{x^2 + 1}\right) \operatorname{csgn}(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((1+x)^2)^(1/2)/(x^2+1)^(1/2),x)

[Out] csgn(1+x)\*(arcsinh(x)+(x^2+1)^(1/2))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x+1)^2}}{\sqrt{x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1+x)^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt((x+1)^2)/sqrt(x^2+1),x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{(x+1)^2}}{\sqrt{x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x+1)^2)^(1/2)/(x^2+1)^(1/2),x)

[Out] int(((x+1)^2)^(1/2)/(x^2+1)^(1/2),x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x+1)^2}}{\sqrt{x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1+x)\*\*2)\*\*(1/2)/(x\*\*2+1)\*\*(1/2),x)

[Out] Integral(sqrt((x+1)\*\*2)/sqrt(x\*\*2+1),x)

$$3.12 \quad \int \frac{1}{(-1+x^2)^2 \sqrt{-1+x+x^2}} dx$$

Optimal. Leaf size=70

$$\frac{\sqrt{x^2+x-1}}{2(1-x^2)} - \frac{1}{8} \tan^{-1}\left(\frac{x+3}{2\sqrt{x^2+x-1}}\right) - \frac{5}{8} \tanh^{-1}\left(\frac{1-3x}{2\sqrt{x^2+x-1}}\right)$$

[Out]  $-1/8*\arctan(1/2*(3+x)/(x^2+x-1)^{(1/2)})-5/8*\operatorname{arctanh}(1/2*(1-3*x)/(x^2+x-1)^{(1/2}))+1/2*(x^2+x-1)^{(1/2)/(-x^2+1)$

**Rubi [A]** time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {976, 1033, 724, 206, 204}

$$\frac{\sqrt{x^2+x-1}}{2(1-x^2)} - \frac{1}{8} \tan^{-1}\left(\frac{x+3}{2\sqrt{x^2+x-1}}\right) - \frac{5}{8} \tanh^{-1}\left(\frac{1-3x}{2\sqrt{x^2+x-1}}\right)$$

Antiderivative was successfully verified.

[In] `Int[1/((-1 + x^2)^2*Sqrt[-1 + x + x^2]),x]`

[Out] `Sqrt[-1 + x + x^2]/(2*(1 - x^2)) - ArcTan[(3 + x)/(2*Sqrt[-1 + x + x^2])]/8 - (5*ArcTanh[(1 - 3*x)/(2*Sqrt[-1 + x + x^2])])/8`

#### Rule 204

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])`

#### Rule 206

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

#### Rule 724

`Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]`

Rule 976

```

Int[((a_.) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x
_Symbol] := Simp[(((2*a*c^2*e + c*(2*c^2*d - c*(2*a*f))*x)*(a + c*x^2)^(p +
1)*(d + e*x + f*x^2)^(q + 1))/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)),
x] - Dist[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^(
p + 1)*(d + e*x + f*x^2)^q*Simp[2*c*((c*d - a*f)^2 - (-(a*e))*(c*e))*(p +
1) - (2*c^2*d - c*(2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(-2*a*c^2*e)*(p
+ q + 2) + (2*f*(2*a*c^2*e)*(p + q + 2) - (2*c^2*d - c*(2*a*f))*(-(c*e*(2*p
+ q + 4)))]*x + c*f*(2*c^2*d - c*(2*a*f))*(2*p + 2*q + 5)*x^2, x], x]
/; FreeQ[{a, c, d, e, f, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ
[a*c*e^2 + (c*d - a*f)^2, 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[
q, 0]

```

Rule 1033

```

Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(-1+x^2)^2 \sqrt{-1+x+x^2}} dx &= \frac{\sqrt{-1+x+x^2}}{2(1-x^2)} - \frac{1}{4} \int \frac{3+2x}{(-1+x^2)\sqrt{-1+x+x^2}} dx \\
&= \frac{\sqrt{-1+x+x^2}}{2(1-x^2)} + \frac{1}{8} \int \frac{1}{(1+x)\sqrt{-1+x+x^2}} dx - \frac{5}{8} \int \frac{1}{(-1+x)\sqrt{-1+x+x^2}} dx \\
&= \frac{\sqrt{-1+x+x^2}}{2(1-x^2)} - \frac{1}{4} \text{Subst} \left( \int \frac{1}{-4-x^2} dx, x, \frac{-3-x}{\sqrt{-1+x+x^2}} \right) + \frac{5}{4} \text{Subst} \left( \int \frac{1}{4-x^2} dx, x, \frac{-3-x}{\sqrt{-1+x+x^2}} \right) \\
&= \frac{\sqrt{-1+x+x^2}}{2(1-x^2)} - \frac{1}{8} \tan^{-1} \left( \frac{3+x}{2\sqrt{-1+x+x^2}} \right) - \frac{5}{8} \tanh^{-1} \left( \frac{1-3x}{2\sqrt{-1+x+x^2}} \right)
\end{aligned}$$

**Mathematica [A]** time = 0.09, size = 66, normalized size = 0.94

$$\frac{1}{8} \left( -\frac{4\sqrt{x^2+x-1}}{x^2-1} - \tan^{-1} \left( \frac{x+3}{2\sqrt{x^2+x-1}} \right) - 5 \tanh^{-1} \left( \frac{1-3x}{2\sqrt{x^2+x-1}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/((-1 + x^2)^2\*Sqrt[-1 + x + x^2]),x]

[Out] ((-4\*Sqrt[-1 + x + x^2])/(-1 + x^2) - ArcTan[(3 + x)/(2\*Sqrt[-1 + x + x^2])]  
] - 5\*ArcTanh[(1 - 3\*x)/(2\*Sqrt[-1 + x + x^2])])/8

**fricas** [A] time = 0.91, size = 82, normalized size = 1.17

$$\frac{2(x^2 - 1) \arctan(-x + \sqrt{x^2 + x - 1} - 1) + 5(x^2 - 1) \log(-x + \sqrt{x^2 + x - 1} + 2) - 5(x^2 - 1) \log(-x + \sqrt{x^2 + x - 1})}{8(x^2 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2-1)^2/(x^2+x-1)^(1/2),x, algorithm="fricas")

[Out] 1/8\*(2\*(x^2 - 1)\*arctan(-x + sqrt(x^2 + x - 1) - 1) + 5\*(x^2 - 1)\*log(-x +  
sqrt(x^2 + x - 1) + 2) - 5\*(x^2 - 1)\*log(-x + sqrt(x^2 + x - 1)) - 4\*sqrt(x  
^2 + x - 1))/(x^2 - 1)

**giac** [B] time = 0.26, size = 143, normalized size = 2.04

$$\frac{2(x - \sqrt{x^2 + x - 1})^3 + 3(x - \sqrt{x^2 + x - 1})^2 - x + \sqrt{x^2 + x - 1} - 1}{2\left(\left(x - \sqrt{x^2 + x - 1}\right)^4 - 2\left(x - \sqrt{x^2 + x - 1}\right)^2 - 4x + 4\sqrt{x^2 + x - 1}\right)} + \frac{1}{4} \arctan(-x + \sqrt{x^2 + x - 1} - 1) + \frac{5}{8} \log$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2-1)^2/(x^2+x-1)^(1/2),x, algorithm="giac")

[Out] 1/2\*(2\*(x - sqrt(x^2 + x - 1))^3 + 3\*(x - sqrt(x^2 + x - 1))^2 - x + sqrt(x  
^2 + x - 1) - 1)/((x - sqrt(x^2 + x - 1))^4 - 2\*(x - sqrt(x^2 + x - 1))^2 -  
4\*x + 4\*sqrt(x^2 + x - 1)) + 1/4\*arctan(-x + sqrt(x^2 + x - 1) - 1) + 5/8\*  
log(abs(-x + sqrt(x^2 + x - 1) + 2)) - 5/8\*log(abs(-x + sqrt(x^2 + x - 1)))

**maple** [A] time = 0.02, size = 84, normalized size = 1.20

$$\frac{5 \operatorname{arctanh}\left(\frac{3x-1}{2\sqrt{3x+(x-1)^2-2}}\right)}{8} + \frac{\operatorname{arctan}\left(\frac{-x-3}{2\sqrt{-x+(x+1)^2-2}}\right)}{8} + \frac{\sqrt{-x+(x+1)^2-2}}{4x+4} - \frac{\sqrt{3x+(x-1)^2-2}}{4(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2-1)^2/(x^2+x-1)^(1/2),x)

[Out] 1/4/(x+1)\*((x+1)^2-2-x)^(1/2)+1/8\*arctan(1/2\*(-3-x)/((x+1)^2-2-x)^(1/2))+5/  
8\*arctanh(1/2\*(-1+3\*x)/((x-1)^2+3\*x-2)^(1/2))-1/4/(x-1)\*((x-1)^2+3\*x-2)^(1/  
2)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2 + x - 1} (x^2 - 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2-1)^2/(x^2+x-1)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(x^2 + x - 1)\*(x^2 - 1)^2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(x^2 - 1)^2 \sqrt{x^2 + x - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((x^2 - 1)^2\*(x + x^2 - 1)^(1/2)),x)

[Out] int(1/((x^2 - 1)^2\*(x + x^2 - 1)^(1/2)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x - 1)^2 (x + 1)^2 \sqrt{x^2 + x - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x\*\*2-1)\*\*2/(x\*\*2+x-1)\*\*(1/2),x)

[Out] Integral(1/((x - 1)\*\*2\*(x + 1)\*\*2\*sqrt(x\*\*2 + x - 1)), x)





1.00, number of steps used = 3, number of rules used = 3, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}}$   
 = 0.115, Rules used = {993, 936, 1103}

$$\sqrt[4]{db^2 + \sqrt{b^2 - 4ac} db - 2a(cd - af)} \left( b + 2cx + \sqrt{b^2 - 4ac} \right)^{3/2} \sqrt{2a + \left( b + \sqrt{b^2 - 4ac} \right) x} \sqrt{\frac{\left( 4ac - \left( b + \sqrt{b^2 - 4ac} \right) x \right)^2}{\left( 4fa^2 + \left( b + \sqrt{b^2 - 4ac} \right) x \right)^2 d}}$$


---


$$\left( 4ac - \left( b + \sqrt{b^2 - 4ac} \right) x \right)$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + b\*x + c\*x^2]\*Sqrt[d + f\*x^2]),x]

[Out] -(((b^2\*d + b\*Sqrt[b^2 - 4\*a\*c]\*d - 2\*a\*(c\*d - a\*f))^(1/4)\*(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)^(3/2)\*Sqrt[2\*a + (b + Sqrt[b^2 - 4\*a\*c])\*x]\*Sqrt[((4\*a\*c - (b + Sqrt[b^2 - 4\*a\*c])^2)\*(d + f\*x^2))/(((b + Sqrt[b^2 - 4\*a\*c])^2\*d + 4\*a^2\*f)\*(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)^2)]\*(1 + (Sqrt[2\*c^2\*d - 2\*a\*c\*f + b\*(b + Sqrt[b^2 - 4\*a\*c])\*f]\*(2\*a + (b + Sqrt[b^2 - 4\*a\*c])\*x)))/(Sqrt[b^2\*d + b\*Sqrt[b^2 - 4\*a\*c]\*d - 2\*a\*(c\*d - a\*f)]\*(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)))\*Sqrt[(1 - (4\*(b + Sqrt[b^2 - 4\*a\*c])\*(c\*d + a\*f)\*(2\*a + (b + Sqrt[b^2 - 4\*a\*c])\*x)))/(((b + Sqrt[b^2 - 4\*a\*c])^2\*d + 4\*a^2\*f)\*(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)) + ((4\*c^2\*d + (b + Sqrt[b^2 - 4\*a\*c])^2\*f)\*(2\*a + (b + Sqrt[b^2 - 4\*a\*c])\*x)^2)/(((b + Sqrt[b^2 - 4\*a\*c])^2\*d + 4\*a^2\*f)\*(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)^2)))/(1 + (Sqrt[2\*c^2\*d - 2\*a\*c\*f + b\*(b + Sqrt[b^2 - 4\*a\*c])\*f]\*(2\*a + (b + Sqrt[b^2 - 4\*a\*c])\*x))/(Sqrt[b^2\*d + b\*Sqrt[b^2 - 4\*a\*c]\*d - 2\*a\*(c\*d - a\*f)]\*(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)))^2]\*EllipticF[2\*ArcTan[(2\*c^2\*d - 2\*a\*c\*f + b\*(b + Sqrt[b^2 - 4\*a\*c])\*f)^(1/4)\*Sqrt[2\*a + (b + Sqrt[b^2 - 4\*a\*c])\*x])/((b^2\*d + b\*Sqrt[b^2 - 4\*a\*c]\*d - 2\*a\*(c\*d - a\*f))^(1/4)\*Sqrt[b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x])], (1 + ((b + Sqrt[b^2 - 4\*a\*c])\*(c\*d + a\*f))/(Sqrt[2\*c^2\*d - 2\*a\*c\*f + b\*(b + Sqrt[b^2 - 4\*a\*c])\*f]\*Sqrt[b^2\*d + b\*Sqrt[b^2 - 4\*a\*c]\*d - 2\*a\*(c\*d - a\*f)]))/2)/((4\*a\*c - (b + Sqrt[b^2 - 4\*a\*c])^2)\*(2\*c^2\*d - 2\*a\*c\*f + b\*(b + Sqrt[b^2 - 4\*a\*c])\*f)^(1/4)\*Sqrt[a + b\*x + c\*x^2]\*Sqrt[d + f\*x^2]\*Sqrt[1 - (4\*(b + Sqrt[b^2 - 4\*a\*c])\*(c\*d + a\*f)\*(2\*a + (b + Sqrt[b^2 - 4\*a\*c])\*x)))/(((b + Sqrt[b^2 - 4\*a\*c])^2\*d + 4\*a^2\*f)\*(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)) + ((4\*c^2\*d + (b + Sqrt[b^2 - 4\*a\*c])^2\*f)\*(2\*a + (b + Sqrt[b^2 - 4\*a\*c])\*x)^2)/(((b + Sqrt[b^2 - 4\*a\*c])^2\*d + 4\*a^2\*f)\*(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)^2)))]

### Rule 936

Int[1/(Sqrt[(d\_.) + (e\_.)\*(x\_.)]\*Sqrt[(f\_.) + (g\_.)\*(x\_.)]\*Sqrt[(a\_.) + (c\_.)\*(x\_.)^2]), x\_Symbol] := Dist[(-2\*(d + e\*x)\*Sqrt[((e\*f - d\*g)^2\*(a + c\*x^2))/

```
((c*f^2 + a*g^2)*(d + e*x)^2)]/((e*f - d*g)*Sqrt[a + c*x^2]), Subst[Int[1/
Sqrt[1 - ((2*c*d*f + 2*a*e*g)*x^2)/(c*f^2 + a*g^2) + ((c*d^2 + a*e^2)*x^4)/
(c*f^2 + a*g^2)], x], x, Sqrt[f + g*x]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0]
```

### Rule 993

```
Int[1/(Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]*Sqrt[(d_) + (f_.)*(x_)^2]), x
_Symbol] := With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[b + r + 2*c*x]*Sqrt[2
*a + (b + r)*x])/Sqrt[a + b*x + c*x^2], Int[1/(Sqrt[b + r + 2*c*x]*Sqrt[2*a
+ (b + r)*x]*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, f}, x] && NeQ
[b^2 - 4*a*c, 0]
```

### Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)]/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

### Rubi steps

$$\int \frac{1}{\sqrt{a+bx+cx^2}\sqrt{d+fx^2}} dx = \frac{\left(\sqrt{b+\sqrt{b^2-4ac}}+2cx\sqrt{2a+(b+\sqrt{b^2-4ac})x}\right) \int \frac{1}{\sqrt{b+\sqrt{b^2-4ac}+2cx}\sqrt{2a+(b+\sqrt{b^2-4ac})x}}}{\sqrt{a+bx+cx^2}}$$

$$= \frac{\left(2(b+\sqrt{b^2-4ac}+2cx)^{3/2}\sqrt{2a+(b+\sqrt{b^2-4ac})x}\sqrt{\frac{(4ac-(b+\sqrt{b^2-4ac})^2)}{((b+\sqrt{b^2-4ac})^2d+4a^2f)}}\right)}{(4ac-(b+\sqrt{b^2-4ac})^2)}$$

$$= \frac{\sqrt[4]{b^2d+b\sqrt{b^2-4ac}d-2a(cd-af)}(b+\sqrt{b^2-4ac}+2cx)^{3/2}\sqrt{2a+(b+\sqrt{b^2-4ac})x}}{\sqrt{a+bx+cx^2}\sqrt{d+fx^2}}$$

**Mathematica [C]** time = 1.64, size = 600, normalized size = 0.56

$$\frac{2\sqrt{2}(\sqrt{f}x-i\sqrt{d})(\sqrt{b^2-4ac}-b-2cx)\sqrt{-\frac{c\sqrt{b^2-4ac}(\sqrt{f}x+i\sqrt{d})}{(\sqrt{b^2-4ac}-b-2cx)(\sqrt{f}(\sqrt{b^2-4ac}+b)-2ic\sqrt{d})}}}{\sqrt{d+fx^2}\sqrt{a+x(b+cx)}(\sqrt{f}(\sqrt{b^2-4ac}-b)-2ic\sqrt{d})}\sqrt{\frac{c(-i\sqrt{d}(\sqrt{b^2-4ac}+2cx)+\sqrt{f}x)}{(\sqrt{b^2-4ac}-b-2cx)(\sqrt{f}(\sqrt{b^2-4ac}+b)-2ic\sqrt{d})}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[a + b\*x + c\*x^2]\*Sqrt[d + f\*x^2]), x]

[Out] (-2\*Sqrt[2]\*(-b + Sqrt[b^2 - 4\*a\*c] - 2\*c\*x)\*((-I)\*Sqrt[d] + Sqrt[f]\*x)\*Sqrt[-((c\*Sqrt[b^2 - 4\*a\*c]\*(I\*Sqrt[d] + Sqrt[f]\*x))/((-2\*I)\*c\*Sqrt[d] + (b + Sqrt[b^2 - 4\*a\*c])\*Sqrt[f]))\*(-b + Sqrt[b^2 - 4\*a\*c] - 2\*c\*x)])/Sqrt[(c\*((-I)\*Sqrt[d]\*(Sqrt[b^2 - 4\*a\*c] + 2\*c\*x) + Sqrt[f]\*(-2\*a + Sqrt[b^2 - 4\*a\*c]\*x) + b\*((-I)\*Sqrt[d] - Sqrt[f]\*x)))/((2\*I)\*c\*Sqrt[d] + (b + Sqrt[b^2 - 4\*a\*c])\*Sqrt[f])]

```
a*c])*Sqrt[f))*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)))*EllipticF[ArcSin[Sqrt[(((
-2*I)*c*Sqrt[d] + (-b + Sqrt[b^2 - 4*a*c])*Sqrt[f]))*(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x)))/(((2*I)*c*Sqrt[d] + (b + Sqrt[b^2 - 4*a*c])*Sqrt[f]))*(-b + Sqrt[b
^2 - 4*a*c] - 2*c*x))]], (c*d - I*Sqrt[b^2 - 4*a*c]*Sqrt[d]*Sqrt[f] + a*f)/
(c*d + I*Sqrt[b^2 - 4*a*c]*Sqrt[d]*Sqrt[f] + a*f)))/(((2*I)*c*Sqrt[d] + (-
b + Sqrt[b^2 - 4*a*c])*Sqrt[f])*Sqrt[(I*c*Sqrt[b^2 - 4*a*c]*(Sqrt[d] + I*Sq
rt[f]*x)))/(((2*I)*c*Sqrt[d] + (b + Sqrt[b^2 - 4*a*c])*Sqrt[f]))*(-b + Sqrt[b
^2 - 4*a*c] - 2*c*x)))]*Sqrt[d + f*x^2]*Sqrt[a + x*(b + c*x))]
```

**fricas** [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{\sqrt{cx^2 + bx + a} \sqrt{fx^2 + d}}{cfx^4 + bfx^3 + bdx + (cd + af)x^2 + ad}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*sqrt(f*x^2 + d)/(c*f*x^4 + b*f*x^3 + b*d*x +
(c*d + a*f)*x^2 + a*d), x)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + bx + a} \sqrt{fx^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(f*x^2 + d)), x)
```

**maple** [A] time = 0.25, size = 661, normalized size = 0.61

$$4 \left( b f^2 x^2 - 4 c d f x + 2 \sqrt{-d f} c f x^2 + \sqrt{-4 a c + b^2} f^2 x^2 - b d f + 2 \sqrt{-d f} b f x - 2 \sqrt{-d f} c d - \sqrt{-4 a c + b^2} d f + 2 \sqrt{-d f} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+d)^(1/2),x)
```

```
[Out] 4*(b*f^2*x^2+2*x^2*c*f*(-d*f)^(1/2)+x^2*f^2*(-4*a*c+b^2)^(1/2)+2*x*b*f*(-d*
f)^(1/2)-4*c*x*f*d+2*x*f*(-4*a*c+b^2)^(1/2)*(-d*f)^(1/2)-b*d*f-2*c*d*(-d*f)
```



$$3.14 \quad \int \frac{\sqrt{-3-4x-x^2}}{3+4x+2x^2} dx$$

Optimal. Leaf size=98

$$-\frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{1}{2} \tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right) - \frac{1}{2} \sin^{-1}(x+2)$$

[Out]  $-1/2*\arcsin(2+x)-1/2*\operatorname{arctanh}(x/(-x^2-4*x-3)^{(1/2)})-1/2*\operatorname{arctan}(1/2*(1+(-3-x)/(-x^2-4*x-3)^{(1/2)})*2^{(1/2)})*2^{(1/2)}+1/2*\operatorname{arctan}(1/2*(1+(3+x)/(-x^2-4*x-3)^{(1/2)})*2^{(1/2)})*2^{(1/2)}$

**Rubi [A]** time = 0.21, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$ , Rules used = {989, 619, 216, 1028, 986, 12, 1026, 1161, 618, 204, 1027, 206}

$$-\frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{1}{2} \tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right) - \frac{1}{2} \sin^{-1}(x+2)$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[-3 - 4*x - x^2]/(3 + 4*x + 2*x^2), x]`

[Out]  $-\operatorname{ArcSin}[2 + x]/2 - \operatorname{ArcTan}[(1 - (3 + x)/\operatorname{Sqrt}[-3 - 4*x - x^2])/\operatorname{Sqrt}[2]]/\operatorname{Sqrt}[2] + \operatorname{ArcTan}[(1 + (3 + x)/\operatorname{Sqrt}[-3 - 4*x - x^2])/\operatorname{Sqrt}[2]]/\operatorname{Sqrt}[2] - \operatorname{ArcTanh}[x/\operatorname{Sqrt}[-3 - 4*x - x^2]]/2$

### Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]`

### Rule 204

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])`

### Rule 206

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt`

$Q[a, 0] \parallel LtQ[b, 0]$

### Rule 216

$\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_)^2], x\_Symbol] \rightarrow \text{Simp}[\text{ArcSin}[(\text{Rt}[-b, 2]*x)/\text{Sqrt}[a]]/\text{Rt}[-b, 2], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{GtQ}[a, 0] \ \&\& \ \text{NegQ}[b]$

### Rule 618

$\text{Int}[(a_) + (b_)*(x_) + (c_)*(x_)^2]^{-1}, x\_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

### Rule 619

$\text{Int}[(a_) + (b_)*(x_) + (c_)*(x_)^2]^{(p_)}, x\_Symbol] \rightarrow \text{Dist}[1/(2*c*((-4*c)/(b^2 - 4*a*c))^{p}), \text{Subst}[\text{Int}[\text{Simp}[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; \text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{GtQ}[4*a - b^2/c, 0]$

### Rule 986

$\text{Int}[1/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*\text{Sqrt}[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)], 2\}, \text{Dist}[1/(2*q), \text{Int}[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)*\text{Sqrt}[d + e*x + f*x^2]), x], x] - \text{Dist}[1/(2*q), \text{Int}[(c*d - a*f - q + (c*e - b*f)*x)/((a + b*x + c*x^2)*\text{Sqrt}[d + e*x + f*x^2]), x], x]] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[e^2 - 4*d*f, 0] \ \&\& \ \text{NeQ}[c*e - b*f, 0] \ \&\& \ \text{NegQ}[b^2 - 4*a*c]$

### Rule 989

$\text{Int}[\text{Sqrt}[(a_) + (b_)*(x_) + (c_)*(x_)^2]/((d_) + (e_)*(x_) + (f_)*(x_)^2), x\_Symbol] \rightarrow \text{Dist}[c/f, \text{Int}[1/\text{Sqrt}[a + b*x + c*x^2], x], x] - \text{Dist}[1/f, \text{Int}[(c*d - a*f + (c*e - b*f)*x)/(\text{Sqrt}[a + b*x + c*x^2]*(d + e*x + f*x^2)), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[e^2 - 4*d*f, 0]$

### Rule 1026

$\text{Int}[(x_)/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*\text{Sqrt}[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x\_Symbol] \rightarrow \text{Dist}[-2*e, \text{Subst}[\text{Int}[(1 - d*x^2)/(c*e - b*f - e*(2*c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4], x], x, (1 + ((e + \text{Sqrt}[e^2 - 4*d*f])*x)/(2*d))/\text{Sqrt}[d + e*x + f*x^2]], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[e^2 - 4*d*f, 0] \ \&\& \ \text{EqQ}[b*d - a*e, 0]$

Rule 1027

```
Int[((g_) + (h_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[g, Subst[Int[1/(a + (c*d - a*f)*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] && EqQ[2*h*d - g*e, 0]
```

Rule 1028

```
Int[((g_) + (h_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := -Dist[(2*h*d - g*e)/e, Int[1/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/e, Int[(2*d + e*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] && NeQ[2*h*d - g*e, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2], 0]))
```

Rubi steps



$$\begin{aligned}
\int \frac{\sqrt{-3-4x-x^2}}{3+4x+2x^2} dx &= -\left(\frac{1}{2} \int \frac{1}{\sqrt{-3-4x-x^2}} dx\right) - \frac{1}{2} \int \frac{3+4x}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx \\
&= \frac{1}{4} \text{Subst} \left( \int \frac{1}{\sqrt{1-\frac{x^2}{4}}} dx, x, -4-2x \right) + \frac{1}{2} \int \frac{-6-4x}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx + \frac{3}{2} \int \frac{1}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx \\
&= -\frac{1}{2} \sin^{-1}(2+x) - \frac{1}{4} \int \frac{-6-4x}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx + \frac{1}{4} \int \frac{4x}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx \\
&= -\frac{1}{2} \sin^{-1}(2+x) - \tanh^{-1} \left( \frac{x}{\sqrt{-3-4x-x^2}} \right) + \frac{3}{2} \text{Subst} \left( \int \frac{1}{3-3x^2} dx, x, \frac{x}{\sqrt{-3-4x-x^2}} \right) \\
&= -\frac{1}{2} \sin^{-1}(2+x) - \frac{1}{2} \tanh^{-1} \left( \frac{x}{\sqrt{-3-4x-x^2}} \right) - 8 \text{Subst} \left( \int \frac{1+3x^2}{-4-8x^2-36x^4} dx, x, \frac{x}{\sqrt{-3-4x-x^2}} \right) \\
&= -\frac{1}{2} \sin^{-1}(2+x) - \frac{1}{2} \tanh^{-1} \left( \frac{x}{\sqrt{-3-4x-x^2}} \right) + \frac{1}{3} \text{Subst} \left( \int \frac{1}{\frac{1}{3} - \frac{2x}{3} + x^2} dx, x, \frac{x}{\sqrt{-3-4x-x^2}} \right) \\
&= -\frac{1}{2} \sin^{-1}(2+x) - \frac{1}{2} \tanh^{-1} \left( \frac{x}{\sqrt{-3-4x-x^2}} \right) - \frac{2}{3} \text{Subst} \left( \int \frac{1}{-\frac{8}{9} - x^2} dx, x, \frac{2}{3} \left( -1 + \frac{x}{\sqrt{-3-4x-x^2}} \right) \right) \\
&= -\frac{1}{2} \sin^{-1}(2+x) - \frac{\tan^{-1} \left( \frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}} \right)}{\sqrt{2}} + \frac{\tan^{-1} \left( \frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}} \right)}{\sqrt{2}} - \frac{1}{2} \tanh^{-1} \left( \frac{x}{\sqrt{-3-4x-x^2}} \right)
\end{aligned}$$

**Mathematica [C]** time = 0.39, size = 159, normalized size = 1.62

$$\frac{1}{4} \left( -i\sqrt{1-2i\sqrt{2}} \tanh^{-1} \left( \frac{i\sqrt{2}x+2x+2i\sqrt{2}+2}{\sqrt{2-4i\sqrt{2}} \sqrt{-x^2-4x-3}} \right) + i\sqrt{1+2i\sqrt{2}} \tanh^{-1} \left( \frac{(2-i\sqrt{2})x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}} \sqrt{-x^2-4x-3}} \right) \right) - 2$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-3 - 4\*x - x^2]/(3 + 4\*x + 2\*x^2), x]

[Out] (-2\*ArcSin[2 + x] - I\*Sqrt[1 - (2\*I)\*Sqrt[2]]\*ArcTanh[(2 + (2\*I)\*Sqrt[2] + 2\*x + I\*Sqrt[2]\*x)/(Sqrt[2 - (4\*I)\*Sqrt[2]]\*Sqrt[-3 - 4\*x - x^2]]) + I\*Sqrt[1 + (2\*I)\*Sqrt[2]]\*ArcTanh[(2 - (2\*I)\*Sqrt[2] + (2 - I\*Sqrt[2])\*x)/(Sqrt[2 + (4\*I)\*Sqrt[2]]\*Sqrt[-3 - 4\*x - x^2]])/4

**fricas [A]** time = 0.96, size = 161, normalized size = 1.64

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}x+3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right)-\frac{1}{4}\sqrt{2}\arctan\left(-\frac{\sqrt{2}x-3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right)+\frac{1}{2}\arctan\left(\frac{\sqrt{-x^2-4x-3}}{x+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2-4\*x-3)^(1/2)/(2\*x^2+4\*x+3),x, algorithm="fricas")

[Out] -1/4\*sqrt(2)\*arctan(1/2\*(sqrt(2)\*x + 3\*sqrt(2)\*sqrt(-x^2 - 4\*x - 3))/(2\*x + 3)) - 1/4\*sqrt(2)\*arctan(-1/2\*(sqrt(2)\*x - 3\*sqrt(2)\*sqrt(-x^2 - 4\*x - 3))/(2\*x + 3)) + 1/2\*arctan(sqrt(-x^2 - 4\*x - 3)\*(x + 2)/(x^2 + 4\*x + 3)) + 1/8\*log(-(2\*sqrt(-x^2 - 4\*x - 3)\*x + 4\*x + 3)/x^2) - 1/8\*log((2\*sqrt(-x^2 - 4\*x - 3)\*x - 4\*x - 3)/x^2)

**giac [B]** time = 0.21, size = 171, normalized size = 1.74

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3(\sqrt{-x^2-4x-3}-1)}{x+2}+1\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right)-\frac{1}{2}\arcsin\left(\frac{\sqrt{-x^2-4x-3}}{x+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2-4\*x-3)^(1/2)/(2\*x^2+4\*x+3),x, algorithm="giac")

[Out] -1/2\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(3\*(sqrt(-x^2 - 4\*x - 3) - 1)/(x + 2) + 1)) - 1/2\*sqrt(2)\*arctan(1/2\*sqrt(2)\*((sqrt(-x^2 - 4\*x - 3) - 1)/(x + 2) + 1)) - 1/2\*arcsin(x + 2) - 1/4\*log(2\*(sqrt(-x^2 - 4\*x - 3) - 1)/(x + 2) + 3\*(sqrt(-x^2 - 4\*x - 3) - 1)^2/(x + 2)^2 + 1) + 1/4\*log(2\*(sqrt(-x^2 - 4\*x - 3) - 1)/(x + 2) + (sqrt(-x^2 - 4\*x - 3) - 1)^2/(x + 2)^2 + 3)

**maple [B]** time = 0.03, size = 341, normalized size = 3.48

$$\frac{\arcsin(x+2)}{2} + \frac{\sqrt{3}\sqrt{4}\sqrt{\frac{3x^2}{(-x-\frac{3}{2})^2}-12}\sqrt{2}\arctan\left(\frac{\sqrt{\frac{3x^2}{(-x-\frac{3}{2})^2}-12}\sqrt{2}}{6}\right) + \sqrt{3}\sqrt{4}\sqrt{\frac{3x^2}{(-x-\frac{3}{2})^2}-12}\left(-\operatorname{arctanh}\left(\frac{\sqrt{\frac{3x^2}{(-x-\frac{3}{2})^2}-12}}{(-x-\frac{3}{2})}\right)\right)}{3\sqrt{\frac{\frac{x^2}{(-x-\frac{3}{2})^2}-4}{\left(\frac{x}{-x-\frac{3}{2}}+1\right)^2}}\left(\frac{x}{-x-\frac{3}{2}}+1\right)} + \frac{12\sqrt{\frac{\frac{x^2}{(-x-\frac{3}{2})^2}-4}{\left(\frac{x}{-x-\frac{3}{2}}+1\right)^2}}}{\sqrt{\frac{\frac{x^2}{(-x-\frac{3}{2})^2}-4}{\left(\frac{x}{-x-\frac{3}{2}}+1\right)^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2-4\*x-3)^(1/2)/(2\*x^2+4\*x+3),x)

```
[Out] -1/2*arcsin(2+x)+1/12*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/2)*
arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))-arctanh(3*x/(-3/2-x)/(3*x^2
/(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+x/(-
3/2-x))-1/3*3^(1/2)*4^(1/2)/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+
x/(-3/2-x))*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2)*arctan(1/6*(3*x^2/(-3/2-x)^
2-12)^(1/2)*2^(1/2))+1/6*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/
2)*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))+arctanh(3*x/(-3/2-x)/(3*
x^2/(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+x
/(-3/2-x))
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^2 - 4x - 3}}{2x^2 + 4x + 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-4*x-3)^(1/2)/(2*x^2+4*x+3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 - 4*x - 3)/(2*x^2 + 4*x + 3), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{-x^2 - 4x - 3}}{2x^2 + 4x + 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- 4*x - x^2 - 3)^(1/2)/(4*x + 2*x^2 + 3),x)
```

```
[Out] int((- 4*x - x^2 - 3)^(1/2)/(4*x + 2*x^2 + 3), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(x+1)(x+3)}}{2x^2 + 4x + 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2-4*x-3)**(1/2)/(2*x**2+4*x+3),x)
```

```
[Out] Integral(sqrt(-(x + 1)*(x + 3))/(2*x**2 + 4*x + 3), x)
```

$$3.15 \quad \int (3 - x + 2x^2)(2 + 3x + 5x^2)^4 dx$$

**Optimal.** Leaf size=68

$$\frac{1250x^{11}}{11} + \frac{475x^{10}}{2} + \frac{5075x^9}{9} + \frac{3415x^8}{4} + 1176x^7 + \frac{2377x^6}{2} + \frac{5099x^5}{5} + 656x^4 + \frac{1064x^3}{3} + 136x^2 + 48x$$

[Out] 48\*x+136\*x^2+1064/3\*x^3+656\*x^4+5099/5\*x^5+2377/2\*x^6+1176\*x^7+3415/4\*x^8+5075/9\*x^9+475/2\*x^10+1250/11\*x^11

**Rubi [A]** time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$ , Rules used = {1657}

$$\frac{1250x^{11}}{11} + \frac{475x^{10}}{2} + \frac{5075x^9}{9} + \frac{3415x^8}{4} + 1176x^7 + \frac{2377x^6}{2} + \frac{5099x^5}{5} + 656x^4 + \frac{1064x^3}{3} + 136x^2 + 48x$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)\*(2 + 3\*x + 5\*x^2)^4,x]

[Out] 48\*x + 136\*x^2 + (1064\*x^3)/3 + 656\*x^4 + (5099\*x^5)/5 + (2377\*x^6)/2 + 1176\*x^7 + (3415\*x^8)/4 + (5075\*x^9)/9 + (475\*x^10)/2 + (1250\*x^11)/11

Rule 1657

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[Expand Integrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (3 - x + 2x^2)(2 + 3x + 5x^2)^4 dx &= \int (48 + 272x + 1064x^2 + 2624x^3 + 5099x^4 + 7131x^5 + 8232x^6 + 6830x^7 \\ &\quad + 48x + 136x^2 + \frac{1064x^3}{3} + 656x^4 + \frac{5099x^5}{5} + \frac{2377x^6}{2} + 1176x^7 + \frac{3415x^8}{4} \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 68, normalized size = 1.00

$$\frac{1250x^{11}}{11} + \frac{475x^{10}}{2} + \frac{5075x^9}{9} + \frac{3415x^8}{4} + 1176x^7 + \frac{2377x^6}{2} + \frac{5099x^5}{5} + 656x^4 + \frac{1064x^3}{3} + 136x^2 + 48x$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)\*(2 + 3\*x + 5\*x^2)^4,x]

[Out]  $48*x + 136*x^2 + (1064*x^3)/3 + 656*x^4 + (5099*x^5)/5 + (2377*x^6)/2 + 1176*x^7 + (3415*x^8)/4 + (5075*x^9)/9 + (475*x^{10})/2 + (1250*x^{11})/11$

**fricas** [A] time = 0.72, size = 54, normalized size = 0.79

$$\frac{1250}{11}x^{11} + \frac{475}{2}x^{10} + \frac{5075}{9}x^9 + \frac{3415}{4}x^8 + 1176x^7 + \frac{2377}{2}x^6 + \frac{5099}{5}x^5 + 656x^4 + \frac{1064}{3}x^3 + 136x^2 + 48x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)\*(5\*x^2+3\*x+2)^4,x, algorithm="fricas")

[Out]  $1250/11*x^{11} + 475/2*x^{10} + 5075/9*x^9 + 3415/4*x^8 + 1176*x^7 + 2377/2*x^6 + 5099/5*x^5 + 656*x^4 + 1064/3*x^3 + 136*x^2 + 48*x$

**giac** [A] time = 0.21, size = 54, normalized size = 0.79

$$\frac{1250}{11}x^{11} + \frac{475}{2}x^{10} + \frac{5075}{9}x^9 + \frac{3415}{4}x^8 + 1176x^7 + \frac{2377}{2}x^6 + \frac{5099}{5}x^5 + 656x^4 + \frac{1064}{3}x^3 + 136x^2 + 48x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)\*(5\*x^2+3\*x+2)^4,x, algorithm="giac")

[Out]  $1250/11*x^{11} + 475/2*x^{10} + 5075/9*x^9 + 3415/4*x^8 + 1176*x^7 + 2377/2*x^6 + 5099/5*x^5 + 656*x^4 + 1064/3*x^3 + 136*x^2 + 48*x$

**maple** [A] time = 0.00, size = 55, normalized size = 0.81

$$\frac{1250}{11}x^{11} + \frac{475}{2}x^{10} + \frac{5075}{9}x^9 + \frac{3415}{4}x^8 + 1176x^7 + \frac{2377}{2}x^6 + \frac{5099}{5}x^5 + 656x^4 + \frac{1064}{3}x^3 + 136x^2 + 48x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)\*(5\*x^2+3\*x+2)^4,x)

[Out]  $48*x + 136*x^2 + 1064/3*x^3 + 656*x^4 + 5099/5*x^5 + 2377/2*x^6 + 1176*x^7 + 3415/4*x^8 + 5075/9*x^9 + 475/2*x^{10} + 1250/11*x^{11}$

**maxima** [A] time = 0.46, size = 54, normalized size = 0.79

$$\frac{1250}{11}x^{11} + \frac{475}{2}x^{10} + \frac{5075}{9}x^9 + \frac{3415}{4}x^8 + 1176x^7 + \frac{2377}{2}x^6 + \frac{5099}{5}x^5 + 656x^4 + \frac{1064}{3}x^3 + 136x^2 + 48x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)\*(5\*x^2+3\*x+2)^4,x, algorithm="maxima")

[Out]  $1250/11*x^{11} + 475/2*x^{10} + 5075/9*x^9 + 3415/4*x^8 + 1176*x^7 + 2377/2*x^6 + 5099/5*x^5 + 656*x^4 + 1064/3*x^3 + 136*x^2 + 48*x$

**mupad [B]** time = 0.06, size = 54, normalized size = 0.79

$$\frac{1250x^{11}}{11} + \frac{475x^{10}}{2} + \frac{5075x^9}{9} + \frac{3415x^8}{4} + 1176x^7 + \frac{2377x^6}{2} + \frac{5099x^5}{5} + 656x^4 + \frac{1064x^3}{3} + 136x^2 + 48x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2 - x + 3)*(3*x + 5*x^2 + 2)^4, x)`

[Out]  $48*x + 136*x^2 + (1064*x^3)/3 + 656*x^4 + (5099*x^5)/5 + (2377*x^6)/2 + 1176*x^7 + (3415*x^8)/4 + (5075*x^9)/9 + (475*x^{10})/2 + (1250*x^{11})/11$

**sympy [A]** time = 0.09, size = 65, normalized size = 0.96

$$\frac{1250x^{11}}{11} + \frac{475x^{10}}{2} + \frac{5075x^9}{9} + \frac{3415x^8}{4} + 1176x^7 + \frac{2377x^6}{2} + \frac{5099x^5}{5} + 656x^4 + \frac{1064x^3}{3} + 136x^2 + 48x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**2-x+3)*(5*x**2+3*x+2)**4, x)`

[Out]  $1250*x^{11}/11 + 475*x^{10}/2 + 5075*x^9/9 + 3415*x^8/4 + 1176*x^7 + 2377*x^6/2 + 5099*x^5/5 + 656*x^4 + 1064*x^3/3 + 136*x^2 + 48*x$

$$3.16 \quad \int (3 - x + 2x^2) (2 + 3x + 5x^2)^3 dx$$

Optimal. Leaf size=56

$$\frac{250x^9}{9} + \frac{325x^8}{8} + \frac{720x^7}{7} + 134x^6 + \frac{876x^5}{5} + \frac{579x^4}{4} + \frac{322x^3}{3} + 50x^2 + 24x$$

[Out] 24\*x+50\*x^2+322/3\*x^3+579/4\*x^4+876/5\*x^5+134\*x^6+720/7\*x^7+325/8\*x^8+250/9\*x^9

Rubi [A] time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$ , Rules used = {1657}

$$\frac{250x^9}{9} + \frac{325x^8}{8} + \frac{720x^7}{7} + 134x^6 + \frac{876x^5}{5} + \frac{579x^4}{4} + \frac{322x^3}{3} + 50x^2 + 24x$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)\*(2 + 3\*x + 5\*x^2)^3,x]

[Out] 24\*x + 50\*x^2 + (322\*x^3)/3 + (579\*x^4)/4 + (876\*x^5)/5 + 134\*x^6 + (720\*x^7)/7 + (325\*x^8)/8 + (250\*x^9)/9

Rule 1657

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[Expand Integrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (3 - x + 2x^2) (2 + 3x + 5x^2)^3 dx &= \int (24 + 100x + 322x^2 + 579x^3 + 876x^4 + 804x^5 + 720x^6 + 325x^7 + 250x^8 + 24x^9) dx \\ &= 24x + 50x^2 + \frac{322x^3}{3} + \frac{579x^4}{4} + \frac{876x^5}{5} + 134x^6 + \frac{720x^7}{7} + \frac{325x^8}{8} + \frac{250x^9}{9} \end{aligned}$$

Mathematica [A] time = 0.00, size = 56, normalized size = 1.00

$$\frac{250x^9}{9} + \frac{325x^8}{8} + \frac{720x^7}{7} + 134x^6 + \frac{876x^5}{5} + \frac{579x^4}{4} + \frac{322x^3}{3} + 50x^2 + 24x$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)\*(2 + 3\*x + 5\*x^2)^3,x]

[Out] 24\*x + 50\*x^2 + (322\*x^3)/3 + (579\*x^4)/4 + (876\*x^5)/5 + 134\*x^6 + (720\*x^7)/7 + (325\*x^8)/8 + (250\*x^9)/9

**fricas** [A] time = 0.72, size = 44, normalized size = 0.79

$$\frac{250}{9}x^9 + \frac{325}{8}x^8 + \frac{720}{7}x^7 + 134x^6 + \frac{876}{5}x^5 + \frac{579}{4}x^4 + \frac{322}{3}x^3 + 50x^2 + 24x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)\*(5\*x^2+3\*x+2)^3,x, algorithm="fricas")

[Out] 250/9\*x^9 + 325/8\*x^8 + 720/7\*x^7 + 134\*x^6 + 876/5\*x^5 + 579/4\*x^4 + 322/3\*x^3 + 50\*x^2 + 24\*x

**giac** [A] time = 0.18, size = 44, normalized size = 0.79

$$\frac{250}{9}x^9 + \frac{325}{8}x^8 + \frac{720}{7}x^7 + 134x^6 + \frac{876}{5}x^5 + \frac{579}{4}x^4 + \frac{322}{3}x^3 + 50x^2 + 24x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)\*(5\*x^2+3\*x+2)^3,x, algorithm="giac")

[Out] 250/9\*x^9 + 325/8\*x^8 + 720/7\*x^7 + 134\*x^6 + 876/5\*x^5 + 579/4\*x^4 + 322/3\*x^3 + 50\*x^2 + 24\*x

**maple** [A] time = 0.00, size = 45, normalized size = 0.80

$$\frac{250}{9}x^9 + \frac{325}{8}x^8 + \frac{720}{7}x^7 + 134x^6 + \frac{876}{5}x^5 + \frac{579}{4}x^4 + \frac{322}{3}x^3 + 50x^2 + 24x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)\*(5\*x^2+3\*x+2)^3,x)

[Out] 24\*x+50\*x^2+322/3\*x^3+579/4\*x^4+876/5\*x^5+134\*x^6+720/7\*x^7+325/8\*x^8+250/9\*x^9

**maxima** [A] time = 0.45, size = 44, normalized size = 0.79

$$\frac{250}{9}x^9 + \frac{325}{8}x^8 + \frac{720}{7}x^7 + 134x^6 + \frac{876}{5}x^5 + \frac{579}{4}x^4 + \frac{322}{3}x^3 + 50x^2 + 24x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)\*(5\*x^2+3\*x+2)^3,x, algorithm="maxima")



[Out]  $250/9*x^9 + 325/8*x^8 + 720/7*x^7 + 134*x^6 + 876/5*x^5 + 579/4*x^4 + 322/3*x^3 + 50*x^2 + 24*x$

**mupad [B]** time = 0.03, size = 44, normalized size = 0.79

$$\frac{250x^9}{9} + \frac{325x^8}{8} + \frac{720x^7}{7} + 134x^6 + \frac{876x^5}{5} + \frac{579x^4}{4} + \frac{322x^3}{3} + 50x^2 + 24x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2 - x + 3)*(3*x + 5*x^2 + 2)^3,x)`

[Out]  $24*x + 50*x^2 + (322*x^3)/3 + (579*x^4)/4 + (876*x^5)/5 + 134*x^6 + (720*x^7)/7 + (325*x^8)/8 + (250*x^9)/9$

**sympy [A]** time = 0.08, size = 53, normalized size = 0.95

$$\frac{250x^9}{9} + \frac{325x^8}{8} + \frac{720x^7}{7} + 134x^6 + \frac{876x^5}{5} + \frac{579x^4}{4} + \frac{322x^3}{3} + 50x^2 + 24x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**2-x+3)*(5*x**2+3*x+2)**3,x)`

[Out]  $250*x**9/9 + 325*x**8/8 + 720*x**7/7 + 134*x**6 + 876*x**5/5 + 579*x**4/4 + 322*x**3/3 + 50*x**2 + 24*x$

$$3.17 \quad \int (3 - x + 2x^2) (2 + 3x + 5x^2)^2 dx$$

Optimal. Leaf size=44

$$\frac{50x^7}{7} + \frac{35x^6}{6} + \frac{103x^5}{5} + \frac{85x^4}{4} + \frac{83x^3}{3} + 16x^2 + 12x$$

[Out] 12\*x+16\*x^2+83/3\*x^3+85/4\*x^4+103/5\*x^5+35/6\*x^6+50/7\*x^7

Rubi [A] time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$ , Rules used = {1657}

$$\frac{50x^7}{7} + \frac{35x^6}{6} + \frac{103x^5}{5} + \frac{85x^4}{4} + \frac{83x^3}{3} + 16x^2 + 12x$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)\*(2 + 3\*x + 5\*x^2)^2,x]

[Out] 12\*x + 16\*x^2 + (83\*x^3)/3 + (85\*x^4)/4 + (103\*x^5)/5 + (35\*x^6)/6 + (50\*x^7)/7

Rule 1657

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[Expand Integrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (3 - x + 2x^2) (2 + 3x + 5x^2)^2 dx &= \int (12 + 32x + 83x^2 + 85x^3 + 103x^4 + 35x^5 + 50x^6) dx \\ &= 12x + 16x^2 + \frac{83x^3}{3} + \frac{85x^4}{4} + \frac{103x^5}{5} + \frac{35x^6}{6} + \frac{50x^7}{7} \end{aligned}$$

Mathematica [A] time = 0.00, size = 44, normalized size = 1.00

$$\frac{50x^7}{7} + \frac{35x^6}{6} + \frac{103x^5}{5} + \frac{85x^4}{4} + \frac{83x^3}{3} + 16x^2 + 12x$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)\*(2 + 3\*x + 5\*x^2)^2,x]

[Out] 12\*x + 16\*x^2 + (83\*x^3)/3 + (85\*x^4)/4 + (103\*x^5)/5 + (35\*x^6)/6 + (50\*x^7)/7

**fricas** [A] time = 0.69, size = 34, normalized size = 0.77

$$\frac{50}{7}x^7 + \frac{35}{6}x^6 + \frac{103}{5}x^5 + \frac{85}{4}x^4 + \frac{83}{3}x^3 + 16x^2 + 12x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)\*(5\*x^2+3\*x+2)^2,x, algorithm="fricas")

[Out] 50/7\*x^7 + 35/6\*x^6 + 103/5\*x^5 + 85/4\*x^4 + 83/3\*x^3 + 16\*x^2 + 12\*x

**giac** [A] time = 0.21, size = 34, normalized size = 0.77

$$\frac{50}{7}x^7 + \frac{35}{6}x^6 + \frac{103}{5}x^5 + \frac{85}{4}x^4 + \frac{83}{3}x^3 + 16x^2 + 12x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)\*(5\*x^2+3\*x+2)^2,x, algorithm="giac")

[Out] 50/7\*x^7 + 35/6\*x^6 + 103/5\*x^5 + 85/4\*x^4 + 83/3\*x^3 + 16\*x^2 + 12\*x

**maple** [A] time = 0.00, size = 35, normalized size = 0.80

$$\frac{50}{7}x^7 + \frac{35}{6}x^6 + \frac{103}{5}x^5 + \frac{85}{4}x^4 + \frac{83}{3}x^3 + 16x^2 + 12x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)\*(5\*x^2+3\*x+2)^2,x)

[Out] 12\*x+16\*x^2+83/3\*x^3+85/4\*x^4+103/5\*x^5+35/6\*x^6+50/7\*x^7

**maxima** [A] time = 0.43, size = 34, normalized size = 0.77

$$\frac{50}{7}x^7 + \frac{35}{6}x^6 + \frac{103}{5}x^5 + \frac{85}{4}x^4 + \frac{83}{3}x^3 + 16x^2 + 12x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)\*(5\*x^2+3\*x+2)^2,x, algorithm="maxima")

[Out] 50/7\*x^7 + 35/6\*x^6 + 103/5\*x^5 + 85/4\*x^4 + 83/3\*x^3 + 16\*x^2 + 12\*x

**mupad** [B] time = 0.03, size = 34, normalized size = 0.77

$$\frac{50x^7}{7} + \frac{35x^6}{6} + \frac{103x^5}{5} + \frac{85x^4}{4} + \frac{83x^3}{3} + 16x^2 + 12x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2 - x + 3)*(3*x + 5*x^2 + 2)^2,x)`

[Out] `12*x + 16*x^2 + (83*x^3)/3 + (85*x^4)/4 + (103*x^5)/5 + (35*x^6)/6 + (50*x^7)/7`

**sympy** [A] time = 0.08, size = 41, normalized size = 0.93

$$\frac{50x^7}{7} + \frac{35x^6}{6} + \frac{103x^5}{5} + \frac{85x^4}{4} + \frac{83x^3}{3} + 16x^2 + 12x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**2-x+3)*(5*x**2+3*x+2)**2,x)`

[Out] `50*x**7/7 + 35*x**6/6 + 103*x**5/5 + 85*x**4/4 + 83*x**3/3 + 16*x**2 + 12*x`

$$3.18 \quad \int (3 - x + 2x^2)(2 + 3x + 5x^2) dx$$

Optimal. Leaf size=30

$$2x^5 + \frac{x^4}{4} + \frac{16x^3}{3} + \frac{7x^2}{2} + 6x$$

[Out] 6\*x+7/2\*x^2+16/3\*x^3+1/4\*x^4+2\*x^5

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {1657}

$$2x^5 + \frac{x^4}{4} + \frac{16x^3}{3} + \frac{7x^2}{2} + 6x$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)\*(2 + 3\*x + 5\*x^2), x]

[Out] 6\*x + (7\*x^2)/2 + (16\*x^3)/3 + x^4/4 + 2\*x^5

Rule 1657

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[Expand Integrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (3 - x + 2x^2)(2 + 3x + 5x^2) dx &= \int (6 + 7x + 16x^2 + x^3 + 10x^4) dx \\ &= 6x + \frac{7x^2}{2} + \frac{16x^3}{3} + \frac{x^4}{4} + 2x^5 \end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$2x^5 + \frac{x^4}{4} + \frac{16x^3}{3} + \frac{7x^2}{2} + 6x$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)\*(2 + 3\*x + 5\*x^2), x]

[Out] 6\*x + (7\*x^2)/2 + (16\*x^3)/3 + x^4/4 + 2\*x^5

**fricas** [A] time = 0.69, size = 24, normalized size = 0.80

$$2x^5 + \frac{1}{4}x^4 + \frac{16}{3}x^3 + \frac{7}{2}x^2 + 6x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)\*(5\*x^2+3\*x+2),x, algorithm="fricas")

[Out] 2\*x^5 + 1/4\*x^4 + 16/3\*x^3 + 7/2\*x^2 + 6\*x

**giac** [A] time = 0.18, size = 24, normalized size = 0.80

$$2x^5 + \frac{1}{4}x^4 + \frac{16}{3}x^3 + \frac{7}{2}x^2 + 6x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)\*(5\*x^2+3\*x+2),x, algorithm="giac")

[Out] 2\*x^5 + 1/4\*x^4 + 16/3\*x^3 + 7/2\*x^2 + 6\*x

**maple** [A] time = 0.00, size = 25, normalized size = 0.83

$$2x^5 + \frac{1}{4}x^4 + \frac{16}{3}x^3 + \frac{7}{2}x^2 + 6x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)\*(5\*x^2+3\*x+2),x)

[Out] 6\*x+7/2\*x^2+16/3\*x^3+1/4\*x^4+2\*x^5

**maxima** [A] time = 0.43, size = 24, normalized size = 0.80

$$2x^5 + \frac{1}{4}x^4 + \frac{16}{3}x^3 + \frac{7}{2}x^2 + 6x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)\*(5\*x^2+3\*x+2),x, algorithm="maxima")

[Out] 2\*x^5 + 1/4\*x^4 + 16/3\*x^3 + 7/2\*x^2 + 6\*x

**mupad** [B] time = 0.02, size = 24, normalized size = 0.80

$$2x^5 + \frac{x^4}{4} + \frac{16x^3}{3} + \frac{7x^2}{2} + 6x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2 - x + 3)*(3*x + 5*x^2 + 2), x)`

[Out] `6*x + (7*x^2)/2 + (16*x^3)/3 + x^4/4 + 2*x^5`

sympy [A] `time = 0.06, size = 26, normalized size = 0.87`

$$2x^5 + \frac{x^4}{4} + \frac{16x^3}{3} + \frac{7x^2}{2} + 6x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**2-x+3)*(5*x**2+3*x+2), x)`

[Out] `2*x**5 + x**4/4 + 16*x**3/3 + 7*x**2/2 + 6*x`

$$3.19 \quad \int \frac{3-x+2x^2}{2+3x+5x^2} dx$$

Optimal. Leaf size=42

$$-\frac{11}{50} \log(5x^2 + 3x + 2) + \frac{2x}{5} + \frac{143 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{25\sqrt{31}}$$

[Out] 2/5\*x-11/50\*ln(5\*x^2+3\*x+2)+143/775\*arctan(1/31\*(3+10\*x)\*31^(1/2))\*31^(1/2)

**Rubi [A]** time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$ , Rules used = {1657, 634, 618, 204, 628}

$$-\frac{11}{50} \log(5x^2 + 3x + 2) + \frac{2x}{5} + \frac{143 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{25\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)/(2 + 3\*x + 5\*x^2), x]

[Out] (2\*x)/5 + (143\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(25\*Sqrt[31]) - (11\*Log[2 + 3\*x + 5\*x^2])/50

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634



```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

### Rubi steps

$$\begin{aligned}
 \int \frac{3-x+2x^2}{2+3x+5x^2} dx &= \int \left( \frac{2}{5} + \frac{11(1-x)}{5(2+3x+5x^2)} \right) dx \\
 &= \frac{2x}{5} + \frac{11}{5} \int \frac{1-x}{2+3x+5x^2} dx \\
 &= \frac{2x}{5} - \frac{11}{50} \int \frac{3+10x}{2+3x+5x^2} dx + \frac{143}{50} \int \frac{1}{2+3x+5x^2} dx \\
 &= \frac{2x}{5} - \frac{11}{50} \log(2+3x+5x^2) - \frac{143}{25} \operatorname{Subst} \left( \int \frac{1}{-31-x^2} dx, x, 3+10x \right) \\
 &= \frac{2x}{5} + \frac{143 \tan^{-1} \left( \frac{3+10x}{\sqrt{31}} \right)}{25\sqrt{31}} - \frac{11}{50} \log(2+3x+5x^2)
 \end{aligned}$$

**Mathematica** [A] time = 0.02, size = 42, normalized size = 1.00

$$-\frac{11}{50} \log(5x^2 + 3x + 2) + \frac{2x}{5} + \frac{143 \tan^{-1} \left( \frac{10x+3}{\sqrt{31}} \right)}{25\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2), x]
```

```
[Out] (2*x)/5 + (143*ArcTan[(3 + 10*x)/Sqrt[31]])/(25*Sqrt[31]) - (11*Log[2 + 3*x + 5*x^2])/50
```

**fricas** [A] time = 0.65, size = 33, normalized size = 0.79

$$\frac{143}{775} \sqrt{31} \arctan \left( \frac{1}{31} \sqrt{31} (10x + 3) \right) + \frac{2}{5} x - \frac{11}{50} \log(5x^2 + 3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)/(5\*x^2+3\*x+2),x, algorithm="fricas")

[Out] 143/775\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 2/5\*x - 11/50\*log(5\*x^2 + 3\*x + 2)

**giac** [A] time = 0.18, size = 33, normalized size = 0.79

$$\frac{143}{775} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x + 3)\right) + \frac{2}{5} x - \frac{11}{50} \log(5x^2 + 3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)/(5\*x^2+3\*x+2),x, algorithm="giac")

[Out] 143/775\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 2/5\*x - 11/50\*log(5\*x^2 + 3\*x + 2)

**maple** [A] time = 0.00, size = 34, normalized size = 0.81

$$\frac{2x}{5} + \frac{143\sqrt{31} \arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{775} - \frac{11 \ln(5x^2 + 3x + 2)}{50}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)/(5\*x^2+3\*x+2),x)

[Out] 2/5\*x-11/50\*ln(5\*x^2+3\*x+2)+143/775\*arctan(1/31\*(3+10\*x)\*31^(1/2))\*31^(1/2)

**maxima** [A] time = 0.97, size = 33, normalized size = 0.79

$$\frac{143}{775} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x + 3)\right) + \frac{2}{5} x - \frac{11}{50} \log(5x^2 + 3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)/(5\*x^2+3\*x+2),x, algorithm="maxima")

[Out] 143/775\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 2/5\*x - 11/50\*log(5\*x^2 + 3\*x + 2)

**mupad** [B] time = 3.39, size = 35, normalized size = 0.83

$$\frac{2x}{5} - \frac{11 \ln(5x^2 + 3x + 2)}{50} + \frac{143 \sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{775}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2 - x + 3)/(3*x + 5*x^2 + 2), x)`

[Out]  $(2*x)/5 - (11*\log(3*x + 5*x^2 + 2))/50 + (143*31^{(1/2)}*\operatorname{atan}((10*31^{(1/2)}*x)/31 + (3*31^{(1/2)})/31))/775$

**sympy** [A] time = 0.14, size = 49, normalized size = 1.17

$$\frac{2x}{5} - \frac{11 \log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{50} + \frac{143\sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{775}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**2-x+3)/(5*x**2+3*x+2), x)`

[Out]  $2*x/5 - 11*\log(x**2 + 3*x/5 + 2/5)/50 + 143*\operatorname{sqrt}(31)*\operatorname{atan}(10*\operatorname{sqrt}(31)*x/31 + 3*\operatorname{sqrt}(31)/31)/775$

$$3.20 \quad \int \frac{3-x+2x^2}{(2+3x+5x^2)^2} dx$$

Optimal. Leaf size=43

$$\frac{11(13x+7)}{155(5x^2+3x+2)} + \frac{82 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{31\sqrt{31}}$$

[Out] 11/155\*(7+13\*x)/(5\*x^2+3\*x+2)+82/961\*arctan(1/31\*(3+10\*x)\*31^(1/2))\*31^(1/2)

**Rubi [A]** time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$ , Rules used = {1660, 12, 618, 204}

$$\frac{11(13x+7)}{155(5x^2+3x+2)} + \frac{82 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{31\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)/(2 + 3\*x + 5\*x^2)^2,x]

[Out] (11\*(7 + 13\*x))/(155\*(2 + 3\*x + 5\*x^2)) + (82\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(31\*Sqrt[31])

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]
```

### Rubi steps

$$\begin{aligned} \int \frac{3 - x + 2x^2}{(2 + 3x + 5x^2)^2} dx &= \frac{11(7 + 13x)}{155(2 + 3x + 5x^2)} + \frac{1}{31} \int \frac{41}{2 + 3x + 5x^2} dx \\ &= \frac{11(7 + 13x)}{155(2 + 3x + 5x^2)} + \frac{41}{31} \int \frac{1}{2 + 3x + 5x^2} dx \\ &= \frac{11(7 + 13x)}{155(2 + 3x + 5x^2)} - \frac{82}{31} \text{Subst} \left( \int \frac{1}{-31 - x^2} dx, x, 3 + 10x \right) \\ &= \frac{11(7 + 13x)}{155(2 + 3x + 5x^2)} + \frac{82 \tan^{-1} \left( \frac{3 + 10x}{\sqrt{31}} \right)}{31\sqrt{31}} \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 43, normalized size = 1.00

$$\frac{11(13x + 7)}{155(5x^2 + 3x + 2)} + \frac{82 \tan^{-1} \left( \frac{10x + 3}{\sqrt{31}} \right)}{31\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)/(2 + 3\*x + 5\*x^2)^2, x]

[Out] (11\*(7 + 13\*x))/(155\*(2 + 3\*x + 5\*x^2)) + (82\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(31\*Sqrt[31])

**fricas [A]** time = 0.73, size = 45, normalized size = 1.05

$$\frac{410 \sqrt{31} (5x^2 + 3x + 2) \arctan \left( \frac{1}{31} \sqrt{31} (10x + 3) \right) + 4433x + 2387}{4805(5x^2 + 3x + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)/(5\*x^2+3\*x+2)^2,x, algorithm="fricas")

[Out] 1/4805\*(410\*sqrt(31)\*(5\*x^2 + 3\*x + 2)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 4433\*x + 2387)/(5\*x^2 + 3\*x + 2)

**giac** [A] time = 0.21, size = 36, normalized size = 0.84

$$\frac{82}{961} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x + 3)\right) + \frac{11(13x + 7)}{155(5x^2 + 3x + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)/(5\*x^2+3\*x+2)^2,x, algorithm="giac")

[Out] 82/961\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 11/155\*(13\*x + 7)/(5\*x^2 + 3\*x + 2)

**maple** [A] time = 0.00, size = 34, normalized size = 0.79

$$\frac{82\sqrt{31} \arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{961} + \frac{\frac{143x}{775} + \frac{77}{775}}{x^2 + \frac{3}{5}x + \frac{2}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)/(5\*x^2+3\*x+2)^2,x)

[Out] (143/775\*x+77/775)/(x^2+3/5\*x+2/5)+82/961\*31^(1/2)\*arctan(1/31\*(10\*x+3)\*31^(1/2))

**maxima** [A] time = 0.97, size = 36, normalized size = 0.84

$$\frac{82}{961} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x + 3)\right) + \frac{11(13x + 7)}{155(5x^2 + 3x + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)/(5\*x^2+3\*x+2)^2,x, algorithm="maxima")

[Out] 82/961\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 11/155\*(13\*x + 7)/(5\*x^2 + 3\*x + 2)

**mupad** [B] time = 0.04, size = 35, normalized size = 0.81

$$\frac{\frac{143x}{775} + \frac{77}{775}}{x^2 + \frac{3x}{5} + \frac{2}{5}} + \frac{82 \sqrt{31} \operatorname{atan}\left(\frac{10 \sqrt{31} x}{31} + \frac{3 \sqrt{31}}{31}\right)}{961}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2 - x + 3)/(3*x + 5*x^2 + 2)^2,x)`

[Out]  $((143*x)/775 + 77/775)/((3*x)/5 + x^2 + 2/5) + (82*31^{(1/2)}*atan((10*31^{(1/2)}*x)/31 + (3*31^{(1/2)})/31))/961$

**sympy** [A] time = 0.16, size = 42, normalized size = 0.98

$$\frac{143x + 77}{775x^2 + 465x + 310} + \frac{82\sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{961}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**2-x+3)/(5*x**2+3*x+2)**2,x)`

[Out]  $(143*x + 77)/(775*x**2 + 465*x + 310) + 82*\operatorname{sqrt}(31)*\operatorname{atan}(10*\operatorname{sqrt}(31)*x/31 + 3*\operatorname{sqrt}(31)/31)/961$

$$3.21 \quad \int \frac{3-x+2x^2}{(2+3x+5x^2)^3} dx$$

Optimal. Leaf size=64

$$\frac{553(10x+3)}{9610(5x^2+3x+2)} + \frac{11(13x+7)}{310(5x^2+3x+2)^2} + \frac{1106 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{961\sqrt{31}}$$

[Out] 11/310\*(7+13\*x)/(5\*x^2+3\*x+2)^2+553/9610\*(3+10\*x)/(5\*x^2+3\*x+2)+1106/29791\*arctan(1/31\*(3+10\*x)\*31^(1/2))\*31^(1/2)

**Rubi [A]** time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$ , Rules used = {1660, 12, 614, 618, 204}

$$\frac{553(10x+3)}{9610(5x^2+3x+2)} + \frac{11(13x+7)}{310(5x^2+3x+2)^2} + \frac{1106 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{961\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)/(2 + 3\*x + 5\*x^2)^3,x]

[Out] (11\*(7 + 13\*x))/(310\*(2 + 3\*x + 5\*x^2)^2) + (553\*(3 + 10\*x))/(9610\*(2 + 3\*x + 5\*x^2)) + (1106\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(961\*Sqrt[31])

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 614

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] - Dist[(2\*c\*(2\*p + 3))/((p + 1)\*(b^2 - 4\*a\*c)), Int[(a + b\*x + c\*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int



egerQ[4\*p]

### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 1660

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 1]}, Simp[((b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] + Dist[1/((p + 1)\*(b^2 - 4\*a\*c)), Int[(a + b\*x + c\*x^2)^(p + 1)\*ExpandToSum[(p + 1)\*(b^2 - 4\*a\*c)\*Q - (2\*p + 3)\*(2\*c\*f - b\*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && LtQ[p, -1]

### Rubi steps

$$\begin{aligned}
 \int \frac{3-x+2x^2}{(2+3x+5x^2)^3} dx &= \frac{11(7+13x)}{310(2+3x+5x^2)^2} + \frac{1}{62} \int \frac{553}{5(2+3x+5x^2)^2} dx \\
 &= \frac{11(7+13x)}{310(2+3x+5x^2)^2} + \frac{553}{310} \int \frac{1}{(2+3x+5x^2)^2} dx \\
 &= \frac{11(7+13x)}{310(2+3x+5x^2)^2} + \frac{553(3+10x)}{9610(2+3x+5x^2)} + \frac{553}{961} \int \frac{1}{2+3x+5x^2} dx \\
 &= \frac{11(7+13x)}{310(2+3x+5x^2)^2} + \frac{553(3+10x)}{9610(2+3x+5x^2)} - \frac{1106}{961} \text{Subst} \left( \int \frac{1}{-31-x^2} dx, x, 3+10x \right) \\
 &= \frac{11(7+13x)}{310(2+3x+5x^2)^2} + \frac{553(3+10x)}{9610(2+3x+5x^2)} + \frac{1106 \tan^{-1} \left( \frac{3+10x}{\sqrt{31}} \right)}{961\sqrt{31}}
 \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 53, normalized size = 0.83

$$\frac{31(5530x^3+4977x^2+4094x+1141)}{(5x^2+3x+2)^2} + 2212\sqrt{31} \tan^{-1} \left( \frac{10x+3}{\sqrt{31}} \right)$$


---

59582

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)/(2 + 3\*x + 5\*x^2)^3, x]

[Out] ((31\*(1141 + 4094\*x + 4977\*x^2 + 5530\*x^3))/(2 + 3\*x + 5\*x^2)^2 + 2212\*sqrt[31]\*ArcTan[(3 + 10\*x)/sqrt[31]])/59582

**fricas** [A] time = 0.80, size = 75, normalized size = 1.17

$$\frac{171430x^3 + 2212\sqrt{31}(25x^4 + 30x^3 + 29x^2 + 12x + 4)\arctan\left(\frac{1}{31}\sqrt{31}(10x + 3)\right) + 154287x^2 + 126914x + 35371}{59582(25x^4 + 30x^3 + 29x^2 + 12x + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)/(5\*x^2+3\*x+2)^3,x, algorithm="fricas")

[Out] 1/59582\*(171430\*x^3 + 2212\*sqrt(31)\*(25\*x^4 + 30\*x^3 + 29\*x^2 + 12\*x + 4)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 154287\*x^2 + 126914\*x + 35371)/(25\*x^4 + 30\*x^3 + 29\*x^2 + 12\*x + 4)

**giac** [A] time = 0.17, size = 46, normalized size = 0.72

$$\frac{1106}{29791}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x + 3)\right) + \frac{5530x^3 + 4977x^2 + 4094x + 1141}{1922(5x^2 + 3x + 2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)/(5\*x^2+3\*x+2)^3,x, algorithm="giac")

[Out] 1106/29791\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 1/1922\*(5530\*x^3 + 4977\*x^2 + 4094\*x + 1141)/(5\*x^2 + 3\*x + 2)^2

**maple** [A] time = 0.00, size = 47, normalized size = 0.73

$$\frac{1106\sqrt{31}\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{29791} + \frac{\frac{2765}{961}x^3 + \frac{4977}{1922}x^2 + \frac{2047}{961}x + \frac{1141}{1922}}{(5x^2 + 3x + 2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)/(5\*x^2+3\*x+2)^3,x)

[Out] 25\*(553/4805\*x^3+4977/48050\*x^2+2047/24025\*x+1141/48050)/(5\*x^2+3\*x+2)^2+1106/29791\*31^(1/2)\*arctan(1/31\*(10\*x+3)\*31^(1/2))

**maxima [A]** time = 0.96, size = 56, normalized size = 0.88

$$\frac{1106}{29791} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x + 3)\right) + \frac{5530x^3 + 4977x^2 + 4094x + 1141}{1922(25x^4 + 30x^3 + 29x^2 + 12x + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)/(5\*x^2+3\*x+2)^3,x, algorithm="maxima")

[Out] 1106/29791\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 1/1922\*(5530\*x^3 + 4977\*x^2 + 4094\*x + 1141)/(25\*x^4 + 30\*x^3 + 29\*x^2 + 12\*x + 4)

**mupad [B]** time = 0.05, size = 55, normalized size = 0.86

$$\frac{1106 \sqrt{31} \operatorname{atan}\left(\frac{10 \sqrt{31} x}{31} + \frac{3 \sqrt{31}}{31}\right)}{29791} + \frac{\frac{553x^3}{4805} + \frac{4977x^2}{48050} + \frac{2047x}{24025} + \frac{1141}{48050}}{x^4 + \frac{6x^3}{5} + \frac{29x^2}{25} + \frac{12x}{25} + \frac{4}{25}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)/(3\*x + 5\*x^2 + 2)^3,x)

[Out] (1106\*31^(1/2)\*atan((10\*31^(1/2)\*x)/31 + (3\*31^(1/2))/31))/29791 + ((2047\*x)/24025 + (4977\*x^2)/48050 + (553\*x^3)/4805 + 1141/48050)/((12\*x)/25 + (29\*x^2)/25 + (6\*x^3)/5 + x^4 + 4/25)

**sympy [A]** time = 0.18, size = 63, normalized size = 0.98

$$\frac{5530x^3 + 4977x^2 + 4094x + 1141}{48050x^4 + 57660x^3 + 55738x^2 + 23064x + 7688} + \frac{1106\sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{29791}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2-x+3)/(5\*x\*\*2+3\*x+2)\*\*3,x)

[Out] (5530\*x\*\*3 + 4977\*x\*\*2 + 4094\*x + 1141)/(48050\*x\*\*4 + 57660\*x\*\*3 + 55738\*x\*\*2 + 23064\*x + 7688) + 1106\*sqrt(31)\*atan(10\*sqrt(31)\*x/31 + 3\*sqrt(31)/31)/29791

$$3.22 \quad \int (3 - x + 2x^2)^2 (2 + 3x + 5x^2)^4 dx$$

**Optimal.** Leaf size=80

$$\frac{2500x^{13}}{13} + \frac{875x^{12}}{3} + \frac{11525x^{11}}{11} + 1571x^{10} + \frac{24859x^9}{9} + 3315x^8 + \frac{27763x^7}{7} + \frac{10771x^6}{3} + \frac{14801x^5}{5} + 1838x^4 + \frac{3016x^3}{3} + 384x^2 + 144x$$

[Out] 144\*x+384\*x^2+3016/3\*x^3+1838\*x^4+14801/5\*x^5+10771/3\*x^6+27763/7\*x^7+3315\*x^8+24859/9\*x^9+1571\*x^10+11525/11\*x^11+875/3\*x^12+2500/13\*x^13

**Rubi [A]** time = 0.06, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$ , Rules used = {1657}

$$\frac{2500x^{13}}{13} + \frac{875x^{12}}{3} + \frac{11525x^{11}}{11} + 1571x^{10} + \frac{24859x^9}{9} + 3315x^8 + \frac{27763x^7}{7} + \frac{10771x^6}{3} + \frac{14801x^5}{5} + 1838x^4 + \frac{3016x^3}{3} + 384x^2 + 144x$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^2\*(2 + 3\*x + 5\*x^2)^4,x]

[Out] 144\*x + 384\*x^2 + (3016\*x^3)/3 + 1838\*x^4 + (14801\*x^5)/5 + (10771\*x^6)/3 + (27763\*x^7)/7 + 3315\*x^8 + (24859\*x^9)/9 + 1571\*x^10 + (11525\*x^11)/11 + (875\*x^12)/3 + (2500\*x^13)/13

**Rule 1657**

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[Expand Integrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

**Rubi steps**

$$\begin{aligned} \int (3 - x + 2x^2)^2 (2 + 3x + 5x^2)^4 dx &= \int (144 + 768x + 3016x^2 + 7352x^3 + 14801x^4 + 21542x^5 + 27763x^6 + 21542x^7 + 7352x^8 + 3016x^9 + 768x^{10} + 144x^{11}) (2 + 3x + 5x^2)^4 dx \\ &= 144x + 384x^2 + \frac{3016x^3}{3} + 1838x^4 + \frac{14801x^5}{5} + \frac{10771x^6}{3} + \frac{27763x^7}{7} + 3315x^8 + \frac{24859x^9}{9} + 1571x^{10} + \frac{11525x^{11}}{11} + \frac{2500x^{13}}{13} \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 80, normalized size = 1.00

$$\frac{2500x^{13}}{13} + \frac{875x^{12}}{3} + \frac{11525x^{11}}{11} + 1571x^{10} + \frac{24859x^9}{9} + 3315x^8 + \frac{27763x^7}{7} + \frac{10771x^6}{3} + \frac{14801x^5}{5} + 1838x^4 + \frac{3016x^3}{3} + 384x^2 + 144x$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)^2\*(2 + 3\*x + 5\*x^2)^4, x]

[Out] 144\*x + 384\*x^2 + (3016\*x^3)/3 + 1838\*x^4 + (14801\*x^5)/5 + (10771\*x^6)/3 + (27763\*x^7)/7 + 3315\*x^8 + (24859\*x^9)/9 + 1571\*x^10 + (11525\*x^11)/11 + (875\*x^12)/3 + (2500\*x^13)/13

**fricas** [A] time = 0.77, size = 64, normalized size = 0.80

$$\frac{2500}{13}x^{13} + \frac{875}{3}x^{12} + \frac{11525}{11}x^{11} + 1571x^{10} + \frac{24859}{9}x^9 + 3315x^8 + \frac{27763}{7}x^7 + \frac{10771}{3}x^6 + \frac{14801}{5}x^5 + 1838x^4 + \frac{3016}{3}x^3 + 384x^2 + 144x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^2\*(5\*x^2+3\*x+2)^4, x, algorithm="fricas")

[Out] 2500/13\*x^13 + 875/3\*x^12 + 11525/11\*x^11 + 1571\*x^10 + 24859/9\*x^9 + 3315\*x^8 + 27763/7\*x^7 + 10771/3\*x^6 + 14801/5\*x^5 + 1838\*x^4 + 3016/3\*x^3 + 384\*x^2 + 144\*x

**giac** [A] time = 0.20, size = 64, normalized size = 0.80

$$\frac{2500}{13}x^{13} + \frac{875}{3}x^{12} + \frac{11525}{11}x^{11} + 1571x^{10} + \frac{24859}{9}x^9 + 3315x^8 + \frac{27763}{7}x^7 + \frac{10771}{3}x^6 + \frac{14801}{5}x^5 + 1838x^4 + \frac{3016}{3}x^3 + 384x^2 + 144x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^2\*(5\*x^2+3\*x+2)^4, x, algorithm="giac")

[Out] 2500/13\*x^13 + 875/3\*x^12 + 11525/11\*x^11 + 1571\*x^10 + 24859/9\*x^9 + 3315\*x^8 + 27763/7\*x^7 + 10771/3\*x^6 + 14801/5\*x^5 + 1838\*x^4 + 3016/3\*x^3 + 384\*x^2 + 144\*x

**maple** [A] time = 0.00, size = 65, normalized size = 0.81

$$\frac{2500}{13}x^{13} + \frac{875}{3}x^{12} + \frac{11525}{11}x^{11} + 1571x^{10} + \frac{24859}{9}x^9 + 3315x^8 + \frac{27763}{7}x^7 + \frac{10771}{3}x^6 + \frac{14801}{5}x^5 + 1838x^4 + \frac{3016}{3}x^3 + 384x^2 + 144x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^2\*(5\*x^2+3\*x+2)^4, x)

[Out] 144\*x+384\*x^2+3016/3\*x^3+1838\*x^4+14801/5\*x^5+10771/3\*x^6+27763/7\*x^7+3315\*x^8+24859/9\*x^9+1571\*x^10+11525/11\*x^11+875/3\*x^12+2500/13\*x^13

**maxima** [A] time = 0.45, size = 64, normalized size = 0.80

$$\frac{2500}{13}x^{13} + \frac{875}{3}x^{12} + \frac{11525}{11}x^{11} + 1571x^{10} + \frac{24859}{9}x^9 + 3315x^8 + \frac{27763}{7}x^7 + \frac{10771}{3}x^6 + \frac{14801}{5}x^5 + 1838x^4 + \frac{3016}{3}x^3 + 384x^2 + 144x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^2\*(5\*x^2+3\*x+2)^4,x, algorithm="maxima")

[Out] 2500/13\*x^13 + 875/3\*x^12 + 11525/11\*x^11 + 1571\*x^10 + 24859/9\*x^9 + 3315\*x^8 + 27763/7\*x^7 + 10771/3\*x^6 + 14801/5\*x^5 + 1838\*x^4 + 3016/3\*x^3 + 384\*x^2 + 144\*x

**mupad [B]** time = 0.08, size = 64, normalized size = 0.80

$$\frac{2500x^{13}}{13} + \frac{875x^{12}}{3} + \frac{11525x^{11}}{11} + 1571x^{10} + \frac{24859x^9}{9} + 3315x^8 + \frac{27763x^7}{7} + \frac{10771x^6}{3} + \frac{14801x^5}{5} + 1838x^4 + \frac{3016x^3}{3} + 384x^2 + 144x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)^2\*(3\*x + 5\*x^2 + 2)^4,x)

[Out] 144\*x + 384\*x^2 + (3016\*x^3)/3 + 1838\*x^4 + (14801\*x^5)/5 + (10771\*x^6)/3 + (27763\*x^7)/7 + 3315\*x^8 + (24859\*x^9)/9 + 1571\*x^10 + (11525\*x^11)/11 + (875\*x^12)/3 + (2500\*x^13)/13

**sympy [A]** time = 0.10, size = 76, normalized size = 0.95

$$\frac{2500x^{13}}{13} + \frac{875x^{12}}{3} + \frac{11525x^{11}}{11} + 1571x^{10} + \frac{24859x^9}{9} + 3315x^8 + \frac{27763x^7}{7} + \frac{10771x^6}{3} + \frac{14801x^5}{5} + 1838x^4 + \frac{3016x^3}{3} + 384x^2 + 144x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2-x+3)\*\*2\*(5\*x\*\*2+3\*x+2)\*\*4,x)

[Out] 2500\*x\*\*13/13 + 875\*x\*\*12/3 + 11525\*x\*\*11/11 + 1571\*x\*\*10 + 24859\*x\*\*9/9 + 3315\*x\*\*8 + 27763\*x\*\*7/7 + 10771\*x\*\*6/3 + 14801\*x\*\*5/5 + 1838\*x\*\*4 + 3016\*x\*\*3/3 + 384\*x\*\*2 + 144\*x

$$3.23 \quad \int (3 - x + 2x^2)^2 (2 + 3x + 5x^2)^3 dx$$

**Optimal.** Leaf size=66

$$\frac{500x^{11}}{11} + 40x^{10} + \frac{1865x^9}{9} + \frac{1863x^8}{8} + 444x^7 + 449x^6 + \frac{2693x^5}{5} + \frac{1615x^4}{4} + \frac{914x^3}{3} + 138x^2 + 72x$$

[Out] 72\*x+138\*x^2+914/3\*x^3+1615/4\*x^4+2693/5\*x^5+449\*x^6+444\*x^7+1863/8\*x^8+1865/9\*x^9+40\*x^10+500/11\*x^11

**Rubi [A]** time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$ , Rules used = {1657}

$$\frac{500x^{11}}{11} + 40x^{10} + \frac{1865x^9}{9} + \frac{1863x^8}{8} + 444x^7 + 449x^6 + \frac{2693x^5}{5} + \frac{1615x^4}{4} + \frac{914x^3}{3} + 138x^2 + 72x$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^2\*(2 + 3\*x + 5\*x^2)^3,x]

[Out] 72\*x + 138\*x^2 + (914\*x^3)/3 + (1615\*x^4)/4 + (2693\*x^5)/5 + 449\*x^6 + 444\*x^7 + (1863\*x^8)/8 + (1865\*x^9)/9 + 40\*x^10 + (500\*x^11)/11

**Rule 1657**

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[Expand Integrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

**Rubi steps**

$$\begin{aligned} \int (3 - x + 2x^2)^2 (2 + 3x + 5x^2)^3 dx &= \int (72 + 276x + 914x^2 + 1615x^3 + 2693x^4 + 2694x^5 + 3108x^6 + 1863x^7 \\ &\quad + 72x + 138x^2 + \frac{914x^3}{3} + \frac{1615x^4}{4} + \frac{2693x^5}{5} + 449x^6 + 444x^7 + \frac{1863x^8}{8} \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 66, normalized size = 1.00

$$\frac{500x^{11}}{11} + 40x^{10} + \frac{1865x^9}{9} + \frac{1863x^8}{8} + 444x^7 + 449x^6 + \frac{2693x^5}{5} + \frac{1615x^4}{4} + \frac{914x^3}{3} + 138x^2 + 72x$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)^2\*(2 + 3\*x + 5\*x^2)^3,x]

[Out] 72\*x + 138\*x^2 + (914\*x^3)/3 + (1615\*x^4)/4 + (2693\*x^5)/5 + 449\*x^6 + 444\*x^7 + (1863\*x^8)/8 + (1865\*x^9)/9 + 40\*x^10 + (500\*x^11)/11

**fricas** [A] time = 0.51, size = 54, normalized size = 0.82

$$\frac{500}{11}x^{11} + 40x^{10} + \frac{1865}{9}x^9 + \frac{1863}{8}x^8 + 444x^7 + 449x^6 + \frac{2693}{5}x^5 + \frac{1615}{4}x^4 + \frac{914}{3}x^3 + 138x^2 + 72x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^2\*(5\*x^2+3\*x+2)^3,x, algorithm="fricas")

[Out] 500/11\*x^11 + 40\*x^10 + 1865/9\*x^9 + 1863/8\*x^8 + 444\*x^7 + 449\*x^6 + 2693/5\*x^5 + 1615/4\*x^4 + 914/3\*x^3 + 138\*x^2 + 72\*x

**giac** [A] time = 0.21, size = 54, normalized size = 0.82

$$\frac{500}{11}x^{11} + 40x^{10} + \frac{1865}{9}x^9 + \frac{1863}{8}x^8 + 444x^7 + 449x^6 + \frac{2693}{5}x^5 + \frac{1615}{4}x^4 + \frac{914}{3}x^3 + 138x^2 + 72x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^2\*(5\*x^2+3\*x+2)^3,x, algorithm="giac")

[Out] 500/11\*x^11 + 40\*x^10 + 1865/9\*x^9 + 1863/8\*x^8 + 444\*x^7 + 449\*x^6 + 2693/5\*x^5 + 1615/4\*x^4 + 914/3\*x^3 + 138\*x^2 + 72\*x

**maple** [A] time = 0.00, size = 55, normalized size = 0.83

$$\frac{500}{11}x^{11} + 40x^{10} + \frac{1865}{9}x^9 + \frac{1863}{8}x^8 + 444x^7 + 449x^6 + \frac{2693}{5}x^5 + \frac{1615}{4}x^4 + \frac{914}{3}x^3 + 138x^2 + 72x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^2\*(5\*x^2+3\*x+2)^3,x)

[Out] 72\*x+138\*x^2+914/3\*x^3+1615/4\*x^4+2693/5\*x^5+449\*x^6+444\*x^7+1863/8\*x^8+1865/9\*x^9+40\*x^10+500/11\*x^11

**maxima** [A] time = 0.44, size = 54, normalized size = 0.82

$$\frac{500}{11}x^{11} + 40x^{10} + \frac{1865}{9}x^9 + \frac{1863}{8}x^8 + 444x^7 + 449x^6 + \frac{2693}{5}x^5 + \frac{1615}{4}x^4 + \frac{914}{3}x^3 + 138x^2 + 72x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^2\*(5\*x^2+3\*x+2)^3,x, algorithm="maxima")



[Out]  $500/11*x^{11} + 40*x^{10} + 1865/9*x^9 + 1863/8*x^8 + 444*x^7 + 449*x^6 + 2693/5*x^5 + 1615/4*x^4 + 914/3*x^3 + 138*x^2 + 72*x$

**mupad [B]** time = 0.05, size = 54, normalized size = 0.82

$$\frac{500x^{11}}{11} + 40x^{10} + \frac{1865x^9}{9} + \frac{1863x^8}{8} + 444x^7 + 449x^6 + \frac{2693x^5}{5} + \frac{1615x^4}{4} + \frac{914x^3}{3} + 138x^2 + 72x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2 - x + 3)^2*(3*x + 5*x^2 + 2)^3,x)`

[Out]  $72*x + 138*x^2 + (914*x^3)/3 + (1615*x^4)/4 + (2693*x^5)/5 + 449*x^6 + 444*x^7 + (1863*x^8)/8 + (1865*x^9)/9 + 40*x^{10} + (500*x^{11})/11$

**sympy [A]** time = 0.09, size = 63, normalized size = 0.95

$$\frac{500x^{11}}{11} + 40x^{10} + \frac{1865x^9}{9} + \frac{1863x^8}{8} + 444x^7 + 449x^6 + \frac{2693x^5}{5} + \frac{1615x^4}{4} + \frac{914x^3}{3} + 138x^2 + 72x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**2-x+3)**2*(5*x**2+3*x+2)**3,x)`

[Out]  $500*x^{11}/11 + 40*x^{10} + 1865*x^9/9 + 1863*x^8/8 + 444*x^7 + 449*x^6 + 2693*x^5/5 + 1615*x^4/4 + 914*x^3/3 + 138*x^2 + 72*x$

$$3.24 \quad \int (3 - x + 2x^2)^2 (2 + 3x + 5x^2)^2 dx$$

Optimal. Leaf size=54

$$\frac{100x^9}{9} + \frac{5x^8}{2} + \frac{321x^7}{7} + \frac{86x^6}{3} + 78x^5 + 59x^4 + \frac{241x^3}{3} + 42x^2 + 36x$$

[Out] 36\*x+42\*x^2+241/3\*x^3+59\*x^4+78\*x^5+86/3\*x^6+321/7\*x^7+5/2\*x^8+100/9\*x^9

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$ , Rules used = {1657}

$$\frac{100x^9}{9} + \frac{5x^8}{2} + \frac{321x^7}{7} + \frac{86x^6}{3} + 78x^5 + 59x^4 + \frac{241x^3}{3} + 42x^2 + 36x$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^2\*(2 + 3\*x + 5\*x^2)^2,x]

[Out] 36\*x + 42\*x^2 + (241\*x^3)/3 + 59\*x^4 + 78\*x^5 + (86\*x^6)/3 + (321\*x^7)/7 + (5\*x^8)/2 + (100\*x^9)/9

Rule 1657

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[Expand Integrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (3 - x + 2x^2)^2 (2 + 3x + 5x^2)^2 dx &= \int (36 + 84x + 241x^2 + 236x^3 + 390x^4 + 172x^5 + 321x^6 + 20x^7 + 100x^8) \\ &= 36x + 42x^2 + \frac{241x^3}{3} + 59x^4 + 78x^5 + \frac{86x^6}{3} + \frac{321x^7}{7} + \frac{5x^8}{2} + \frac{100x^9}{9} \end{aligned}$$

Mathematica [A] time = 0.00, size = 54, normalized size = 1.00

$$\frac{100x^9}{9} + \frac{5x^8}{2} + \frac{321x^7}{7} + \frac{86x^6}{3} + 78x^5 + 59x^4 + \frac{241x^3}{3} + 42x^2 + 36x$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)^2\*(2 + 3\*x + 5\*x^2)^2,x]

[Out] 36\*x + 42\*x^2 + (241\*x^3)/3 + 59\*x^4 + 78\*x^5 + (86\*x^6)/3 + (321\*x^7)/7 + (5\*x^8)/2 + (100\*x^9)/9

**fricas** [A] time = 0.98, size = 44, normalized size = 0.81

$$\frac{100}{9}x^9 + \frac{5}{2}x^8 + \frac{321}{7}x^7 + \frac{86}{3}x^6 + 78x^5 + 59x^4 + \frac{241}{3}x^3 + 42x^2 + 36x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^2\*(5\*x^2+3\*x+2)^2,x, algorithm="fricas")

[Out] 100/9\*x^9 + 5/2\*x^8 + 321/7\*x^7 + 86/3\*x^6 + 78\*x^5 + 59\*x^4 + 241/3\*x^3 + 42\*x^2 + 36\*x

**giac** [A] time = 0.20, size = 44, normalized size = 0.81

$$\frac{100}{9}x^9 + \frac{5}{2}x^8 + \frac{321}{7}x^7 + \frac{86}{3}x^6 + 78x^5 + 59x^4 + \frac{241}{3}x^3 + 42x^2 + 36x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^2\*(5\*x^2+3\*x+2)^2,x, algorithm="giac")

[Out] 100/9\*x^9 + 5/2\*x^8 + 321/7\*x^7 + 86/3\*x^6 + 78\*x^5 + 59\*x^4 + 241/3\*x^3 + 42\*x^2 + 36\*x

**maple** [A] time = 0.00, size = 45, normalized size = 0.83

$$\frac{100}{9}x^9 + \frac{5}{2}x^8 + \frac{321}{7}x^7 + \frac{86}{3}x^6 + 78x^5 + 59x^4 + \frac{241}{3}x^3 + 42x^2 + 36x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^2\*(5\*x^2+3\*x+2)^2,x)

[Out] 36\*x+42\*x^2+241/3\*x^3+59\*x^4+78\*x^5+86/3\*x^6+321/7\*x^7+5/2\*x^8+100/9\*x^9

**maxima** [A] time = 0.44, size = 44, normalized size = 0.81

$$\frac{100}{9}x^9 + \frac{5}{2}x^8 + \frac{321}{7}x^7 + \frac{86}{3}x^6 + 78x^5 + 59x^4 + \frac{241}{3}x^3 + 42x^2 + 36x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^2\*(5\*x^2+3\*x+2)^2,x, algorithm="maxima")

[Out]  $100/9*x^9 + 5/2*x^8 + 321/7*x^7 + 86/3*x^6 + 78*x^5 + 59*x^4 + 241/3*x^3 + 42*x^2 + 36*x$

mupad [B] time = 0.03, size = 44, normalized size = 0.81

$$\frac{100x^9}{9} + \frac{5x^8}{2} + \frac{321x^7}{7} + \frac{86x^6}{3} + 78x^5 + 59x^4 + \frac{241x^3}{3} + 42x^2 + 36x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2 - x + 3)^2*(3*x + 5*x^2 + 2)^2,x)`

[Out]  $36*x + 42*x^2 + (241*x^3)/3 + 59*x^4 + 78*x^5 + (86*x^6)/3 + (321*x^7)/7 + (5*x^8)/2 + (100*x^9)/9$

sympy [A] time = 0.08, size = 51, normalized size = 0.94

$$\frac{100x^9}{9} + \frac{5x^8}{2} + \frac{321x^7}{7} + \frac{86x^6}{3} + 78x^5 + 59x^4 + \frac{241x^3}{3} + 42x^2 + 36x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**2-x+3)**2*(5*x**2+3*x+2)**2,x)`

[Out]  $100*x**9/9 + 5*x**8/2 + 321*x**7/7 + 86*x**6/3 + 78*x**5 + 59*x**4 + 241*x**3/3 + 42*x**2 + 36*x$

$$3.25 \quad \int (3 - x + 2x^2)^2 (2 + 3x + 5x^2) dx$$

Optimal. Leaf size=46

$$\frac{20x^7}{7} - \frac{4x^6}{3} + \frac{61x^5}{5} + \frac{x^4}{4} + \frac{53x^3}{3} + \frac{15x^2}{2} + 18x$$

[Out] 18\*x+15/2\*x^2+53/3\*x^3+1/4\*x^4+61/5\*x^5-4/3\*x^6+20/7\*x^7

Rubi [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$ , Rules used = {1657}

$$\frac{20x^7}{7} - \frac{4x^6}{3} + \frac{61x^5}{5} + \frac{x^4}{4} + \frac{53x^3}{3} + \frac{15x^2}{2} + 18x$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^2\*(2 + 3\*x + 5\*x^2), x]

[Out] 18\*x + (15\*x^2)/2 + (53\*x^3)/3 + x^4/4 + (61\*x^5)/5 - (4\*x^6)/3 + (20\*x^7)/7

Rule 1657

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[Expand Integrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (3 - x + 2x^2)^2 (2 + 3x + 5x^2) dx &= \int (18 + 15x + 53x^2 + x^3 + 61x^4 - 8x^5 + 20x^6) dx \\ &= 18x + \frac{15x^2}{2} + \frac{53x^3}{3} + \frac{x^4}{4} + \frac{61x^5}{5} - \frac{4x^6}{3} + \frac{20x^7}{7} \end{aligned}$$

Mathematica [A] time = 0.00, size = 46, normalized size = 1.00

$$\frac{20x^7}{7} - \frac{4x^6}{3} + \frac{61x^5}{5} + \frac{x^4}{4} + \frac{53x^3}{3} + \frac{15x^2}{2} + 18x$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)^2\*(2 + 3\*x + 5\*x^2),x]

[Out] 18\*x + (15\*x^2)/2 + (53\*x^3)/3 + x^4/4 + (61\*x^5)/5 - (4\*x^6)/3 + (20\*x^7)/7

**fricas** [A] time = 1.11, size = 34, normalized size = 0.74

$$\frac{20}{7}x^7 - \frac{4}{3}x^6 + \frac{61}{5}x^5 + \frac{1}{4}x^4 + \frac{53}{3}x^3 + \frac{15}{2}x^2 + 18x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^2\*(5\*x^2+3\*x+2),x, algorithm="fricas")

[Out] 20/7\*x^7 - 4/3\*x^6 + 61/5\*x^5 + 1/4\*x^4 + 53/3\*x^3 + 15/2\*x^2 + 18\*x

**giac** [A] time = 0.19, size = 34, normalized size = 0.74

$$\frac{20}{7}x^7 - \frac{4}{3}x^6 + \frac{61}{5}x^5 + \frac{1}{4}x^4 + \frac{53}{3}x^3 + \frac{15}{2}x^2 + 18x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^2\*(5\*x^2+3\*x+2),x, algorithm="giac")

[Out] 20/7\*x^7 - 4/3\*x^6 + 61/5\*x^5 + 1/4\*x^4 + 53/3\*x^3 + 15/2\*x^2 + 18\*x

**maple** [A] time = 0.00, size = 35, normalized size = 0.76

$$\frac{20}{7}x^7 - \frac{4}{3}x^6 + \frac{61}{5}x^5 + \frac{1}{4}x^4 + \frac{53}{3}x^3 + \frac{15}{2}x^2 + 18x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^2\*(5\*x^2+3\*x+2),x)

[Out] 18\*x+15/2\*x^2+53/3\*x^3+1/4\*x^4+61/5\*x^5-4/3\*x^6+20/7\*x^7

**maxima** [A] time = 0.44, size = 34, normalized size = 0.74

$$\frac{20}{7}x^7 - \frac{4}{3}x^6 + \frac{61}{5}x^5 + \frac{1}{4}x^4 + \frac{53}{3}x^3 + \frac{15}{2}x^2 + 18x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^2\*(5\*x^2+3\*x+2),x, algorithm="maxima")

[Out] 20/7\*x^7 - 4/3\*x^6 + 61/5\*x^5 + 1/4\*x^4 + 53/3\*x^3 + 15/2\*x^2 + 18\*x

**mupad** [B] time = 0.03, size = 34, normalized size = 0.74

$$\frac{20x^7}{7} - \frac{4x^6}{3} + \frac{61x^5}{5} + \frac{x^4}{4} + \frac{53x^3}{3} + \frac{15x^2}{2} + 18x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2 - x + 3)^2*(3*x + 5*x^2 + 2), x)`

[Out] `18*x + (15*x^2)/2 + (53*x^3)/3 + x^4/4 + (61*x^5)/5 - (4*x^6)/3 + (20*x^7)/7`

**sympy** [A] time = 0.07, size = 41, normalized size = 0.89

$$\frac{20x^7}{7} - \frac{4x^6}{3} + \frac{61x^5}{5} + \frac{x^4}{4} + \frac{53x^3}{3} + \frac{15x^2}{2} + 18x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**2-x+3)**2*(5*x**2+3*x+2), x)`

[Out] `20*x**7/7 - 4*x**6/3 + 61*x**5/5 + x**4/4 + 53*x**3/3 + 15*x**2/2 + 18*x`

$$3.26 \quad \int \frac{(3-x+2x^2)^2}{2+3x+5x^2} dx$$

Optimal. Leaf size=56

$$\frac{4x^3}{15} - \frac{16x^2}{25} - \frac{1573 \log(5x^2 + 3x + 2)}{1250} + \frac{381x}{125} + \frac{8349 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{625\sqrt{31}}$$

[Out] 381/125\*x-16/25\*x^2+4/15\*x^3-1573/1250\*ln(5\*x^2+3\*x+2)+8349/19375\*arctan(1/31\*(3+10\*x)\*31^(1/2))\*31^(1/2)

**Rubi [A]** time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {1657, 634, 618, 204, 628}

$$\frac{4x^3}{15} - \frac{16x^2}{25} - \frac{1573 \log(5x^2 + 3x + 2)}{1250} + \frac{381x}{125} + \frac{8349 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{625\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^2/(2 + 3\*x + 5\*x^2), x]

[Out] (381\*x)/125 - (16\*x^2)/25 + (4\*x^3)/15 + (8349\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(625\*Sqrt[31]) - (1573\*Log[2 + 3\*x + 5\*x^2])/1250

Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 634



```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

### Rubi steps

$$\begin{aligned} \int \frac{(3-x+2x^2)^2}{2+3x+5x^2} dx &= \int \left( \frac{381}{125} - \frac{32x}{25} + \frac{4x^2}{5} + \frac{121(3-13x)}{125(2+3x+5x^2)} \right) dx \\ &= \frac{381x}{125} - \frac{16x^2}{25} + \frac{4x^3}{15} + \frac{121}{125} \int \frac{3-13x}{2+3x+5x^2} dx \\ &= \frac{381x}{125} - \frac{16x^2}{25} + \frac{4x^3}{15} - \frac{1573}{1250} \int \frac{3+10x}{2+3x+5x^2} dx + \frac{8349}{1250} \int \frac{1}{2+3x+5x^2} dx \\ &= \frac{381x}{125} - \frac{16x^2}{25} + \frac{4x^3}{15} - \frac{1573 \log(2+3x+5x^2)}{1250} - \frac{8349}{625} \text{Subst} \left( \int \frac{1}{-31-x^2} dx, x, 3+10x \right) \\ &= \frac{381x}{125} - \frac{16x^2}{25} + \frac{4x^3}{15} + \frac{8349 \tan^{-1} \left( \frac{3+10x}{\sqrt{31}} \right)}{625\sqrt{31}} - \frac{1573 \log(2+3x+5x^2)}{1250} \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 53, normalized size = 0.95

$$\frac{10x(100x^2 - 240x + 1143) - 4719 \log(5x^2 + 3x + 2)}{3750} + \frac{8349 \tan^{-1} \left( \frac{10x+3}{\sqrt{31}} \right)}{625\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2), x]
```

```
[Out] (8349*ArcTan[(3 + 10*x)/Sqrt[31]])/(625*Sqrt[31]) + (10*x*(1143 - 240*x + 100*x^2) - 4719*Log[2 + 3*x + 5*x^2])/3750
```

**fricas [A]** time = 0.71, size = 43, normalized size = 0.77

$$\frac{4}{15} x^3 - \frac{16}{25} x^2 + \frac{8349}{19375} \sqrt{31} \arctan \left( \frac{1}{31} \sqrt{31} (10x + 3) \right) + \frac{381}{125} x - \frac{1573}{1250} \log(5x^2 + 3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^2/(5\*x^2+3\*x+2),x, algorithm="fricas")

[Out]  $\frac{4}{15}x^3 - \frac{16}{25}x^2 + \frac{8349}{19375}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) + \frac{381}{125}x - \frac{1573}{1250}\log(5x^2+3x+2)$

**giac** [A] time = 0.21, size = 43, normalized size = 0.77

$$\frac{4}{15}x^3 - \frac{16}{25}x^2 + \frac{8349}{19375}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) + \frac{381}{125}x - \frac{1573}{1250}\log(5x^2+3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^2/(5\*x^2+3\*x+2),x, algorithm="giac")

[Out]  $\frac{4}{15}x^3 - \frac{16}{25}x^2 + \frac{8349}{19375}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) + \frac{381}{125}x - \frac{1573}{1250}\log(5x^2+3x+2)$

**maple** [A] time = 0.00, size = 44, normalized size = 0.79

$$\frac{4x^3}{15} - \frac{16x^2}{25} + \frac{381x}{125} + \frac{8349\sqrt{31}\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{19375} - \frac{1573\ln(5x^2+3x+2)}{1250}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^2/(5\*x^2+3\*x+2),x)

[Out]  $\frac{381}{125}x - \frac{16}{25}x^2 + \frac{4}{15}x^3 - \frac{1573}{1250}\ln(5x^2+3x+2) + \frac{8349}{19375}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)$

**maxima** [A] time = 0.97, size = 43, normalized size = 0.77

$$\frac{4}{15}x^3 - \frac{16}{25}x^2 + \frac{8349}{19375}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) + \frac{381}{125}x - \frac{1573}{1250}\log(5x^2+3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^2/(5\*x^2+3\*x+2),x, algorithm="maxima")

[Out]  $\frac{4}{15}x^3 - \frac{16}{25}x^2 + \frac{8349}{19375}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) + \frac{381}{125}x - \frac{1573}{1250}\log(5x^2+3x+2)$

**mupad** [B] time = 3.45, size = 45, normalized size = 0.80

$$\frac{381x}{125} - \frac{1573\ln(5x^2+3x+2)}{1250} + \frac{8349\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{19375} - \frac{16x^2}{25} + \frac{4x^3}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2 - x + 3)^2/(3*x + 5*x^2 + 2), x)`

[Out]  $(381*x)/125 - (1573*\log(3*x + 5*x^2 + 2))/1250 + (8349*31^{(1/2)}*atan((10*31^{(1/2)}*x)/31 + (3*31^{(1/2)})/31))/19375 - (16*x^2)/25 + (4*x^3)/15$

**sympy** [A] time = 0.15, size = 63, normalized size = 1.12

$$\frac{4x^3}{15} - \frac{16x^2}{25} + \frac{381x}{125} - \frac{1573 \log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{1250} + \frac{8349\sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{19375}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**2-x+3)**2/(5*x**2+3*x+2), x)`

[Out]  $4*x**3/15 - 16*x**2/25 + 381*x/125 - 1573*\log(x**2 + 3*x/5 + 2/5)/1250 + 8349*\sqrt{31}*atan(10*\sqrt{31}*x/31 + 3*\sqrt{31}/31)/19375$

$$3.27 \quad \int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^2} dx$$

Optimal. Leaf size=63

$$\frac{121(69x+61)}{3875(5x^2+3x+2)} - \frac{22}{125} \log(5x^2+3x+2) + \frac{4x}{25} + \frac{41932 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{3875\sqrt{31}}$$

[Out] 4/25\*x+121/3875\*(61+69\*x)/(5\*x^2+3\*x+2)-22/125\*ln(5\*x^2+3\*x+2)+41932/120125\*arctan(1/31\*(3+10\*x)\*31^(1/2))\*31^(1/2)

**Rubi [A]** time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$ , Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{121(69x+61)}{3875(5x^2+3x+2)} - \frac{22}{125} \log(5x^2+3x+2) + \frac{4x}{25} + \frac{41932 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{3875\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^2/(2 + 3\*x + 5\*x^2)^2,x]

[Out] (4\*x)/25 + (121\*(61 + 69\*x))/(3875\*(2 + 3\*x + 5\*x^2)) + (41932\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(3875\*Sqrt[31]) - (22\*Log[2 + 3\*x + 5\*x^2])/125

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^2} dx &= \frac{121(61+69x)}{3875(2+3x+5x^2)} + \frac{1}{31} \int \frac{\frac{4032}{25} - \frac{992x}{25} + \frac{124x^2}{5}}{2+3x+5x^2} dx \\
&= \frac{121(61+69x)}{3875(2+3x+5x^2)} + \frac{1}{31} \int \left( \frac{124}{25} + \frac{44(86-31x)}{25(2+3x+5x^2)} \right) dx \\
&= \frac{4x}{25} + \frac{121(61+69x)}{3875(2+3x+5x^2)} + \frac{44}{775} \int \frac{86-31x}{2+3x+5x^2} dx \\
&= \frac{4x}{25} + \frac{121(61+69x)}{3875(2+3x+5x^2)} - \frac{22}{125} \int \frac{3+10x}{2+3x+5x^2} dx + \frac{20966}{3875} \int \frac{1}{2+3x+5x^2} dx \\
&= \frac{4x}{25} + \frac{121(61+69x)}{3875(2+3x+5x^2)} - \frac{22}{125} \log(2+3x+5x^2) - \frac{41932 \operatorname{Subst}\left(\int \frac{1}{-31-x^2} dx, x, 3+\right)}{3875} \\
&= \frac{4x}{25} + \frac{121(61+69x)}{3875(2+3x+5x^2)} + \frac{41932 \tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{3875\sqrt{31}} - \frac{22}{125} \log(2+3x+5x^2)
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 59, normalized size = 0.94

$$\frac{\frac{3751(69x+61)}{5x^2+3x+2} - 21142 \log(5x^2 + 3x + 2) + 19220x + 41932\sqrt{31} \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{120125}$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)^2/(2 + 3\*x + 5\*x^2)^2,x]

[Out] (19220\*x + (3751\*(61 + 69\*x))/(2 + 3\*x + 5\*x^2) + 41932\*sqrt[31]\*ArcTan[(3 + 10\*x)/sqrt[31]] - 21142\*Log[2 + 3\*x + 5\*x^2])/120125

**fricas [A]** time = 0.87, size = 78, normalized size = 1.24

$$\frac{96100x^3 + 41932\sqrt{31}(5x^2 + 3x + 2) \arctan\left(\frac{1}{31}\sqrt{31}(10x + 3)\right) + 57660x^2 - 21142(5x^2 + 3x + 2) \log(5x^2 + 3x + 2)}{120125(5x^2 + 3x + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^2/(5\*x^2+3\*x+2)^2,x, algorithm="fricas")

[Out]  $\frac{1}{120125}(96100x^3 + 41932\sqrt{31})(5x^2 + 3x + 2)\arctan\left(\frac{1}{31}\sqrt{31}\right) + 57660x^2 - 21142(5x^2 + 3x + 2)\log(5x^2 + 3x + 2) + 297259x + 228811)/(5x^2 + 3x + 2)$

**giac** [A] time = 0.17, size = 52, normalized size = 0.83

$$\frac{41932}{120125} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x + 3)\right) + \frac{4}{25} x + \frac{121(69x + 61)}{3875(5x^2 + 3x + 2)} - \frac{22}{125} \log(5x^2 + 3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x, algorithm="giac")`

[Out]  $\frac{41932}{120125}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\right)(10x + 3) + \frac{4}{25}x + \frac{121}{3875}(69x + 61)/(5x^2 + 3x + 2) - \frac{22}{125}\log(5x^2 + 3x + 2)$

**maple** [A] time = 0.01, size = 51, normalized size = 0.81

$$\frac{4x}{25} + \frac{41932\sqrt{31} \arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{120125} - \frac{22 \ln(5x^2 + 3x + 2)}{125} - \frac{11\left(-\frac{759x}{775} - \frac{671}{775}\right)}{25\left(x^2 + \frac{3}{5}x + \frac{2}{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x)`

[Out]  $\frac{4}{25}x - \frac{11}{25}\left(-\frac{759}{775}x - \frac{671}{775}\right)/(x^2 + \frac{3}{5}x + \frac{2}{5}) - \frac{22}{125}\ln(5x^2 + 3x + 2) + \frac{41932}{120125}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\right)$

**maxima** [A] time = 0.96, size = 52, normalized size = 0.83

$$\frac{41932}{120125} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x + 3)\right) + \frac{4}{25} x + \frac{121(69x + 61)}{3875(5x^2 + 3x + 2)} - \frac{22}{125} \log(5x^2 + 3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x, algorithm="maxima")`

[Out]  $\frac{41932}{120125}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\right)(10x + 3) + \frac{4}{25}x + \frac{121}{3875}(69x + 61)/(5x^2 + 3x + 2) - \frac{22}{125}\log(5x^2 + 3x + 2)$

**mupad** [B] time = 0.05, size = 51, normalized size = 0.81

$$\frac{4x}{25} - \frac{22 \ln(5x^2 + 3x + 2)}{125} + \frac{\frac{8349x}{19375} + \frac{7381}{19375}}{x^2 + \frac{3x}{5} + \frac{2}{5}} + \frac{41932 \sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{120125}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2 - x + 3)^2/(3*x + 5*x^2 + 2)^2,x)`

[Out]  $(4*x)/25 - (22*\log(3*x + 5*x^2 + 2))/125 + ((8349*x)/19375 + 7381/19375)/((3*x)/5 + x^2 + 2/5) + (41932*31^{(1/2)}*atan((10*31^{(1/2)}*x)/31 + (3*31^{(1/2)})/31))/120125$

**sympy** [A] time = 0.20, size = 65, normalized size = 1.03

$$\frac{4x}{25} + \frac{8349x + 7381}{19375x^2 + 11625x + 7750} - \frac{22 \log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{125} + \frac{41932\sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{120125}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**2-x+3)**2/(5*x**2+3*x+2)**2,x)`

[Out]  $4*x/25 + (8349*x + 7381)/(19375*x**2 + 11625*x + 7750) - 22*\log(x**2 + 3*x/5 + 2/5)/125 + 41932*\sqrt{31}*atan(10*\sqrt{31}*x/31 + 3*\sqrt{31}/31)/120125$



$$3.28 \quad \int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^3} dx$$

Optimal. Leaf size=64

$$\frac{121(69x+61)}{7750(5x^2+3x+2)^2} + \frac{11(45710x+17557)}{240250(5x^2+3x+2)} + \frac{4330 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{961\sqrt{31}}$$

[Out] 121/7750\*(61+69\*x)/(5\*x^2+3\*x+2)^2+11/240250\*(17557+45710\*x)/(5\*x^2+3\*x+2)+4330/29791\*arctan(1/31\*(3+10\*x)\*31^(1/2))\*31^(1/2)

**Rubi [A]** time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$ , Rules used = {1660, 12, 618, 204}

$$\frac{121(69x+61)}{7750(5x^2+3x+2)^2} + \frac{11(45710x+17557)}{240250(5x^2+3x+2)} + \frac{4330 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{961\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^2/(2 + 3\*x + 5\*x^2)^3, x]

[Out] (121\*(61 + 69\*x))/(7750\*(2 + 3\*x + 5\*x^2)^2) + (11\*(17557 + 45710\*x))/(240250\*(2 + 3\*x + 5\*x^2)) + (4330\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(961\*Sqrt[31])

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^3} dx &= \frac{121(61+69x)}{7750(2+3x+5x^2)^2} + \frac{1}{62} \int \frac{\frac{48669}{125} - \frac{1984x}{25} + \frac{248x^2}{5}}{(2+3x+5x^2)^2} dx \\
&= \frac{121(61+69x)}{7750(2+3x+5x^2)^2} + \frac{11(17557+45710x)}{240250(2+3x+5x^2)} + \frac{\int \frac{4330}{2+3x+5x^2} dx}{1922} \\
&= \frac{121(61+69x)}{7750(2+3x+5x^2)^2} + \frac{11(17557+45710x)}{240250(2+3x+5x^2)} + \frac{2165}{961} \int \frac{1}{2+3x+5x^2} dx \\
&= \frac{121(61+69x)}{7750(2+3x+5x^2)^2} + \frac{11(17557+45710x)}{240250(2+3x+5x^2)} - \frac{4330}{961} \text{Subst}\left(\int \frac{1}{-31-x^2} dx, x, 3+10x\right) \\
&= \frac{121(61+69x)}{7750(2+3x+5x^2)^2} + \frac{11(17557+45710x)}{240250(2+3x+5x^2)} + \frac{4330 \tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{961\sqrt{31}}
\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 53, normalized size = 0.83

$$\frac{11(45710x^3 + 44983x^2 + 33524x + 11183)}{48050(5x^2 + 3x + 2)^2} + \frac{4330 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{961\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (11*(11183 + 33524*x + 44983*x^2 + 45710*x^3))/(48050*(2 + 3*x + 5*x^2)^2)
+ (4330*ArcTan[(3 + 10*x)/Sqrt[31]])/(961*Sqrt[31])
```

**fricas** [A] time = 0.84, size = 75, normalized size = 1.17

$$\frac{15587110x^3 + 216500\sqrt{31}(25x^4 + 30x^3 + 29x^2 + 12x + 4)\arctan\left(\frac{1}{31}\sqrt{31}(10x + 3)\right) + 15339203x^2 + 1143}{1489550(25x^4 + 30x^3 + 29x^2 + 12x + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^2/(5\*x^2+3\*x+2)^3,x, algorithm="fricas")

[Out] 1/1489550\*(15587110\*x^3 + 216500\*sqrt(31)\*(25\*x^4 + 30\*x^3 + 29\*x^2 + 12\*x + 4)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 15339203\*x^2 + 11431684\*x + 3813403)/(25\*x^4 + 30\*x^3 + 29\*x^2 + 12\*x + 4)

**giac** [A] time = 0.18, size = 46, normalized size = 0.72

$$\frac{4330}{29791}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x + 3)\right) + \frac{11(45710x^3 + 44983x^2 + 33524x + 11183)}{48050(5x^2 + 3x + 2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^2/(5\*x^2+3\*x+2)^3,x, algorithm="giac")

[Out] 4330/29791\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 11/48050\*(45710\*x^3 + 44983\*x^2 + 33524\*x + 11183)/(5\*x^2 + 3\*x + 2)^2

**maple** [A] time = 0.01, size = 47, normalized size = 0.73

$$\frac{4330\sqrt{31}\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{29791} + \frac{\frac{50281}{4805}x^3 + \frac{494813}{48050}x^2 + \frac{184382}{24025}x + \frac{123013}{48050}}{(5x^2 + 3x + 2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^2/(5\*x^2+3\*x+2)^3,x)

[Out] 25\*(50281/120125\*x^3+494813/1201250\*x^2+184382/600625\*x+123013/1201250)/(5\*x^2+3\*x+2)^2+4330/29791\*31^(1/2)\*arctan(1/31\*(10\*x+3)\*31^(1/2))

**maxima** [A] time = 0.97, size = 56, normalized size = 0.88

$$\frac{4330}{29791}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x + 3)\right) + \frac{11(45710x^3 + 44983x^2 + 33524x + 11183)}{48050(25x^4 + 30x^3 + 29x^2 + 12x + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^2/(5\*x^2+3\*x+2)^3,x, algorithm="maxima")

[Out] 4330/29791\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 11/48050\*(45710\*x^3 + 44983\*x^2 + 33524\*x + 11183)/(25\*x^4 + 30\*x^3 + 29\*x^2 + 12\*x + 4)

mupad [B] time = 3.44, size = 55, normalized size = 0.86

$$\frac{4330\sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{29791} + \frac{\frac{50281x^3}{120125} + \frac{494813x^2}{1201250} + \frac{184382x}{600625} + \frac{123013}{1201250}}{x^4 + \frac{6x^3}{5} + \frac{29x^2}{25} + \frac{12x}{25} + \frac{4}{25}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)^2/(3\*x + 5\*x^2 + 2)^3,x)

[Out] (4330\*31^(1/2)\*atan((10\*31^(1/2)\*x)/31 + (3\*31^(1/2))/31))/29791 + ((184382\*x)/600625 + (494813\*x^2)/1201250 + (50281\*x^3)/120125 + 123013/1201250)/((12\*x)/25 + (29\*x^2)/25 + (6\*x^3)/5 + x^4 + 4/25)

sympy [A] time = 0.20, size = 63, normalized size = 0.98

$$\frac{502810x^3 + 494813x^2 + 368764x + 123013}{1201250x^4 + 1441500x^3 + 1393450x^2 + 576600x + 192200} + \frac{4330\sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{29791}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2-x+3)\*\*2/(5\*x\*\*2+3\*x+2)\*\*3,x)

[Out] (502810\*x\*\*3 + 494813\*x\*\*2 + 368764\*x + 123013)/(1201250\*x\*\*4 + 1441500\*x\*\*3 + 1393450\*x\*\*2 + 576600\*x + 192200) + 4330\*sqrt(31)\*atan(10\*sqrt(31)\*x/31 + 3\*sqrt(31)/31)/29791

$$3.29 \quad \int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^4} dx$$

**Optimal.** Leaf size=85

$$\frac{16688(10x+3)}{148955(5x^2+3x+2)} + \frac{11(12060x+4579)}{120125(5x^2+3x+2)^2} + \frac{121(69x+61)}{11625(5x^2+3x+2)^3} + \frac{66752 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{29791\sqrt{31}}$$

[Out] 121/11625\*(61+69\*x)/(5\*x^2+3\*x+2)^3+11/120125\*(4579+12060\*x)/(5\*x^2+3\*x+2)^2+16688/148955\*(3+10\*x)/(5\*x^2+3\*x+2)+66752/923521\*arctan(1/31\*(3+10\*x)\*31^(1/2))\*31^(1/2)

**Rubi [A]** time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {1660, 12, 614, 618, 204}

$$\frac{16688(10x+3)}{148955(5x^2+3x+2)} + \frac{11(12060x+4579)}{120125(5x^2+3x+2)^2} + \frac{121(69x+61)}{11625(5x^2+3x+2)^3} + \frac{66752 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{29791\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^2/(2 + 3\*x + 5\*x^2)^4, x]

[Out] (121\*(61 + 69\*x))/(11625\*(2 + 3\*x + 5\*x^2)^3) + (11\*(4579 + 12060\*x))/(120125\*(2 + 3\*x + 5\*x^2)^2) + (16688\*(3 + 10\*x))/(148955\*(2 + 3\*x + 5\*x^2)) + (66752\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(29791\*Sqrt[31])

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 614

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] - Dist[(2\*c\*(2\*p +

3))/((p + 1)\*(b^2 - 4\*a\*c)), Int[(a + b\*x + c\*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4\*p]

### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 1660

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 1]}, Simp[((b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] + Dist[1/((p + 1)\*(b^2 - 4\*a\*c)), Int[(a + b\*x + c\*x^2)^(p + 1)\*ExpandToSum[(p + 1)\*(b^2 - 4\*a\*c)\*Q - (2\*p + 3)\*(2\*c\*f - b\*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && LtQ[p, -1]

### Rubi steps

$$\begin{aligned}
\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^4} dx &= \frac{121(61+69x)}{11625(2+3x+5x^2)^3} + \frac{1}{93} \int \frac{\frac{77178}{125} - \frac{2976x}{25} + \frac{372x^2}{5}}{(2+3x+5x^2)^3} dx \\
&= \frac{121(61+69x)}{11625(2+3x+5x^2)^3} + \frac{11(4579+12060x)}{120125(2+3x+5x^2)^2} + \frac{\int \frac{100128}{5(2+3x+5x^2)^2} dx}{5766} \\
&= \frac{121(61+69x)}{11625(2+3x+5x^2)^3} + \frac{11(4579+12060x)}{120125(2+3x+5x^2)^2} + \frac{16688 \int \frac{1}{(2+3x+5x^2)^2} dx}{4805} \\
&= \frac{121(61+69x)}{11625(2+3x+5x^2)^3} + \frac{11(4579+12060x)}{120125(2+3x+5x^2)^2} + \frac{16688(3+10x)}{148955(2+3x+5x^2)} + \frac{33376}{2} \int \frac{1}{2+3x+5x^2} dx \\
&= \frac{121(61+69x)}{11625(2+3x+5x^2)^3} + \frac{11(4579+12060x)}{120125(2+3x+5x^2)^2} + \frac{16688(3+10x)}{148955(2+3x+5x^2)} - \frac{66752}{29791} \tan^{-1} \left( \frac{10x+3}{\sqrt{31}} \right) \\
&= \frac{121(61+69x)}{11625(2+3x+5x^2)^3} + \frac{11(4579+12060x)}{120125(2+3x+5x^2)^2} + \frac{16688(3+10x)}{148955(2+3x+5x^2)} + \frac{66752}{29791} \tan^{-1} \left( \frac{10x+3}{\sqrt{31}} \right)
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 63, normalized size = 0.74

$$\frac{12516000x^5 + 18774000x^4 + 21491796x^3 + 12780597x^2 + 5674908x + 1259239}{446865(5x^2 + 3x + 2)^3} + \frac{66752 \tan^{-1} \left( \frac{10x+3}{\sqrt{31}} \right)}{29791\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)^2/(2 + 3\*x + 5\*x^2)^4, x]

[Out] (1259239 + 5674908\*x + 12780597\*x^2 + 21491796\*x^3 + 18774000\*x^4 + 12516000\*x^5)/(446865\*(2 + 3\*x + 5\*x^2)^3) + (66752\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(29791\*Sqrt[31])

**fricas [A]** time = 0.84, size = 105, normalized size = 1.24

$$\frac{387996000x^5 + 581994000x^4 + 666245676x^3 + 1001280\sqrt{31}(125x^6 + 225x^5 + 285x^4 + 207x^3 + 114x^2 + 30x + 1)}{13852815(125x^6 + 225x^5 + 285x^4 + 207x^3 + 114x^2 + 30x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^2/(5\*x^2+3\*x+2)^4,x, algorithm="fricas")

[Out] 1/13852815\*(387996000\*x^5 + 581994000\*x^4 + 666245676\*x^3 + 1001280\*sqrt(31)  
)\*(125\*x^6 + 225\*x^5 + 285\*x^4 + 207\*x^3 + 114\*x^2 + 36\*x + 8)\*arctan(1/31\*  
sqrt(31)\*(10\*x + 3)) + 396198507\*x^2 + 175922148\*x + 39036409)/(125\*x^6 + 2  
25\*x^5 + 285\*x^4 + 207\*x^3 + 114\*x^2 + 36\*x + 8)

**giac** [A] time = 0.18, size = 56, normalized size = 0.66

$$\frac{66752}{923521} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x + 3)\right) + \frac{12516000x^5 + 18774000x^4 + 21491796x^3 + 12780597x^2 + 5674908x + 1259239}{446865(5x^2 + 3x + 2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^2/(5\*x^2+3\*x+2)^4,x, algorithm="giac")

[Out] 66752/923521\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 1/446865\*(12516000  
\*x^5 + 18774000\*x^4 + 21491796\*x^3 + 12780597\*x^2 + 5674908\*x + 1259239)/(5  
\*x^2 + 3\*x + 2)^3

**maple** [A] time = 0.01, size = 57, normalized size = 0.67

$$\frac{66752\sqrt{31} \arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right) + \frac{834400}{29791}x^5 + \frac{1251600}{29791}x^4 + \frac{7163932}{148955}x^3 + \frac{4260199}{148955}x^2 + \frac{1891636}{148955}x + \frac{1259239}{446865}}{923521(5x^2 + 3x + 2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^2/(5\*x^2+3\*x+2)^4,x)

[Out] 125\*(33376/148955\*x^5+50064/148955\*x^4+7163932/18619375\*x^3+4260199/1861937  
5\*x^2+1891636/18619375\*x+1259239/55858125)/(5\*x^2+3\*x+2)^3+66752/923521\*31^  
(1/2)\*arctan(1/31\*(10\*x+3)\*31^(1/2))

**maxima** [A] time = 0.96, size = 76, normalized size = 0.89

$$\frac{66752}{923521} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x + 3)\right) + \frac{12516000x^5 + 18774000x^4 + 21491796x^3 + 12780597x^2 + 5674908x + 1259239}{446865(125x^6 + 225x^5 + 285x^4 + 207x^3 + 114x^2 + 36x + 8)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^2/(5\*x^2+3\*x+2)^4,x, algorithm="maxima")



[Out]  $66752/923521*\sqrt{31}*\arctan(1/31*\sqrt{31}*(10*x + 3)) + 1/446865*(12516000*x^5 + 18774000*x^4 + 21491796*x^3 + 12780597*x^2 + 5674908*x + 1259239)/(125*x^6 + 225*x^5 + 285*x^4 + 207*x^3 + 114*x^2 + 36*x + 8)$

**mupad [B]** time = 3.47, size = 75, normalized size = 0.88

$$\frac{66752 \sqrt{31} \operatorname{atan}\left(\frac{10 \sqrt{31} x}{31} + \frac{3 \sqrt{31}}{31}\right)}{923521} + \frac{\frac{33376 x^5}{148955} + \frac{50064 x^4}{148955} + \frac{7163932 x^3}{18619375} + \frac{4260199 x^2}{18619375} + \frac{1891636 x}{18619375} + \frac{1259239}{55858125}}{x^6 + \frac{9 x^5}{5} + \frac{57 x^4}{25} + \frac{207 x^3}{125} + \frac{114 x^2}{125} + \frac{36 x}{125} + \frac{8}{125}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{int}((2*x^2 - x + 3)^2/(3*x + 5*x^2 + 2)^4, x)$

[Out]  $(66752*31^{(1/2)}*\operatorname{atan}((10*31^{(1/2)}*x)/31 + (3*31^{(1/2)})/31))/923521 + ((1891636*x)/18619375 + (4260199*x^2)/18619375 + (7163932*x^3)/18619375 + (50064*x^4)/148955 + (33376*x^5)/148955 + 1259239/55858125)/((36*x)/125 + (114*x^2)/125 + (207*x^3)/125 + (57*x^4)/25 + (9*x^5)/5 + x^6 + 8/125)$

**sympy [A]** time = 0.23, size = 83, normalized size = 0.98

$$\frac{12516000x^5 + 18774000x^4 + 21491796x^3 + 12780597x^2 + 5674908x + 1259239}{55858125x^6 + 100544625x^5 + 127356525x^4 + 92501055x^3 + 50942610x^2 + 16087140x + 3574920} + \frac{66752\sqrt{31}}{923521} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{integrate}((2*x**2-x+3)**2/(5*x**2+3*x+2)**4, x)$

[Out]  $(12516000*x**5 + 18774000*x**4 + 21491796*x**3 + 12780597*x**2 + 5674908*x + 1259239)/(55858125*x**6 + 100544625*x**5 + 127356525*x**4 + 92501055*x**3 + 50942610*x**2 + 16087140*x + 3574920) + 66752*\sqrt{31}*\operatorname{atan}(10*\sqrt{31}*x/31 + 3*\sqrt{31}/31)/923521$

$$3.30 \quad \int (3 - x + 2x^2)^3 (2 + 3x + 5x^2)^4 dx$$

**Optimal.** Leaf size=96

$$\frac{1000x^{15}}{3} + \frac{2250x^{14}}{7} + \frac{27050x^{13}}{13} + \frac{30395x^{12}}{12} + \frac{68583x^{11}}{11} + \frac{75311x^{10}}{10} + \frac{103583x^9}{9} + \frac{94881x^8}{8} + \frac{91349x^7}{7} + \frac{64529x^6}{6} + \frac{432x^5}{5}$$

[Out] 432\*x+1080\*x^2+2856\*x^3+5144\*x^4+43083/5\*x^5+64529/6\*x^6+91349/7\*x^7+94881/8\*x^8+103583/9\*x^9+75311/10\*x^10+68583/11\*x^11+30395/12\*x^12+27050/13\*x^13+2250/7\*x^14+1000/3\*x^15

**Rubi [A]** time = 0.07, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$ , Rules used = {1657}

$$\frac{1000x^{15}}{3} + \frac{2250x^{14}}{7} + \frac{27050x^{13}}{13} + \frac{30395x^{12}}{12} + \frac{68583x^{11}}{11} + \frac{75311x^{10}}{10} + \frac{103583x^9}{9} + \frac{94881x^8}{8} + \frac{91349x^7}{7} + \frac{64529x^6}{6} + \frac{432x^5}{5}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^3\*(2 + 3\*x + 5\*x^2)^4,x]

[Out] 432\*x + 1080\*x^2 + 2856\*x^3 + 5144\*x^4 + (43083\*x^5)/5 + (64529\*x^6)/6 + (91349\*x^7)/7 + (94881\*x^8)/8 + (103583\*x^9)/9 + (75311\*x^10)/10 + (68583\*x^11)/11 + (30395\*x^12)/12 + (27050\*x^13)/13 + (2250\*x^14)/7 + (1000\*x^15)/3

**Rule 1657**

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Int[Expand Integrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

**Rubi steps**

$$\begin{aligned} \int (3 - x + 2x^2)^3 (2 + 3x + 5x^2)^4 dx &= \int (432 + 2160x + 8568x^2 + 20576x^3 + 43083x^4 + 64529x^5 + 91349x^6 + 94881x^7 + 103583x^8 + 75311x^9 + 30395x^{10} + 27050x^{11} + 2250x^{12} + 1000x^{13}) dx \\ &= 432x + 1080x^2 + 2856x^3 + 5144x^4 + \frac{43083x^5}{5} + \frac{64529x^6}{6} + \frac{91349x^7}{7} + \frac{94881x^8}{8} + \frac{103583x^9}{9} + \frac{75311x^{10}}{10} + \frac{30395x^{11}}{11} + \frac{27050x^{12}}{12} + \frac{1000x^{13}}{13} \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 96, normalized size = 1.00

$$\frac{1000x^{15}}{3} + \frac{2250x^{14}}{7} + \frac{27050x^{13}}{13} + \frac{30395x^{12}}{12} + \frac{68583x^{11}}{11} + \frac{75311x^{10}}{10} + \frac{103583x^9}{9} + \frac{94881x^8}{8} + \frac{91349x^7}{7} + \frac{64529x^6}{6} + \frac{432x^5}{5}$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)^3\*(2 + 3\*x + 5\*x^2)^4, x]

[Out] 432\*x + 1080\*x^2 + 2856\*x^3 + 5144\*x^4 + (43083\*x^5)/5 + (64529\*x^6)/6 + (91349\*x^7)/7 + (94881\*x^8)/8 + (103583\*x^9)/9 + (75311\*x^10)/10 + (68583\*x^11)/11 + (30395\*x^12)/12 + (27050\*x^13)/13 + (2250\*x^14)/7 + (1000\*x^15)/3

**fricas** [A] time = 0.78, size = 74, normalized size = 0.77

$$\frac{1000}{3}x^{15} + \frac{2250}{7}x^{14} + \frac{27050}{13}x^{13} + \frac{30395}{12}x^{12} + \frac{68583}{11}x^{11} + \frac{75311}{10}x^{10} + \frac{103583}{9}x^9 + \frac{94881}{8}x^8 + \frac{91349}{7}x^7 + \frac{64529}{6}x^6 + 432x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^3\*(5\*x^2+3\*x+2)^4, x, algorithm="fricas")

[Out] 1000/3\*x^15 + 2250/7\*x^14 + 27050/13\*x^13 + 30395/12\*x^12 + 68583/11\*x^11 + 75311/10\*x^10 + 103583/9\*x^9 + 94881/8\*x^8 + 91349/7\*x^7 + 64529/6\*x^6 + 43083/5\*x^5 + 5144\*x^4 + 2856\*x^3 + 1080\*x^2 + 432\*x

**giac** [A] time = 0.18, size = 74, normalized size = 0.77

$$\frac{1000}{3}x^{15} + \frac{2250}{7}x^{14} + \frac{27050}{13}x^{13} + \frac{30395}{12}x^{12} + \frac{68583}{11}x^{11} + \frac{75311}{10}x^{10} + \frac{103583}{9}x^9 + \frac{94881}{8}x^8 + \frac{91349}{7}x^7 + \frac{64529}{6}x^6 + 432x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^3\*(5\*x^2+3\*x+2)^4, x, algorithm="giac")

[Out] 1000/3\*x^15 + 2250/7\*x^14 + 27050/13\*x^13 + 30395/12\*x^12 + 68583/11\*x^11 + 75311/10\*x^10 + 103583/9\*x^9 + 94881/8\*x^8 + 91349/7\*x^7 + 64529/6\*x^6 + 43083/5\*x^5 + 5144\*x^4 + 2856\*x^3 + 1080\*x^2 + 432\*x

**maple** [A] time = 0.00, size = 75, normalized size = 0.78

$$\frac{1000}{3}x^{15} + \frac{2250}{7}x^{14} + \frac{27050}{13}x^{13} + \frac{30395}{12}x^{12} + \frac{68583}{11}x^{11} + \frac{75311}{10}x^{10} + \frac{103583}{9}x^9 + \frac{94881}{8}x^8 + \frac{91349}{7}x^7 + \frac{64529}{6}x^6 + 432x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^3\*(5\*x^2+3\*x+2)^4, x)

[Out] 432\*x+1080\*x^2+2856\*x^3+5144\*x^4+43083/5\*x^5+64529/6\*x^6+91349/7\*x^7+94881/8\*x^8+103583/9\*x^9+75311/10\*x^10+68583/11\*x^11+30395/12\*x^12+27050/13\*x^13+2250/7\*x^14+1000/3\*x^15

**maxima** [A] time = 0.43, size = 74, normalized size = 0.77

$$\frac{1000}{3}x^{15} + \frac{2250}{7}x^{14} + \frac{27050}{13}x^{13} + \frac{30395}{12}x^{12} + \frac{68583}{11}x^{11} + \frac{75311}{10}x^{10} + \frac{103583}{9}x^9 + \frac{94881}{8}x^8 + \frac{91349}{7}x^7 + \frac{64529}{6}x^6 + 432x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^3\*(5\*x^2+3\*x+2)^4,x, algorithm="maxima")

[Out] 1000/3\*x^15 + 2250/7\*x^14 + 27050/13\*x^13 + 30395/12\*x^12 + 68583/11\*x^11 + 75311/10\*x^10 + 103583/9\*x^9 + 94881/8\*x^8 + 91349/7\*x^7 + 64529/6\*x^6 + 43083/5\*x^5 + 5144\*x^4 + 2856\*x^3 + 1080\*x^2 + 432\*x

**mupad [B]** time = 0.12, size = 74, normalized size = 0.77

$$\frac{1000x^{15}}{3} + \frac{2250x^{14}}{7} + \frac{27050x^{13}}{13} + \frac{30395x^{12}}{12} + \frac{68583x^{11}}{11} + \frac{75311x^{10}}{10} + \frac{103583x^9}{9} + \frac{94881x^8}{8} + \frac{91349x^7}{7} + \frac{64529x^6}{6} + 43083x^5 + 5144x^4 + 2856x^3 + 1080x^2 + 432x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)^3\*(3\*x + 5\*x^2 + 2)^4,x)

[Out] 432\*x + 1080\*x^2 + 2856\*x^3 + 5144\*x^4 + (43083\*x^5)/5 + (64529\*x^6)/6 + (91349\*x^7)/7 + (94881\*x^8)/8 + (103583\*x^9)/9 + (75311\*x^10)/10 + (68583\*x^11)/11 + (30395\*x^12)/12 + (27050\*x^13)/13 + (2250\*x^14)/7 + (1000\*x^15)/3

**sympy [A]** time = 0.10, size = 92, normalized size = 0.96

$$\frac{1000x^{15}}{3} + \frac{2250x^{14}}{7} + \frac{27050x^{13}}{13} + \frac{30395x^{12}}{12} + \frac{68583x^{11}}{11} + \frac{75311x^{10}}{10} + \frac{103583x^9}{9} + \frac{94881x^8}{8} + \frac{91349x^7}{7} + \frac{64529x^6}{6} + 43083x^5 + 5144x^4 + 2856x^3 + 1080x^2 + 432x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2-x+3)\*\*3\*(5\*x\*\*2+3\*x+2)\*\*4,x)

[Out] 1000\*x\*\*15/3 + 2250\*x\*\*14/7 + 27050\*x\*\*13/13 + 30395\*x\*\*12/12 + 68583\*x\*\*11/11 + 75311\*x\*\*10/10 + 103583\*x\*\*9/9 + 94881\*x\*\*8/8 + 91349\*x\*\*7/7 + 64529\*x\*\*6/6 + 43083\*x\*\*5/5 + 5144\*x\*\*4 + 2856\*x\*\*3 + 1080\*x\*\*2 + 432\*x

$$3.31 \quad \int (3 - x + 2x^2)^3 (2 + 3x + 5x^2)^3 dx$$

**Optimal.** Leaf size=82

$$\frac{1000x^{13}}{13} + 25x^{12} + \frac{4830x^{11}}{11} + \frac{3061x^{10}}{10} + \frac{3316x^9}{3} + \frac{7869x^8}{8} + \frac{12016x^7}{7} + \frac{2873x^6}{2} + \frac{8292x^5}{5} + \frac{4483x^4}{4} + 870x^3 + 378x^2 + 216x + 378$$

[Out] 216\*x+378\*x^2+870\*x^3+4483/4\*x^4+8292/5\*x^5+2873/2\*x^6+12016/7\*x^7+7869/8\*x^8+3316/3\*x^9+3061/10\*x^10+4830/11\*x^11+25\*x^12+1000/13\*x^13

**Rubi [A]** time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$ , Rules used = {1657}

$$\frac{1000x^{13}}{13} + 25x^{12} + \frac{4830x^{11}}{11} + \frac{3061x^{10}}{10} + \frac{3316x^9}{3} + \frac{7869x^8}{8} + \frac{12016x^7}{7} + \frac{2873x^6}{2} + \frac{8292x^5}{5} + \frac{4483x^4}{4} + 870x^3 + 378x^2 + 216x + 378$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^3\*(2 + 3\*x + 5\*x^2)^3,x]

[Out] 216\*x + 378\*x^2 + 870\*x^3 + (4483\*x^4)/4 + (8292\*x^5)/5 + (2873\*x^6)/2 + (12016\*x^7)/7 + (7869\*x^8)/8 + (3316\*x^9)/3 + (3061\*x^10)/10 + (4830\*x^11)/11 + 25\*x^12 + (1000\*x^13)/13

**Rule 1657**

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[Expand Integrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

**Rubi steps**

$$\begin{aligned} \int (3 - x + 2x^2)^3 (2 + 3x + 5x^2)^3 dx &= \int (216 + 756x + 2610x^2 + 4483x^3 + 8292x^4 + 8619x^5 + 12016x^6 + 7869x^7 + 3061x^8 + 4830x^9 + 25x^{10} + 1000x^{11} + 13x^{12}) (2 + 3x + 5x^2)^3 dx \\ &= 216x + 378x^2 + 870x^3 + \frac{4483x^4}{4} + \frac{8292x^5}{5} + \frac{2873x^6}{2} + \frac{12016x^7}{7} + \frac{7869x^8}{8} + \frac{3316x^9}{3} + \frac{3061x^{10}}{10} + \frac{4830x^{11}}{11} + 25x^{12} + \frac{1000x^{13}}{13} \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 82, normalized size = 1.00

$$\frac{1000x^{13}}{13} + 25x^{12} + \frac{4830x^{11}}{11} + \frac{3061x^{10}}{10} + \frac{3316x^9}{3} + \frac{7869x^8}{8} + \frac{12016x^7}{7} + \frac{2873x^6}{2} + \frac{8292x^5}{5} + \frac{4483x^4}{4} + 870x^3 + 378x^2 + 216x + 378$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)^3\*(2 + 3\*x + 5\*x^2)^3,x]

[Out] 216\*x + 378\*x^2 + 870\*x^3 + (4483\*x^4)/4 + (8292\*x^5)/5 + (2873\*x^6)/2 + (12016\*x^7)/7 + (7869\*x^8)/8 + (3316\*x^9)/3 + (3061\*x^10)/10 + (4830\*x^11)/11 + 25\*x^12 + (1000\*x^13)/13

**fricas** [A] time = 0.80, size = 64, normalized size = 0.78

$$\frac{1000}{13}x^{13} + 25x^{12} + \frac{4830}{11}x^{11} + \frac{3061}{10}x^{10} + \frac{3316}{3}x^9 + \frac{7869}{8}x^8 + \frac{12016}{7}x^7 + \frac{2873}{2}x^6 + \frac{8292}{5}x^5 + \frac{4483}{4}x^4 + 870x^3 + 378x^2 + 216x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^3\*(5\*x^2+3\*x+2)^3,x, algorithm="fricas")

[Out] 1000/13\*x^13 + 25\*x^12 + 4830/11\*x^11 + 3061/10\*x^10 + 3316/3\*x^9 + 7869/8\*x^8 + 12016/7\*x^7 + 2873/2\*x^6 + 8292/5\*x^5 + 4483/4\*x^4 + 870\*x^3 + 378\*x^2 + 216\*x

**giac** [A] time = 0.21, size = 64, normalized size = 0.78

$$\frac{1000}{13}x^{13} + 25x^{12} + \frac{4830}{11}x^{11} + \frac{3061}{10}x^{10} + \frac{3316}{3}x^9 + \frac{7869}{8}x^8 + \frac{12016}{7}x^7 + \frac{2873}{2}x^6 + \frac{8292}{5}x^5 + \frac{4483}{4}x^4 + 870x^3 + 378x^2 + 216x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^3\*(5\*x^2+3\*x+2)^3,x, algorithm="giac")

[Out] 1000/13\*x^13 + 25\*x^12 + 4830/11\*x^11 + 3061/10\*x^10 + 3316/3\*x^9 + 7869/8\*x^8 + 12016/7\*x^7 + 2873/2\*x^6 + 8292/5\*x^5 + 4483/4\*x^4 + 870\*x^3 + 378\*x^2 + 216\*x

**maple** [A] time = 0.00, size = 65, normalized size = 0.79

$$\frac{1000}{13}x^{13} + 25x^{12} + \frac{4830}{11}x^{11} + \frac{3061}{10}x^{10} + \frac{3316}{3}x^9 + \frac{7869}{8}x^8 + \frac{12016}{7}x^7 + \frac{2873}{2}x^6 + \frac{8292}{5}x^5 + \frac{4483}{4}x^4 + 870x^3 + 378x^2 + 216x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^3\*(5\*x^2+3\*x+2)^3,x)

[Out] 216\*x+378\*x^2+870\*x^3+4483/4\*x^4+8292/5\*x^5+2873/2\*x^6+12016/7\*x^7+7869/8\*x^8+3316/3\*x^9+3061/10\*x^10+4830/11\*x^11+25\*x^12+1000/13\*x^13

**maxima** [A] time = 0.43, size = 64, normalized size = 0.78

$$\frac{1000}{13}x^{13} + 25x^{12} + \frac{4830}{11}x^{11} + \frac{3061}{10}x^{10} + \frac{3316}{3}x^9 + \frac{7869}{8}x^8 + \frac{12016}{7}x^7 + \frac{2873}{2}x^6 + \frac{8292}{5}x^5 + \frac{4483}{4}x^4 + 870x^3 + 378x^2 + 216x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^3\*(5\*x^2+3\*x+2)^3,x, algorithm="maxima")

[Out] 1000/13\*x^13 + 25\*x^12 + 4830/11\*x^11 + 3061/10\*x^10 + 3316/3\*x^9 + 7869/8\*x^8 + 12016/7\*x^7 + 2873/2\*x^6 + 8292/5\*x^5 + 4483/4\*x^4 + 870\*x^3 + 378\*x^2 + 216\*x

**mupad [B]** time = 0.08, size = 64, normalized size = 0.78

$$\frac{1000x^{13}}{13} + 25x^{12} + \frac{4830x^{11}}{11} + \frac{3061x^{10}}{10} + \frac{3316x^9}{3} + \frac{7869x^8}{8} + \frac{12016x^7}{7} + \frac{2873x^6}{2} + \frac{8292x^5}{5} + \frac{4483x^4}{4} + 870x^3 + 378x^2 + 216x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)^3\*(3\*x + 5\*x^2 + 2)^3,x)

[Out] 216\*x + 378\*x^2 + 870\*x^3 + (4483\*x^4)/4 + (8292\*x^5)/5 + (2873\*x^6)/2 + (12016\*x^7)/7 + (7869\*x^8)/8 + (3316\*x^9)/3 + (3061\*x^10)/10 + (4830\*x^11)/11 + 25\*x^12 + (1000\*x^13)/13

**sympy [A]** time = 0.09, size = 78, normalized size = 0.95

$$\frac{1000x^{13}}{13} + 25x^{12} + \frac{4830x^{11}}{11} + \frac{3061x^{10}}{10} + \frac{3316x^9}{3} + \frac{7869x^8}{8} + \frac{12016x^7}{7} + \frac{2873x^6}{2} + \frac{8292x^5}{5} + \frac{4483x^4}{4} + 870x^3 + 378x^2 + 216x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2-x+3)\*\*3\*(5\*x\*\*2+3\*x+2)\*\*3,x)

[Out] 1000\*x\*\*13/13 + 25\*x\*\*12 + 4830\*x\*\*11/11 + 3061\*x\*\*10/10 + 3316\*x\*\*9/3 + 7869\*x\*\*8/8 + 12016\*x\*\*7/7 + 2873\*x\*\*6/2 + 8292\*x\*\*5/5 + 4483\*x\*\*4/4 + 870\*x\*\*3 + 378\*x\*\*2 + 216\*x

$$3.32 \quad \int (3 - x + 2x^2)^3 (2 + 3x + 5x^2)^2 dx$$

**Optimal.** Leaf size=68

$$\frac{200x^{11}}{11} - 6x^{10} + \frac{922x^9}{9} + \frac{83x^8}{8} + \frac{1571x^7}{7} + \frac{299x^6}{3} + \frac{1416x^5}{5} + \frac{635x^4}{4} + 237x^3 + 108x^2 + 108x$$

[Out] 108\*x+108\*x^2+237\*x^3+635/4\*x^4+1416/5\*x^5+299/3\*x^6+1571/7\*x^7+83/8\*x^8+922/9\*x^9-6\*x^10+200/11\*x^11

**Rubi [A]** time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$ , Rules used = {1657}

$$\frac{200x^{11}}{11} - 6x^{10} + \frac{922x^9}{9} + \frac{83x^8}{8} + \frac{1571x^7}{7} + \frac{299x^6}{3} + \frac{1416x^5}{5} + \frac{635x^4}{4} + 237x^3 + 108x^2 + 108x$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^3\*(2 + 3\*x + 5\*x^2)^2,x]

[Out] 108\*x + 108\*x^2 + 237\*x^3 + (635\*x^4)/4 + (1416\*x^5)/5 + (299\*x^6)/3 + (1571\*x^7)/7 + (83\*x^8)/8 + (922\*x^9)/9 - 6\*x^10 + (200\*x^11)/11

Rule 1657

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[Expand Integrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (3 - x + 2x^2)^3 (2 + 3x + 5x^2)^2 dx &= \int (108 + 216x + 711x^2 + 635x^3 + 1416x^4 + 598x^5 + 1571x^6 + 83x^7 + 922x^8 + 200x^9 - 6x^{10} + 200x^{11}) dx \\ &= 108x + 108x^2 + 237x^3 + \frac{635x^4}{4} + \frac{1416x^5}{5} + \frac{299x^6}{3} + \frac{1571x^7}{7} + \frac{83x^8}{8} + \frac{200x^9}{9} - 6x^{10} + \frac{200x^{11}}{11} \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 68, normalized size = 1.00

$$\frac{200x^{11}}{11} - 6x^{10} + \frac{922x^9}{9} + \frac{83x^8}{8} + \frac{1571x^7}{7} + \frac{299x^6}{3} + \frac{1416x^5}{5} + \frac{635x^4}{4} + 237x^3 + 108x^2 + 108x$$

Antiderivative was successfully verified.



[In] Integrate[(3 - x + 2\*x^2)^3\*(2 + 3\*x + 5\*x^2)^2,x]

[Out] 108\*x + 108\*x^2 + 237\*x^3 + (635\*x^4)/4 + (1416\*x^5)/5 + (299\*x^6)/3 + (1571\*x^7)/7 + (83\*x^8)/8 + (922\*x^9)/9 - 6\*x^10 + (200\*x^11)/11

**fricas** [A] time = 0.77, size = 54, normalized size = 0.79

$$\frac{200}{11}x^{11} - 6x^{10} + \frac{922}{9}x^9 + \frac{83}{8}x^8 + \frac{1571}{7}x^7 + \frac{299}{3}x^6 + \frac{1416}{5}x^5 + \frac{635}{4}x^4 + 237x^3 + 108x^2 + 108x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^3\*(5\*x^2+3\*x+2)^2,x, algorithm="fricas")

[Out] 200/11\*x^11 - 6\*x^10 + 922/9\*x^9 + 83/8\*x^8 + 1571/7\*x^7 + 299/3\*x^6 + 1416/5\*x^5 + 635/4\*x^4 + 237\*x^3 + 108\*x^2 + 108\*x

**giac** [A] time = 0.20, size = 54, normalized size = 0.79

$$\frac{200}{11}x^{11} - 6x^{10} + \frac{922}{9}x^9 + \frac{83}{8}x^8 + \frac{1571}{7}x^7 + \frac{299}{3}x^6 + \frac{1416}{5}x^5 + \frac{635}{4}x^4 + 237x^3 + 108x^2 + 108x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^3\*(5\*x^2+3\*x+2)^2,x, algorithm="giac")

[Out] 200/11\*x^11 - 6\*x^10 + 922/9\*x^9 + 83/8\*x^8 + 1571/7\*x^7 + 299/3\*x^6 + 1416/5\*x^5 + 635/4\*x^4 + 237\*x^3 + 108\*x^2 + 108\*x

**maple** [A] time = 0.00, size = 55, normalized size = 0.81

$$\frac{200}{11}x^{11} - 6x^{10} + \frac{922}{9}x^9 + \frac{83}{8}x^8 + \frac{1571}{7}x^7 + \frac{299}{3}x^6 + \frac{1416}{5}x^5 + \frac{635}{4}x^4 + 237x^3 + 108x^2 + 108x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^3\*(5\*x^2+3\*x+2)^2,x)

[Out] 108\*x+108\*x^2+237\*x^3+635/4\*x^4+1416/5\*x^5+299/3\*x^6+1571/7\*x^7+83/8\*x^8+922/9\*x^9-6\*x^10+200/11\*x^11

**maxima** [A] time = 0.43, size = 54, normalized size = 0.79

$$\frac{200}{11}x^{11} - 6x^{10} + \frac{922}{9}x^9 + \frac{83}{8}x^8 + \frac{1571}{7}x^7 + \frac{299}{3}x^6 + \frac{1416}{5}x^5 + \frac{635}{4}x^4 + 237x^3 + 108x^2 + 108x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^3\*(5\*x^2+3\*x+2)^2,x, algorithm="maxima")

[Out]  $200/11*x^{11} - 6*x^{10} + 922/9*x^9 + 83/8*x^8 + 1571/7*x^7 + 299/3*x^6 + 1416/5*x^5 + 635/4*x^4 + 237*x^3 + 108*x^2 + 108*x$

**mupad [B]** time = 0.05, size = 54, normalized size = 0.79

$$\frac{200x^{11}}{11} - 6x^{10} + \frac{922x^9}{9} + \frac{83x^8}{8} + \frac{1571x^7}{7} + \frac{299x^6}{3} + \frac{1416x^5}{5} + \frac{635x^4}{4} + 237x^3 + 108x^2 + 108x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2 - x + 3)^3*(3*x + 5*x^2 + 2)^2,x)`

[Out]  $108*x + 108*x^2 + 237*x^3 + (635*x^4)/4 + (1416*x^5)/5 + (299*x^6)/3 + (1571*x^7)/7 + (83*x^8)/8 + (922*x^9)/9 - 6*x^{10} + (200*x^{11})/11$

**sympy [A]** time = 0.08, size = 65, normalized size = 0.96

$$\frac{200x^{11}}{11} - 6x^{10} + \frac{922x^9}{9} + \frac{83x^8}{8} + \frac{1571x^7}{7} + \frac{299x^6}{3} + \frac{1416x^5}{5} + \frac{635x^4}{4} + 237x^3 + 108x^2 + 108x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**2-x+3)**3*(5*x**2+3*x+2)**2,x)`

[Out]  $200*x^{11}/11 - 6*x^{10} + 922*x^9/9 + 83*x^8/8 + 1571*x^7/7 + 299*x^6/3 + 1416*x^5/5 + 635*x^4/4 + 237*x^3 + 108*x^2 + 108*x$

$$3.33 \quad \int (3 - x + 2x^2)^3 (2 + 3x + 5x^2) dx$$

Optimal. Leaf size=56

$$\frac{40x^9}{9} - \frac{9x^8}{2} + \frac{190x^7}{7} - \frac{83x^6}{6} + \frac{288x^5}{5} - 5x^4 + 60x^3 + \frac{27x^2}{2} + 54x$$

[Out] 54\*x+27/2\*x^2+60\*x^3-5\*x^4+288/5\*x^5-83/6\*x^6+190/7\*x^7-9/2\*x^8+40/9\*x^9

Rubi [A] time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$ , Rules used = {1657}

$$\frac{40x^9}{9} - \frac{9x^8}{2} + \frac{190x^7}{7} - \frac{83x^6}{6} + \frac{288x^5}{5} - 5x^4 + 60x^3 + \frac{27x^2}{2} + 54x$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^3\*(2 + 3\*x + 5\*x^2), x]

[Out] 54\*x + (27\*x^2)/2 + 60\*x^3 - 5\*x^4 + (288\*x^5)/5 - (83\*x^6)/6 + (190\*x^7)/7 - (9\*x^8)/2 + (40\*x^9)/9

Rule 1657

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[Expand Integrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (3 - x + 2x^2)^3 (2 + 3x + 5x^2) dx &= \int (54 + 27x + 180x^2 - 20x^3 + 288x^4 - 83x^5 + 190x^6 - 36x^7 + 40x^8) dx \\ &= 54x + \frac{27x^2}{2} + 60x^3 - 5x^4 + \frac{288x^5}{5} - \frac{83x^6}{6} + \frac{190x^7}{7} - \frac{9x^8}{2} + \frac{40x^9}{9} \end{aligned}$$

Mathematica [A] time = 0.00, size = 56, normalized size = 1.00

$$\frac{40x^9}{9} - \frac{9x^8}{2} + \frac{190x^7}{7} - \frac{83x^6}{6} + \frac{288x^5}{5} - 5x^4 + 60x^3 + \frac{27x^2}{2} + 54x$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)^3\*(2 + 3\*x + 5\*x^2),x]

[Out] 54\*x + (27\*x^2)/2 + 60\*x^3 - 5\*x^4 + (288\*x^5)/5 - (83\*x^6)/6 + (190\*x^7)/7 - (9\*x^8)/2 + (40\*x^9)/9

**fricas** [A] time = 0.83, size = 44, normalized size = 0.79

$$\frac{40}{9}x^9 - \frac{9}{2}x^8 + \frac{190}{7}x^7 - \frac{83}{6}x^6 + \frac{288}{5}x^5 - 5x^4 + 60x^3 + \frac{27}{2}x^2 + 54x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^3\*(5\*x^2+3\*x+2),x, algorithm="fricas")

[Out] 40/9\*x^9 - 9/2\*x^8 + 190/7\*x^7 - 83/6\*x^6 + 288/5\*x^5 - 5\*x^4 + 60\*x^3 + 27/2\*x^2 + 54\*x

**giac** [A] time = 0.20, size = 44, normalized size = 0.79

$$\frac{40}{9}x^9 - \frac{9}{2}x^8 + \frac{190}{7}x^7 - \frac{83}{6}x^6 + \frac{288}{5}x^5 - 5x^4 + 60x^3 + \frac{27}{2}x^2 + 54x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^3\*(5\*x^2+3\*x+2),x, algorithm="giac")

[Out] 40/9\*x^9 - 9/2\*x^8 + 190/7\*x^7 - 83/6\*x^6 + 288/5\*x^5 - 5\*x^4 + 60\*x^3 + 27/2\*x^2 + 54\*x

**maple** [A] time = 0.00, size = 45, normalized size = 0.80

$$\frac{40}{9}x^9 - \frac{9}{2}x^8 + \frac{190}{7}x^7 - \frac{83}{6}x^6 + \frac{288}{5}x^5 - 5x^4 + 60x^3 + \frac{27}{2}x^2 + 54x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^3\*(5\*x^2+3\*x+2),x)

[Out] 54\*x+27/2\*x^2+60\*x^3-5\*x^4+288/5\*x^5-83/6\*x^6+190/7\*x^7-9/2\*x^8+40/9\*x^9

**maxima** [A] time = 0.43, size = 44, normalized size = 0.79

$$\frac{40}{9}x^9 - \frac{9}{2}x^8 + \frac{190}{7}x^7 - \frac{83}{6}x^6 + \frac{288}{5}x^5 - 5x^4 + 60x^3 + \frac{27}{2}x^2 + 54x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^3\*(5\*x^2+3\*x+2),x, algorithm="maxima")

[Out]  $40/9*x^9 - 9/2*x^8 + 190/7*x^7 - 83/6*x^6 + 288/5*x^5 - 5*x^4 + 60*x^3 + 27/2*x^2 + 54*x$

**mupad [B]** time = 0.03, size = 44, normalized size = 0.79

$$\frac{40x^9}{9} - \frac{9x^8}{2} + \frac{190x^7}{7} - \frac{83x^6}{6} + \frac{288x^5}{5} - 5x^4 + 60x^3 + \frac{27x^2}{2} + 54x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2 - x + 3)^3*(3*x + 5*x^2 + 2), x)`

[Out]  $54*x + (27*x^2)/2 + 60*x^3 - 5*x^4 + (288*x^5)/5 - (83*x^6)/6 + (190*x^7)/7 - (9*x^8)/2 + (40*x^9)/9$

**sympy [A]** time = 0.08, size = 53, normalized size = 0.95

$$\frac{40x^9}{9} - \frac{9x^8}{2} + \frac{190x^7}{7} - \frac{83x^6}{6} + \frac{288x^5}{5} - 5x^4 + 60x^3 + \frac{27x^2}{2} + 54x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**2-x+3)**3*(5*x**2+3*x+2), x)`

[Out]  $40*x**9/9 - 9*x**8/2 + 190*x**7/7 - 83*x**6/6 + 288*x**5/5 - 5*x**4 + 60*x**3 + 27*x**2/2 + 54*x$

$$3.34 \quad \int \frac{(3-x+2x^2)^3}{2+3x+5x^2} dx$$

Optimal. Leaf size=70

$$\frac{8x^5}{25} - \frac{21x^4}{25} + \frac{1222x^3}{375} - \frac{7451x^2}{1250} - \frac{158389 \log(5x^2 + 3x + 2)}{31250} + \frac{49508x}{3125} + \frac{328757 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{15625\sqrt{31}}$$

[Out] 49508/3125\*x-7451/1250\*x^2+1222/375\*x^3-21/25\*x^4+8/25\*x^5-158389/31250\*ln(5\*x^2+3\*x+2)+328757/484375\*arctan(1/31\*(3+10\*x)\*31^(1/2))\*31^(1/2)

**Rubi [A]** time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {1657, 634, 618, 204, 628}

$$\frac{8x^5}{25} - \frac{21x^4}{25} + \frac{1222x^3}{375} - \frac{7451x^2}{1250} - \frac{158389 \log(5x^2 + 3x + 2)}{31250} + \frac{49508x}{3125} + \frac{328757 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{15625\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^3/(2 + 3\*x + 5\*x^2), x]

[Out] (49508\*x)/3125 - (7451\*x^2)/1250 + (1222\*x^3)/375 - (21\*x^4)/25 + (8\*x^5)/25 + (328757\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(15625\*Sqrt[31]) - (158389\*Log[2 + 3\*x + 5\*x^2])/31250

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[Expand[Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned} \int \frac{(3-x+2x^2)^3}{2+3x+5x^2} dx &= \int \left( \frac{49508}{3125} - \frac{7451x}{625} + \frac{1222x^2}{125} - \frac{84x^3}{25} + \frac{8x^4}{5} - \frac{1331(11+119x)}{3125(2+3x+5x^2)} \right) dx \\ &= \frac{49508x}{3125} - \frac{7451x^2}{1250} + \frac{1222x^3}{375} - \frac{21x^4}{25} + \frac{8x^5}{25} - \frac{1331 \int \frac{11+119x}{2+3x+5x^2} dx}{3125} \\ &= \frac{49508x}{3125} - \frac{7451x^2}{1250} + \frac{1222x^3}{375} - \frac{21x^4}{25} + \frac{8x^5}{25} - \frac{158389 \int \frac{3+10x}{2+3x+5x^2} dx}{31250} + \frac{328757 \int \frac{1}{2+3x+5x^2} dx}{31250} \\ &= \frac{49508x}{3125} - \frac{7451x^2}{1250} + \frac{1222x^3}{375} - \frac{21x^4}{25} + \frac{8x^5}{25} - \frac{158389 \log(2+3x+5x^2)}{31250} - \frac{328757 \operatorname{Subst}(\int \frac{1}{2+3x+5x^2} dx, \sqrt{31})}{31250} \\ &= \frac{49508x}{3125} - \frac{7451x^2}{1250} + \frac{1222x^3}{375} - \frac{21x^4}{25} + \frac{8x^5}{25} + \frac{328757 \tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{15625\sqrt{31}} - \frac{158389 \log(2+3x+5x^2)}{31250} \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 63, normalized size = 0.90

$$\frac{31 \left( 5x \left( 6000x^4 - 15750x^3 + 61100x^2 - 111765x + 297048 \right) - 475167 \log \left( 5x^2 + 3x + 2 \right) \right) + 1972542\sqrt{31} \tan^{-1} \left( \frac{3+10x}{\sqrt{31}} \right)}{2906250}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2), x]
```

```
[Out] (1972542*Sqrt[31]*ArcTan[(3 + 10*x)/Sqrt[31]] + 31*(5*x*(297048 - 111765*x + 61100*x^2 - 15750*x^3 + 6000*x^4) - 475167*Log[2 + 3*x + 5*x^2]))/2906250
```

**fricas** [A] time = 0.87, size = 53, normalized size = 0.76

$$\frac{8}{25}x^5 - \frac{21}{25}x^4 + \frac{1222}{375}x^3 - \frac{7451}{1250}x^2 + \frac{328757}{484375}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) + \frac{49508}{3125}x - \frac{158389}{31250}\log(5x^2+3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^3/(5\*x^2+3\*x+2),x, algorithm="fricas")

[Out] 8/25\*x^5 - 21/25\*x^4 + 1222/375\*x^3 - 7451/1250\*x^2 + 328757/484375\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 49508/3125\*x - 158389/31250\*log(5\*x^2 + 3\*x + 2)

**giac** [A] time = 0.21, size = 53, normalized size = 0.76

$$\frac{8}{25}x^5 - \frac{21}{25}x^4 + \frac{1222}{375}x^3 - \frac{7451}{1250}x^2 + \frac{328757}{484375}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) + \frac{49508}{3125}x - \frac{158389}{31250}\log(5x^2+3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^3/(5\*x^2+3\*x+2),x, algorithm="giac")

[Out] 8/25\*x^5 - 21/25\*x^4 + 1222/375\*x^3 - 7451/1250\*x^2 + 328757/484375\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 49508/3125\*x - 158389/31250\*log(5\*x^2 + 3\*x + 2)

**maple** [A] time = 0.00, size = 54, normalized size = 0.77

$$\frac{8x^5}{25} - \frac{21x^4}{25} + \frac{1222x^3}{375} - \frac{7451x^2}{1250} + \frac{49508x}{3125} + \frac{328757\sqrt{31}\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{484375} - \frac{158389\ln(5x^2+3x+2)}{31250}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^3/(5\*x^2+3\*x+2),x)

[Out] 49508/3125\*x-7451/1250\*x^2+1222/375\*x^3-21/25\*x^4+8/25\*x^5-158389/31250\*ln(5\*x^2+3\*x+2)+328757/484375\*31^(1/2)\*arctan(1/31\*(10\*x+3)\*31^(1/2))

**maxima** [A] time = 0.96, size = 53, normalized size = 0.76

$$\frac{8}{25}x^5 - \frac{21}{25}x^4 + \frac{1222}{375}x^3 - \frac{7451}{1250}x^2 + \frac{328757}{484375}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) + \frac{49508}{3125}x - \frac{158389}{31250}\log(5x^2+3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^3/(5\*x^2+3\*x+2),x, algorithm="maxima")



[Out]  $8/25*x^5 - 21/25*x^4 + 1222/375*x^3 - 7451/1250*x^2 + 328757/484375*\sqrt{31}$   
 $(1/31*\sqrt{31}*(10*x + 3)) + 49508/3125*x - 158389/31250*\log(5*x^2$   
 $+ 3*x + 2)$

**mupad [B]** time = 0.04, size = 55, normalized size = 0.79

$$\frac{49508x}{3125} - \frac{158389 \ln(5x^2 + 3x + 2)}{31250} + \frac{328757 \sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{484375} - \frac{7451x^2}{1250} + \frac{1222x^3}{375} - \frac{21x^4}{25} + \frac{8x^5}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{int}((2*x^2 - x + 3)^3/(3*x + 5*x^2 + 2), x)$

[Out]  $(49508*x)/3125 - (158389*\log(3*x + 5*x^2 + 2))/31250 + (328757*31^{(1/2)}*\operatorname{atan}$   
 $\operatorname{n}((10*31^{(1/2)}*x)/31 + (3*31^{(1/2)})/31))/484375 - (7451*x^2)/1250 + (1222*x$   
 $^3)/375 - (21*x^4)/25 + (8*x^5)/25$

**sympy [A]** time = 0.16, size = 76, normalized size = 1.09

$$\frac{8x^5}{25} - \frac{21x^4}{25} + \frac{1222x^3}{375} - \frac{7451x^2}{1250} + \frac{49508x}{3125} - \frac{158389 \log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{31250} + \frac{328757\sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{484375}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{integrate}((2*x**2-x+3)**3/(5*x**2+3*x+2), x)$

[Out]  $8*x**5/25 - 21*x**4/25 + 1222*x**3/375 - 7451*x**2/1250 + 49508*x/3125 - 15$   
 $8389*\log(x**2 + 3*x/5 + 2/5)/31250 + 328757*\sqrt{31}*\operatorname{atan}(10*\sqrt{31}*x/31$   
 $+ 3*\sqrt{31}/31)/484375$

$$3.35 \quad \int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^2} dx$$

**Optimal.** Leaf size=77

$$\frac{8x^3}{75} - \frac{54x^2}{125} + \frac{1331(247x + 443)}{96875(5x^2 + 3x + 2)} - \frac{10769 \log(5x^2 + 3x + 2)}{6250} + \frac{1466x}{625} + \frac{3819607 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{96875\sqrt{31}}$$

[Out] 1466/625\*x-54/125\*x^2+8/75\*x^3+1331/96875\*(443+247\*x)/(5\*x^2+3\*x+2)-10769/6250\*ln(5\*x^2+3\*x+2)+3819607/3003125\*arctan(1/31\*(3+10\*x)\*31^(1/2))\*31^(1/2)

**Rubi [A]** time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$ , Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{8x^3}{75} - \frac{54x^2}{125} + \frac{1331(247x + 443)}{96875(5x^2 + 3x + 2)} - \frac{10769 \log(5x^2 + 3x + 2)}{6250} + \frac{1466x}{625} + \frac{3819607 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{96875\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^3/(2 + 3\*x + 5\*x^2)^2,x]

[Out] (1466\*x)/625 - (54\*x^2)/125 + (8\*x^3)/75 + (1331\*(443 + 247\*x))/(96875\*(2 + 3\*x + 5\*x^2)) + (3819607\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(96875\*Sqrt[31]) - (10769\*Log[2 + 3\*x + 5\*x^2])/6250

Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x, x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d}, x]

e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

### Rule 1657

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Int[ExpandIntegrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

### Rule 1660

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 1]}, Simp[((b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] + Dist[1/((p + 1)\*(b^2 - 4\*a\*c)), Int[(a + b\*x + c\*x^2)^(p + 1)\*ExpandToSum[(p + 1)\*(b^2 - 4\*a\*c)\*Q - (2\*p + 3)\*(2\*c\*f - b\*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && LtQ[p, -1]

### Rubi steps

$$\begin{aligned}
\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^2} dx &= \frac{1331(443+247x)}{96875(2+3x+5x^2)} + \frac{1}{31} \int \frac{\frac{372701}{625} - \frac{230981x}{625} + \frac{37882x^2}{125} - \frac{2604x^3}{25} + \frac{248x^4}{5}}{2+3x+5x^2} dx \\
&= \frac{1331(443+247x)}{96875(2+3x+5x^2)} + \frac{1}{31} \int \left( \frac{45446}{625} - \frac{3348x}{125} + \frac{248x^2}{25} + \frac{121(2329-2759x)}{625(2+3x+5x^2)} \right) dx \\
&= \frac{1466x}{625} - \frac{54x^2}{125} + \frac{8x^3}{75} + \frac{1331(443+247x)}{96875(2+3x+5x^2)} + \frac{121 \int \frac{2329-2759x}{2+3x+5x^2} dx}{19375} \\
&= \frac{1466x}{625} - \frac{54x^2}{125} + \frac{8x^3}{75} + \frac{1331(443+247x)}{96875(2+3x+5x^2)} - \frac{10769 \int \frac{3+10x}{2+3x+5x^2} dx}{6250} + \frac{3819607 \int \frac{1}{2+3x+5x^2} dx}{193750} \\
&= \frac{1466x}{625} - \frac{54x^2}{125} + \frac{8x^3}{75} + \frac{1331(443+247x)}{96875(2+3x+5x^2)} - \frac{10769 \log(2+3x+5x^2)}{6250} - \frac{3819607 \operatorname{Sqrt}[2+3x+5x^2]}{193750} \\
&= \frac{1466x}{625} - \frac{54x^2}{125} + \frac{8x^3}{75} + \frac{1331(443+247x)}{96875(2+3x+5x^2)} + \frac{3819607 \tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{96875\sqrt{31}} - \frac{10769 \log(2+3x+5x^2)}{6250}
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 77, normalized size = 1.00

$$\frac{8x^3}{75} - \frac{54x^2}{125} + \frac{1331(247x+443)}{96875(5x^2+3x+2)} - \frac{10769 \log(5x^2+3x+2)}{6250} + \frac{1466x}{625} + \frac{3819607 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{96875\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)^3/(2 + 3\*x + 5\*x^2)^2,x]

[Out] (1466\*x)/625 - (54\*x^2)/125 + (8\*x^3)/75 + (1331\*(443 + 247\*x))/(96875\*(2 + 3\*x + 5\*x^2)) + (3819607\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(96875\*Sqrt[31]) - (10769\*Log[2 + 3\*x + 5\*x^2])/6250

**fricas [A]** time = 0.85, size = 88, normalized size = 1.14

$$\frac{9610000x^5 - 33154500x^4 + 191815600x^3 + 22917642\sqrt{31}(5x^2 + 3x + 2)\arctan\left(\frac{1}{31}\sqrt{31}(10x + 3)\right) + 111226}{18018750(5x^2 + 3x + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^3/(5\*x^2+3\*x+2)^2,x, algorithm="fricas")

[Out]  $\frac{1}{18018750}(9610000x^5 - 33154500x^4 + 191815600x^3 + 22917642\sqrt{31})(5x^2 + 3x + 2)\arctan\left(\frac{1}{31}\sqrt{31}(10x + 3)\right) + 111226140x^2 - 31047027(5x^2 + 3x + 2)\log(5x^2 + 3x + 2) + 145678362x + 109671738)/(5x^2 + 3x + 2)$

**giac** [A] time = 0.21, size = 62, normalized size = 0.81

$$\frac{8}{75}x^3 - \frac{54}{125}x^2 + \frac{3819607}{3003125}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x + 3)\right) + \frac{1466}{625}x + \frac{1331(247x + 443)}{96875(5x^2 + 3x + 2)} - \frac{10769}{6250}\log(5x^2 + 3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x, algorithm="giac")`

[Out]  $\frac{8}{75}x^3 - \frac{54}{125}x^2 + \frac{3819607}{3003125}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x + 3)\right) + 1466/625x + 1331/96875(247x + 443)/(5x^2 + 3x + 2) - 10769/6250\log(5x^2 + 3x + 2)$

**maple** [A] time = 0.01, size = 61, normalized size = 0.79

$$\frac{8x^3}{75} - \frac{54x^2}{125} + \frac{1466x}{625} + \frac{3819607\sqrt{31}\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{3003125} - \frac{10769\ln(5x^2 + 3x + 2)}{6250} - \frac{121\left(-\frac{2717x}{775} - \frac{4873}{775}\right)}{625\left(x^2 + \frac{3}{5}x + \frac{2}{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x)`

[Out]  $\frac{8}{75}x^3 - \frac{54}{125}x^2 + \frac{1466}{625}x - \frac{121}{625}\left(-\frac{2717}{775}x - \frac{4873}{775}\right)/(x^2 + \frac{3}{5}x + \frac{2}{5}) - \frac{10769}{6250}\ln(5x^2 + 3x + 2) + \frac{3819607}{3003125}31^{1/2}\arctan\left(\frac{1}{31}\sqrt{31}(10x + 3)\right)$

**maxima** [A] time = 0.96, size = 62, normalized size = 0.81

$$\frac{8}{75}x^3 - \frac{54}{125}x^2 + \frac{3819607}{3003125}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x + 3)\right) + \frac{1466}{625}x + \frac{1331(247x + 443)}{96875(5x^2 + 3x + 2)} - \frac{10769}{6250}\log(5x^2 + 3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x, algorithm="maxima")`

[Out]  $\frac{8}{75}x^3 - \frac{54}{125}x^2 + \frac{3819607}{3003125}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x + 3)\right) + 1466/625x + 1331/96875(247x + 443)/(5x^2 + 3x + 2) - 10769/6250\log(5x^2 + 3x + 2)$

**mupad [B]** time = 3.43, size = 61, normalized size = 0.79

$$\frac{1466x}{625} - \frac{10769 \ln(5x^2 + 3x + 2)}{6250} + \frac{\frac{328757x}{484375} + \frac{589633}{484375}}{x^2 + \frac{3x}{5} + \frac{2}{5}} + \frac{3819607 \sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{3003125} - \frac{54x^2}{125} + \frac{8x^3}{75}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2 - x + 3)^3/(3*x + 5*x^2 + 2)^2,x)`

[Out]  $(1466*x)/625 - (10769*\log(3*x + 5*x^2 + 2))/6250 + ((328757*x)/484375 + 589633/484375)/((3*x)/5 + x^2 + 2/5) + (3819607*31^{(1/2)}*\operatorname{atan}((10*31^{(1/2)}*x)/31 + (3*31^{(1/2)})/31))/3003125 - (54*x^2)/125 + (8*x^3)/75$

**sympy [A]** time = 0.19, size = 78, normalized size = 1.01

$$\frac{8x^3}{75} - \frac{54x^2}{125} + \frac{1466x}{625} + \frac{328757x + 589633}{484375x^2 + 290625x + 193750} - \frac{10769 \log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{6250} + \frac{3819607\sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{3003125}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**2-x+3)**3/(5*x**2+3*x+2)**2,x)`

[Out]  $8*x**3/75 - 54*x**2/125 + 1466*x/625 + (328757*x + 589633)/(484375*x**2 + 290625*x + 193750) - 10769*\log(x**2 + 3*x/5 + 2/5)/6250 + 3819607*\sqrt{31}*\operatorname{atan}(10*\sqrt{31}*x/31 + 3*\sqrt{31}/31)/3003125$

$$3.36 \quad \int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^3} dx$$

**Optimal.** Leaf size=84

$$\frac{121(342840x + 188381)}{6006250(5x^2 + 3x + 2)} + \frac{1331(247x + 443)}{193750(5x^2 + 3x + 2)^2} - \frac{66}{625} \log(5x^2 + 3x + 2) + \frac{8x}{125} + \frac{11341176 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{600625\sqrt{31}}$$

[Out] 8/125\*x+1331/193750\*(443+247\*x)/(5\*x^2+3\*x+2)^2+121/6006250\*(188381+342840\*x)/(5\*x^2+3\*x+2)-66/625\*ln(5\*x^2+3\*x+2)+11341176/18619375\*arctan(1/31\*(3+10\*x)\*31^(1/2))\*31^(1/2)

**Rubi [A]** time = 0.09, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$ , Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{121(342840x + 188381)}{6006250(5x^2 + 3x + 2)} + \frac{1331(247x + 443)}{193750(5x^2 + 3x + 2)^2} - \frac{66}{625} \log(5x^2 + 3x + 2) + \frac{8x}{125} + \frac{11341176 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{600625\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^3/(2 + 3\*x + 5\*x^2)^3,x]

[Out] (8\*x)/125 + (1331\*(443 + 247\*x))/(193750\*(2 + 3\*x + 5\*x^2)^2) + (121\*(188381 + 342840\*x))/(6006250\*(2 + 3\*x + 5\*x^2)) + (11341176\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(600625\*Sqrt[31]) - (66\*Log[2 + 3\*x + 5\*x^2])/625

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,

e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

### Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

### Rubi steps



$$\begin{aligned}
\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^3} dx &= \frac{1331(443+247x)}{193750(2+3x+5x^2)^2} + \frac{1}{62} \int \frac{\frac{4055767}{3125} - \frac{461962x}{625} + \frac{75764x^2}{125} - \frac{5208x^3}{25} + \frac{496x^4}{5}}{(2+3x+5x^2)^2} dx \\
&= \frac{1331(443+247x)}{193750(2+3x+5x^2)^2} + \frac{121(188381+342840x)}{6006250(2+3x+5x^2)} + \frac{\int \frac{\frac{2222876}{125} - \frac{207576x}{125} + \frac{15376x^2}{25}}{2+3x+5x^2} dx}{1922} \\
&= \frac{1331(443+247x)}{193750(2+3x+5x^2)^2} + \frac{121(188381+342840x)}{6006250(2+3x+5x^2)} + \frac{\int \left( \frac{15376}{125} + \frac{132(16607-1922x)}{125(2+3x+5x^2)} \right) dx}{1922} \\
&= \frac{8x}{125} + \frac{1331(443+247x)}{193750(2+3x+5x^2)^2} + \frac{121(188381+342840x)}{6006250(2+3x+5x^2)} + \frac{66 \int \frac{16607-1922x}{2+3x+5x^2} dx}{120125} \\
&= \frac{8x}{125} + \frac{1331(443+247x)}{193750(2+3x+5x^2)^2} + \frac{121(188381+342840x)}{6006250(2+3x+5x^2)} - \frac{66}{625} \int \frac{3+10x}{2+3x+5x^2} dx + \\
&= \frac{8x}{125} + \frac{1331(443+247x)}{193750(2+3x+5x^2)^2} + \frac{121(188381+342840x)}{6006250(2+3x+5x^2)} - \frac{66}{625} \log(2+3x+5x^2) - \\
&= \frac{8x}{125} + \frac{1331(443+247x)}{193750(2+3x+5x^2)^2} + \frac{121(188381+342840x)}{6006250(2+3x+5x^2)} + \frac{11341176 \tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{600625\sqrt{31}}
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 78, normalized size = 0.93

$$\frac{3751(342840x+188381)}{5x^2+3x+2} + \frac{1279091(247x+443)}{(5x^2+3x+2)^2} - 19662060 \log(5x^2+3x+2) + 11916400x + 113411760\sqrt{31} \tan^{-1}\left(\frac{10x}{\sqrt{31}}\right)$$


---

186193750

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)^3/(2 + 3\*x + 5\*x^2)^3,x]

[Out] (11916400\*x + (1279091\*(443 + 247\*x))/(2 + 3\*x + 5\*x^2) + (3751\*(188381 + 342840\*x))/(2 + 3\*x + 5\*x^2) + 113411760\*sqrt[31]\*ArcTan[(3 + 10\*x)/sqrt[31]] - 19662060\*Log[2 + 3\*x + 5\*x^2])/186193750

**fricas** [A] time = 0.88, size = 118, normalized size = 1.40

$$\frac{59582000x^5 + 71498400x^4 + 1355107960x^3 + 22682352\sqrt{31}(25x^4 + 30x^3 + 29x^2 + 12x + 4)\arctan\left(\frac{1}{31}\sqrt{31}\right)}{37238750(25x^2 + 3x + 2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^3/(5\*x^2+3\*x+2)^3,x, algorithm="fricas")

[Out] 1/37238750\*(59582000\*x^5 + 71498400\*x^4 + 1355107960\*x^3 + 22682352\*sqrt(31)\*(25\*x^4 + 30\*x^3 + 29\*x^2 + 12\*x + 4)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 1506812195\*x^2 - 3932412\*(25\*x^4 + 30\*x^3 + 29\*x^2 + 12\*x + 4)\*log(5\*x^2 + 3\*x + 2) + 1011087630\*x + 395974315)/(25\*x^4 + 30\*x^3 + 29\*x^2 + 12\*x + 4)

**giac** [A] time = 0.19, size = 62, normalized size = 0.74

$$\frac{11341176}{18619375}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) + \frac{8}{125}x + \frac{121(68568x^3 + 78817x^2 + 53402x + 21113)}{240250(5x^2 + 3x + 2)^2} - \frac{66}{625}\log(5x^2 + 3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^3/(5\*x^2+3\*x+2)^3,x, algorithm="giac")

[Out] 11341176/18619375\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 8/125\*x + 121/240250\*(68568\*x^3 + 78817\*x^2 + 53402\*x + 21113)/(5\*x^2 + 3\*x + 2)^2 - 66/625\*log(5\*x^2 + 3\*x + 2)

**maple** [A] time = 0.01, size = 63, normalized size = 0.75

$$\frac{8x}{125} + \frac{11341176\sqrt{31}\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{18619375} - \frac{66\ln(5x^2 + 3x + 2)}{625} - \frac{11\left(-\frac{377124}{24025}x^3 - \frac{866987}{48050}x^2 - \frac{293711}{24025}x - \frac{232243}{48050}\right)}{5(5x^2 + 3x + 2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^3/(5\*x^2+3\*x+2)^3,x)

[Out] 8/125\*x-11/5\*(-377124/24025\*x^3-866987/48050\*x^2-293711/24025\*x-232243/48050)/(5\*x^2+3\*x+2)^2-66/625\*ln(5\*x^2+3\*x+2)+11341176/18619375\*31^(1/2)\*arctan(1/31\*(10\*x+3)\*31^(1/2))

**maxima** [A] time = 0.97, size = 72, normalized size = 0.86

$$\frac{11341176}{18619375}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) + \frac{8}{125}x + \frac{121(68568x^3 + 78817x^2 + 53402x + 21113)}{240250(25x^4 + 30x^3 + 29x^2 + 12x + 4)} - \frac{66}{625}\log(5x^2 + 3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^3/(5\*x^2+3\*x+2)^3,x, algorithm="maxima")

[Out] 11341176/18619375\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 8/125\*x + 121/240250\*(68568\*x^3 + 78817\*x^2 + 53402\*x + 21113)/(25\*x^4 + 30\*x^3 + 29\*x^2 + 12\*x + 4) - 66/625\*log(5\*x^2 + 3\*x + 2)

**mupad [B]** time = 3.43, size = 71, normalized size = 0.85

$$\frac{8x}{125} - \frac{66 \ln(5x^2 + 3x + 2)}{625} + \frac{11341176 \sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{18619375} + \frac{\frac{4148364x^3}{3003125} + \frac{9536857x^2}{6006250} + \frac{3230821x}{3003125} + \frac{2554673}{6006250}}{x^4 + \frac{6x^3}{5} + \frac{29x^2}{25} + \frac{12x}{25} + \frac{4}{25}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)^3/(3\*x + 5\*x^2 + 2)^3,x)

[Out] (8\*x)/125 - (66\*log(3\*x + 5\*x^2 + 2))/625 + (11341176\*31^(1/2)\*atan((10\*31^(1/2)\*x)/31 + (3\*31^(1/2))/31))/18619375 + ((3230821\*x)/3003125 + (9536857\*x^2)/6006250 + (4148364\*x^3)/3003125 + 2554673/6006250)/((12\*x)/25 + (29\*x^2)/25 + (6\*x^3)/5 + x^4 + 4/25)

**sympy [A]** time = 0.23, size = 85, normalized size = 1.01

$$\frac{8x}{125} + \frac{8296728x^3 + 9536857x^2 + 6461642x + 2554673}{6006250x^4 + 7207500x^3 + 6967250x^2 + 2883000x + 961000} - \frac{66 \log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{625} + \frac{11341176\sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{18619375}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2-x+3)\*\*3/(5\*x\*\*2+3\*x+2)\*\*3,x)

[Out] 8\*x/125 + (8296728\*x\*\*3 + 9536857\*x\*\*2 + 6461642\*x + 2554673)/(6006250\*x\*\*4 + 7207500\*x\*\*3 + 6967250\*x\*\*2 + 2883000\*x + 961000) - 66\*log(x\*\*2 + 3\*x/5 + 2/5)/625 + 11341176\*sqrt(31)\*atan(10\*sqrt(31)\*x/31 + 3\*sqrt(31)/31)/18619375

$$3.37 \quad \int \frac{(2+3x+5x^2)^4}{3-x+2x^2} dx$$

Optimal. Leaf size=84

$$\frac{625x^7}{14} + \frac{3625x^6}{24} + \frac{1855x^5}{8} + \frac{6245x^4}{64} - \frac{21229x^3}{96} - \frac{28747x^2}{128} + \frac{307461}{512} \log(2x^2 - x + 3) + \frac{122691x}{128} + \frac{1156639 \tan^{-1}\left(\frac{1-\sqrt{23}}{\sqrt{23}}\right)}{256\sqrt{23}}$$

[Out] 122691/128\*x-28747/128\*x^2-21229/96\*x^3+6245/64\*x^4+1855/8\*x^5+3625/24\*x^6+625/14\*x^7+307461/512\*ln(2\*x^2-x+3)+1156639/5888\*arctan(1/23\*(1-4\*x)\*23^(1/2))\*23^(1/2)

**Rubi [A]** time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {1657, 634, 618, 204, 628}

$$\frac{625x^7}{14} + \frac{3625x^6}{24} + \frac{1855x^5}{8} + \frac{6245x^4}{64} - \frac{21229x^3}{96} - \frac{28747x^2}{128} + \frac{307461}{512} \log(2x^2 - x + 3) + \frac{122691x}{128} + \frac{1156639 \tan^{-1}\left(\frac{1-\sqrt{23}}{\sqrt{23}}\right)}{256\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3\*x + 5\*x^2)^4/(3 - x + 2\*x^2),x]

[Out] (122691\*x)/128 - (28747\*x^2)/128 - (21229\*x^3)/96 + (6245\*x^4)/64 + (1855\*x^5)/8 + (3625\*x^6)/24 + (625\*x^7)/14 + (1156639\*ArcTan[(1 - 4\*x)/Sqrt[23]])/(256\*Sqrt[23]) + (307461\*Log[3 - x + 2\*x^2])/512

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d},

e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

### Rule 1657

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Int[Expand[Integrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

### Rubi steps

$$\begin{aligned} \int \frac{(2 + 3x + 5x^2)^4}{3 - x + 2x^2} dx &= \int \left( \frac{122691}{128} - \frac{28747x}{64} - \frac{21229x^2}{32} + \frac{6245x^3}{16} + \frac{9275x^4}{8} + \frac{3625x^5}{4} + \frac{625x^6}{2} - \frac{14641(2 - x)}{128} \right) dx \\ &= \frac{122691x}{128} - \frac{28747x^2}{128} - \frac{21229x^3}{96} + \frac{6245x^4}{64} + \frac{1855x^5}{8} + \frac{3625x^6}{24} + \frac{625x^7}{14} - \frac{14641}{128} \int \frac{2 - x}{3 - x + 2x^2} dx \\ &= \frac{122691x}{128} - \frac{28747x^2}{128} - \frac{21229x^3}{96} + \frac{6245x^4}{64} + \frac{1855x^5}{8} + \frac{3625x^6}{24} + \frac{625x^7}{14} + \frac{307461}{512} \int \frac{2 - x}{3 - x + 2x^2} dx \\ &= \frac{122691x}{128} - \frac{28747x^2}{128} - \frac{21229x^3}{96} + \frac{6245x^4}{64} + \frac{1855x^5}{8} + \frac{3625x^6}{24} + \frac{625x^7}{14} + \frac{307461}{512} \log|2x^2 - x + 3| \\ &= \frac{122691x}{128} - \frac{28747x^2}{128} - \frac{21229x^3}{96} + \frac{6245x^4}{64} + \frac{1855x^5}{8} + \frac{3625x^6}{24} + \frac{625x^7}{14} + \frac{1156639}{256} \tan^{-1} \left( \frac{2x - 1}{\sqrt{5}} \right) \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 72, normalized size = 0.86

$$\frac{307461}{512} \log(2x^2 - x + 3) + \frac{x(120000x^6 + 406000x^5 + 623280x^4 + 262290x^3 - 594412x^2 - 603687x + 257651)}{2688}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x + 5\*x^2)^4/(3 - x + 2\*x^2), x]

[Out]  $(x*(2576511 - 603687*x - 594412*x^2 + 262290*x^3 + 623280*x^4 + 406000*x^5 + 120000*x^6))/2688 - (1156639*\text{ArcTan}[(-1 + 4*x)/\text{Sqrt}[23]])/(256*\text{Sqrt}[23]) + (307461*\text{Log}[3 - x + 2*x^2])/512$

**fricas** [A] time = 0.80, size = 63, normalized size = 0.75

$$\frac{625}{14}x^7 + \frac{3625}{24}x^6 + \frac{1855}{8}x^5 + \frac{6245}{64}x^4 - \frac{21229}{96}x^3 - \frac{28747}{128}x^2 - \frac{1156639}{5888}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{122691}{128}x + \frac{307461}{512}\log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x+2)^4/(2*x^2-x+3),x, algorithm="fricas")`

[Out]  $625/14*x^7 + 3625/24*x^6 + 1855/8*x^5 + 6245/64*x^4 - 21229/96*x^3 - 28747/128*x^2 - 1156639/5888*\text{sqrt}(23)*\text{arctan}(1/23*\text{sqrt}(23)*(4*x - 1)) + 122691/128*x + 307461/512*\text{log}(2*x^2 - x + 3)$

**giac** [A] time = 0.21, size = 63, normalized size = 0.75

$$\frac{625}{14}x^7 + \frac{3625}{24}x^6 + \frac{1855}{8}x^5 + \frac{6245}{64}x^4 - \frac{21229}{96}x^3 - \frac{28747}{128}x^2 - \frac{1156639}{5888}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{122691}{128}x + \frac{307461}{512}\log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x+2)^4/(2*x^2-x+3),x, algorithm="giac")`

[Out]  $625/14*x^7 + 3625/24*x^6 + 1855/8*x^5 + 6245/64*x^4 - 21229/96*x^3 - 28747/128*x^2 - 1156639/5888*\text{sqrt}(23)*\text{arctan}(1/23*\text{sqrt}(23)*(4*x - 1)) + 122691/128*x + 307461/512*\text{log}(2*x^2 - x + 3)$

**maple** [A] time = 0.01, size = 64, normalized size = 0.76

$$\frac{625x^7}{14} + \frac{3625x^6}{24} + \frac{1855x^5}{8} + \frac{6245x^4}{64} - \frac{21229x^3}{96} - \frac{28747x^2}{128} + \frac{122691x}{128} - \frac{1156639\sqrt{23}\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{5888} + \frac{307461\ln(2x^2-x+3)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^2+3*x+2)^4/(2*x^2-x+3),x)`

[Out]  $625/14*x^7+3625/24*x^6+1855/8*x^5+6245/64*x^4-21229/96*x^3-28747/128*x^2+122691/128*x+307461/512*\text{ln}(2*x^2-x+3)-1156639/5888*23^{(1/2)}*\text{arctan}(1/23*(4*x-1)*23^{(1/2)})$

**maxima** [A] time = 0.98, size = 63, normalized size = 0.75

$$\frac{625}{14}x^7 + \frac{3625}{24}x^6 + \frac{1855}{8}x^5 + \frac{6245}{64}x^4 - \frac{21229}{96}x^3 - \frac{28747}{128}x^2 - \frac{1156639}{5888}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{122691}{128}x + \frac{307461}{512}\log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^4/(2\*x^2-x+3),x, algorithm="maxima")

[Out] 625/14\*x^7 + 3625/24\*x^6 + 1855/8\*x^5 + 6245/64\*x^4 - 21229/96\*x^3 - 28747/128\*x^2 - 1156639/5888\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 122691/128\*x + 307461/512\*log(2\*x^2 - x + 3)

**mupad [B]** time = 3.44, size = 65, normalized size = 0.77

$$\frac{122691}{128}x + \frac{307461}{512}\ln(2x^2 - x + 3) - \frac{1156639\sqrt{23}}{5888}\operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right) - \frac{28747x^2}{128} - \frac{21229x^3}{96} + \frac{6245x^4}{64} + \frac{1855x^5}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x + 5\*x^2 + 2)^4/(2\*x^2 - x + 3),x)

[Out] (122691\*x)/128 + (307461\*log(2\*x^2 - x + 3))/512 - (1156639\*23^(1/2)\*atan((4\*23^(1/2)\*x)/23 - 23^(1/2)/23))/5888 - (28747\*x^2)/128 - (21229\*x^3)/96 + (6245\*x^4)/64 + (1855\*x^5)/8 + (3625\*x^6)/24 + (625\*x^7)/14

**sympy [A]** time = 0.17, size = 87, normalized size = 1.04

$$\frac{625x^7}{14} + \frac{3625x^6}{24} + \frac{1855x^5}{8} + \frac{6245x^4}{64} - \frac{21229x^3}{96} - \frac{28747x^2}{128} + \frac{122691x}{128} + \frac{307461\log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{512} - \frac{1156639\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{5888}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*2+3\*x+2)\*\*4/(2\*x\*\*2-x+3),x)

[Out] 625\*x\*\*7/14 + 3625\*x\*\*6/24 + 1855\*x\*\*5/8 + 6245\*x\*\*4/64 - 21229\*x\*\*3/96 - 28747\*x\*\*2/128 + 122691\*x/128 + 307461\*log(x\*\*2 - x/2 + 3/2)/512 - 1156639\*sqrt(23)\*atan(4\*sqrt(23)\*x/23 - sqrt(23)/23)/5888

$$3.38 \quad \int \frac{(2+3x+5x^2)^3}{3-x+2x^2} dx$$

Optimal. Leaf size=70

$$\frac{25x^5}{2} + \frac{575x^4}{16} + \frac{965x^3}{24} - \frac{829x^2}{32} + \frac{1331}{128} \log(2x^2 - x + 3) - \frac{4795x}{32} - \frac{59895 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{23}}$$

[Out]  $-4795/32*x-829/32*x^2+965/24*x^3+575/16*x^4+25/2*x^5+1331/128*\ln(2*x^2-x+3)-59895/1472*\arctan(1/23*(1-4*x)*23^{(1/2)})*23^{(1/2)}$

**Rubi [A]** time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {1657, 634, 618, 204, 628}

$$\frac{25x^5}{2} + \frac{575x^4}{16} + \frac{965x^3}{24} - \frac{829x^2}{32} + \frac{1331}{128} \log(2x^2 - x + 3) - \frac{4795x}{32} - \frac{59895 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3\*x + 5\*x^2)^3/(3 - x + 2\*x^2), x]

[Out]  $(-4795*x)/32 - (829*x^2)/32 + (965*x^3)/24 + (575*x^4)/16 + (25*x^5)/2 - (59895*ArcTan[(1 - 4*x)/Sqrt[23]])/(64*Sqrt[23]) + (1331*Log[3 - x + 2*x^2])/128$

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]



Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

Rule 1657

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Int[Expand[Integrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{(2 + 3x + 5x^2)^3}{3 - x + 2x^2} dx &= \int \left( -\frac{4795}{32} - \frac{829x}{16} + \frac{965x^2}{8} + \frac{575x^3}{4} + \frac{125x^4}{2} + \frac{1331(11 + x)}{32(3 - x + 2x^2)} \right) dx \\ &= -\frac{4795x}{32} - \frac{829x^2}{32} + \frac{965x^3}{24} + \frac{575x^4}{16} + \frac{25x^5}{2} + \frac{1331}{32} \int \frac{11 + x}{3 - x + 2x^2} dx \\ &= -\frac{4795x}{32} - \frac{829x^2}{32} + \frac{965x^3}{24} + \frac{575x^4}{16} + \frac{25x^5}{2} + \frac{1331}{128} \int \frac{-1 + 4x}{3 - x + 2x^2} dx + \frac{59895}{128} \int \frac{1}{3 - x + 2x^2} dx \\ &= -\frac{4795x}{32} - \frac{829x^2}{32} + \frac{965x^3}{24} + \frac{575x^4}{16} + \frac{25x^5}{2} + \frac{1331}{128} \log(3 - x + 2x^2) - \frac{59895}{64} \operatorname{Subst} \left[ \frac{1}{3 - x + 2x^2}, x, \frac{1 - 4x}{\sqrt{23}} \right] \\ &= -\frac{4795x}{32} - \frac{829x^2}{32} + \frac{965x^3}{24} + \frac{575x^4}{16} + \frac{25x^5}{2} - \frac{59895 \tan^{-1} \left( \frac{1 - 4x}{\sqrt{23}} \right)}{64\sqrt{23}} + \frac{1331}{128} \log(3 - x + 2x^2) \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 63, normalized size = 0.90

$$\frac{1}{384} \left( 3993 \log(2x^2 - x + 3) + 4x(1200x^4 + 3450x^3 + 3860x^2 - 2487x - 14385) \right) + \frac{59895 \tan^{-1} \left( \frac{4x-1}{\sqrt{23}} \right)}{64\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x + 5\*x^2)^3/(3 - x + 2\*x^2), x]

[Out] (59895\*ArcTan[(-1 + 4\*x)/Sqrt[23]])/(64\*Sqrt[23]) + (4\*x\*(-14385 - 2487\*x + 3860\*x^2 + 3450\*x^3 + 1200\*x^4) + 3993\*Log[3 - x + 2\*x^2])/384

**fricas** [A] time = 1.02, size = 53, normalized size = 0.76

$$\frac{25}{2}x^5 + \frac{575}{16}x^4 + \frac{965}{24}x^3 - \frac{829}{32}x^2 + \frac{59895}{1472}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{4795}{32}x + \frac{1331}{128}\log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^3/(2\*x^2-x+3),x, algorithm="fricas")

[Out] 25/2\*x^5 + 575/16\*x^4 + 965/24\*x^3 - 829/32\*x^2 + 59895/1472\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) - 4795/32\*x + 1331/128\*log(2\*x^2 - x + 3)

**giac** [A] time = 0.21, size = 53, normalized size = 0.76

$$\frac{25}{2}x^5 + \frac{575}{16}x^4 + \frac{965}{24}x^3 - \frac{829}{32}x^2 + \frac{59895}{1472}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{4795}{32}x + \frac{1331}{128}\log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^3/(2\*x^2-x+3),x, algorithm="giac")

[Out] 25/2\*x^5 + 575/16\*x^4 + 965/24\*x^3 - 829/32\*x^2 + 59895/1472\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) - 4795/32\*x + 1331/128\*log(2\*x^2 - x + 3)

**maple** [A] time = 0.00, size = 54, normalized size = 0.77

$$\frac{25x^5}{2} + \frac{575x^4}{16} + \frac{965x^3}{24} - \frac{829x^2}{32} - \frac{4795x}{32} + \frac{59895\sqrt{23}\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{1472} + \frac{1331\ln(2x^2 - x + 3)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+3\*x+2)^3/(2\*x^2-x+3),x)

[Out] 25/2\*x^5+575/16\*x^4+965/24\*x^3-829/32\*x^2-4795/32\*x+1331/128\*ln(2\*x^2-x+3)+59895/1472\*23^(1/2)\*arctan(1/23\*(4\*x-1)\*23^(1/2))

**maxima** [A] time = 0.97, size = 53, normalized size = 0.76

$$\frac{25}{2}x^5 + \frac{575}{16}x^4 + \frac{965}{24}x^3 - \frac{829}{32}x^2 + \frac{59895}{1472}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{4795}{32}x + \frac{1331}{128}\log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^3/(2\*x^2-x+3),x, algorithm="maxima")

[Out] 25/2\*x^5 + 575/16\*x^4 + 965/24\*x^3 - 829/32\*x^2 + 59895/1472\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) - 4795/32\*x + 1331/128\*log(2\*x^2 - x + 3)

**mupad [B]** time = 0.04, size = 55, normalized size = 0.79

$$\frac{1331 \ln(2x^2 - x + 3)}{128} - \frac{4795x}{32} + \frac{59895\sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{1472} - \frac{829x^2}{32} + \frac{965x^3}{24} + \frac{575x^4}{16} + \frac{25x^5}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3), x)`

[Out]  $(1331*\log(2*x^2 - x + 3))/128 - (4795*x)/32 + (59895*23^{(1/2)}*\operatorname{atan}((4*23^{(1/2)}*x)/23 - 23^{(1/2)}/23))/1472 - (829*x^2)/32 + (965*x^3)/24 + (575*x^4)/16 + (25*x^5)/2$

**sympy [A]** time = 0.15, size = 73, normalized size = 1.04

$$\frac{25x^5}{2} + \frac{575x^4}{16} + \frac{965x^3}{24} - \frac{829x^2}{32} - \frac{4795x}{32} + \frac{1331 \log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{128} + \frac{59895\sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{1472}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**2+3*x+2)**3/(2*x**2-x+3), x)`

[Out]  $25*x**5/2 + 575*x**4/16 + 965*x**3/24 - 829*x**2/32 - 4795*x/32 + 1331*\log(x**2 - x/2 + 3/2)/128 + 59895*\sqrt{23}*\operatorname{atan}(4*\sqrt{23}*x/23 - \sqrt{23}/23)/1472$

$$3.39 \quad \int \frac{(2+3x+5x^2)^2}{3-x+2x^2} dx$$

Optimal. Leaf size=56

$$\frac{25x^3}{6} + \frac{85x^2}{8} - \frac{363}{32} \log(2x^2 - x + 3) + \frac{51x}{8} + \frac{847 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{23}}$$

[Out] 51/8\*x+85/8\*x^2+25/6\*x^3-363/32\*ln(2\*x^2-x+3)+847/368\*arctan(1/23\*(1-4\*x)\*23^(1/2))\*23^(1/2)

**Rubi [A]** time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {1657, 634, 618, 204, 628}

$$\frac{25x^3}{6} + \frac{85x^2}{8} - \frac{363}{32} \log(2x^2 - x + 3) + \frac{51x}{8} + \frac{847 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3\*x + 5\*x^2)^2/(3 - x + 2\*x^2),x]

[Out] (51\*x)/8 + (85\*x^2)/8 + (25\*x^3)/6 + (847\*ArcTan[(1 - 4\*x)/Sqrt[23]])/(16\*Sqrt[23]) - (363\*Log[3 - x + 2\*x^2])/32

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

### Rubi steps

$$\begin{aligned} \int \frac{(2+3x+5x^2)^2}{3-x+2x^2} dx &= \int \left( \frac{51}{8} + \frac{85x}{4} + \frac{25x^2}{2} - \frac{121(1+3x)}{8(3-x+2x^2)} \right) dx \\ &= \frac{51x}{8} + \frac{85x^2}{8} + \frac{25x^3}{6} - \frac{121}{8} \int \frac{1+3x}{3-x+2x^2} dx \\ &= \frac{51x}{8} + \frac{85x^2}{8} + \frac{25x^3}{6} - \frac{363}{32} \int \frac{-1+4x}{3-x+2x^2} dx - \frac{847}{32} \int \frac{1}{3-x+2x^2} dx \\ &= \frac{51x}{8} + \frac{85x^2}{8} + \frac{25x^3}{6} - \frac{363}{32} \log(3-x+2x^2) + \frac{847}{16} \text{Subst} \left( \int \frac{1}{-23-x^2} dx, x, -1+4x \right) \\ &= \frac{51x}{8} + \frac{85x^2}{8} + \frac{25x^3}{6} + \frac{847 \tan^{-1} \left( \frac{1-4x}{\sqrt{23}} \right)}{16\sqrt{23}} - \frac{363}{32} \log(3-x+2x^2) \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 52, normalized size = 0.93

$$\frac{1}{24}x(100x^2 + 255x + 153) - \frac{363}{32} \log(2x^2 - x + 3) - \frac{847 \tan^{-1} \left( \frac{4x-1}{\sqrt{23}} \right)}{16\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2), x]
```

```
[Out] (x*(153 + 255*x + 100*x^2))/24 - (847*ArcTan[(-1 + 4*x)/Sqrt[23]])/(16*Sqrt
[23]) - (363*Log[3 - x + 2*x^2])/32
```

**fricas [A]** time = 0.85, size = 43, normalized size = 0.77

$$\frac{25}{6}x^3 + \frac{85}{8}x^2 - \frac{847}{368}\sqrt{23} \arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{51}{8}x - \frac{363}{32} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^2/(2\*x^2-x+3),x, algorithm="fricas")

[Out]  $\frac{25}{6}x^3 + \frac{85}{8}x^2 - \frac{847}{368}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{51}{8}x - \frac{363}{32}\log(2x^2-x+3)$

**giac** [A] time = 0.18, size = 43, normalized size = 0.77

$$\frac{25}{6}x^3 + \frac{85}{8}x^2 - \frac{847}{368}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{51}{8}x - \frac{363}{32}\log(2x^2-x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^2/(2\*x^2-x+3),x, algorithm="giac")

[Out]  $\frac{25}{6}x^3 + \frac{85}{8}x^2 - \frac{847}{368}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{51}{8}x - \frac{363}{32}\log(2x^2-x+3)$

**maple** [A] time = 0.00, size = 44, normalized size = 0.79

$$\frac{25x^3}{6} + \frac{85x^2}{8} + \frac{51x}{8} - \frac{847\sqrt{23}\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{368} - \frac{363\ln(2x^2-x+3)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+3\*x+2)^2/(2\*x^2-x+3),x)

[Out]  $\frac{25}{6}x^3 + \frac{85}{8}x^2 + \frac{51}{8}x - \frac{363}{32}\ln(2x^2-x+3) - \frac{847}{368}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)$

**maxima** [A] time = 0.96, size = 43, normalized size = 0.77

$$\frac{25}{6}x^3 + \frac{85}{8}x^2 - \frac{847}{368}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{51}{8}x - \frac{363}{32}\log(2x^2-x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^2/(2\*x^2-x+3),x, algorithm="maxima")

[Out]  $\frac{25}{6}x^3 + \frac{85}{8}x^2 - \frac{847}{368}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{51}{8}x - \frac{363}{32}\log(2x^2-x+3)$

**mupad** [B] time = 3.44, size = 45, normalized size = 0.80

$$\frac{51x}{8} - \frac{363\ln(2x^2-x+3)}{32} - \frac{847\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{368} + \frac{85x^2}{8} + \frac{25x^3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3), x)`

[Out]  $(51*x)/8 - (363*\log(2*x^2 - x + 3))/32 - (847*23^{(1/2)}*atan((4*23^{(1/2)}*x)/23 - 23^{(1/2)}/23))/368 + (85*x^2)/8 + (25*x^3)/6$

**sympy** [A] time = 0.15, size = 60, normalized size = 1.07

$$\frac{25x^3}{6} + \frac{85x^2}{8} + \frac{51x}{8} - \frac{363 \log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{32} - \frac{847\sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{368}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**2+3*x+2)**2/(2*x**2-x+3), x)`

[Out]  $25*x**3/6 + 85*x**2/8 + 51*x/8 - 363*\log(x**2 - x/2 + 3/2)/32 - 847*\sqrt{23} *atan(4*\sqrt{23}*x/23 - \sqrt{23}/23)/368$

$$3.40 \quad \int \frac{2+3x+5x^2}{3-x+2x^2} dx$$

Optimal. Leaf size=42

$$\frac{11}{8} \log(2x^2 - x + 3) + \frac{5x}{2} + \frac{33 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{23}}$$

[Out] 5/2\*x+11/8\*ln(2\*x^2-x+3)+33/92\*arctan(1/23\*(1-4\*x)\*23^(1/2))\*23^(1/2)

**Rubi [A]** time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$ , Rules used = {1657, 634, 618, 204, 628}

$$\frac{11}{8} \log(2x^2 - x + 3) + \frac{5x}{2} + \frac{33 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3\*x + 5\*x^2)/(3 - x + 2\*x^2), x]

[Out] (5\*x)/2 + (33\*ArcTan[(1 - 4\*x)/Sqrt[23]])/(4\*Sqrt[23]) + (11\*Log[3 - x + 2\*x^2])/8

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634



```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

### Rubi steps

$$\begin{aligned}
 \int \frac{2 + 3x + 5x^2}{3 - x + 2x^2} dx &= \int \left( \frac{5}{2} - \frac{11(1-x)}{2(3-x+2x^2)} \right) dx \\
 &= \frac{5x}{2} - \frac{11}{2} \int \frac{1-x}{3-x+2x^2} dx \\
 &= \frac{5x}{2} + \frac{11}{8} \int \frac{-1+4x}{3-x+2x^2} dx - \frac{33}{8} \int \frac{1}{3-x+2x^2} dx \\
 &= \frac{5x}{2} + \frac{11}{8} \log(3-x+2x^2) + \frac{33}{4} \text{Subst} \left( \int \frac{1}{-23-x^2} dx, x, -1+4x \right) \\
 &= \frac{5x}{2} + \frac{33 \tan^{-1} \left( \frac{1-4x}{\sqrt{23}} \right)}{4\sqrt{23}} + \frac{11}{8} \log(3-x+2x^2)
 \end{aligned}$$

**Mathematica** [A] time = 0.01, size = 42, normalized size = 1.00

$$\frac{11}{8} \log(2x^2 - x + 3) + \frac{5x}{2} - \frac{33 \tan^{-1} \left( \frac{4x-1}{\sqrt{23}} \right)}{4\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2), x]
```

```
[Out] (5*x)/2 - (33*ArcTan[(-1 + 4*x)/Sqrt[23]])/(4*Sqrt[23]) + (11*Log[3 - x + 2*x^2])/8
```

**fricas** [A] time = 0.86, size = 33, normalized size = 0.79

$$-\frac{33}{92} \sqrt{23} \arctan \left( \frac{1}{23} \sqrt{23} (4x-1) \right) + \frac{5}{2} x + \frac{11}{8} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)/(2\*x^2-x+3),x, algorithm="fricas")

[Out]  $-33/92\sqrt{23}\arctan(1/23\sqrt{23}(4x-1)) + 5/2x + 11/8\log(2x^2 - x + 3)$

**giac** [A] time = 0.21, size = 33, normalized size = 0.79

$$-\frac{33}{92}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{5}{2}x + \frac{11}{8}\log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)/(2\*x^2-x+3),x, algorithm="giac")

[Out]  $-33/92\sqrt{23}\arctan(1/23\sqrt{23}(4x-1)) + 5/2x + 11/8\log(2x^2 - x + 3)$

**maple** [A] time = 0.00, size = 34, normalized size = 0.81

$$\frac{5x}{2} - \frac{33\sqrt{23}\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{92} + \frac{11\ln(2x^2 - x + 3)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+3\*x+2)/(2\*x^2-x+3),x)

[Out]  $5/2x + 11/8\ln(2x^2 - x + 3) - 33/92\sqrt{23}\arctan(1/23\sqrt{23}(4x-1))$

**maxima** [A] time = 0.95, size = 33, normalized size = 0.79

$$-\frac{33}{92}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{5}{2}x + \frac{11}{8}\log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)/(2\*x^2-x+3),x, algorithm="maxima")

[Out]  $-33/92\sqrt{23}\arctan(1/23\sqrt{23}(4x-1)) + 5/2x + 11/8\log(2x^2 - x + 3)$

**mupad** [B] time = 0.04, size = 35, normalized size = 0.83

$$\frac{5x}{2} + \frac{11\ln(2x^2 - x + 3)}{8} - \frac{33\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x - \sqrt{23}}{23}\right)}{92}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x + 5*x^2 + 2)/(2*x^2 - x + 3), x)`

[Out]  $(5*x)/2 + (11*\log(2*x^2 - x + 3))/8 - (33*23^{(1/2)}*\operatorname{atan}((4*23^{(1/2)}*x)/23 - 23^{(1/2)}/23))/92$

**sympy [A]** time = 0.14, size = 46, normalized size = 1.10

$$\frac{5x}{2} + \frac{11 \log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{8} - \frac{33\sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{92}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**2+3*x+2)/(2*x**2-x+3), x)`

[Out]  $5*x/2 + 11*\log(x**2 - x/2 + 3/2)/8 - 33*\operatorname{sqrt}(23)*\operatorname{atan}(4*\operatorname{sqrt}(23)*x/23 - \operatorname{sqrt}(23)/23)/92$

$$3.41 \quad \int \frac{1}{(3-x+2x^2)(2+3x+5x^2)} dx$$

Optimal. Leaf size=73

$$-\frac{1}{44} \log(2x^2 - x + 3) + \frac{1}{44} \log(5x^2 + 3x + 2) + \frac{3 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{22\sqrt{23}} + \frac{13 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{22\sqrt{31}}$$

[Out]  $-1/44*\ln(2*x^2-x+3)+1/44*\ln(5*x^2+3*x+2)+3/506*\arctan(1/23*(1-4*x)*23^{(1/2)})*23^{(1/2)}+13/682*\arctan(1/31*(3+10*x)*31^{(1/2)})*31^{(1/2)}$

**Rubi [A]** time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {980, 634, 618, 204, 628}

$$-\frac{1}{44} \log(2x^2 - x + 3) + \frac{1}{44} \log(5x^2 + 3x + 2) + \frac{3 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{22\sqrt{23}} + \frac{13 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{22\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - x + 2\*x^2)\*(2 + 3\*x + 5\*x^2)),x]

[Out]  $(3*\text{ArcTan}[(1 - 4*x)/\text{Sqrt}[23]])/(22*\text{Sqrt}[23]) + (13*\text{ArcTan}[(3 + 10*x)/\text{Sqrt}[31]])/(22*\text{Sqrt}[31]) - \text{Log}[3 - x + 2*x^2]/44 + \text{Log}[2 + 3*x + 5*x^2]/44$

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 980

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] := With[{q = c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(c^2*d - b*c*e + b^2*f - a*c*f - (c^2*e - b*c*f)*x)/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[(c*e^2 - c*d*f - b*e*f + a*f^2 + (c*e*f - b*f^2)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{1}{(3-x+2x^2)(2+3x+5x^2)} dx &= \frac{1}{242} \int \frac{-11-22x}{3-x+2x^2} dx + \frac{1}{242} \int \frac{88+55x}{2+3x+5x^2} dx \\ &= -\left(\frac{1}{44} \int \frac{-1+4x}{3-x+2x^2} dx\right) + \frac{1}{44} \int \frac{3+10x}{2+3x+5x^2} dx - \frac{3}{44} \int \frac{1}{3-x+2x^2} dx \\ &= -\frac{1}{44} \log(3-x+2x^2) + \frac{1}{44} \log(2+3x+5x^2) + \frac{3}{22} \text{Subst}\left(\int \frac{1}{-23-x^2} dx\right) \\ &= \frac{3 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{22\sqrt{23}} + \frac{13 \tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{22\sqrt{31}} - \frac{1}{44} \log(3-x+2x^2) + \frac{1}{44} \log(2+3x+5x^2) \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 73, normalized size = 1.00

$$-\frac{1}{44} \log(2x^2 - x + 3) + \frac{1}{44} \log(5x^2 + 3x + 2) - \frac{3 \tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{22\sqrt{23}} + \frac{13 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{22\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)),x]
```

```
[Out] (-3*ArcTan[(-1 + 4*x)/Sqrt[23]])/(22*Sqrt[23]) + (13*ArcTan[(3 + 10*x)/Sqrt[31]])/(22*Sqrt[31]) - Log[3 - x + 2*x^2]/44 + Log[2 + 3*x + 5*x^2]/44
```

**fricas** [A] time = 0.81, size = 59, normalized size = 0.81

$$\frac{13}{682} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x + 3)\right) - \frac{3}{506} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4x - 1)\right) + \frac{1}{44} \log(5x^2 + 3x + 2) - \frac{1}{44} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)/(5\*x^2+3\*x+2),x, algorithm="fricas")

[Out] 13/682\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) - 3/506\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 1/44\*log(5\*x^2 + 3\*x + 2) - 1/44\*log(2\*x^2 - x + 3)

**giac** [A] time = 0.19, size = 59, normalized size = 0.81

$$\frac{13}{682} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x + 3)\right) - \frac{3}{506} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4x - 1)\right) + \frac{1}{44} \log(5x^2 + 3x + 2) - \frac{1}{44} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)/(5\*x^2+3\*x+2),x, algorithm="giac")

[Out] 13/682\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) - 3/506\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 1/44\*log(5\*x^2 + 3\*x + 2) - 1/44\*log(2\*x^2 - x + 3)

**maple** [A] time = 0.01, size = 60, normalized size = 0.82

$$\frac{13\sqrt{31} \arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right) - 3\sqrt{23} \arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right) - \ln(2x^2 - x + 3) + \ln(5x^2 + 3x + 2)}{682} - \frac{3\sqrt{23} \arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{506} - \frac{\ln(2x^2 - x + 3)}{44} + \frac{\ln(5x^2 + 3x + 2)}{44}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2\*x^2-x+3)/(5\*x^2+3\*x+2),x)

[Out] 1/44\*ln(5\*x^2+3\*x+2)+13/682\*31^(1/2)\*arctan(1/31\*(10\*x+3)\*31^(1/2))-1/44\*ln(2\*x^2-x+3)-3/506\*23^(1/2)\*arctan(1/23\*(4\*x-1)\*23^(1/2))

**maxima** [A] time = 0.96, size = 59, normalized size = 0.81

$$\frac{13}{682} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x + 3)\right) - \frac{3}{506} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4x - 1)\right) + \frac{1}{44} \log(5x^2 + 3x + 2) - \frac{1}{44} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)/(5\*x^2+3\*x+2),x, algorithm="maxima")

[Out]  $13/682*\sqrt{31}*\arctan(1/31*\sqrt{31}*(10*x + 3)) - 3/506*\sqrt{23}*\arctan(1/23*\sqrt{23}*(4*x - 1)) + 1/44*\log(5*x^2 + 3*x + 2) - 1/44*\log(2*x^2 - x + 3)$   
)

**mupad [B]** time = 0.19, size = 79, normalized size = 1.08

$$\ln\left(x - \frac{1}{4} - \frac{\sqrt{23} 1i}{4}\right)\left(-\frac{1}{44} + \frac{\sqrt{23} 3i}{1012}\right) - \ln\left(x - \frac{1}{4} + \frac{\sqrt{23} 1i}{4}\right)\left(\frac{1}{44} + \frac{\sqrt{23} 3i}{1012}\right) - \ln\left(x + \frac{3}{10} - \frac{\sqrt{31} 1i}{10}\right)\left(-\frac{1}{44} + \frac{\sqrt{31} 13i}{1364}\right) - \ln\left(x + \frac{3}{10} + \frac{\sqrt{31} 1i}{10}\right)\left(\frac{1}{44} + \frac{\sqrt{31} 13i}{1364}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((2*x^2 - x + 3)*(3*x + 5*x^2 + 2)),x)`

[Out]  $\log(x - (23^{(1/2)*1i})/4 - 1/4)*((23^{(1/2)*3i})/1012 - 1/44) - \log(x + (23^{(1/2)*1i})/4 - 1/4)*((23^{(1/2)*3i})/1012 + 1/44) - \log(x - (31^{(1/2)*1i})/10 + 3/10)*((31^{(1/2)*13i})/1364 - 1/44) + \log(x + (31^{(1/2)*1i})/10 + 3/10)*((31^{(1/2)*13i})/1364 + 1/44)$

**sympy [A]** time = 0.24, size = 83, normalized size = 1.14

$$-\frac{\log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{44} + \frac{\log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{44} - \frac{3\sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{506} + \frac{13\sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{682}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2*x**2-x+3)/(5*x**2+3*x+2),x)`

[Out]  $-\log(x**2 - x/2 + 3/2)/44 + \log(x**2 + 3*x/5 + 2/5)/44 - 3*\sqrt{23}*\operatorname{atan}(4*\sqrt{23}*x/23 - \sqrt{23}/23)/506 + 13*\sqrt{31}*\operatorname{atan}(10*\sqrt{31}*x/31 + 3*\sqrt{31}/31)/682$

$$3.42 \quad \int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^2} dx$$

Optimal. Leaf size=94

$$\frac{65x+4}{682(5x^2+3x+2)} + \frac{3}{968} \log(2x^2-x+3) - \frac{3}{968} \log(5x^2+3x+2) + \frac{7 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{484\sqrt{23}} + \frac{2891 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{15004\sqrt{31}}$$

[Out] 1/682\*(4+65\*x)/(5\*x^2+3\*x+2)+3/968\*ln(2\*x^2-x+3)-3/968\*ln(5\*x^2+3\*x+2)+7/11132\*arctan(1/23\*(1-4\*x)\*23^(1/2))\*23^(1/2)+2891/465124\*arctan(1/31\*(3+10\*x)\*31^(1/2))\*31^(1/2)

**Rubi [A]** time = 0.09, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$ , Rules used = {974, 1072, 634, 618, 204, 628}

$$\frac{65x+4}{682(5x^2+3x+2)} + \frac{3}{968} \log(2x^2-x+3) - \frac{3}{968} \log(5x^2+3x+2) + \frac{7 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{484\sqrt{23}} + \frac{2891 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{15004\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - x + 2\*x^2)\*(2 + 3\*x + 5\*x^2)^2), x]

[Out] (4 + 65\*x)/(682\*(2 + 3\*x + 5\*x^2)) + (7\*ArcTan[(1 - 4\*x)/Sqrt[23]])/(484\*Sqrt[23]) + (2891\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(15004\*Sqrt[31]) + (3\*Log[3 - x + 2\*x^2])/968 - (3\*Log[2 + 3\*x + 5\*x^2])/968

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d},



e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

### Rule 974

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_)\*((d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2)^(q\_), x\_Symbol] := Simp[((2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f) + c\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*x)\*(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^(q + 1))/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1)), x] - Dist[1/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1)), Int[(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^q\*Simp[2\*c\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1) - (2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(a\*f\*(p + 1) - c\*d\*(p + 2)) - e\*(b^2\*c\*e - 2\*a\*c^2\*e - b^3\*f - b\*c\*(c\*d - 3\*a\*f))\*(p + q + 2) + (2\*f\*(2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f))\*(p + q + 2) - (2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(b\*f\*(p + 1) - c\*e\*(2\*p + q + 4)))\*x + c\*f\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(2\*p + 2\*q + 5)\*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && LtQ[p, -1] && NeQ[(c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]

### Rule 1072

Int[((A\_.) + (B\_.)\*(x\_) + (C\_.)\*(x\_)^2)/(((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*((d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2)), x\_Symbol] := With[{q = c^2\*d^2 - b\*c\*d\*e + a\*c\*e^2 + b^2\*d\*f - 2\*a\*c\*d\*f - a\*b\*e\*f + a^2\*f^2}, Dist[1/q, Int[(A\*c^2\*d - a\*c\*C\*d - A\*b\*c\*e + a\*B\*c\*e + A\*b^2\*f - a\*b\*B\*f - a\*A\*c\*f + a^2\*C\*f + c\*(B\*c\*d - b\*C\*d - A\*c\*e + a\*C\*e + A\*b\*f - a\*B\*f)\*x)/(a + b\*x + c\*x^2), x], x] + Dist[1/q, Int[(c\*C\*d^2 - B\*c\*d\*e + A\*c\*e^2 + b\*B\*d\*f - A\*c\*d\*f - a\*C\*d\*f - A\*b\*e\*f + a\*A\*f^2 - f\*(B\*c\*d - b\*C\*d - A\*c\*e + a\*C\*e + A\*b\*f - a\*B\*f)\*x)/(d + e\*x + f\*x^2), x], x] /; NeQ[q, 0] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0]

### Rubi steps

$$\begin{aligned}
\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^2} dx &= \frac{4+65x}{682(2+3x+5x^2)} - \frac{\int \frac{-1804+1397x-1430x^2}{(3-x+2x^2)(2+3x+5x^2)} dx}{7502} \\
&= \frac{4+65x}{682(2+3x+5x^2)} - \frac{\int \frac{18755-22506x}{3-x+2x^2} dx}{1815484} - \frac{\int \frac{-158026+56265x}{2+3x+5x^2} dx}{1815484} \\
&= \frac{4+65x}{682(2+3x+5x^2)} + \frac{3}{968} \int \frac{-1+4x}{3-x+2x^2} dx - \frac{3}{968} \int \frac{3+10x}{2+3x+5x^2} dx - \\
&= \frac{4+65x}{682(2+3x+5x^2)} + \frac{3}{968} \log(3-x+2x^2) - \frac{3}{968} \log(2+3x+5x^2) + \\
&= \frac{4+65x}{682(2+3x+5x^2)} + \frac{7 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{484\sqrt{23}} + \frac{2891 \tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{15004\sqrt{31}} + \frac{3}{968} \log(3-
\end{aligned}$$

**Mathematica [A]** time = 0.08, size = 94, normalized size = 1.00

$$\frac{65x+4}{682(5x^2+3x+2)} + \frac{3}{968} \log(2x^2-x+3) - \frac{3}{968} \log(5x^2+3x+2) - \frac{7 \tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{484\sqrt{23}} + \frac{2891 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{15004\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((3-x+2\*x^2)\*(2+3\*x+5\*x^2)^2),x]

[Out] (4+65\*x)/(682\*(2+3\*x+5\*x^2)) - (7\*ArcTan[(-1+4\*x)/Sqrt[23]])/(484\*Sqrt[23]) + (2891\*ArcTan[(3+10\*x)/Sqrt[31]])/(15004\*Sqrt[31]) + (3\*Log[3-x+2\*x^2])/968 - (3\*Log[2+3\*x+5\*x^2])/968

**fricas [A]** time = 0.96, size = 117, normalized size = 1.24

$$\frac{132986 \sqrt{31} (5x^2 + 3x + 2) \arctan\left(\frac{1}{31} \sqrt{31} (10x + 3)\right) - 13454 \sqrt{23} (5x^2 + 3x + 2) \arctan\left(\frac{1}{23} \sqrt{23} (4x - 1)\right) - 66}{21395704 (5x^2 + 3x + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)/(5\*x^2+3\*x+2)^2,x, algorithm="fricas")

[Out] 1/21395704\*(132986\*sqrt(31)\*(5\*x^2+3\*x+2)\*arctan(1/31\*sqrt(31)\*(10\*x+3)) - 13454\*sqrt(23)\*(5\*x^2+3\*x+2)\*arctan(1/23\*sqrt(23)\*(4\*x-1)) - 66

$309*(5*x^2 + 3*x + 2)*\log(5*x^2 + 3*x + 2) + 66309*(5*x^2 + 3*x + 2)*\log(2*x^2 - x + 3) + 2039180*x + 125488)/(5*x^2 + 3*x + 2)$

**giac [A]** time = 0.20, size = 78, normalized size = 0.83

$$\frac{2891}{465124} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x + 3)\right) - \frac{7}{11132} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4x - 1)\right) + \frac{65x + 4}{682(5x^2 + 3x + 2)} - \frac{3}{968} \log$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)/(5\*x^2+3\*x+2)^2,x, algorithm="giac")

[Out] 2891/465124\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) - 7/11132\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 1/682\*(65\*x + 4)/(5\*x^2 + 3\*x + 2) - 3/968\*log(5\*x^2 + 3\*x + 2) + 3/968\*log(2\*x^2 - x + 3)

**maple [A]** time = 0.01, size = 77, normalized size = 0.82

$$\frac{2891\sqrt{31} \arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right) - 7\sqrt{23} \arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right) + 3 \ln(2x^2 - x + 3) - 3 \ln(5x^2 + 3x + 2) - \frac{286x}{31}}{465124 - \frac{11132}{968} + \frac{968}{968} - \frac{484}{484} \left(x^2 + \dots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2\*x^2-x+3)/(5\*x^2+3\*x+2)^2,x)

[Out] -1/484\*(-286/31\*x-88/155)/(x^2+3/5\*x+2/5)-3/968\*ln(5\*x^2+3\*x+2)+2891/465124\*31^(1/2)\*arctan(1/31\*(10\*x+3)\*31^(1/2))+3/968\*ln(2\*x^2-x+3)-7/11132\*23^(1/2)\*arctan(1/23\*(4\*x-1)\*23^(1/2))

**maxima [A]** time = 0.96, size = 78, normalized size = 0.83

$$\frac{2891}{465124} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x + 3)\right) - \frac{7}{11132} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4x - 1)\right) + \frac{65x + 4}{682(5x^2 + 3x + 2)} - \frac{3}{968} \log$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)/(5\*x^2+3\*x+2)^2,x, algorithm="maxima")

[Out] 2891/465124\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) - 7/11132\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 1/682\*(65\*x + 4)/(5\*x^2 + 3\*x + 2) - 3/968\*log(5\*x^2 + 3\*x + 2) + 3/968\*log(2\*x^2 - x + 3)

**mupad [B]** time = 3.57, size = 95, normalized size = 1.01

$$\frac{\frac{13x}{682} + \frac{2}{1705}}{x^2 + \frac{3x}{5} + \frac{2}{5}} + \ln\left(x - \frac{1}{4} - \frac{\sqrt{23} 1i}{4}\right) \left(\frac{3}{968} + \frac{\sqrt{23} 7i}{22264}\right) - \ln\left(x - \frac{1}{4} + \frac{\sqrt{23} 1i}{4}\right) \left(-\frac{3}{968} + \frac{\sqrt{23} 7i}{22264}\right) - \ln\left(x + \frac{3}{10} - \frac{\sqrt{3}}{1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((2*x^2 - x + 3)*(3*x + 5*x^2 + 2)^2),x)`

[Out]  $((13x)/682 + 2/1705)/((3x)/5 + x^2 + 2/5) + \log(x - (23^{(1/2)}i)/4 - 1/4) * ((23^{(1/2)}i)/22264 + 3/968) - \log(x + (23^{(1/2)}i)/4 - 1/4) * ((23^{(1/2)}i)/22264 - 3/968) - \log(x - (31^{(1/2)}i)/10 + 3/10) * ((31^{(1/2)}i)/930248 + 3/968) + \log(x + (31^{(1/2)}i)/10 + 3/10) * ((31^{(1/2)}i)/930248 - 3/968)$

**sympy [A]** time = 0.32, size = 102, normalized size = 1.09

$$\frac{65x + 4}{3410x^2 + 2046x + 1364} + \frac{3 \log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{968} - \frac{3 \log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{968} - \frac{7\sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{11132} + \frac{2891\sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{465124}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2*x**2-x+3)/(5*x**2+3*x+2)**2,x)`

[Out]  $(65x + 4)/(3410x^2 + 2046x + 1364) + 3 \log(x^2 - x/2 + 3/2)/968 - 3 \log(x^2 + 3x/5 + 2/5)/968 - 7 \sqrt{23} \operatorname{atan}(4 \sqrt{23} x / 23 - \sqrt{23} / 23) / 11132 + 2891 \sqrt{31} \operatorname{atan}(10 \sqrt{31} x / 31 + 3 \sqrt{31} / 31) / 465124$

$$3.43 \quad \int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^3} dx$$

Optimal. Leaf size=115

$$\frac{65x+4}{1364(5x^2+3x+2)^2} + \frac{21605x+7923}{465124(5x^2+3x+2)} - \frac{\log(2x^2-x+3)}{21296} + \frac{\log(5x^2+3x+2)}{21296} - \frac{45 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{10648\sqrt{23}} + \frac{847793}{10648\sqrt{23}}$$

[Out] 1/1364\*(4+65\*x)/(5\*x^2+3\*x+2)^2+1/465124\*(7923+21605\*x)/(5\*x^2+3\*x+2)-1/21296\*ln(2\*x^2-x+3)+1/21296\*ln(5\*x^2+3\*x+2)-45/244904\*arctan(1/23\*(1-4\*x)\*23^(1/2))\*23^(1/2)+847793/317214568\*arctan(1/31\*(3+10\*x)\*31^(1/2))\*31^(1/2)

**Rubi [A]** time = 0.12, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$ , Rules used = {974, 1060, 1072, 634, 618, 204, 628}

$$\frac{65x+4}{1364(5x^2+3x+2)^2} + \frac{21605x+7923}{465124(5x^2+3x+2)} - \frac{\log(2x^2-x+3)}{21296} + \frac{\log(5x^2+3x+2)}{21296} - \frac{45 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{10648\sqrt{23}} + \frac{847793}{10648\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Int[1/((3-x+2\*x^2)\*(2+3\*x+5\*x^2)^3),x]

[Out] (4+65\*x)/(1364\*(2+3\*x+5\*x^2)^2) + (7923+21605\*x)/(465124\*(2+3\*x+5\*x^2)) - (45\*ArcTan[(1-4\*x)/Sqrt[23]])/(10648\*Sqrt[23]) + (847793\*ArcTan[(3+10\*x)/Sqrt[31]])/(10232728\*Sqrt[31]) - Log[3-x+2\*x^2]/21296 + Log[2+3\*x+5\*x^2]/21296

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2-4\*a\*c-x^2, x], x], x, b+2\*c\*x, x] /; FreeQ[{a, b, c}, x] && NeQ[b^2-4\*a\*c, 0]

#### Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 974

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((d_) + (e_)*(x_) + (f_)*(x
_)^2)^(q_), x_Symbol] := Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q,
0]
```

### Rule 1060

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((A_) + (B_)*(x_) + (C_)*(x
_)^2)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x +
c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
+ q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
```

```

A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f))*
(b*f*(p + 1) - c*e*(2*p + q + 4))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && !LtQ[q, -1]) && !
IGtQ[q, 0]

```

### Rule 1072

```

Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
+ c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x]/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x]/(d + e*x + f*x^2), x], x] /; NeQ[q, 0] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]

```

### Rubi steps

$$\begin{aligned}
\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^3} dx &= \frac{4+65x}{1364(2+3x+5x^2)^2} - \frac{\int \frac{-5753+3509x-4290x^2}{(3-x+2x^2)(2+3x+5x^2)^2} dx}{15004} \\
&= \frac{4+65x}{1364(2+3x+5x^2)^2} + \frac{7923+21605x}{465124(2+3x+5x^2)} - \frac{\int \frac{-14522420+3833038x-1045}{(3-x+2x^2)(2+3x+5x^2)} dx}{112560008} \\
&= \frac{4+65x}{1364(2+3x+5x^2)^2} + \frac{7923+21605x}{465124(2+3x+5x^2)} - \frac{\int \frac{-58838186+5116364x}{3-x+2x^2} dx}{27239521936} \\
&= \frac{4+65x}{1364(2+3x+5x^2)^2} + \frac{7923+21605x}{465124(2+3x+5x^2)} - \frac{\int \frac{-1+4x}{3-x+2x^2} dx}{21296} + \frac{\int \frac{3+10}{2+3x+5x^2} dx}{21296} \\
&= \frac{4+65x}{1364(2+3x+5x^2)^2} + \frac{7923+21605x}{465124(2+3x+5x^2)} - \frac{\log(3-x+2x^2)}{21296} + \frac{\log(2+3x+5x^2)}{21296} \\
&= \frac{4+65x}{1364(2+3x+5x^2)^2} + \frac{7923+21605x}{465124(2+3x+5x^2)} - \frac{45 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{10648\sqrt{23}} + \frac{8477}{10648\sqrt{23}}
\end{aligned}$$

**Mathematica [A]** time = 0.16, size = 104, normalized size = 0.90

$$\frac{31 \left( -961 \log(2x^2 - x + 3) + 961 \log(5x^2 + 3x + 2) + \frac{44(108025x^3 + 104430x^2 + 89144x + 17210)}{(5x^2 + 3x + 2)^2} \right) + 1695586\sqrt{31} \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{634429136}$$

Antiderivative was successfully verified.

[In] Integrate[1/((3 - x + 2\*x^2)\*(2 + 3\*x + 5\*x^2)^3), x]

[Out] (45\*ArcTan[(-1 + 4\*x)/Sqrt[23]])/(10648\*Sqrt[23]) + (1695586\*Sqrt[31]\*ArcTan[(3 + 10\*x)/Sqrt[31]] + 31\*((44\*(17210 + 89144\*x + 104430\*x^2 + 108025\*x^3))/(2 + 3\*x + 5\*x^2)^2 - 961\*Log[3 - x + 2\*x^2] + 961\*Log[2 + 3\*x + 5\*x^2]))/634429136

**fricas [A]** time = 1.02, size = 177, normalized size = 1.54

$$\frac{3388960300 x^3 + 38998478 \sqrt{31} (25 x^4 + 30 x^3 + 29 x^2 + 12 x + 4) \arctan\left(\frac{1}{31} \sqrt{31} (10 x + 3)\right) + 2681190 \sqrt{23} (2 + 3 x + 5 x^2)}{634429136}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)/(5\*x^2+3\*x+2)^3,x, algorithm="fricas")

[Out] 1/14591870128\*(3388960300\*x^3 + 38998478\*sqrt(31)\*(25\*x^4 + 30\*x^3 + 29\*x^2 + 12\*x + 4)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 2681190\*sqrt(23)\*(25\*x^4 + 30\*x^3 + 29\*x^2 + 12\*x + 4)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 3276177960\*x^2 + 685193\*(25\*x^4 + 30\*x^3 + 29\*x^2 + 12\*x + 4)\*log(5\*x^2 + 3\*x + 2) - 685193\*(25\*x^4 + 30\*x^3 + 29\*x^2 + 12\*x + 4)\*log(2\*x^2 - x + 3) + 2796625568\*x + 539912120)/(25\*x^4 + 30\*x^3 + 29\*x^2 + 12\*x + 4)

**giac** [A] time = 0.18, size = 88, normalized size = 0.77

$$\frac{847793}{317214568} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x + 3)\right) + \frac{45}{244904} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4x - 1)\right) + \frac{108025x^3 + 104430x^2 - 89144x + 17210}{465124(5x^2 + 3x + 2)^2} + \frac{1}{21296} \log(5x^2 + 3x + 2) - \frac{1}{21296} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)/(5\*x^2+3\*x+2)^3,x, algorithm="giac")

[Out] 847793/317214568\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 45/244904\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 1/465124\*(108025\*x^3 + 104430\*x^2 + 89144\*x + 17210)/(5\*x^2 + 3\*x + 2)^2 + 1/21296\*log(5\*x^2 + 3\*x + 2) - 1/21296\*log(2\*x^2 - x + 3)

**maple** [A] time = 0.01, size = 89, normalized size = 0.77

$$\frac{847793\sqrt{31} \arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right) + 45\sqrt{23} \arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right) + \ln(2x^2 - x + 3) + \ln(5x^2 + 3x + 2) + \frac{108025}{465124}x^3 + \frac{104430}{465124}x^2 - \frac{89144}{465124}x + \frac{17210}{465124}}{317214568 + \frac{45}{244904} + \frac{1}{21296} + \frac{1}{21296}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2\*x^2-x+3)/(5\*x^2+3\*x+2)^3,x)

[Out] 25/10648\*(95062/961\*x^3+459492/4805\*x^2+1961168/24025\*x+75724/4805)/(5\*x^2+3\*x+2)^2+1/21296\*ln(5\*x^2+3\*x+2)+847793/317214568\*31^(1/2)\*arctan(1/31\*(10\*x+3)\*31^(1/2))-1/21296\*ln(2\*x^2-x+3)+45/244904\*23^(1/2)\*arctan(1/23\*(4\*x-1)\*23^(1/2))

**maxima** [A] time = 0.96, size = 98, normalized size = 0.85

$$\frac{847793}{317214568} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x + 3)\right) + \frac{45}{244904} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4x - 1)\right) + \frac{108025x^3 + 104430x^2 - 89144x + 17210}{465124(25x^4 + 30x^3 + 29x^2 + 12x + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)/(5\*x^2+3\*x+2)^3,x, algorithm="maxima")

[Out] 847793/317214568\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 45/244904\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 1/465124\*(108025\*x^3 + 104430\*x^2 + 89144\*x + 17210)/(25\*x^4 + 30\*x^3 + 29\*x^2 + 12\*x + 4) + 1/21296\*log(5\*x^2 + 3\*x + 2) - 1/21296\*log(2\*x^2 - x + 3)

**mupad [B]** time = 0.18, size = 115, normalized size = 1.00

$$\frac{\frac{4321x^3}{465124} + \frac{10443x^2}{1162810} + \frac{2026x}{264275} + \frac{1721}{1162810}}{x^4 + \frac{6x^3}{5} + \frac{29x^2}{25} + \frac{12x}{25} + \frac{4}{25}} + \ln\left(x - \frac{1}{4} + \frac{\sqrt{23}1i}{4}\right) \left(-\frac{1}{21296} + \frac{\sqrt{23}45i}{489808}\right) - \ln\left(x + \frac{3}{10} - \frac{\sqrt{31}1i}{10}\right) \left(-\frac{1}{21296}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((2\*x^2 - x + 3)\*(3\*x + 5\*x^2 + 2)^3),x)

[Out] log(x + (23^(1/2)\*1i)/4 - 1/4)\*((23^(1/2)\*45i)/489808 - 1/21296) - log(x - (23^(1/2)\*1i)/4 - 1/4)\*((23^(1/2)\*45i)/489808 + 1/21296) - log(x - (31^(1/2)\*1i)/10 + 3/10)\*((31^(1/2)\*847793i)/634429136 - 1/21296) + log(x + (31^(1/2)\*1i)/10 + 3/10)\*((31^(1/2)\*847793i)/634429136 + 1/21296) + ((2026\*x)/264275 + (10443\*x^2)/1162810 + (4321\*x^3)/465124 + 1721/1162810)/((12\*x)/25 + (29\*x^2)/25 + (6\*x^3)/5 + x^4 + 4/25)

**sympy [A]** time = 0.36, size = 119, normalized size = 1.03

$$\frac{108025x^3 + 104430x^2 + 89144x + 17210}{11628100x^4 + 13953720x^3 + 13488596x^2 + 5581488x + 1860496} - \frac{\log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{21296} + \frac{\log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{21296} + \frac{45\sqrt{23}}{21296}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x\*\*2-x+3)/(5\*x\*\*2+3\*x+2)\*\*3,x)

[Out] (108025\*x\*\*3 + 104430\*x\*\*2 + 89144\*x + 17210)/(11628100\*x\*\*4 + 13953720\*x\*\*3 + 13488596\*x\*\*2 + 5581488\*x + 1860496) - log(x\*\*2 - x/2 + 3/2)/21296 + log(x\*\*2 + 3\*x/5 + 2/5)/21296 + 45\*sqrt(23)\*atan(4\*sqrt(23)\*x/23 - sqrt(23)/3)/244904 + 847793\*sqrt(31)\*atan(10\*sqrt(31)\*x/31 + 3\*sqrt(31)/31)/31721456

8

$$3.44 \quad \int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^2} dx$$

Optimal. Leaf size=91

$$\frac{125x^5}{4} + \frac{2125x^4}{16} + \frac{9775x^3}{48} - \frac{1185x^2}{8} - \frac{14641(79x+101)}{2944(2x^2-x+3)} - \frac{30613}{128} \log(2x^2-x+3) - \frac{89359x}{64} - \frac{13292697 \tan^{-1}\left(\frac{1-\sqrt{23}}{\sqrt{23}}\right)}{1472\sqrt{23}}$$

[Out] -89359/64\*x-1185/8\*x^2+9775/48\*x^3+2125/16\*x^4+125/4\*x^5-14641/2944\*(101+79\*x)/(2\*x^2-x+3)-30613/128\*ln(2\*x^2-x+3)-13292697/33856\*arctan(1/23\*(1-4\*x)\*23^(1/2))\*23^(1/2)

**Rubi** [A] time = 0.09, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$ , Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{125x^5}{4} + \frac{2125x^4}{16} + \frac{9775x^3}{48} - \frac{1185x^2}{8} - \frac{14641(79x+101)}{2944(2x^2-x+3)} - \frac{30613}{128} \log(2x^2-x+3) - \frac{89359x}{64} - \frac{13292697 \tan^{-1}\left(\frac{1-\sqrt{23}}{\sqrt{23}}\right)}{1472\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3\*x + 5\*x^2)^4/(3 - x + 2\*x^2)^2,x]

[Out] (-89359\*x)/64 - (1185\*x^2)/8 + (9775\*x^3)/48 + (2125\*x^4)/16 + (125\*x^5)/4 - (14641\*(101 + 79\*x))/(2944\*(3 - x + 2\*x^2)) - (13292697\*ArcTan[(1 - 4\*x)/Sqrt[23]])/(1472\*Sqrt[23]) - (30613\*Log[3 - x + 2\*x^2])/128

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,

e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

### Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^2} dx &= -\frac{14641(101+79x)}{2944(3-x+2x^2)} + \frac{1}{23} \int \frac{\frac{832627}{64} - \frac{661181x}{64} - \frac{488267x^2}{32} + \frac{143635x^3}{16} + \frac{213325x^4}{8} + \frac{83375x^5}{4}}{3-x+2x^2} \\
&= -\frac{14641(101+79x)}{2944(3-x+2x^2)} + \frac{1}{23} \int \left( -\frac{2055257}{64} - \frac{27255x}{4} + \frac{224825x^2}{16} + \frac{48875x^3}{4} + \frac{14375x^4}{4} \right. \\
&= -\frac{89359x}{64} - \frac{1185x^2}{8} + \frac{9775x^3}{48} + \frac{2125x^4}{16} + \frac{125x^5}{4} - \frac{14641(101+79x)}{2944(3-x+2x^2)} + \frac{1331}{736} \int \frac{262}{3-x+2x^2} \\
&= -\frac{89359x}{64} - \frac{1185x^2}{8} + \frac{9775x^3}{48} + \frac{2125x^4}{16} + \frac{125x^5}{4} - \frac{14641(101+79x)}{2944(3-x+2x^2)} - \frac{30613}{128} \int \frac{1}{3-x+2x^2} \\
&= -\frac{89359x}{64} - \frac{1185x^2}{8} + \frac{9775x^3}{48} + \frac{2125x^4}{16} + \frac{125x^5}{4} - \frac{14641(101+79x)}{2944(3-x+2x^2)} - \frac{30613}{128} \log \left( \frac{4x-1}{\sqrt{23}} \right) \\
&= -\frac{89359x}{64} - \frac{1185x^2}{8} + \frac{9775x^3}{48} + \frac{2125x^4}{16} + \frac{125x^5}{4} - \frac{14641(101+79x)}{2944(3-x+2x^2)} - \frac{13292697 \tan^{-1} \left( \frac{4x-1}{\sqrt{23}} \right)}{1472}
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 91, normalized size = 1.00

$$\frac{125x^5}{4} + \frac{2125x^4}{16} + \frac{9775x^3}{48} - \frac{1185x^2}{8} - \frac{14641(79x+101)}{2944(2x^2-x+3)} - \frac{30613}{128} \log(2x^2-x+3) - \frac{89359x}{64} + \frac{13292697 \tan^{-1} \left( \frac{4x-1}{\sqrt{23}} \right)}{1472\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x + 5\*x^2)^4/(3 - x + 2\*x^2)^2,x]

[Out] (-89359\*x)/64 - (1185\*x^2)/8 + (9775\*x^3)/48 + (2125\*x^4)/16 + (125\*x^5)/4 - (14641\*(101 + 79\*x))/(2944\*(3 - x + 2\*x^2)) + (13292697\*ArcTan[(-1 + 4\*x)/Sqrt[23]])/(1472\*Sqrt[23]) - (30613\*Log[3 - x + 2\*x^2])/128

**fricas [A]** time = 1.17, size = 98, normalized size = 1.08

$$\frac{12696000x^7 + 47610000x^6 + 74800600x^5 - 20609840x^4 - 413058012x^3 + 79756182\sqrt{23}(2x^2 - x + 3) \arctan\left(\frac{4x-1}{\sqrt{23}}\right) - 13292697 \arctan\left(\frac{4x-1}{\sqrt{23}}\right) - 30613 \log(2x^2 - x + 3)}{1472\sqrt{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^4/(2\*x^2-x+3)^2,x, algorithm="fricas")

[Out]  $\frac{1}{203136}(12696000x^7 + 47610000x^6 + 74800600x^5 - 20609840x^4 - 413058012x^3 + 79756182\sqrt{23}(2x^2 - x + 3)\arctan(1/23\sqrt{23}(4x - 1)) + 193356906x^2 - 48582831(2x^2 - x + 3)\log(2x^2 - x + 3) - 930684489x - 102033129)/(2x^2 - x + 3)$

**giac** [A] time = 0.18, size = 72, normalized size = 0.79

$$\frac{125}{4}x^5 + \frac{2125}{16}x^4 + \frac{9775}{48}x^3 - \frac{1185}{8}x^2 + \frac{13292697}{33856}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{89359}{64}x - \frac{14641(79x+101)}{2944(2x^2-x+3)} - \frac{30613}{128}\log(2x^2-x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^4/(2\*x^2-x+3)^2,x, algorithm="giac")

[Out]  $\frac{125}{4}x^5 + \frac{2125}{16}x^4 + \frac{9775}{48}x^3 - \frac{1185}{8}x^2 + \frac{13292697}{33856}\sqrt{23}\arctan(1/23\sqrt{23}(4x-1)) - \frac{89359}{64}x - \frac{14641(79x+101)}{2944(2x^2-x+3)} - \frac{30613}{128}\log(2x^2-x+3)$

**maple** [A] time = 0.01, size = 71, normalized size = 0.78

$$\frac{125x^5}{4} + \frac{2125x^4}{16} + \frac{9775x^3}{48} - \frac{1185x^2}{8} - \frac{89359x}{64} + \frac{13292697\sqrt{23}\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{33856} - \frac{30613\ln(2x^2-x+3)}{128} - \frac{1331}{64}\left(\frac{869}{92}x + \frac{111}{92}\right)/(x^2-1/2x+3/2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+3\*x+2)^4/(2\*x^2-x+3)^2,x)

[Out]  $\frac{125}{4}x^5 + \frac{2125}{16}x^4 + \frac{9775}{48}x^3 - \frac{1185}{8}x^2 - \frac{89359}{64}x - \frac{1331}{64}\left(\frac{869}{92}x + \frac{111}{92}\right)/(x^2-1/2x+3/2) - \frac{30613}{128}\ln(2x^2-x+3) + \frac{13292697}{33856}23^{1/2}\arctan(1/23(4x-1)23^{1/2})$

**maxima** [A] time = 0.96, size = 72, normalized size = 0.79

$$\frac{125}{4}x^5 + \frac{2125}{16}x^4 + \frac{9775}{48}x^3 - \frac{1185}{8}x^2 + \frac{13292697}{33856}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{89359}{64}x - \frac{14641(79x+101)}{2944(2x^2-x+3)} - \frac{30613}{128}\log(2x^2-x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^4/(2\*x^2-x+3)^2,x, algorithm="maxima")

[Out]  $\frac{125}{4}x^5 + \frac{2125}{16}x^4 + \frac{9775}{48}x^3 - \frac{1185}{8}x^2 + \frac{13292697}{33856}\sqrt{23}\arctan(1/23\sqrt{23}(4x-1)) - \frac{89359}{64}x - \frac{14641(79x+101)}{2944(2x^2-x+3)} - \frac{30613}{128}\log(2x^2-x+3)$

**mupad [B]** time = 3.46, size = 72, normalized size = 0.79

$$\frac{13292697 \sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{33856} - \frac{30613 \ln(2x^2 - x + 3)}{128} - \frac{\frac{1156639x}{5888} + \frac{1478741}{5888}}{x^2 - \frac{x}{2} + \frac{3}{2}} - \frac{89359x}{64} - \frac{1185x^2}{8} + \frac{9775x^3}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3)^2,x)`

[Out]  $(13292697 \cdot 23^{1/2} \cdot \operatorname{atan}((4 \cdot 23^{1/2} \cdot x)/23 - 23^{1/2}/23))/33856 - (30613 \cdot \log(2 \cdot x^2 - x + 3))/128 - ((1156639 \cdot x)/5888 + 1478741/5888)/(x^2 - x/2 + 3/2) - (89359 \cdot x)/64 - (1185 \cdot x^2)/8 + (9775 \cdot x^3)/48 + (2125 \cdot x^4)/16 + (125 \cdot x^5)/4$

**sympy [A]** time = 0.20, size = 90, normalized size = 0.99

$$\frac{125x^5}{4} + \frac{2125x^4}{16} + \frac{9775x^3}{48} - \frac{1185x^2}{8} - \frac{89359x}{64} + \frac{-1156639x - 1478741}{5888x^2 - 2944x + 8832} - \frac{30613 \log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{128} + \frac{13292697\sqrt{23}}{33856}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**2+3*x+2)**4/(2*x**2-x+3)**2,x)`

[Out]  $125 \cdot x^{5/4} + 2125 \cdot x^{4/16} + 9775 \cdot x^{3/48} - 1185 \cdot x^{2/8} - 89359 \cdot x/64 + (-1156639 \cdot x - 1478741)/(5888 \cdot x^{2/2} - 2944 \cdot x + 8832) - 30613 \cdot \log(x^{2/2} - x/2 + 3/2)/128 + 13292697 \cdot \operatorname{sqrt}(23) \cdot \operatorname{atan}(4 \cdot \operatorname{sqrt}(23) \cdot x/23 - \operatorname{sqrt}(23)/23)/33856$

$$3.45 \quad \int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^2} dx$$

**Optimal.** Leaf size=77

$$\frac{125x^3}{12} + \frac{175x^2}{4} - \frac{1331(17-45x)}{736(2x^2-x+3)} - \frac{2057}{32} \log(2x^2-x+3) + \frac{915x}{16} + \frac{223971 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{368\sqrt{23}}$$

[Out] 915/16\*x+175/4\*x^2+125/12\*x^3-1331/736\*(17-45\*x)/(2\*x^2-x+3)-2057/32\*ln(2\*x^2-x+3)+223971/8464\*arctan(1/23\*(1-4\*x)\*23^(1/2))\*23^(1/2)

**Rubi [A]** time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$ , Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{125x^3}{12} + \frac{175x^2}{4} - \frac{1331(17-45x)}{736(2x^2-x+3)} - \frac{2057}{32} \log(2x^2-x+3) + \frac{915x}{16} + \frac{223971 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{368\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3\*x + 5\*x^2)^3/(3 - x + 2\*x^2)^2,x]

[Out] (915\*x)/16 + (175\*x^2)/4 + (125\*x^3)/12 - (1331\*(17 - 45\*x))/(736\*(3 - x + 2\*x^2)) + (223971\*ArcTan[(1 - 4\*x)/Sqrt[23]])/(368\*Sqrt[23]) - (2057\*Log[3 - x + 2\*x^2])/32

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x, x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d}, x]



e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

### Rule 1657

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Int[ExpandIntegrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

### Rule 1660

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 1]}, Simp[((b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] + Dist[1/((p + 1)\*(b^2 - 4\*a\*c)), Int[(a + b\*x + c\*x^2)^(p + 1)\*ExpandToSum[(p + 1)\*(b^2 - 4\*a\*c)\*Q - (2\*p + 3)\*(2\*c\*f - b\*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && LtQ[p, -1]

### Rubi steps

$$\begin{aligned}
\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^2} dx &= -\frac{1331(17-45x)}{736(3-x+2x^2)} + \frac{1}{23} \int \frac{-\frac{25195}{16} - \frac{19067x}{16} + \frac{22195x^2}{8} + \frac{13225x^3}{4} + \frac{2875x^4}{2}}{3-x+2x^2} dx \\
&= -\frac{1331(17-45x)}{736(3-x+2x^2)} + \frac{1}{23} \int \left( \frac{21045}{16} + \frac{4025x}{2} + \frac{2875x^2}{4} - \frac{121(365+391x)}{8(3-x+2x^2)} \right) dx \\
&= \frac{915x}{16} + \frac{175x^2}{4} + \frac{125x^3}{12} - \frac{1331(17-45x)}{736(3-x+2x^2)} - \frac{121}{184} \int \frac{365+391x}{3-x+2x^2} dx \\
&= \frac{915x}{16} + \frac{175x^2}{4} + \frac{125x^3}{12} - \frac{1331(17-45x)}{736(3-x+2x^2)} - \frac{2057}{32} \int \frac{-1+4x}{3-x+2x^2} dx - \frac{223971}{736} \int \frac{1}{3-x+2x^2} dx \\
&= \frac{915x}{16} + \frac{175x^2}{4} + \frac{125x^3}{12} - \frac{1331(17-45x)}{736(3-x+2x^2)} - \frac{2057}{32} \log(3-x+2x^2) + \frac{223971}{368} \operatorname{Subst} \int \frac{1}{3-x+2x^2} dx \\
&= \frac{915x}{16} + \frac{175x^2}{4} + \frac{125x^3}{12} - \frac{1331(17-45x)}{736(3-x+2x^2)} + \frac{223971 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{368\sqrt{23}} - \frac{2057}{32} \log(3-x+2x^2)
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 77, normalized size = 1.00

$$\frac{125x^3}{12} + \frac{175x^2}{4} + \frac{1331(45x-17)}{736(2x^2-x+3)} - \frac{2057}{32} \log(2x^2-x+3) + \frac{915x}{16} - \frac{223971 \tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{368\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x + 5\*x^2)^3/(3 - x + 2\*x^2)^2,x]

[Out] (915\*x)/16 + (175\*x^2)/4 + (125\*x^3)/12 + (1331\*(-17 + 45\*x))/(736\*(3 - x + 2\*x^2)) - (223971\*ArcTan[(-1 + 4\*x)/Sqrt[23]])/(368\*Sqrt[23]) - (2057\*Log[3 - x + 2\*x^2])/32

**fricas [A]** time = 0.73, size = 88, normalized size = 1.14

$$\frac{1058000x^5 + 3914600x^4 + 5173620x^3 - 1343826\sqrt{23}(2x^2 - x + 3) \arctan\left(\frac{1}{23}\sqrt{23}(4x - 1)\right) + 3761190x^2 - 3761190x + 1058000}{50784(2x^2 - x + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^3/(2\*x^2-x+3)^2,x, algorithm="fricas")

[Out]  $\frac{1}{50784} \cdot (1058000x^5 + 3914600x^4 + 5173620x^3 - 1343826\sqrt{23})(2x^2 - x + 3) \cdot \arctan\left(\frac{1}{23}\sqrt{23}(4x - 1)\right) + 3761190x^2 - 3264459(2x^2 - x + 3) \cdot \log(2x^2 - x + 3) + 12845385x - 1561263)/(2x^2 - x + 3)$

**giac** [A] time = 0.19, size = 62, normalized size = 0.81

$$\frac{125}{12}x^3 + \frac{175}{4}x^2 - \frac{223971}{8464}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x - 1)\right) + \frac{915}{16}x + \frac{1331(45x - 17)}{736(2x^2 - x + 3)} - \frac{2057}{32}\log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^2,x, algorithm="giac")`

[Out]  $\frac{125}{12}x^3 + \frac{175}{4}x^2 - \frac{223971}{8464}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x - 1)\right) + \frac{915}{16}x + \frac{1331}{736}(45x - 17)/(2x^2 - x + 3) - \frac{2057}{32}\log(2x^2 - x + 3)$

**maple** [A] time = 0.01, size = 61, normalized size = 0.79

$$\frac{125x^3}{12} + \frac{175x^2}{4} + \frac{915x}{16} - \frac{223971\sqrt{23}\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{8464} - \frac{2057\ln(2x^2 - x + 3)}{32} - \frac{121\left(-\frac{495x}{92} + \frac{187}{92}\right)}{16\left(x^2 - \frac{1}{2}x + \frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^2+3*x+2)^3/(2*x^2-x+3)^2,x)`

[Out]  $\frac{125}{12}x^3 + \frac{175}{4}x^2 + \frac{915}{16}x - \frac{121}{16}\left(-\frac{495}{92}x + \frac{187}{92}\right)/(x^2 - \frac{1}{2}x + \frac{3}{2}) - \frac{2057}{32}\ln(2x^2 - x + 3) - \frac{223971}{8464} \cdot 23^{(1/2)} \cdot \arctan\left(\frac{1}{23}(4x - 1) \cdot 23^{(1/2)}\right)$

**maxima** [A] time = 0.96, size = 62, normalized size = 0.81

$$\frac{125}{12}x^3 + \frac{175}{4}x^2 - \frac{223971}{8464}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x - 1)\right) + \frac{915}{16}x + \frac{1331(45x - 17)}{736(2x^2 - x + 3)} - \frac{2057}{32}\log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^2,x, algorithm="maxima")`

[Out]  $\frac{125}{12}x^3 + \frac{175}{4}x^2 - \frac{223971}{8464}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x - 1)\right) + \frac{915}{16}x + \frac{1331}{736}(45x - 17)/(2x^2 - x + 3) - \frac{2057}{32}\log(2x^2 - x + 3)$

**mupad** [B] time = 3.42, size = 61, normalized size = 0.79

$$\frac{915x}{16} - \frac{2057\ln(2x^2 - x + 3)}{32} + \frac{\frac{59895x}{1472} - \frac{22627}{1472}}{x^2 - \frac{x}{2} + \frac{3}{2}} - \frac{223971\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{8464} + \frac{175x^2}{4} + \frac{125x^3}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^2,x)`

[Out]  $(915*x)/16 - (2057*\log(2*x^2 - x + 3))/32 + ((59895*x)/1472 - 22627/1472)/(x^2 - x/2 + 3/2) - (223971*23^{(1/2)}*\operatorname{atan}((4*23^{(1/2)}*x)/23 - 23^{(1/2)}/23))/8464 + (175*x^2)/4 + (125*x^3)/12$

**sympy** [A] time = 0.19, size = 75, normalized size = 0.97

$$\frac{125x^3}{12} + \frac{175x^2}{4} + \frac{915x}{16} + \frac{59895x - 22627}{1472x^2 - 736x + 2208} - \frac{2057 \log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{32} - \frac{223971\sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{8464}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**2+3*x+2)**3/(2*x**2-x+3)**2,x)`

[Out]  $125*x**3/12 + 175*x**2/4 + 915*x/16 + (59895*x - 22627)/(1472*x**2 - 736*x + 2208) - 2057*\log(x**2 - x/2 + 3/2)/32 - 223971*\operatorname{sqrt}(23)*\operatorname{atan}(4*\operatorname{sqrt}(23)*x/23 - \operatorname{sqrt}(23)/23)/8464$

$$3.46 \quad \int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^2} dx$$

Optimal. Leaf size=63

$$\frac{121(19-7x)}{184(2x^2-x+3)} + \frac{55}{8} \log(2x^2-x+3) + \frac{25x}{4} + \frac{1859 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{92\sqrt{23}}$$

[Out] 25/4\*x+121/184\*(19-7\*x)/(2\*x^2-x+3)+55/8\*ln(2\*x^2-x+3)+1859/2116\*arctan(1/23\*(1-4\*x)\*23^(1/2))\*23^(1/2)

**Rubi [A]** time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$ , Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{121(19-7x)}{184(2x^2-x+3)} + \frac{55}{8} \log(2x^2-x+3) + \frac{25x}{4} + \frac{1859 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{92\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3\*x + 5\*x^2)^2/(3 - x + 2\*x^2)^2,x]

[Out] (25\*x)/4 + (121\*(19 - 7\*x))/(184\*(3 - x + 2\*x^2)) + (1859\*ArcTan[(1 - 4\*x)/Sqrt[23]])/(92\*Sqrt[23]) + (55\*Log[3 - x + 2\*x^2])/8

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2 + 3x + 5x^2)^2}{(3 - x + 2x^2)^2} dx &= \frac{121(19 - 7x)}{184(3 - x + 2x^2)} + \frac{1}{23} \int \frac{\frac{163}{4} + \frac{1955x}{4} + \frac{575x^2}{2}}{3 - x + 2x^2} dx \\
&= \frac{121(19 - 7x)}{184(3 - x + 2x^2)} + \frac{1}{23} \int \left( \frac{575}{4} - \frac{11(71 - 115x)}{2(3 - x + 2x^2)} \right) dx \\
&= \frac{25x}{4} + \frac{121(19 - 7x)}{184(3 - x + 2x^2)} - \frac{11}{46} \int \frac{71 - 115x}{3 - x + 2x^2} dx \\
&= \frac{25x}{4} + \frac{121(19 - 7x)}{184(3 - x + 2x^2)} + \frac{55}{8} \int \frac{-1 + 4x}{3 - x + 2x^2} dx - \frac{1859}{184} \int \frac{1}{3 - x + 2x^2} dx \\
&= \frac{25x}{4} + \frac{121(19 - 7x)}{184(3 - x + 2x^2)} + \frac{55}{8} \log(3 - x + 2x^2) + \frac{1859}{92} \text{Subst} \left( \int \frac{1}{-23 - x^2} dx, x, -1 \right) \\
&= \frac{25x}{4} + \frac{121(19 - 7x)}{184(3 - x + 2x^2)} + \frac{1859 \tan^{-1} \left( \frac{1-4x}{\sqrt{23}} \right)}{92\sqrt{23}} + \frac{55}{8} \log(3 - x + 2x^2)
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 63, normalized size = 1.00

$$-\frac{121(7x - 19)}{184(2x^2 - x + 3)} + \frac{55}{8} \log(2x^2 - x + 3) + \frac{25x}{4} - \frac{1859 \tan^{-1} \left( \frac{4x-1}{\sqrt{23}} \right)}{92\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x + 5\*x^2)^2/(3 - x + 2\*x^2)^2,x]

[Out] (25\*x)/4 - (121\*(-19 + 7\*x))/(184\*(3 - x + 2\*x^2)) - (1859\*ArcTan[(-1 + 4\*x)/Sqrt[23]])/(92\*Sqrt[23]) + (55\*Log[3 - x + 2\*x^2])/8

**fricas [A]** time = 0.97, size = 78, normalized size = 1.24

$$\frac{52900x^3 - 3718\sqrt{23}(2x^2 - x + 3)\arctan\left(\frac{1}{23}\sqrt{23}(4x - 1)\right) - 26450x^2 + 29095(2x^2 - x + 3)\log(2x^2 - x + 3)}{4232(2x^2 - x + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^2/(2\*x^2-x+3)^2,x, algorithm="fricas")

[Out] 1/4232\*(52900\*x^3 - 3718\*sqrt(23)\*(2\*x^2 - x + 3)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) - 26450\*x^2 + 29095\*(2\*x^2 - x + 3)\*log(2\*x^2 - x + 3) + 59869\*x + 52877)/(2\*x^2 - x + 3)

**giac** [A] time = 0.17, size = 52, normalized size = 0.83

$$-\frac{1859}{2116} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4x-1)\right) + \frac{25}{4} x - \frac{121(7x-19)}{184(2x^2-x+3)} + \frac{55}{8} \log(2x^2-x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^2/(2\*x^2-x+3)^2,x, algorithm="giac")

[Out] -1859/2116\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 25/4\*x - 121/184\*(7\*x - 19)/(2\*x^2 - x + 3) + 55/8\*log(2\*x^2 - x + 3)

**maple** [A] time = 0.01, size = 51, normalized size = 0.81

$$\frac{25x}{4} - \frac{1859\sqrt{23} \arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{2116} + \frac{55 \ln(2x^2 - x + 3)}{8} + \frac{-\frac{847x}{368} + \frac{2299}{368}}{x^2 - \frac{1}{2}x + \frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+3\*x+2)^2/(2\*x^2-x+3)^2,x)

[Out] 25/4\*x+11/4\*(-77/92\*x+209/92)/(x^2-1/2\*x+3/2)+55/8\*ln(2\*x^2-x+3)-1859/2116\*23^(1/2)\*arctan(1/23\*(4\*x-1)\*23^(1/2))

**maxima** [A] time = 0.97, size = 52, normalized size = 0.83

$$-\frac{1859}{2116} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4x-1)\right) + \frac{25}{4} x - \frac{121(7x-19)}{184(2x^2-x+3)} + \frac{55}{8} \log(2x^2-x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^2/(2\*x^2-x+3)^2,x, algorithm="maxima")

[Out] -1859/2116\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 25/4\*x - 121/184\*(7\*x - 19)/(2\*x^2 - x + 3) + 55/8\*log(2\*x^2 - x + 3)

**mupad** [B] time = 3.40, size = 52, normalized size = 0.83

$$\frac{25x}{4} + \frac{55 \ln(2x^2 - x + 3)}{8} - \frac{\frac{847x}{368} - \frac{2299}{368}}{x^2 - \frac{x}{2} + \frac{3}{2}} - \frac{1859 \sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{2116}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x + 5\*x^2 + 2)^2/(2\*x^2 - x + 3)^2,x)



[Out]  $(25*x)/4 + (55*\log(2*x^2 - x + 3))/8 - ((847*x)/368 - 2299/368)/(x^2 - x/2 + 3/2) - (1859*23^{(1/2)}*\operatorname{atan}((4*23^{(1/2)}*x)/23 - 23^{(1/2)}/23))/2116$

sympy [A] time = 0.19, size = 61, normalized size = 0.97

$$\frac{25x}{4} + \frac{2299 - 847x}{368x^2 - 184x + 552} + \frac{55 \log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{8} - \frac{1859\sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{2116}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**2+3*x+2)**2/(2*x**2-x+3)**2,x)`

[Out]  $25*x/4 + (2299 - 847*x)/(368*x**2 - 184*x + 552) + 55*\log(x**2 - x/2 + 3/2)/8 - 1859*\sqrt{23}*\operatorname{atan}(4*\sqrt{23}*x/23 - \sqrt{23}/23)/2116$

$$3.47 \quad \int \frac{2+3x+5x^2}{(3-x+2x^2)^2} dx$$

**Optimal.** Leaf size=43

$$-\frac{11(3x+5)}{46(2x^2-x+3)} - \frac{82 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{23\sqrt{23}}$$

[Out]  $-11/46*(5+3*x)/(2*x^2-x+3)-82/529*\arctan(1/23*(1-4*x)*23^{(1/2)})*23^{(1/2)}$

**Rubi [A]** time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$ , Rules used = {1660, 12, 618, 204}

$$-\frac{11(3x+5)}{46(2x^2-x+3)} - \frac{82 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{23\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3\*x + 5\*x^2)/(3 - x + 2\*x^2)^2, x]

[Out]  $(-11*(5 + 3*x))/(46*(3 - x + 2*x^2)) - (82*ArcTan[(1 - 4*x)/Sqrt[23]])/(23*Sqrt[23])$

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{2 + 3x + 5x^2}{(3 - x + 2x^2)^2} dx &= -\frac{11(5 + 3x)}{46(3 - x + 2x^2)} + \frac{1}{23} \int \frac{41}{3 - x + 2x^2} dx \\
&= -\frac{11(5 + 3x)}{46(3 - x + 2x^2)} + \frac{41}{23} \int \frac{1}{3 - x + 2x^2} dx \\
&= -\frac{11(5 + 3x)}{46(3 - x + 2x^2)} - \frac{82}{23} \text{Subst}\left(\int \frac{1}{-23 - x^2} dx, x, -1 + 4x\right) \\
&= -\frac{11(5 + 3x)}{46(3 - x + 2x^2)} - \frac{82 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{23\sqrt{23}}
\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 43, normalized size = 1.00

$$\frac{82 \tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{23\sqrt{23}} - \frac{11(3x+5)}{46(2x^2-x+3)}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x + 5\*x^2)/(3 - x + 2\*x^2)^2, x]

[Out] (-11\*(5 + 3\*x))/(46\*(3 - x + 2\*x^2)) + (82\*ArcTan[(-1 + 4\*x)/Sqrt[23]])/(23\*Sqrt[23])

**fricas [A]** time = 0.95, size = 45, normalized size = 1.05

$$\frac{164\sqrt{23}(2x^2 - x + 3) \arctan\left(\frac{1}{23}\sqrt{23}(4x - 1)\right) - 759x - 1265}{1058(2x^2 - x + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)/(2\*x^2-x+3)^2,x, algorithm="fricas")

[Out] 1/1058\*(164\*sqrt(23)\*(2\*x^2 - x + 3)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) - 759\*x - 1265)/(2\*x^2 - x + 3)

**giac** [A] time = 0.19, size = 36, normalized size = 0.84

$$\frac{82}{529} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4x - 1)\right) - \frac{11(3x + 5)}{46(2x^2 - x + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)/(2\*x^2-x+3)^2,x, algorithm="giac")

[Out] 82/529\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) - 11/46\*(3\*x + 5)/(2\*x^2 - x + 3)

**maple** [A] time = 0.00, size = 34, normalized size = 0.79

$$\frac{82\sqrt{23} \arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{529} + \frac{-\frac{33x}{92} - \frac{55}{92}}{x^2 - \frac{1}{2}x + \frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+3\*x+2)/(2\*x^2-x+3)^2,x)

[Out] (-33/92\*x-55/92)/(x^2-1/2\*x+3/2)+82/529\*23^(1/2)\*arctan(1/23\*(4\*x-1)\*23^(1/2))

**maxima** [A] time = 0.96, size = 36, normalized size = 0.84

$$\frac{82}{529} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4x - 1)\right) - \frac{11(3x + 5)}{46(2x^2 - x + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)/(2\*x^2-x+3)^2,x, algorithm="maxima")

[Out] 82/529\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) - 11/46\*(3\*x + 5)/(2\*x^2 - x + 3)

**mupad** [B] time = 0.04, size = 36, normalized size = 0.84

$$\frac{82 \sqrt{23} \operatorname{atan}\left(\frac{4 \sqrt{23} x}{23} - \frac{\sqrt{23}}{23}\right)}{529} - \frac{\frac{33x}{92} + \frac{55}{92}}{x^2 - \frac{x}{2} + \frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x + 5*x^2 + 2)/(2*x^2 - x + 3)^2, x)`

[Out]  $(82 \cdot 23^{1/2} \cdot \operatorname{atan}((4 \cdot 23^{1/2} \cdot x)/23 - 23^{1/2}/23))/529 - ((33 \cdot x)/92 + 55/92)/(x^2 - x/2 + 3/2)$

**sympy [A]** time = 0.15, size = 42, normalized size = 0.98

$$\frac{-33x - 55}{92x^2 - 46x + 138} + \frac{82\sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{529}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**2+3*x+2)/(2*x**2-x+3)**2, x)`

[Out]  $(-33x - 55)/(92x^2 - 46x + 138) + 82 \cdot \sqrt{23} \cdot \operatorname{atan}(4 \cdot \sqrt{23} \cdot x/23 - \sqrt{23}/23)/529$

$$3.48 \quad \int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)} dx$$

**Optimal.** Leaf size=94

$$\frac{13-6x}{506(2x^2-x+3)} - \frac{13}{968} \log(2x^2-x+3) + \frac{13}{968} \log(5x^2+3x+2) + \frac{241 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{11132\sqrt{23}} + \frac{69 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{484\sqrt{31}}$$

[Out] 1/506\*(13-6\*x)/(2\*x^2-x+3)-13/968\*ln(2\*x^2-x+3)+13/968\*ln(5\*x^2+3\*x+2)+241/256036\*arctan(1/23\*(1-4\*x)\*23^(1/2))\*23^(1/2)+69/15004\*arctan(1/31\*(3+10\*x)\*31^(1/2))\*31^(1/2)

**Rubi [A]** time = 0.09, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$ , Rules used = {974, 1072, 634, 618, 204, 628}

$$\frac{13-6x}{506(2x^2-x+3)} - \frac{13}{968} \log(2x^2-x+3) + \frac{13}{968} \log(5x^2+3x+2) + \frac{241 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{11132\sqrt{23}} + \frac{69 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{484\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - x + 2\*x^2)^2\*(2 + 3\*x + 5\*x^2)), x]

[Out] (13 - 6\*x)/(506\*(3 - x + 2\*x^2)) + (241\*ArcTan[(1 - 4\*x)/Sqrt[23]])/(11132\*Sqrt[23]) + (69\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(484\*Sqrt[31]) - (13\*Log[3 - x + 2\*x^2])/968 + (13\*Log[2 + 3\*x + 5\*x^2])/968

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d}, x]

e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

### Rule 974

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_)\*((d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2)^(q\_), x\_Symbol] := Simp[((2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f) + c\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*x)\*(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^(q + 1))/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1)), x] - Dist[1/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1)), Int[(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^q\*Simp[2\*c\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1) - (2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(a\*f\*(p + 1) - c\*d\*(p + 2)) - e\*(b^2\*c\*e - 2\*a\*c^2\*e - b^3\*f - b\*c\*(c\*d - 3\*a\*f))\*(p + q + 2) + (2\*f\*(2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f))\*(p + q + 2) - (2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(b\*f\*(p + 1) - c\*e\*(2\*p + q + 4)))\*x + c\*f\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(2\*p + 2\*q + 5)\*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && LtQ[p, -1] && NeQ[(c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]

### Rule 1072

Int[((A\_.) + (B\_.)\*(x\_) + (C\_.)\*(x\_)^2)/(((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*((d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2)), x\_Symbol] := With[{q = c^2\*d^2 - b\*c\*d\*e + a\*c\*e^2 + b^2\*d\*f - 2\*a\*c\*d\*f - a\*b\*e\*f + a^2\*f^2}, Dist[1/q, Int[(A\*c^2\*d - a\*c\*C\*d - A\*b\*c\*e + a\*B\*c\*e + A\*b^2\*f - a\*b\*B\*f - a\*A\*c\*f + a^2\*C\*f + c\*(B\*c\*d - b\*C\*d - A\*c\*e + a\*C\*e + A\*b\*f - a\*B\*f)\*x)/(a + b\*x + c\*x^2), x], x] + Dist[1/q, Int[(c\*C\*d^2 - B\*c\*d\*e + A\*c\*e^2 + b\*B\*d\*f - A\*c\*d\*f - a\*C\*d\*f - A\*b\*e\*f + a\*A\*f^2 - f\*(B\*c\*d - b\*C\*d - A\*c\*e + a\*C\*e + A\*b\*f - a\*B\*f)\*x)/(d + e\*x + f\*x^2), x], x] /; NeQ[q, 0] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0]

### Rubi steps

$$\begin{aligned}
\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)} dx &= \frac{13-6x}{506(3-x+2x^2)} - \frac{\int \frac{-1892-1067x+330x^2}{(3-x+2x^2)(2+3x+5x^2)} dx}{5566} \\
&= \frac{13-6x}{506(3-x+2x^2)} - \frac{\int \frac{-3509+72358x}{3-x+2x^2} dx}{1346972} - \frac{\int \frac{-150282-180895x}{2+3x+5x^2} dx}{1346972} \\
&= \frac{13-6x}{506(3-x+2x^2)} - \frac{241 \int \frac{1}{3-x+2x^2} dx}{22264} - \frac{13}{968} \int \frac{-1+4x}{3-x+2x^2} dx + \frac{13}{968} \int \frac{24}{2+3x+5x^2} dx \\
&= \frac{13-6x}{506(3-x+2x^2)} - \frac{13}{968} \log(3-x+2x^2) + \frac{13}{968} \log(2+3x+5x^2) + \frac{24}{968} \int \frac{1}{2+3x+5x^2} dx \\
&= \frac{13-6x}{506(3-x+2x^2)} + \frac{241 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{11132\sqrt{23}} + \frac{69 \tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{484\sqrt{31}} - \frac{13}{968} \log(3-x+2x^2) + \frac{13}{968} \log(2+3x+5x^2) + \frac{24}{968} \int \frac{1}{2+3x+5x^2} dx
\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 94, normalized size = 1.00

$$\frac{13-6x}{506(2x^2-x+3)} - \frac{13}{968} \log(2x^2-x+3) + \frac{13}{968} \log(5x^2+3x+2) - \frac{241 \tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{11132\sqrt{23}} + \frac{69 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{484\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((3-x+2\*x^2)^2\*(2+3\*x+5\*x^2)),x]

[Out] (13-6\*x)/(506\*(3-x+2\*x^2)) - (241\*ArcTan[(-1+4\*x)/Sqrt[23]])/(11132\*Sqrt[23]) + (69\*ArcTan[(3+10\*x)/Sqrt[31]])/(484\*Sqrt[31]) - (13\*Log[3-x+2\*x^2])/968 + (13\*Log[2+3\*x+5\*x^2])/968

**fricas [A]** time = 0.93, size = 117, normalized size = 1.24

$$\frac{73002 \sqrt{31} (2x^2 - x + 3) \arctan\left(\frac{1}{31} \sqrt{31} (10x + 3)\right) - 14942 \sqrt{23} (2x^2 - x + 3) \arctan\left(\frac{1}{23} \sqrt{23} (4x - 1)\right) + 213187}{15874232 (2x^2 - x + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^2/(5\*x^2+3\*x+2),x, algorithm="fricas")

[Out] 1/15874232\*(73002\*sqrt(31)\*(2\*x^2-x+3)\*arctan(1/31\*sqrt(31)\*(10\*x+3)) - 14942\*sqrt(23)\*(2\*x^2-x+3)\*arctan(1/23\*sqrt(23)\*(4\*x-1)) + 213187\*



$$(2x^2 - x + 3) \cdot \log(5x^2 + 3x + 2) - 213187(2x^2 - x + 3) \cdot \log(2x^2 - x + 3) - 188232x + 407836 / (2x^2 - x + 3)$$

**giac [A]** time = 0.19, size = 78, normalized size = 0.83

$$\frac{69}{15004} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x + 3)\right) - \frac{241}{256036} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4x - 1)\right) - \frac{6x - 13}{506(2x^2 - x + 3)} + \frac{13}{968} \log$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^2/(5\*x^2+3\*x+2),x, algorithm="giac")

[Out] 69/15004\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) - 241/256036\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) - 1/506\*(6\*x - 13)/(2\*x^2 - x + 3) + 13/968\*log(5\*x^2 + 3\*x + 2) - 13/968\*log(2\*x^2 - x + 3)

**maple [A]** time = 0.01, size = 77, normalized size = 0.82

$$\frac{69\sqrt{31} \arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right) - 241\sqrt{23} \arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right) - 13 \ln(2x^2 - x + 3) + \frac{13 \ln(5x^2 + 3x + 2)}{968} - \frac{66x}{484(x^2 - x + 3)}}{15004 - \frac{241\sqrt{23}}{256036} - \frac{13}{968} + \frac{13}{968} - \frac{66x}{484(x^2 - x + 3)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2\*x^2-x+3)^2/(5\*x^2+3\*x+2),x)

[Out] 13/968\*ln(5\*x^2+3\*x+2)+69/15004\*31^(1/2)\*arctan(1/31\*(10\*x+3)\*31^(1/2))-1/484\*(66/23\*x-143/23)/(x^2-1/2\*x+3/2)-13/968\*ln(2\*x^2-x+3)-241/256036\*23^(1/2)\*arctan(1/23\*(4\*x-1)\*23^(1/2))

**maxima [A]** time = 0.96, size = 78, normalized size = 0.83

$$\frac{69}{15004} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x + 3)\right) - \frac{241}{256036} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4x - 1)\right) - \frac{6x - 13}{506(2x^2 - x + 3)} + \frac{13}{968} \log$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^2/(5\*x^2+3\*x+2),x, algorithm="maxima")

[Out] 69/15004\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) - 241/256036\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) - 1/506\*(6\*x - 13)/(2\*x^2 - x + 3) + 13/968\*log(5\*x^2 + 3\*x + 2) - 13/968\*log(2\*x^2 - x + 3)

**mupad [B]** time = 3.58, size = 96, normalized size = 1.02

$$-\frac{\frac{3x}{x^2 - \frac{x}{2} + \frac{3}{2}} - \frac{13}{1012}}{\frac{3x}{x^2 - \frac{x}{2} + \frac{3}{2}} - \frac{13}{1012}} - \ln\left(x + \frac{3}{10} - \frac{\sqrt{31} \operatorname{li}}{10}\right) \left(-\frac{13}{968} + \frac{\sqrt{31} \operatorname{li}}{30008}\right) + \ln\left(x + \frac{3}{10} + \frac{\sqrt{31} \operatorname{li}}{10}\right) \left(\frac{13}{968} + \frac{\sqrt{31} \operatorname{li}}{30008}\right) + \ln\left(x - \frac{1}{4} - \frac{13}{1012}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((2*x^2 - x + 3)^2*(3*x + 5*x^2 + 2)),x)`

[Out]  $\log(x + (31^{1/2}i)/10 + 3/10)*((31^{1/2}i)/30008 + 13/968) - \log(x - (31^{1/2}i)/10 + 3/10)*((31^{1/2}i)/30008 - 13/968) - ((3x)/506 - 13/1012)/(x^2 - x/2 + 3/2) + \log(x - (23^{1/2}i)/4 - 1/4)*((23^{1/2}i)/512072 - 13/968) - \log(x + (23^{1/2}i)/4 - 1/4)*((23^{1/2}i)/512072 + 13/968)$

**sympy [A]** time = 0.32, size = 102, normalized size = 1.09

$$\frac{13 - 6x}{1012x^2 - 506x + 1518} - \frac{13 \log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{968} + \frac{13 \log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{968} - \frac{241\sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{256036} + \frac{69\sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{15004}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2*x**2-x+3)**2/(5*x**2+3*x+2),x)`

[Out]  $(13 - 6x)/(1012x^2 - 506x + 1518) - 13*\log(x^2 - x/2 + 3/2)/968 + 13*\log(x^2 + 3x/5 + 2/5)/968 - 241*\sqrt{23}*\operatorname{atan}(4*\sqrt{23}*x/23 - \sqrt{23}/23)/256036 + 69*\sqrt{31}*\operatorname{atan}(10*\sqrt{31}*x/31 + 3*\sqrt{31}/31)/15004$

$$3.49 \quad \int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^2} dx$$

**Optimal.** Leaf size=127

$$-\frac{25(117-137x)}{172546(5x^2+3x+2)} + \frac{13-6x}{506(2x^2-x+3)(5x^2+3x+2)} + \frac{19 \log(2x^2-x+3)}{10648} - \frac{19 \log(5x^2+3x+2)}{10648} + \frac{2769}{10648}$$

[Out] -25/172546\*(117-137\*x)/(5\*x^2+3\*x+2)+1/506\*(13-6\*x)/(2\*x^2-x+3)/(5\*x^2+3\*x+2)+19/10648\*ln(2\*x^2-x+3)-19/10648\*ln(5\*x^2+3\*x+2)+2769/2816396\*arctan(1/23\*(1-4\*x)\*23^(1/2))\*23^(1/2)+12643/5116364\*arctan(1/31\*(3+10\*x)\*31^(1/2))\*31^(1/2)

**Rubi [A]** time = 0.12, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$ , Rules used = {974, 1060, 1072, 634, 618, 204, 628}

$$-\frac{25(117-137x)}{172546(5x^2+3x+2)} + \frac{13-6x}{506(2x^2-x+3)(5x^2+3x+2)} + \frac{19 \log(2x^2-x+3)}{10648} - \frac{19 \log(5x^2+3x+2)}{10648} + \frac{2769}{10648}$$

Antiderivative was successfully verified.

[In] Int[1/((3-x+2\*x^2)^2\*(2+3\*x+5\*x^2)^2),x]

[Out] (-25\*(117-137\*x))/(172546\*(2+3\*x+5\*x^2)) + (13-6\*x)/(506\*(3-x+2\*x^2)\*(2+3\*x+5\*x^2)) + (2769\*ArcTan[(1-4\*x)/Sqrt[23]])/(122452\*Sqrt[23]) + (12643\*ArcTan[(3+10\*x)/Sqrt[31]])/(165044\*Sqrt[31]) + (19\*Log[3-x+2\*x^2])/10648 - (19\*Log[2+3\*x+5\*x^2])/10648

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2-4\*a\*c-x^2, x], x], x, b+2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2-4\*a\*c, 0]

#### Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 974

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((d_) + (e_)*(x_) + (f_)*(x
_)^2)^(q_), x_Symbol] := Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q,
0]
```

### Rule 1060

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((A_) + (B_)*(x_) + (C_)*(x
_)^2)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x +
c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
+ q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
```

$A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*$   
 $(b*f*(p + 1) - c*e*(2*p + q + 4))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*$   
 $c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*$   
 $p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] \&\&$   
 $NeQ[b^2 - 4*a*c, 0] \&\& NeQ[e^2 - 4*d*f, 0] \&\& LtQ[p, -1] \&\& NeQ[(c*d - a*f$   
 $)^2 - (b*d - a*e)*(c*e - b*f), 0] \&\& !(IntegerQ[p] \&\& ILtQ[q, -1]) \&\& !$   
 $IGtQ[q, 0]$

### Rule 1072

$Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)$   
 $*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*$   
 $d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*$   
 $c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f$   
 $+ c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),$   
 $x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a$   
 $*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B$   
 $*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, e, f,$   
 $A, B, C}, x] \&\& NeQ[b^2 - 4*a*c, 0] \&\& NeQ[e^2 - 4*d*f, 0]$

### Rubi steps

$$\begin{aligned}
 \int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^2} dx &= \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)} - \frac{\int \frac{-2321-2299x+990x^2}{(3-x+2x^2)(2+3x+5x^2)^2} dx}{5566} \\
 &= -\frac{25(117-137x)}{172546(2+3x+5x^2)} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)} - \frac{\int \frac{-30341}{(3-x+2x^2)(2+3x+5x^2)^2} dx}{101} \\
 &= -\frac{25(117-137x)}{172546(2+3x+5x^2)} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)} - \frac{\int \frac{132282}{(3-x+2x^2)(2+3x+5x^2)^2} dx}{101} \\
 &= -\frac{25(117-137x)}{172546(2+3x+5x^2)} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)} + \frac{19 \int \frac{-1}{3-x} dx}{106} \\
 &= -\frac{25(117-137x)}{172546(2+3x+5x^2)} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)} + \frac{19 \log(3-x)}{106} \\
 &= -\frac{25(117-137x)}{172546(2+3x+5x^2)} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)} + \frac{2769 \operatorname{atan}\left(\frac{3-x}{5x^2+3x-3}\right)}{1224}
 \end{aligned}$$

**Mathematica [A]** time = 0.06, size = 106, normalized size = 0.83

$$\frac{9659011 \log(2x^2 - x + 3) - 9659011 \log(5x^2 + 3x + 2) + \frac{31372(6850x^3 - 9275x^2 + 11154x - 4342)}{10x^4 + x^3 + 16x^2 + 7x + 6} - 5322018\sqrt{23} \tan^{-1}\left(\frac{1}{31}\sqrt{31}(10x + 3)\right)}{5413113112}$$

Antiderivative was successfully verified.

[In] Integrate[1/((3 - x + 2\*x^2)^2\*(2 + 3\*x + 5\*x^2)^2), x]

[Out] ((31372\*(-4342 + 11154\*x - 9275\*x^2 + 6850\*x^3))/(6 + 7\*x + 16\*x^2 + x^3 + 10\*x^4) - 5322018\*Sqrt[23]\*ArcTan[(-1 + 4\*x)/Sqrt[23]] + 13376294\*Sqrt[31]\*ArcTan[(3 + 10\*x)/Sqrt[31]] + 9659011\*Log[3 - x + 2\*x^2] - 9659011\*Log[2 + 3\*x + 5\*x^2])/5413113112

**fricas [A]** time = 0.86, size = 167, normalized size = 1.31

$$\frac{214898200x^3 + 13376294\sqrt{31}(10x^4 + x^3 + 16x^2 + 7x + 6) \arctan\left(\frac{1}{31}\sqrt{31}(10x + 3)\right) - 5322018\sqrt{23}(10x^4 + x^3 + 16x^2 + 7x + 6) \arctan\left(\frac{1}{23}\sqrt{23}(4x - 1)\right) - 290975300x^2 - 9659011(10x^4 + x^3 + 16x^2 + 7x + 6) \log(5x^2 + 3x + 2) + 9659011(10x^4 + x^3 + 16x^2 + 7x + 6) \log(2x^2 - x + 3) + 349923288x - 136217224}{(10x^4 + x^3 + 16x^2 + 7x + 6)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^2/(5\*x^2+3\*x+2)^2,x, algorithm="fricas")

[Out] 1/5413113112\*(214898200\*x^3 + 13376294\*sqrt(31)\*(10\*x^4 + x^3 + 16\*x^2 + 7\*x + 6)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) - 5322018\*sqrt(23)\*(10\*x^4 + x^3 + 16\*x^2 + 7\*x + 6)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) - 290975300\*x^2 - 9659011\*(10\*x^4 + x^3 + 16\*x^2 + 7\*x + 6)\*log(5\*x^2 + 3\*x + 2) + 9659011\*(10\*x^4 + x^3 + 16\*x^2 + 7\*x + 6)\*log(2\*x^2 - x + 3) + 349923288\*x - 136217224)/(10\*x^4 + x^3 + 16\*x^2 + 7\*x + 6)

**giac [A]** time = 0.19, size = 96, normalized size = 0.76

$$\frac{12643}{5116364} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x + 3)\right) - \frac{2769}{2816396} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4x - 1)\right) + \frac{6850x^3 - 9275x^2 + 11154x - 4342}{172546(10x^4 + x^3 + 16x^2 + 7x + 6)} - \frac{19}{10648} \log(5x^2 + 3x + 2) + \frac{19}{10648} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^2/(5\*x^2+3\*x+2)^2,x, algorithm="giac")

[Out] 12643/5116364\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) - 2769/2816396\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 1/172546\*(6850\*x^3 - 9275\*x^2 + 11154\*x - 4342)/(10\*x^4 + x^3 + 16\*x^2 + 7\*x + 6) - 19/10648\*log(5\*x^2 + 3\*x + 2) + 19/10648\*log(2\*x^2 - x + 3)

**maple [A]** time = 0.01, size = 94, normalized size = 0.74

$$\frac{12643\sqrt{31} \arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{5116364} - \frac{2769\sqrt{23} \arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{2816396} + \frac{19 \ln(2x^2 - x + 3)}{10648} - \frac{19 \ln(5x^2 + 3x + 2)}{10648} - \frac{532}{532}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2\*x^2-x+3)^2/(5\*x^2+3\*x+2)^2,x)

[Out] -1/5324\*(-759/31\*x+1078/155)/(x^2+3/5\*x+2/5)-19/10648\*ln(5\*x^2+3\*x+2)+12643/5116364\*31^(1/2)\*arctan(1/31\*(10\*x+3)\*31^(1/2))+1/5324\*(-77/23\*x-341/46)/(x^2-1/2\*x+3/2)+19/10648\*ln(2\*x^2-x+3)-2769/2816396\*23^(1/2)\*arctan(1/23\*(4\*x-1)\*23^(1/2))

**maxima [A]** time = 0.96, size = 96, normalized size = 0.76

$$\frac{12643}{5116364} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x + 3)\right) - \frac{2769}{2816396} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4x - 1)\right) + \frac{6850x^3 - 9275x^2 + 1115}{172546(10x^4 + x^3 + 16x^2 + 7x + 6)} - \frac{19}{10648} \log(5x^2 + 3x + 2) + \frac{19}{10648} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^2/(5\*x^2+3\*x+2)^2,x, algorithm="maxima")

[Out] 12643/5116364\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) - 2769/2816396\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 1/172546\*(6850\*x^3 - 9275\*x^2 + 1115 4\*x - 4342)/(10\*x^4 + x^3 + 16\*x^2 + 7\*x + 6) - 19/10648\*log(5\*x^2 + 3\*x + 2) + 19/10648\*log(2\*x^2 - x + 3)

**mupad [B]** time = 0.18, size = 115, normalized size = 0.91

$$\ln\left(x - \frac{1}{4} - \frac{\sqrt{23} 1i}{4}\right) \left(\frac{19}{10648} + \frac{\sqrt{23} 2769i}{5632792}\right) - \ln\left(x - \frac{1}{4} + \frac{\sqrt{23} 1i}{4}\right) \left(-\frac{19}{10648} + \frac{\sqrt{23} 2769i}{5632792}\right) - \ln\left(x + \frac{3}{10} - \frac{\sqrt{31}}{10}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((2\*x^2 - x + 3)^2\*(3\*x + 5\*x^2 + 2)^2),x)

[Out] log(x - (23^(1/2)\*1i)/4 - 1/4)\*((23^(1/2)\*2769i)/5632792 + 19/10648) - log(x + (23^(1/2)\*1i)/4 - 1/4)\*((23^(1/2)\*2769i)/5632792 - 19/10648) - log(x - (31^(1/2)\*1i)/10 + 3/10)\*((31^(1/2)\*12643i)/10232728 + 19/10648) + log(x + (31^(1/2)\*1i)/10 + 3/10)\*((31^(1/2)\*12643i)/10232728 - 19/10648) + ((507\*x)/78430 - (1855\*x^2)/345092 + (685\*x^3)/172546 - 2171/862730)/((7\*x)/10 + (8\*x^2)/5 + x^3/10 + x^4 + 3/5)

sympy [A] time = 0.36, size = 122, normalized size = 0.96

$$\frac{6850x^3 - 9275x^2 + 11154x - 4342}{1725460x^4 + 172546x^3 + 2760736x^2 + 1207822x + 1035276} + \frac{19 \log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{10648} - \frac{19 \log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{10648} - \frac{2769\sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{2816396} + \frac{12643\sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{5116364}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x\*\*2-x+3)\*\*2/(5\*x\*\*2+3\*x+2)\*\*2,x)

[Out] (6850\*x\*\*3 - 9275\*x\*\*2 + 11154\*x - 4342)/(1725460\*x\*\*4 + 172546\*x\*\*3 + 2760736\*x\*\*2 + 1207822\*x + 1035276) + 19\*log(x\*\*2 - x/2 + 3/2)/10648 - 19\*log(x\*\*2 + 3\*x/5 + 2/5)/10648 - 2769\*sqrt(23)\*atan(4\*sqrt(23)\*x/23 - sqrt(23)/23)/2816396 + 12643\*sqrt(31)\*atan(10\*sqrt(31)\*x/31 + 3\*sqrt(31)/31)/5116364



$$3.50 \quad \int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^3} dx$$

**Optimal.** Leaf size=148

$$\frac{13-6x}{506(2x^2-x+3)(5x^2+3x+2)^2} + \frac{3996965x+1765599}{235352744(5x^2+3x+2)} + \frac{5765x-9446}{690184(5x^2+3x+2)^2} + \frac{97 \log(2x^2-x+3)}{468512}$$

[Out] 1/690184\*(-9446+5765\*x)/(5\*x^2+3\*x+2)^2+1/506\*(13-6\*x)/(2\*x^2-x+3)/(5\*x^2+3\*x+2)^2+1/235352744\*(1765599+3996965\*x)/(5\*x^2+3\*x+2)+97/468512\*ln(2\*x^2-x+3)-97/468512\*ln(5\*x^2+3\*x+2)-25557/123921424\*arctan(1/23\*(1-4\*x)\*23^(1/2))\*23^(1/2)+4464079/6978720496\*arctan(1/31\*(3+10\*x)\*31^(1/2))\*31^(1/2)

**Rubi [A]** time = 0.16, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$ , Rules used = {974, 1060, 1072, 634, 618, 204, 628}

$$-\frac{9446-5765x}{690184(5x^2+3x+2)^2} + \frac{3996965x+1765599}{235352744(5x^2+3x+2)} + \frac{13-6x}{506(2x^2-x+3)(5x^2+3x+2)^2} + \frac{97 \log(2x^2-x+3)}{468512}$$

Antiderivative was successfully verified.

[In] Int[1/((3-x+2\*x^2)^2\*(2+3\*x+5\*x^2)^3),x]

[Out] -(9446-5765\*x)/(690184\*(2+3\*x+5\*x^2)^2) + (13-6\*x)/(506\*(3-x+2\*x^2)\*(2+3\*x+5\*x^2)^2) + (1765599+3996965\*x)/(235352744\*(2+3\*x+5\*x^2)) - (25557\*ArcTan[(1-4\*x)/Sqrt[23]])/(5387888\*Sqrt[23]) + (4464079\*ArcTan[(3+10\*x)/Sqrt[31]])/(225120016\*Sqrt[31]) + (97\*Log[3-x+2\*x^2])/468512 - (97\*Log[2+3\*x+5\*x^2])/468512

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2-4\*a\*c-x^2, x], x], x, b+2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2-4\*a\*c, 0]

### Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 974

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((d_) + (e_)*(x_) + (f_)*(x
_)^2)^(q_), x_Symbol] := Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q,
0]
```

### Rule 1060

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((A_) + (B_)*(x_) + (C_)*(x
_)^2)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x +
c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
+ q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
```

```

A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f))*
(b*f*(p + 1) - c*e*(2*p + q + 4))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && !LtQ[q, -1]) && !
IGtQ[q, 0]

```

### Rule 1072

```

Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
+ c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x]/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x]/(d + e*x + f*x^2), x], x] /; NeQ[q, 0] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]

```

### Rubi steps

$$\begin{aligned}
\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^3} dx &= \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)^2} - \frac{\int \frac{-2750-3531x+1650x^2}{(3-x+2x^2)(2+3x+5x^2)^3} dx}{5566} \\
&= -\frac{9446-5765x}{690184(2+3x+5x^2)^2} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)^2} - \frac{\int \frac{-8251}{(3-x+2x^2)(2+3x+5x^2)^3} dx}{5566} \\
&= -\frac{9446-5765x}{690184(2+3x+5x^2)^2} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)^2} + \frac{1765}{235352} \\
&= -\frac{9446-5765x}{690184(2+3x+5x^2)^2} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)^2} + \frac{1765}{235352} \\
&= -\frac{9446-5765x}{690184(2+3x+5x^2)^2} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)^2} + \frac{1765}{235352} \\
&= -\frac{9446-5765x}{690184(2+3x+5x^2)^2} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)^2} + \frac{1765}{235352} \\
&= -\frac{9446-5765x}{690184(2+3x+5x^2)^2} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)^2} + \frac{1765}{235352} \\
&= -\frac{9446-5765x}{690184(2+3x+5x^2)^2} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)^2} + \frac{1765}{235352}
\end{aligned}$$

**Mathematica [A]** time = 0.08, size = 136, normalized size = 0.92

$$\frac{90x-11}{244904(2x^2-x+3)} + \frac{164380x+67573}{10232728(5x^2+3x+2)} + \frac{345x-98}{30008(5x^2+3x+2)^2} + \frac{97 \log(2x^2-x+3)}{468512} - \frac{97 \log(5x^2+3x+2)}{468512}$$

Antiderivative was successfully verified.

[In] Integrate[1/((3-x+2\*x^2)^2\*(2+3\*x+5\*x^2)^3),x]

[Out] (-11+90\*x)/(244904\*(3-x+2\*x^2)) + (-98+345\*x)/(30008\*(2+3\*x+5\*x^2)^2) + (67573+164380\*x)/(10232728\*(2+3\*x+5\*x^2)) + (25557\*ArcTan[(-1+4\*x)/Sqrt[23]])/(5387888\*Sqrt[23]) + (4464079\*ArcTan[(3+10\*x)/Sqrt[31]])/(225120016\*Sqrt[31]) + (97\*Log[3-x+2\*x^2])/468512 - (97\*Log[2+3\*x+5\*x^2])/468512

**fricas** [A] time = 1.07, size = 237, normalized size = 1.60

$$1253927859800 x^5 + 679296504260 x^4 + 2185021181068 x^3 + 4722995582 \sqrt{31} (50 x^6 + 35 x^5 + 103 x^4 + 85 x^3 + 83 x^2 + 32 x + 12)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^2/(5\*x^2+3\*x+2)^3,x, algorithm="fricas")

[Out] 1/7383486284768\*(1253927859800\*x^5 + 679296504260\*x^4 + 2185021181068\*x^3 + 4722995582\*sqrt(31)\*(50\*x^6 + 35\*x^5 + 103\*x^4 + 85\*x^3 + 83\*x^2 + 32\*x + 12)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 1522737174\*sqrt(23)\*(50\*x^6 + 35\*x^5 + 103\*x^4 + 85\*x^3 + 83\*x^2 + 32\*x + 12)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 1500218514344\*x^2 - 1528665583\*(50\*x^6 + 35\*x^5 + 103\*x^4 + 85\*x^3 + 83\*x^2 + 32\*x + 12)\*log(5\*x^2 + 3\*x + 2) + 1528665583\*(50\*x^6 + 35\*x^5 + 103\*x^4 + 85\*x^3 + 83\*x^2 + 32\*x + 12)\*log(2\*x^2 - x + 3) + 1338609358240\*x + 218880812656)/(50\*x^6 + 35\*x^5 + 103\*x^4 + 85\*x^3 + 83\*x^2 + 32\*x + 12)

**giac** [A] time = 0.20, size = 110, normalized size = 0.74

$$\frac{4464079}{6978720496} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x + 3)\right) + \frac{25557}{123921424} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4x - 1)\right) + \frac{39969650x^5 + 21652955x^4 + 69648769x^3 + 47820302x^2 + 42668920x + 6976948}{(5x^2 + 3x + 2)^2(2x^2 - x + 3)} - \frac{97}{468512} \log(5x^2 + 3x + 2) + \frac{97}{468512} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^2/(5\*x^2+3\*x+2)^3,x, algorithm="giac")

[Out] 4464079/6978720496\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 25557/123921424\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 1/235352744\*(39969650\*x^5 + 21652955\*x^4 + 69648769\*x^3 + 47820302\*x^2 + 42668920\*x + 6976948)/((5\*x^2 + 3\*x + 2)^2\*(2\*x^2 - x + 3)) - 97/468512\*log(5\*x^2 + 3\*x + 2) + 97/468512\*log(2\*x^2 - x + 3)

**maple** [A] time = 0.01, size = 106, normalized size = 0.72

$$\frac{4464079\sqrt{31} \arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{6978720496} + \frac{25557\sqrt{23} \arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{123921424} + \frac{97 \ln(2x^2 - x + 3)}{468512} - \frac{97 \ln(5x^2 + 3x + 2)}{468512}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2\*x^2-x+3)^2/(5\*x^2+3\*x+2)^3,x)

[Out] -25/234256\*(-723272/961\*x^3-3656422/4805\*x^2-14280728/24025\*x-2238016/24025)/(5\*x^2+3\*x+2)^2-97/468512\*ln(5\*x^2+3\*x+2)+4464079/6978720496\*31^(1/2)\*arc

$\tan(1/31*(10*x+3)*31^{(1/2)})+1/234256*(990/23*x-121/23)/(x^2-1/2*x+3/2)+97/4$   
 $68512*\ln(2*x^2-x+3)+25557/123921424*23^{(1/2)}*\arctan(1/23*(4*x-1)*23^{(1/2)})$

**maxima** [A] time = 0.97, size = 118, normalized size = 0.80

$$\frac{4464079}{6978720496} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x + 3)\right) + \frac{25557}{123921424} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4x - 1)\right) + \frac{39969650x^5 + 21652955x^4 + 69648769x^3 + 47820302x^2 + 42668920x + 6976948}{235352744}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^2/(5\*x^2+3\*x+2)^3,x, algorithm="maxima")

[Out] 4464079/6978720496\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 25557/123921424\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 1/235352744\*(39969650\*x^5 + 21652955\*x^4 + 69648769\*x^3 + 47820302\*x^2 + 42668920\*x + 6976948)/(50\*x^6 + 35\*x^5 + 103\*x^4 + 85\*x^3 + 83\*x^2 + 32\*x + 12) - 97/468512\*log(5\*x^2 + 3\*x + 2) + 97/468512\*log(2\*x^2 - x + 3)

**mupad** [B] time = 3.59, size = 135, normalized size = 0.91

$$\frac{799393x^5}{235352744} + \frac{4330591x^4}{2353527440} + \frac{69648769x^3}{11767637200} + \frac{23910151x^2}{5883818600} + \frac{1066723x}{294190930} + \frac{158567}{267446300} + \ln\left(x - \frac{1}{4} + \frac{\sqrt{23} \operatorname{li}}{4}\right) \left(\frac{97}{468512} + \frac{\sqrt{23} \operatorname{li}}{247842848}\right)$$

$$x^6 + \frac{7x^5}{10} + \frac{103x^4}{50} + \frac{17x^3}{10} + \frac{83x^2}{50} + \frac{16x}{25} + \frac{6}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((2\*x^2 - x + 3)^2\*(3\*x + 5\*x^2 + 2)^3), x)

[Out] log(x + (23^(1/2)\*1i)/4 - 1/4)\*((23^(1/2)\*25557i)/247842848 + 97/468512) - log(x - (23^(1/2)\*1i)/4 - 1/4)\*((23^(1/2)\*25557i)/247842848 - 97/468512) + ((1066723\*x)/294190930 + (23910151\*x^2)/5883818600 + (69648769\*x^3)/11767637200 + (4330591\*x^4)/2353527440 + (799393\*x^5)/235352744 + 158567/267446300)/((16\*x)/25 + (83\*x^2)/50 + (17\*x^3)/10 + (103\*x^4)/50 + (7\*x^5)/10 + x^6 + 6/25) - log(x - (31^(1/2)\*1i)/10 + 3/10)\*((31^(1/2)\*4464079i)/13957440992 + 97/468512) + log(x + (31^(1/2)\*1i)/10 + 3/10)\*((31^(1/2)\*4464079i)/13957440992 - 97/468512)

**sympy** [A] time = 0.40, size = 143, normalized size = 0.97

$$\frac{39969650x^5 + 21652955x^4 + 69648769x^3 + 47820302x^2 + 42668920x + 6976948}{11767637200x^6 + 8237346040x^5 + 24241332632x^4 + 20004983240x^3 + 19534277752x^2 + 7531287808x + 282429536}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x\*\*2-x+3)\*\*2/(5\*x\*\*2+3\*x+2)\*\*3,x)

```
[Out] (39969650*x**5 + 21652955*x**4 + 69648769*x**3 + 47820302*x**2 + 42668920*x
+ 6976948)/(11767637200*x**6 + 8237346040*x**5 + 24241332632*x**4 + 200049
83240*x**3 + 19534277752*x**2 + 7531287808*x + 2824232928) + 97*log(x**2 -
x/2 + 3/2)/468512 - 97*log(x**2 + 3*x/5 + 2/5)/468512 + 25557*sqrt(23)*atan
(4*sqrt(23)*x/23 - sqrt(23)/23)/123921424 + 4464079*sqrt(31)*atan(10*sqrt(3
1)*x/31 + 3*sqrt(31)/31)/6978720496
```

$$3.51 \quad \int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^3} dx$$

**Optimal.** Leaf size=98

$$\frac{625x^3}{24} + \frac{4875x^2}{32} + \frac{1331(76420x + 5229)}{135424(2x^2 - x + 3)} - \frac{14641(79x + 101)}{5888(2x^2 - x + 3)^2} - \frac{13915}{64} \log(2x^2 - x + 3) + \frac{2725x}{8} + \frac{63799791 \tan^{-1}}{16928\sqrt{2}}$$

[Out] 2725/8\*x+4875/32\*x^2+625/24\*x^3-14641/5888\*(101+79\*x)/(2\*x^2-x+3)^2+1331/135424\*(5229+76420\*x)/(2\*x^2-x+3)-13915/64\*ln(2\*x^2-x+3)+63799791/389344\*arctan(1/23\*(1-4\*x)\*23^(1/2))\*23^(1/2)

**Rubi [A]** time = 0.11, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$ , Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{625x^3}{24} + \frac{4875x^2}{32} + \frac{1331(76420x + 5229)}{135424(2x^2 - x + 3)} - \frac{14641(79x + 101)}{5888(2x^2 - x + 3)^2} - \frac{13915}{64} \log(2x^2 - x + 3) + \frac{2725x}{8} + \frac{63799791 \tan^{-1}}{16928\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3\*x + 5\*x^2)^4/(3 - x + 2\*x^2)^3,x]

[Out] (2725\*x)/8 + (4875\*x^2)/32 + (625\*x^3)/24 - (14641\*(101 + 79\*x))/(5888\*(3 - x + 2\*x^2)^2) + (1331\*(5229 + 76420\*x))/(135424\*(3 - x + 2\*x^2)) + (63799791\*ArcTan[(1 - 4\*x)/Sqrt[23]])/(16928\*Sqrt[23]) - (13915\*Log[3 - x + 2\*x^2])/64

**Rule 204**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 618**

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

**Rule 628**



```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 1657

```
Int[(Pq_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

### Rule 1660

```
Int[(Pq_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^3} dx &= -\frac{14641(101+79x)}{5888(3-x+2x^2)^2} + \frac{1}{46} \int \frac{\frac{2173869}{128} - \frac{661181x}{32} - \frac{488267x^2}{16} + \frac{143635x^3}{8} + \frac{213325x^4}{4} + \frac{83375x^5}{2}}{(3-x+2x^2)^2} \\
&= -\frac{14641(101+79x)}{5888(3-x+2x^2)^2} + \frac{1331(5229+76420x)}{135424(3-x+2x^2)} + \frac{\int \frac{-\frac{5460539}{8} - \frac{626865x}{2} + \frac{5170975x^2}{8} + \frac{1124125x^3}{2} + \frac{330625x^4}{2}}{3-x+2x^2}}{1058} \\
&= -\frac{14641(101+79x)}{5888(3-x+2x^2)^2} + \frac{1331(5229+76420x)}{135424(3-x+2x^2)} + \frac{\int \left( \frac{1441525}{4} + \frac{2578875x}{8} + \frac{330625x^2}{4} - \frac{121(101+79x)}{8} \right)}{1058} \\
&= \frac{2725x}{8} + \frac{4875x^2}{32} + \frac{625x^3}{24} - \frac{14641(101+79x)}{5888(3-x+2x^2)^2} + \frac{1331(5229+76420x)}{135424(3-x+2x^2)} - \frac{121}{84} \int \frac{11660}{3-x+2x^2} \\
&= \frac{2725x}{8} + \frac{4875x^2}{32} + \frac{625x^3}{24} - \frac{14641(101+79x)}{5888(3-x+2x^2)^2} + \frac{1331(5229+76420x)}{135424(3-x+2x^2)} - \frac{13915}{64} \int \frac{1}{3-x+2x^2} \\
&= \frac{2725x}{8} + \frac{4875x^2}{32} + \frac{625x^3}{24} - \frac{14641(101+79x)}{5888(3-x+2x^2)^2} + \frac{1331(5229+76420x)}{135424(3-x+2x^2)} - \frac{13915}{64} \log \left| \frac{2x^2-x+3}{3-x+2x^2} \right| \\
&= \frac{2725x}{8} + \frac{4875x^2}{32} + \frac{625x^3}{24} - \frac{14641(101+79x)}{5888(3-x+2x^2)^2} + \frac{1331(5229+76420x)}{135424(3-x+2x^2)} + \frac{63799791}{16928\sqrt{23}} \operatorname{ArcTan} \left[ \frac{-1+4x}{\sqrt{23}} \right]
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 98, normalized size = 1.00

$$\frac{625x^3}{24} + \frac{4875x^2}{32} + \frac{1331(76420x+5229)}{135424(2x^2-x+3)} - \frac{14641(79x+101)}{5888(2x^2-x+3)^2} - \frac{13915}{64} \log(2x^2-x+3) + \frac{2725x}{8} - \frac{63799791 \tan^{-1} \left( \frac{-1+4x}{\sqrt{23}} \right)}{16928\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x + 5\*x^2)^4/(3 - x + 2\*x^2)^3,x]

[Out] (2725\*x)/8 + (4875\*x^2)/32 + (625\*x^3)/24 - (14641\*(101 + 79\*x))/(5888\*(3 - x + 2\*x^2)^2) + (1331\*(5229 + 76420\*x))/(135424\*(3 - x + 2\*x^2)) - (63799791\*ArcTan[(-1 + 4\*x)/Sqrt[23]])/(16928\*Sqrt[23]) - (13915\*Log[3 - x + 2\*x^2])/64

**fricas** [A] time = 0.82, size = 128, normalized size = 1.31

$$486680000x^7 + 2360398000x^6 + 5100406400x^5 + 2157209100x^4 + 24531516180x^3 - 765597492\sqrt{23}(4x^4 -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^4/(2\*x^2-x+3)^3,x, algorithm="fricas")

[Out] 1/4672128\*(486680000\*x^7 + 2360398000\*x^6 + 5100406400\*x^5 + 2157209100\*x^4 + 24531516180\*x^3 - 765597492\*sqrt(23)\*(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) - 6171678159\*x^2 - 1015822830\*(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)\*log(2\*x^2 - x + 3) + 23692590858\*x - 453041787)/(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)

**giac** [A] time = 0.21, size = 72, normalized size = 0.73

$$\frac{625}{24}x^3 + \frac{4875}{32}x^2 - \frac{63799791}{389344}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{2725}{8}x + \frac{1331(76420x^3 - 32981x^2 + 102022x - 4933)}{67712(2x^2 - x + 3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^4/(2\*x^2-x+3)^3,x, algorithm="giac")

[Out] 625/24\*x^3 + 4875/32\*x^2 - 63799791/389344\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 2725/8\*x + 1331/67712\*(76420\*x^3 - 32981\*x^2 + 102022\*x - 4933)/(2\*x^2 - x + 3)^2 - 13915/64\*log(2\*x^2 - x + 3)

**maple** [A] time = 0.01, size = 73, normalized size = 0.74

$$\frac{625x^3}{24} + \frac{4875x^2}{32} + \frac{2725x}{8} - \frac{63799791\sqrt{23}\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{389344} - \frac{13915\ln(2x^2 - x + 3)}{64} - \frac{121\left(-\frac{210155}{4232}x^3 + \frac{362791}{16928}x^2 - 56\right)}{4(2x^2 - x + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+3\*x+2)^4/(2\*x^2-x+3)^3,x)

[Out] 625/24\*x^3+4875/32\*x^2+2725/8\*x-121/4\*(-210155/4232\*x^3+362791/16928\*x^2-561121/8464\*x+54263/16928)/(2\*x^2-x+3)^2-13915/64\*ln(2\*x^2-x+3)-63799791/389344\*23^(1/2)\*arctan(1/23\*(4\*x-1)\*23^(1/2))

**maxima** [A] time = 0.95, size = 82, normalized size = 0.84

$$\frac{625}{24}x^3 + \frac{4875}{32}x^2 - \frac{63799791}{389344}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{2725}{8}x + \frac{1331(76420x^3 - 32981x^2 + 102022x - 4933)}{67712(4x^4 - 4x^3 + 13x^2 - 6x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^4/(2\*x^2-x+3)^3,x, algorithm="maxima")

[Out]  $625/24*x^3 + 4875/32*x^2 - 63799791/389344*\sqrt{23}*\arctan(1/23*\sqrt{23}*(4*x - 1)) + 2725/8*x + 1331/67712*(76420*x^3 - 32981*x^2 + 102022*x - 4933)/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9) - 13915/64*\log(2*x^2 - x + 3)$

**mupad [B]** time = 0.05, size = 81, normalized size = 0.83

$$\frac{2725x}{8} - \frac{13915 \ln(2x^2 - x + 3)}{64} - \frac{63799791 \sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{389344} + \frac{4875x^2}{32} + \frac{625x^3}{24} + \frac{25428755x^3}{67712} - \frac{43897711x^2}{270848} - \frac{13}{4x^4 - x^3 + 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x + 5\*x^2 + 2)^4/(2\*x^2 - x + 3)^3,x)

[Out]  $(2725*x)/8 - (13915*\log(2*x^2 - x + 3))/64 - (63799791*23^{(1/2)}*\operatorname{atan}((4*23^{(1/2)}*x)/23 - 23^{(1/2)}/23))/389344 + (4875*x^2)/32 + (625*x^3)/24 + ((67895641*x)/135424 - (43897711*x^2)/270848 + (25428755*x^3)/67712 - 6565823/270848)/((13*x^2)/4 - (3*x)/2 - x^3 + x^4 + 9/4)$

**sympy [A]** time = 0.24, size = 95, normalized size = 0.97

$$\frac{625x^3}{24} + \frac{4875x^2}{32} + \frac{2725x}{8} + \frac{101715020x^3 - 43897711x^2 + 135791282x - 6565823}{270848x^4 - 270848x^3 + 880256x^2 - 406272x + 609408} - \frac{13915 \log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{64} - \frac{63799791 \sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{389344}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*2+3\*x+2)\*\*4/(2\*x\*\*2-x+3)\*\*3,x)

[Out]  $625*x**3/24 + 4875*x**2/32 + 2725*x/8 + (101715020*x**3 - 43897711*x**2 + 135791282*x - 6565823)/(270848*x**4 - 270848*x**3 + 880256*x**2 - 406272*x + 609408) - 13915*\log(x**2 - x/2 + 3/2)/64 - 63799791*\sqrt{23}*\operatorname{atan}(4*\sqrt{23}*x/23 - \sqrt{23}/23)/389344$

$$3.52 \quad \int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^3} dx$$

**Optimal.** Leaf size=84

$$\frac{121(21193 - 12828x)}{33856(2x^2 - x + 3)} - \frac{1331(17 - 45x)}{1472(2x^2 - x + 3)^2} + \frac{825}{32} \log(2x^2 - x + 3) + \frac{125x}{8} + \frac{165099 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8464\sqrt{23}}$$

[Out] 125/8\*x-1331/1472\*(17-45\*x)/(2\*x^2-x+3)^2+121/33856\*(21193-12828\*x)/(2\*x^2-x+3)+825/32\*ln(2\*x^2-x+3)+165099/194672\*arctan(1/23\*(1-4\*x)\*23^(1/2))\*23^(1/2)

**Rubi [A]** time = 0.09, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$ , Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{121(21193 - 12828x)}{33856(2x^2 - x + 3)} - \frac{1331(17 - 45x)}{1472(2x^2 - x + 3)^2} + \frac{825}{32} \log(2x^2 - x + 3) + \frac{125x}{8} + \frac{165099 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8464\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3\*x + 5\*x^2)^3/(3 - x + 2\*x^2)^3,x]

[Out] (125\*x)/8 - (1331\*(17 - 45\*x))/(1472\*(3 - x + 2\*x^2)^2) + (121\*(21193 - 12828\*x))/(33856\*(3 - x + 2\*x^2)) + (165099\*ArcTan[(1 - 4\*x)/Sqrt[23]])/(8464\*Sqrt[23]) + (825\*Log[3 - x + 2\*x^2])/32

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,

e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

### Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^3} dx &= -\frac{1331(17-45x)}{1472(3-x+2x^2)^2} + \frac{1}{46} \int \frac{-\frac{40885}{32} - \frac{19067x}{8} + \frac{22195x^2}{4} + \frac{13225x^3}{2} + 2875x^4}{(3-x+2x^2)^2} dx \\
&= -\frac{1331(17-45x)}{1472(3-x+2x^2)^2} + \frac{121(21193-12828x)}{33856(3-x+2x^2)} + \frac{\int \frac{\frac{23997}{2} + 92575x + \frac{66125x^2}{2}}{3-x+2x^2} dx}{1058} \\
&= -\frac{1331(17-45x)}{1472(3-x+2x^2)^2} + \frac{121(21193-12828x)}{33856(3-x+2x^2)} + \frac{\int \left( \frac{66125}{4} - \frac{33(4557-13225x)}{4(3-x+2x^2)} \right) dx}{1058} \\
&= \frac{125x}{8} - \frac{1331(17-45x)}{1472(3-x+2x^2)^2} + \frac{121(21193-12828x)}{33856(3-x+2x^2)} - \frac{33 \int \frac{4557-13225x}{3-x+2x^2} dx}{4232} \\
&= \frac{125x}{8} - \frac{1331(17-45x)}{1472(3-x+2x^2)^2} + \frac{121(21193-12828x)}{33856(3-x+2x^2)} - \frac{165099 \int \frac{1}{3-x+2x^2} dx}{16928} + \frac{825}{32} \int \\
&= \frac{125x}{8} - \frac{1331(17-45x)}{1472(3-x+2x^2)^2} + \frac{121(21193-12828x)}{33856(3-x+2x^2)} + \frac{825}{32} \log(3-x+2x^2) + \frac{165099}{32} \\
&= \frac{125x}{8} - \frac{1331(17-45x)}{1472(3-x+2x^2)^2} + \frac{121(21193-12828x)}{33856(3-x+2x^2)} + \frac{165099 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8464\sqrt{23}} + \frac{825}{32} \log
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 84, normalized size = 1.00

$$-\frac{121(12828x-21193)}{33856(2x^2-x+3)} + \frac{1331(45x-17)}{1472(2x^2-x+3)^2} + \frac{825}{32} \log(2x^2-x+3) + \frac{125x}{8} - \frac{165099 \tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{8464\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x + 5\*x^2)^3/(3 - x + 2\*x^2)^3,x]

[Out] (125\*x)/8 + (1331\*(-17 + 45\*x))/(1472\*(3 - x + 2\*x^2)^2) - (121\*(-21193 + 12828\*x))/(33856\*(3 - x + 2\*x^2)) - (165099\*ArcTan[(-1 + 4\*x)/Sqrt[23]])/(8464\*Sqrt[23]) + (825\*Log[3 - x + 2\*x^2])/32

**fricas** [A] time = 0.82, size = 118, normalized size = 1.40

$$\frac{24334000x^5 - 24334000x^4 + 43385176x^3 - 330198\sqrt{23}(4x^4 - 4x^3 + 13x^2 - 6x + 9)\arctan\left(\frac{1}{23}\sqrt{23}(4x - 1)\right)}{389344(4x^4 - 4x^3 + 13x^2 - 6x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^3/(2\*x^2-x+3)^3,x, algorithm="fricas")

[Out] 1/389344\*(24334000\*x^5 - 24334000\*x^4 + 43385176\*x^3 - 330198\*sqrt(23)\*(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 40329281\*x^2 + 10037775\*(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)\*log(2\*x^2 - x + 3) - 12446818\*x + 82485337)/(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)

**giac** [A] time = 0.18, size = 62, normalized size = 0.74

$$-\frac{165099}{194672}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{125}{8}x - \frac{121(12828x^3 - 27607x^2 + 24146x - 29639)}{16928(2x^2 - x + 3)^2} + \frac{825}{32}\log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^3/(2\*x^2-x+3)^3,x, algorithm="giac")

[Out] -165099/194672\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 125/8\*x - 121/16928\*(12828\*x^3 - 27607\*x^2 + 24146\*x - 29639)/(2\*x^2 - x + 3)^2 + 825/32\*log(2\*x^2 - x + 3)

**maple** [A] time = 0.01, size = 63, normalized size = 0.75

$$\frac{125x}{8} - \frac{165099\sqrt{23}\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{194672} + \frac{825\ln(2x^2 - x + 3)}{32} + \frac{-\frac{388047}{4232}x^3 + \frac{3340447}{16928}x^2 - \frac{1460833}{8464}x + \frac{3586319}{16928}}{(2x^2 - x + 3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+3\*x+2)^3/(2\*x^2-x+3)^3,x)

[Out] 125/8\*x+11/2\*(-35277/2116\*x^3+303677/8464\*x^2-132803/4232\*x+326029/8464)/(2\*x^2-x+3)^2+825/32\*ln(2\*x^2-x+3)-165099/194672\*23^(1/2)\*arctan(1/23\*(4\*x-1)\*23^(1/2))

**maxima** [A] time = 0.96, size = 72, normalized size = 0.86

$$-\frac{165099}{194672}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{125}{8}x - \frac{121(12828x^3 - 27607x^2 + 24146x - 29639)}{16928(4x^4 - 4x^3 + 13x^2 - 6x + 9)} + \frac{825}{32}\log(2x^2 - x + 3)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^3/(2\*x^2-x+3)^3,x, algorithm="maxima")

[Out] -165099/194672\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 125/8\*x - 121/169  
28\*(12828\*x^3 - 27607\*x^2 + 24146\*x - 29639)/(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x  
+ 9) + 825/32\*log(2\*x^2 - x + 3)

**mupad [B]** time = 0.05, size = 72, normalized size = 0.86

$$\frac{125x}{8} + \frac{825 \ln(2x^2 - x + 3)}{32} - \frac{165099 \sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{194672} - \frac{\frac{388047x^3}{16928} - \frac{3340447x^2}{67712} + \frac{1460833x}{33856} - \frac{3586319}{67712}}{x^4 - x^3 + \frac{13x^2}{4} - \frac{3x}{2} + \frac{9}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x + 5\*x^2 + 2)^3/(2\*x^2 - x + 3)^3,x)

[Out] (125\*x)/8 + (825\*log(2\*x^2 - x + 3))/32 - (165099\*23^(1/2)\*atan((4\*23^(1/2)  
\*x)/23 - 23^(1/2)/23))/194672 - ((1460833\*x)/33856 - (3340447\*x^2)/67712 +  
(388047\*x^3)/16928 - 3586319/67712)/((13\*x^2)/4 - (3\*x)/2 - x^3 + x^4 + 9/4  
)

**sympy [A]** time = 0.23, size = 82, normalized size = 0.98

$$\frac{125x}{8} + \frac{-1552188x^3 + 3340447x^2 - 2921666x + 3586319}{67712x^4 - 67712x^3 + 220064x^2 - 101568x + 152352} + \frac{825 \log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{32} - \frac{165099\sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{194672}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*2+3\*x+2)\*\*3/(2\*x\*\*2-x+3)\*\*3,x)

[Out] 125\*x/8 + (-1552188\*x\*\*3 + 3340447\*x\*\*2 - 2921666\*x + 3586319)/(67712\*x\*\*4  
- 67712\*x\*\*3 + 220064\*x\*\*2 - 101568\*x + 152352) + 825\*log(x\*\*2 - x/2 + 3/2)  
/32 - 165099\*sqrt(23)\*atan(4\*sqrt(23)\*x/23 - sqrt(23)/23)/194672

$$3.53 \quad \int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^3} dx$$

Optimal. Leaf size=64

$$\frac{121(19-7x)}{368(2x^2-x+3)^2} - \frac{55(332x+975)}{8464(2x^2-x+3)} - \frac{4330 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{529\sqrt{23}}$$

[Out] 121/368\*(19-7\*x)/(2\*x^2-x+3)^2-55/8464\*(975+332\*x)/(2\*x^2-x+3)-4330/12167\*arctan(1/23\*(1-4\*x)\*23^(1/2))\*23^(1/2)

**Rubi [A]** time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$ , Rules used = {1660, 12, 618, 204}

$$\frac{121(19-7x)}{368(2x^2-x+3)^2} - \frac{55(332x+975)}{8464(2x^2-x+3)} - \frac{4330 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{529\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3\*x + 5\*x^2)^2/(3 - x + 2\*x^2)^3,x]

[Out] (121\*(19 - 7\*x))/(368\*(3 - x + 2\*x^2)^2) - (55\*(975 + 332\*x))/(8464\*(3 - x + 2\*x^2)) - (4330\*ArcTan[(1 - 4\*x)/Sqrt[23]])/(529\*Sqrt[23])

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2 + 3x + 5x^2)^2}{(3 - x + 2x^2)^3} dx &= \frac{121(19 - 7x)}{368(3 - x + 2x^2)^2} + \frac{1}{46} \int \frac{-\frac{195}{8} + \frac{1955x}{2} + 575x^2}{(3 - x + 2x^2)^2} dx \\
&= \frac{121(19 - 7x)}{368(3 - x + 2x^2)^2} - \frac{55(975 + 332x)}{8464(3 - x + 2x^2)} + \frac{\int \frac{4330}{3 - x + 2x^2} dx}{1058} \\
&= \frac{121(19 - 7x)}{368(3 - x + 2x^2)^2} - \frac{55(975 + 332x)}{8464(3 - x + 2x^2)} + \frac{2165}{529} \int \frac{1}{3 - x + 2x^2} dx \\
&= \frac{121(19 - 7x)}{368(3 - x + 2x^2)^2} - \frac{55(975 + 332x)}{8464(3 - x + 2x^2)} - \frac{4330}{529} \operatorname{Subst} \left( \int \frac{1}{-23 - x^2} dx, x, -1 + 4x \right) \\
&= \frac{121(19 - 7x)}{368(3 - x + 2x^2)^2} - \frac{55(975 + 332x)}{8464(3 - x + 2x^2)} - \frac{4330 \tan^{-1} \left( \frac{1 - 4x}{\sqrt{23}} \right)}{529\sqrt{23}}
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 51, normalized size = 0.80

$$\frac{4330 \tan^{-1} \left( \frac{4x-1}{\sqrt{23}} \right)}{529\sqrt{23}} - \frac{11(1660x^3 + 4045x^2 + 938x + 4909)}{4232(-2x^2 + x - 3)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x + 5\*x^2)^2/(3 - x + 2\*x^2)^3, x]

[Out] (-11\*(4909 + 938\*x + 4045\*x^2 + 1660\*x^3))/(4232\*(-3 + x - 2\*x^2)^2) + (4330\*ArcTan[(-1 + 4\*x)/Sqrt[23]])/(529\*Sqrt[23])

**fricas** [A] time = 0.87, size = 75, normalized size = 1.17

$$\frac{419980x^3 - 34640\sqrt{23}(4x^4 - 4x^3 + 13x^2 - 6x + 9)\arctan\left(\frac{1}{23}\sqrt{23}(4x - 1)\right) + 1023385x^2 + 237314x + 1241977}{97336(4x^4 - 4x^3 + 13x^2 - 6x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^2/(2\*x^2-x+3)^3,x, algorithm="fricas")

[Out] -1/97336\*(419980\*x^3 - 34640\*sqrt(23)\*(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 1023385\*x^2 + 237314\*x + 1241977)/(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)

**giac** [A] time = 0.19, size = 46, normalized size = 0.72

$$\frac{4330}{12167}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x - 1)\right) - \frac{11(1660x^3 + 4045x^2 + 938x + 4909)}{4232(2x^2 - x + 3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^2/(2\*x^2-x+3)^3,x, algorithm="giac")

[Out] 4330/12167\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) - 11/4232\*(1660\*x^3 + 4045\*x^2 + 938\*x + 4909)/(2\*x^2 - x + 3)^2

**maple** [A] time = 0.01, size = 47, normalized size = 0.73

$$\frac{4330\sqrt{23}\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{12167} + \frac{-\frac{4565}{1058}x^3 - \frac{44495}{4232}x^2 - \frac{5159}{2116}x - \frac{53999}{4232}}{(2x^2 - x + 3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+3\*x+2)^2/(2\*x^2-x+3)^3,x)

[Out] 4\*(-4565/4232\*x^3-44495/16928\*x^2-5159/8464\*x-53999/16928)/(2\*x^2-x+3)^2+4330/12167\*23^(1/2)\*arctan(1/23\*(4\*x-1)\*23^(1/2))

**maxima** [A] time = 0.96, size = 56, normalized size = 0.88

$$\frac{4330}{12167}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x - 1)\right) - \frac{11(1660x^3 + 4045x^2 + 938x + 4909)}{4232(4x^4 - 4x^3 + 13x^2 - 6x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^2/(2\*x^2-x+3)^3,x, algorithm="maxima")

[Out] 4330/12167\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) - 11/4232\*(1660\*x^3 + 4045\*x^2 + 938\*x + 4909)/(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)

**mupad [B]** time = 3.47, size = 56, normalized size = 0.88

$$\frac{4330\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{12167} - \frac{\frac{4565x^3}{4232} + \frac{44495x^2}{16928} + \frac{5159x}{8464} + \frac{53999}{16928}}{x^4 - x^3 + \frac{13x^2}{4} - \frac{3x}{2} + \frac{9}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x + 5\*x^2 + 2)^2/(2\*x^2 - x + 3)^3,x)

[Out] (4330\*23^(1/2)\*atan((4\*23^(1/2)\*x)/23 - 23^(1/2)/23))/12167 - ((5159\*x)/8464 + (44495\*x^2)/16928 + (4565\*x^3)/4232 + 53999/16928)/((13\*x^2)/4 - (3\*x)/2 - x^3 + x^4 + 9/4)

**sympy [A]** time = 0.20, size = 63, normalized size = 0.98

$$\frac{-18260x^3 - 44495x^2 - 10318x - 53999}{16928x^4 - 16928x^3 + 55016x^2 - 25392x + 38088} + \frac{4330\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{12167}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*2+3\*x+2)\*\*2/(2\*x\*\*2-x+3)\*\*3,x)

[Out] (-18260\*x\*\*3 - 44495\*x\*\*2 - 10318\*x - 53999)/(16928\*x\*\*4 - 16928\*x\*\*3 + 55016\*x\*\*2 - 25392\*x + 38088) + 4330\*sqrt(23)\*atan(4\*sqrt(23)\*x/23 - sqrt(23)/23)/12167

$$3.54 \quad \int \frac{2+3x+5x^2}{(3-x+2x^2)^3} dx$$

**Optimal.** Leaf size=64

$$-\frac{131(1-4x)}{2116(2x^2-x+3)} - \frac{11(3x+5)}{92(2x^2-x+3)^2} - \frac{262 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{529\sqrt{23}}$$

[Out]  $-11/92*(5+3*x)/(2*x^2-x+3)^2-131/2116*(1-4*x)/(2*x^2-x+3)-262/12167*\arctan(1/23*(1-4*x)*23^{(1/2)})*23^{(1/2)}$

**Rubi [A]** time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$ , Rules used = {1660, 12, 614, 618, 204}

$$-\frac{131(1-4x)}{2116(2x^2-x+3)} - \frac{11(3x+5)}{92(2x^2-x+3)^2} - \frac{262 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{529\sqrt{23}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^3, x]$

[Out]  $(-11*(5 + 3*x))/(92*(3 - x + 2*x^2)^2) - (131*(1 - 4*x))/(2116*(3 - x + 2*x^2)) - (262*\text{ArcTan}[(1 - 4*x)/\text{Sqrt}[23]])/(529*\text{Sqrt}[23])$

#### Rule 12

$\text{Int}[(a_*)(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{Match}[\text{Q}[u, (b_)*(v_)] /; \text{FreeQ}[b, x]]$

#### Rule 204

$\text{Int}[((a_*) + (b_.)*(x_)^2)^{-1}, x\_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

#### Rule 614

$\text{Int}[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^{(p_)}, x\_Symbol] \rightarrow \text{Simp}[(b + 2*c*x)*(a + b*x + c*x^2)^{(p+1)}/((p+1)*(b^2 - 4*a*c)), x] - \text{Dist}[(2*c*(2*p+3))/((p+1)*(b^2 - 4*a*c)), \text{Int}[(a + b*x + c*x^2)^{(p+1)}, x], x] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{NeQ}[p, -3/2] \ \&\& \ \text{Int}$

egerQ[4\*p]

### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 1660

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 1]}, Simp[((b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] + Dist[1/((p + 1)\*(b^2 - 4\*a\*c)), Int[(a + b\*x + c\*x^2)^(p + 1)\*ExpandToSum[(p + 1)\*(b^2 - 4\*a\*c)\*Q - (2\*p + 3)\*(2\*c\*f - b\*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && LtQ[p, -1]

### Rubi steps

$$\begin{aligned}
 \int \frac{2 + 3x + 5x^2}{(3 - x + 2x^2)^3} dx &= -\frac{11(5 + 3x)}{92(3 - x + 2x^2)^2} + \frac{1}{46} \int \frac{131}{2(3 - x + 2x^2)^2} dx \\
 &= -\frac{11(5 + 3x)}{92(3 - x + 2x^2)^2} + \frac{131}{92} \int \frac{1}{(3 - x + 2x^2)^2} dx \\
 &= -\frac{11(5 + 3x)}{92(3 - x + 2x^2)^2} - \frac{131(1 - 4x)}{2116(3 - x + 2x^2)} + \frac{131}{529} \int \frac{1}{3 - x + 2x^2} dx \\
 &= -\frac{11(5 + 3x)}{92(3 - x + 2x^2)^2} - \frac{131(1 - 4x)}{2116(3 - x + 2x^2)} - \frac{262}{529} \text{Subst} \left( \int \frac{1}{-23 - x^2} dx, x, -1 + 4x \right) \\
 &= -\frac{11(5 + 3x)}{92(3 - x + 2x^2)^2} - \frac{131(1 - 4x)}{2116(3 - x + 2x^2)} - \frac{262 \tan^{-1} \left( \frac{1-4x}{\sqrt{23}} \right)}{529\sqrt{23}}
 \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 51, normalized size = 0.80

$$\frac{46(524x^3 - 393x^2 + 472x - 829)}{(-2x^2 + x - 3)^2} + 1048\sqrt{23} \tan^{-1} \left( \frac{4x-1}{\sqrt{23}} \right)$$


---

48668

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x + 5\*x^2)/(3 - x + 2\*x^2)^3,x]

[Out] ((46\*(-829 + 472\*x - 393\*x^2 + 524\*x^3))/(-3 + x - 2\*x^2)^2 + 1048\*Sqrt[23]\*ArcTan[(-1 + 4\*x)/Sqrt[23]])/48668

**fricas** [A] time = 0.92, size = 75, normalized size = 1.17

$$\frac{12052x^3 + 524\sqrt{23}(4x^4 - 4x^3 + 13x^2 - 6x + 9)\arctan\left(\frac{1}{23}\sqrt{23}(4x - 1)\right) - 9039x^2 + 10856x - 19067}{24334(4x^4 - 4x^3 + 13x^2 - 6x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)/(2\*x^2-x+3)^3,x, algorithm="fricas")

[Out] 1/24334\*(12052\*x^3 + 524\*sqrt(23)\*(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) - 9039\*x^2 + 10856\*x - 19067)/(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)

**giac** [A] time = 0.21, size = 46, normalized size = 0.72

$$\frac{262}{12167}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x - 1)\right) + \frac{524x^3 - 393x^2 + 472x - 829}{1058(2x^2 - x + 3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)/(2\*x^2-x+3)^3,x, algorithm="giac")

[Out] 262/12167\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 1/1058\*(524\*x^3 - 393\*x^2 + 472\*x - 829)/(2\*x^2 - x + 3)^2

**maple** [A] time = 0.01, size = 47, normalized size = 0.73

$$\frac{262\sqrt{23}\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{12167} + \frac{\frac{262}{529}x^3 - \frac{393}{1058}x^2 + \frac{236}{529}x - \frac{829}{1058}}{(2x^2 - x + 3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+3\*x+2)/(2\*x^2-x+3)^3,x)

[Out] 4\*(131/1058\*x^3-393/4232\*x^2+59/529\*x-829/4232)/(2\*x^2-x+3)^2+262/12167\*23^(1/2)\*arctan(1/23\*(4\*x-1)\*23^(1/2))



**maxima** [A] time = 0.96, size = 56, normalized size = 0.88

$$\frac{262}{12167} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4x - 1)\right) + \frac{524x^3 - 393x^2 + 472x - 829}{1058(4x^4 - 4x^3 + 13x^2 - 6x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)/(2\*x^2-x+3)^3,x, algorithm="maxima")

[Out] 262/12167\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 1/1058\*(524\*x^3 - 393\*x^2 + 472\*x - 829)/(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)

**mupad** [B] time = 0.04, size = 55, normalized size = 0.86

$$\frac{262 \sqrt{23} \operatorname{atan}\left(\frac{4 \sqrt{23} x}{23} - \frac{\sqrt{23}}{23}\right)}{12167} + \frac{\frac{131x^3}{1058} - \frac{393x^2}{4232} + \frac{59x}{529} - \frac{829}{4232}}{x^4 - x^3 + \frac{13x^2}{4} - \frac{3x}{2} + \frac{9}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x + 5\*x^2 + 2)/(2\*x^2 - x + 3)^3,x)

[Out] (262\*23^(1/2)\*atan((4\*23^(1/2)\*x)/23 - 23^(1/2)/23))/12167 + ((59\*x)/529 - (393\*x^2)/4232 + (131\*x^3)/1058 - 829/4232)/((13\*x^2)/4 - (3\*x)/2 - x^3 + x^4 + 9/4)

**sympy** [A] time = 0.18, size = 61, normalized size = 0.95

$$\frac{524x^3 - 393x^2 + 472x - 829}{4232x^4 - 4232x^3 + 13754x^2 - 6348x + 9522} + \frac{262\sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{12167}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*2+3\*x+2)/(2\*x\*\*2-x+3)\*\*3,x)

[Out] (524\*x\*\*3 - 393\*x\*\*2 + 472\*x - 829)/(4232\*x\*\*4 - 4232\*x\*\*3 + 13754\*x\*\*2 - 6348\*x + 9522) + 262\*sqrt(23)\*atan(4\*sqrt(23)\*x/23 - sqrt(23)/23)/12167

$$3.55 \quad \int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)} dx$$

**Optimal.** Leaf size=115

$$\frac{3625 - 746x}{256036(2x^2 - x + 3)} + \frac{13 - 6x}{1012(2x^2 - x + 3)^2} - \frac{119 \log(2x^2 - x + 3)}{21296} + \frac{119 \log(5x^2 + 3x + 2)}{21296} - \frac{53403 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{5632792\sqrt{23}}$$

[Out] 1/1012\*(13-6\*x)/(2\*x^2-x+3)^2+1/256036\*(3625-746\*x)/(2\*x^2-x+3)-119/21296\*ln(2\*x^2-x+3)+119/21296\*ln(5\*x^2+3\*x+2)-53403/129554216\*arctan(1/23\*(1-4\*x)\*23^(1/2))\*23^(1/2)+247/330088\*arctan(1/31\*(3+10\*x)\*31^(1/2))\*31^(1/2)

**Rubi [A]** time = 0.12, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$ , Rules used = {974, 1060, 1072, 634, 618, 204, 628}

$$\frac{3625 - 746x}{256036(2x^2 - x + 3)} + \frac{13 - 6x}{1012(2x^2 - x + 3)^2} - \frac{119 \log(2x^2 - x + 3)}{21296} + \frac{119 \log(5x^2 + 3x + 2)}{21296} - \frac{53403 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{5632792\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - x + 2\*x^2)^3\*(2 + 3\*x + 5\*x^2)),x]

[Out] (13 - 6\*x)/(1012\*(3 - x + 2\*x^2)^2) + (3625 - 746\*x)/(256036\*(3 - x + 2\*x^2)) - (53403\*ArcTan[(1 - 4\*x)/Sqrt[23]])/(5632792\*Sqrt[23]) + (247\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(10648\*Sqrt[31]) - (119\*Log[3 - x + 2\*x^2])/21296 + (119\*Log[2 + 3\*x + 5\*x^2])/21296

**Rule 204**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 618**

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x, x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

**Rule 628**

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 974

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((d_) + (e_)*(x_) + (f_)*(x
_)^2)^(q_), x_Symbol] := Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q,
0]
```

### Rule 1060

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((A_) + (B_)*(x_) + (C_)*(x
_)^2)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x +
c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
+ q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
```

```

A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f))*
(b*f*(p + 1) - c*e*(2*p + q + 4))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !
IGtQ[q, 0]

```

### Rule 1072

```

Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
+ c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x]/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x]/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]

```

### Rubi steps

$$\begin{aligned}
\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)} dx &= \frac{13-6x}{1012(3-x+2x^2)^2} - \frac{\int \frac{-3652-1936x+990x^2}{(3-x+2x^2)^2(2+3x+5x^2)} dx}{11132} \\
&= \frac{13-6x}{1012(3-x+2x^2)^2} + \frac{3625-746x}{256036(3-x+2x^2)} - \frac{\int \frac{-6551908-7779574x+902660}{(3-x+2x^2)(2+3x+5x^2)} dx}{61960712} \\
&= \frac{13-6x}{1012(3-x+2x^2)^2} + \frac{3625-746x}{256036(3-x+2x^2)} - \frac{\int \frac{-154867174+335151124x}{3-x+2x^2} dx}{14994492304} \\
&= \frac{13-6x}{1012(3-x+2x^2)^2} + \frac{3625-746x}{256036(3-x+2x^2)} + \frac{53403 \int \frac{1}{3-x+2x^2} dx}{11265584} - \frac{119}{21296} \log(3-x+2x^2) + \frac{24}{5632792\sqrt{23}} \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.16, size = 99, normalized size = 0.86

$$\frac{713 \left( -62951 \log(2x^2 - x + 3) + 62951 \log(5x^2 + 3x + 2) - \frac{44(1492x^3 - 7996x^2 + 7381x - 14164)}{(-2x^2 + x - 3)^2} \right) + 3310986\sqrt{23} \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8032361392}$$

Antiderivative was successfully verified.

[In] Integrate[1/((3 - x + 2\*x^2)^3\*(2 + 3\*x + 5\*x^2)),x]

[Out] (3310986\*sqrt[23]\*ArcTan[(-1 + 4\*x)/sqrt[23]] + 6010498\*sqrt[31]\*ArcTan[(3 + 10\*x)/sqrt[31]] + 713\*((-44\*(-14164 + 7381\*x - 7996\*x^2 + 1492\*x^3))/(-3 + x - 2\*x^2)^2 - 62951\*Log[3 - x + 2\*x^2] + 62951\*Log[2 + 3\*x + 5\*x^2]))/8032361392

**fricas [A]** time = 0.83, size = 177, normalized size = 1.54

$$\frac{46807024x^3 - 6010498\sqrt{31}(4x^4 - 4x^3 + 13x^2 - 6x + 9) \arctan\left(\frac{1}{31}\sqrt{31}(10x + 3)\right) - 3310986\sqrt{23}(4x^4 - \dots)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^3/(5\*x^2+3\*x+2),x, algorithm="fricas")

[Out] -1/8032361392\*(46807024\*x^3 - 6010498\*sqrt(31)\*(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) - 3310986\*sqrt(23)\*(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) - 250850512\*x^2 - 44884063\*(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)\*log(5\*x^2 + 3\*x + 2) + 44884063\*(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)\*log(2\*x^2 - x + 3) + 231556732\*x - 444353008)/(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)

**giac** [A] time = 0.21, size = 88, normalized size = 0.77

$$\frac{247}{330088} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x + 3)\right) + \frac{53403}{129554216} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4x - 1)\right) - \frac{1492x^3 - 7996x^2 + 7381x - 14164}{256036(2x^2 - x + 3)^2} + \frac{119}{21296} \log(5x^2 + 3x + 2) - \frac{119}{21296} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^3/(5\*x^2+3\*x+2),x, algorithm="giac")

[Out] 247/330088\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 53403/129554216\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) - 1/256036\*(1492\*x^3 - 7996\*x^2 + 7381\*x - 14164)/(2\*x^2 - x + 3)^2 + 119/21296\*log(5\*x^2 + 3\*x + 2) - 119/21296\*log(2\*x^2 - x + 3)

**maple** [A] time = 0.01, size = 89, normalized size = 0.77

$$\frac{247\sqrt{31} \arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{330088} + \frac{53403\sqrt{23} \arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{129554216} - \frac{119 \ln(2x^2 - x + 3)}{21296} + \frac{119 \ln(5x^2 + 3x + 2)}{21296} - \frac{8206x^3 - 43978x^2 + 81191x - 77902}{529(2x^2 - x + 3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2\*x^2-x+3)^3/(5\*x^2+3\*x+2),x)

[Out] 119/21296\*ln(5\*x^2+3\*x+2)+247/330088\*31^(1/2)\*arctan(1/31\*(10\*x+3)\*31^(1/2))-1/2662\*(8206/529\*x^3-43978/529\*x^2+81191/1058\*x-77902/529)/(2\*x^2-x+3)^2-119/21296\*ln(2\*x^2-x+3)+53403/129554216\*23^(1/2)\*arctan(1/23\*(4\*x-1)\*23^(1/2))

**maxima** [A] time = 0.97, size = 98, normalized size = 0.85

$$\frac{247}{330088} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x + 3)\right) + \frac{53403}{129554216} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4x - 1)\right) - \frac{1492x^3 - 7996x^2 + 7381x - 14164}{256036(4x^4 - 4x^3 + 13x^2 - 6x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^3/(5\*x^2+3\*x+2),x, algorithm="maxima")

[Out] 247/330088\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 53403/129554216\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) - 1/256036\*(1492\*x^3 - 7996\*x^2 + 7381\*x - 14164)/(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9) + 119/21296\*log(5\*x^2 + 3\*x + 2) - 119/21296\*log(2\*x^2 - x + 3)

**mupad [B]** time = 3.58, size = 116, normalized size = 1.01

$$-\ln\left(x + \frac{3}{10} - \frac{\sqrt{31} \operatorname{li}}{10}\right)\left(-\frac{119}{21296} + \frac{\sqrt{31} \operatorname{247i}}{660176}\right) + \ln\left(x + \frac{3}{10} + \frac{\sqrt{31} \operatorname{li}}{10}\right)\left(\frac{119}{21296} + \frac{\sqrt{31} \operatorname{247i}}{660176}\right) - \ln\left(x - \frac{1}{4} - \frac{\sqrt{23}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((2\*x^2 - x + 3)^3\*(3\*x + 5\*x^2 + 2)),x)

[Out] log(x + (31^(1/2)\*1i)/10 + 3/10)\*((31^(1/2)\*247i)/660176 + 119/21296) - log(x - (31^(1/2)\*1i)/10 + 3/10)\*((31^(1/2)\*247i)/660176 - 119/21296) - log(x - (23^(1/2)\*1i)/4 - 1/4)\*((23^(1/2)\*53403i)/259108432 + 119/21296) + log(x + (23^(1/2)\*1i)/4 - 1/4)\*((23^(1/2)\*53403i)/259108432 - 119/21296) - ((61\*x)/8464 - (1999\*x^2)/256036 + (373\*x^3)/256036 - 3541/256036)/((13\*x^2)/4 - (3\*x)/2 - x^3 + x^4 + 9/4)

**sympy [A]** time = 0.36, size = 122, normalized size = 1.06

$$\frac{-1492x^3 + 7996x^2 - 7381x + 14164}{1024144x^4 - 1024144x^3 + 3328468x^2 - 1536216x + 2304324} - \frac{119 \log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{21296} + \frac{119 \log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{21296} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x\*\*2-x+3)\*\*3/(5\*x\*\*2+3\*x+2),x)

[Out] (-1492\*x\*\*3 + 7996\*x\*\*2 - 7381\*x + 14164)/(1024144\*x\*\*4 - 1024144\*x\*\*3 + 3328468\*x\*\*2 - 1536216\*x + 2304324) - 119\*log(x\*\*2 - x/2 + 3/2)/21296 + 119\*log(x\*\*2 + 3\*x/5 + 2/5)/21296 + 53403\*sqrt(23)\*atan(4\*sqrt(23)\*x/23 - sqrt(23)/23)/129554216 + 247\*sqrt(31)\*atan(10\*sqrt(31)\*x/31 + 3\*sqrt(31)/31)/330088

$$3.56 \quad \int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)^2} dx$$

**Optimal.** Leaf size=160

$$\frac{-252815x - 2328909}{174616552(5x^2 + 3x + 2)} + \frac{9665 - 1446x}{512072(2x^2 - x + 3)(5x^2 + 3x + 2)} + \frac{13 - 6x}{1012(2x^2 - x + 3)^2(5x^2 + 3x + 2)} + \frac{181 \log(46)}{46}$$

[Out] 1/174616552\*(-2328909-252815\*x)/(5\*x^2+3\*x+2)+1/1012\*(13-6\*x)/(2\*x^2-x+3)^2/(5\*x^2+3\*x+2)+1/512072\*(9665-1446\*x)/(2\*x^2-x+3)/(5\*x^2+3\*x+2)+181/468512\*ln(2\*x^2-x+3)-181/468512\*ln(5\*x^2+3\*x+2)+2038497/2850192752\*arctan(1/23\*(1-4\*x)\*23^(1/2))\*23^(1/2)+246757/225120016\*arctan(1/31\*(3+10\*x)\*31^(1/2))\*31^(1/2)

**Rubi [A]** time = 0.16, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$ , Rules used = {974, 1060, 1072, 634, 618, 204, 628}

$$\frac{9665 - 1446x}{512072(2x^2 - x + 3)(5x^2 + 3x + 2)} - \frac{252815x + 2328909}{174616552(5x^2 + 3x + 2)} + \frac{13 - 6x}{1012(2x^2 - x + 3)^2(5x^2 + 3x + 2)} + \frac{181 \log(46)}{46}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - x + 2\*x^2)^3\*(2 + 3\*x + 5\*x^2)^2), x]

[Out] -(2328909 + 252815\*x)/(174616552\*(2 + 3\*x + 5\*x^2)) + (13 - 6\*x)/(1012\*(3 - x + 2\*x^2)^2\*(2 + 3\*x + 5\*x^2)) + (9665 - 1446\*x)/(512072\*(3 - x + 2\*x^2)\*(2 + 3\*x + 5\*x^2)) + (2038497\*ArcTan[(1 - 4\*x)/Sqrt[23]])/(123921424\*Sqrt[23]) + (246757\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(7261936\*Sqrt[31]) + (181\*Log[3 - x + 2\*x^2])/468512 - (181\*Log[2 + 3\*x + 5\*x^2])/468512

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]



Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]]/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 974

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((d_) + (e_)*(x_) + (f_)*(x
_)^2)^(q_), x_Symbol] := Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q,
0]
```

Rule 1060

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((A_) + (B_)*(x_) + (C_)*(x_
)^2)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x +
c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))]*(p
```

```

+ q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && !LtQ[q, -1]) && !
IGtQ[q, 0]

```

### Rule 1072

```

Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] := With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
+ c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x]/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x]/(d + e*x + f*x^2), x], x] /; NeQ[q, 0] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]

```

### Rubi steps

$$\begin{aligned}
\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)^2} dx &= \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)} - \frac{\int \frac{-4081-3168x+1650x^2}{(3-x+2x^2)^2(2+3x+5x^2)^2} dx}{11132} \\
&= \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)} + \frac{9665-1446x}{512072(3-x+2x^2)(2+3x+5x^2)} \\
&= -\frac{2328909+252815x}{174616552(2+3x+5x^2)} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)} + \frac{181 \log(2x^2-x+3)}{468512} - \frac{181 \log(5x^2+3x+2)}{468512} \\
&= -\frac{2328909+252815x}{174616552(2+3x+5x^2)} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)} + \frac{181 \log(2x^2-x+3)}{468512} - \frac{181 \log(5x^2+3x+2)}{468512} \\
&= -\frac{2328909+252815x}{174616552(2+3x+5x^2)} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)} + \frac{181 \log(2x^2-x+3)}{468512} - \frac{181 \log(5x^2+3x+2)}{468512} \\
&= -\frac{2328909+252815x}{174616552(2+3x+5x^2)} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)} + \frac{181 \log(2x^2-x+3)}{468512} - \frac{181 \log(5x^2+3x+2)}{468512} \\
&= -\frac{2328909+252815x}{174616552(2+3x+5x^2)} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)} + \frac{181 \log(2x^2-x+3)}{468512} - \frac{181 \log(5x^2+3x+2)}{468512}
\end{aligned}$$

**Mathematica [A]** time = 0.12, size = 136, normalized size = 0.85

$$\frac{-2923x-1782}{1408198(2x^2-x+3)} + \frac{1235x-1474}{330088(5x^2+3x+2)} + \frac{-14x-31}{22264(2x^2-x+3)^2} + \frac{181 \log(2x^2-x+3)}{468512} - \frac{181 \log(5x^2+3x+2)}{468512}$$

Antiderivative was successfully verified.

[In] Integrate[1/((3-x+2\*x^2)^3\*(2+3\*x+5\*x^2)^2),x]

[Out] (-31-14\*x)/(22264\*(3-x+2\*x^2)^2) + (-1782-2923\*x)/(1408198\*(3-x+2\*x^2)) + (-1474+1235\*x)/(330088\*(2+3\*x+5\*x^2)) - (2038497\*ArcTan[(-1+4\*x)/Sqrt[23]])/(123921424\*Sqrt[23]) + (246757\*ArcTan[(3+10\*x)/Sqrt[31]])/(7261936\*Sqrt[31]) + (181\*Log[3-x+2\*x^2])/468512 - (181\*Log[2+3\*x+5\*x^2])/468512

**fricas** [A] time = 0.87, size = 227, normalized size = 1.42

$$31725248720x^5 + 260524883872x^4 - 158204886268x^3 - 6004584838\sqrt{31}(20x^6 - 8x^5 + 61x^4 + x^3 + 53x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^3/(5\*x^2+3\*x+2)^2,x, algorithm="fricas")

[Out] -1/5478070469344\*(31725248720\*x^5 + 260524883872\*x^4 - 158204886268\*x^3 - 6004584838\*sqrt(31)\*(20\*x^6 - 8\*x^5 + 61\*x^4 + x^3 + 53\*x^2 + 15\*x + 18)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 3917991234\*sqrt(23)\*(20\*x^6 - 8\*x^5 + 61\*x^4 + x^3 + 53\*x^2 + 15\*x + 18)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 679966484692\*x^2 + 2116340147\*(20\*x^6 - 8\*x^5 + 61\*x^4 + x^3 + 53\*x^2 + 15\*x + 18)\*log(5\*x^2 + 3\*x + 2) - 2116340147\*(20\*x^6 - 8\*x^5 + 61\*x^4 + x^3 + 53\*x^2 + 15\*x + 18)\*log(2\*x^2 - x + 3) - 184712689040\*x + 277008109136)/(20\*x^6 - 8\*x^5 + 61\*x^4 + x^3 + 53\*x^2 + 15\*x + 18)

**giac** [A] time = 0.22, size = 110, normalized size = 0.69

$$\frac{246757}{225120016}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) - \frac{2038497}{2850192752}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{1011260x^5 + 830437}{17$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^3/(5\*x^2+3\*x+2)^2,x, algorithm="giac")

[Out] 246757/225120016\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) - 2038497/2850192752\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) - 1/174616552\*(1011260\*x^5 + 8304376\*x^4 - 5042869\*x^3 + 21674311\*x^2 - 5887820\*x + 8829788)/((5\*x^2 + 3\*x + 2)\*(2\*x^2 - x + 3)^2) - 181/468512\*log(5\*x^2 + 3\*x + 2) + 181/468512\*log(2\*x^2 - x + 3)

**maple** [A] time = 0.01, size = 106, normalized size = 0.66

$$\frac{246757\sqrt{31}\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right) - 2038497\sqrt{23}\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right) + \frac{181\ln(2x^2-x+3)}{468512} - \frac{181\ln(5x^2+3x+2)}{468512}}{225120016 - 2850192752}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2\*x^2-x+3)^3/(5\*x^2+3\*x+2)^2,x)

[Out] -1/234256\*(-5434/31\*x+32428/155)/(x^2+3/5\*x+2/5)-181/468512\*ln(5\*x^2+3\*x+2)+246757/225120016\*31^(1/2)\*arctan(1/31\*(10\*x+3)\*31^(1/2))+1/58564\*(-128612/

529\*x^3-14102/529\*x^2-173195/529\*x-321497/1058)/(2\*x^2-x+3)^2+181/468512\*ln  
 (2\*x^2-x+3)-2038497/2850192752\*23^(1/2)\*arctan(1/23\*(4\*x-1)\*23^(1/2))

**maxima [A]** time = 0.97, size = 116, normalized size = 0.72

$$\frac{246757}{225120016} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x + 3)\right) - \frac{2038497}{2850192752} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4x - 1)\right) - \frac{1011260x^5 + 8304376x^4 - 5042869x^3 + 21674311x^2 - 5887820x + 8829788}{174616552(20x^6 - 8x^5 + 61x^4 + x^3 + 53x^2 + 15x + 18)} - \frac{181}{468512} \log(5x^2 + 3x + 2) + \frac{181}{468512} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^3/(5\*x^2+3\*x+2)^2,x, algorithm="maxima")

[Out] 246757/225120016\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) - 2038497/2850192752\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) - 1/174616552\*(1011260\*x^5 + 8304376\*x^4 - 5042869\*x^3 + 21674311\*x^2 - 5887820\*x + 8829788)/(20\*x^6 - 8\*x^5 + 61\*x^4 + x^3 + 53\*x^2 + 15\*x + 18) - 181/468512\*log(5\*x^2 + 3\*x + 2) + 181/468512\*log(2\*x^2 - x + 3)

**mupad [B]** time = 3.60, size = 136, normalized size = 0.85

$$\frac{\frac{50563x^5}{174616552} + \frac{1038047x^4}{436541380} - \frac{5042869x^3}{3492331040} + \frac{21674311x^2}{3492331040} - \frac{294391x}{174616552} + \frac{200677}{79371160}}{x^6 - \frac{2x^5}{5} + \frac{61x^4}{20} + \frac{x^3}{20} + \frac{53x^2}{20} + \frac{3x}{4} + \frac{9}{10}} - \ln\left(x + \frac{3}{10} - \frac{\sqrt{31}i}{10}\right) \left(\frac{181}{468512} + \frac{\sqrt{31}i}{450240032}\right) - \ln\left(x - \frac{3}{10} - \frac{\sqrt{31}i}{10}\right) \left(\frac{181}{468512} - \frac{\sqrt{31}i}{450240032}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((2\*x^2 - x + 3)^3\*(3\*x + 5\*x^2 + 2)^2),x)

[Out] log(x + (31^(1/2)\*1i)/10 + 3/10)\*((31^(1/2)\*246757i)/450240032 - 181/468512) - log(x - (31^(1/2)\*1i)/10 + 3/10)\*((31^(1/2)\*246757i)/450240032 + 181/468512) - ((21674311\*x^2)/3492331040 - (294391\*x)/174616552 - (5042869\*x^3)/3492331040 + (1038047\*x^4)/436541380 + (50563\*x^5)/174616552 + 200677/79371160)/((3\*x)/4 + (53\*x^2)/20 + x^3/20 + (61\*x^4)/20 - (2\*x^5)/5 + x^6 + 9/10) + log(x - (23^(1/2)\*1i)/4 - 1/4)\*((23^(1/2)\*2038497i)/5700385504 + 181/468512) - log(x + (23^(1/2)\*1i)/4 - 1/4)\*((23^(1/2)\*2038497i)/5700385504 - 181/468512)

**sympy [A]** time = 0.43, size = 143, normalized size = 0.89

$$\frac{-1011260x^5 - 8304376x^4 + 5042869x^3 - 21674311x^2 + 5887820x - 8829788}{3492331040x^6 - 1396932416x^5 + 10651609672x^4 + 174616552x^3 + 9254677256x^2 + 2619248280x + 314309760}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x\*\*2-x+3)\*\*3/(5\*x\*\*2+3\*x+2)\*\*2,x)

```
[Out] (-1011260*x**5 - 8304376*x**4 + 5042869*x**3 - 21674311*x**2 + 5887820*x -
8829788)/(3492331040*x**6 - 1396932416*x**5 + 10651609672*x**4 + 174616552*
x**3 + 9254677256*x**2 + 2619248280*x + 3143097936) + 181*log(x**2 - x/2 +
3/2)/468512 - 181*log(x**2 + 3*x/5 + 2/5)/468512 - 2038497*sqrt(23)*atan(4*
sqrt(23)*x/23 - sqrt(23)/23)/2850192752 + 246757*sqrt(31)*atan(10*sqrt(31)*
x/31 + 3*sqrt(31)/31)/225120016
```

$$3.57 \quad \int \frac{1}{(3-x+2x^2)^3 (2+3x+5x^2)^3} dx$$

**Optimal.** Leaf size=181

$$\frac{5(302 - 35x)}{64009 (2x^2 - x + 3) (5x^2 + 3x + 2)^2} + \frac{15(7140435x + 2618306)}{14886061058 (5x^2 + 3x + 2)} - \frac{5(77020x + 223707)}{87308276 (5x^2 + 3x + 2)^2} + \frac{1}{1012 (2x^2 - x + 3)}$$

[Out]  $-5/87308276*(223707+77020*x)/(5*x^2+3*x+2)^2+1/1012*(13-6*x)/(2*x^2-x+3)^2/(5*x^2+3*x+2)^2+5/64009*(302-35*x)/(2*x^2-x+3)/(5*x^2+3*x+2)^2+15/14886061058*(2618306+7140435*x)/(5*x^2+3*x+2)+405/1288408*\ln(2*x^2-x+3)-405/1288408*\ln(5*x^2+3*x+2)-880575/7838030068*\arctan(1/23*(1-4*x)*23^{(1/2)})*23^{(1/2)}+2768835/19191481364*\arctan(1/31*(3+10*x)*31^{(1/2)})*31^{(1/2)}$

**Rubi [A]** time = 0.20, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$ , Rules used = {974, 1060, 1072, 634, 618, 204, 628}

$$\frac{5(302 - 35x)}{64009 (2x^2 - x + 3) (5x^2 + 3x + 2)^2} + \frac{15(7140435x + 2618306)}{14886061058 (5x^2 + 3x + 2)} - \frac{5(77020x + 223707)}{87308276 (5x^2 + 3x + 2)^2} + \frac{1}{1012 (2x^2 - x + 3)}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - x + 2\*x^2)^3\*(2 + 3\*x + 5\*x^2)^3),x]

[Out]  $(-5*(223707 + 77020*x))/(87308276*(2 + 3*x + 5*x^2)^2) + (13 - 6*x)/(1012*(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^2) + (5*(302 - 35*x))/(64009*(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^2) + (15*(2618306 + 7140435*x))/(14886061058*(2 + 3*x + 5*x^2)) - (880575*ArcTan[(1 - 4*x)/Sqrt[23]])/(340783916*Sqrt[23]) + (2768835*ArcTan[(3 + 10*x)/Sqrt[31]])/(619080044*Sqrt[31]) + (405*Log[3 - x + 2*x^2])/1288408 - (405*Log[2 + 3*x + 5*x^2])/1288408$

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c},

x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 628

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 634

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] :> Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

### Rule 974

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_)\*((d\_) + (e\_)\*(x\_) + (f\_)\*(x\_)^2)^(q\_), x\_Symbol] :> Simp[((2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f) + c\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*x)\*(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^(q + 1))/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))^(p + 1)), x] - Dist[1/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))^(p + 1)), Int[(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^q\*Simp[2\*c\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1) - (2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(a\*f\*(p + 1) - c\*d\*(p + 2)) - e\*(b^2\*c\*e - 2\*a\*c^2\*e - b^3\*f - b\*c\*(c\*d - 3\*a\*f))\*(p + q + 2) + (2\*f\*(2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f))\*(p + q + 2) - (2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(b\*f\*(p + 1) - c\*e\*(2\*p + q + 4)))\*x + c\*f\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(2\*p + 2\*q + 5)\*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && LtQ[p, -1] && NeQ[(c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f), 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]

### Rule 1060

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_)\*((A\_) + (B\_)\*(x\_) + (C\_)\*(x\_)^2)\*((d\_) + (e\_)\*(x\_) + (f\_)\*(x\_)^2)^(q\_), x\_Symbol] :> Simp[((a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^(q + 1)\*((A\*c - a\*C)\*(2\*a\*c\*e - b\*(c\*d + a\*f)) + (A\*b - a\*B)\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f)) + c\*(A\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f)) - B\*(b\*c\*d - 2\*a\*c\*e + a\*b\*f) + C\*(b^2\*d - a\*b\*e - 2\*a\*(c\*d - a\*f))\*x))/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))^(p + 1)), x] + Dist[1/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))^(p + 1)), Int[(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^q\*Simp[(b\*B - 2\*A\*c - 2\*a\*C)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1) + (b^2\*(C\*d + A\*f) - b\*(B\*c\*d + A\*c\*e + a\*C\*e + a\*B\*f) + 2\*(A\*c\*(c\*d - a\*f) - a\*



```
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
+ q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !
IGtQ[q, 0]
```

### Rule 1072

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] := With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
+ c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x]/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x]/(d + e*x + f*x^2), x], x] /; NeQ[q, 0] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)^3} dx &= \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)^2} - \frac{\int \frac{-4510-4400x+2310x^2}{(3-x+2x^2)^2(2+3x+5x^2)^3} dx}{11132} \\
&= \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)^2} + \frac{5(302-35x)}{64009(3-x+2x^2)(2+3x+5x^2)} \\
&= -\frac{5(223707+77020x)}{87308276(2+3x+5x^2)^2} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)^2} + \frac{5(302-35x)}{64009(3-x+2x^2)(2+3x+5x^2)} \\
&= -\frac{5(223707+77020x)}{87308276(2+3x+5x^2)^2} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)^2} + \frac{5(302-35x)}{64009(3-x+2x^2)(2+3x+5x^2)} \\
&= -\frac{5(223707+77020x)}{87308276(2+3x+5x^2)^2} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)^2} + \frac{5(302-35x)}{64009(3-x+2x^2)(2+3x+5x^2)} \\
&= -\frac{5(223707+77020x)}{87308276(2+3x+5x^2)^2} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)^2} + \frac{5(302-35x)}{64009(3-x+2x^2)(2+3x+5x^2)} \\
&= -\frac{5(223707+77020x)}{87308276(2+3x+5x^2)^2} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)^2} + \frac{5(302-35x)}{64009(3-x+2x^2)(2+3x+5x^2)} \\
&= -\frac{5(223707+77020x)}{87308276(2+3x+5x^2)^2} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)^2} + \frac{5(302-35x)}{64009(3-x+2x^2)(2+3x+5x^2)}
\end{aligned}$$

**Mathematica [A]** time = 0.10, size = 151, normalized size = 0.83

$$\frac{405 \log(2x^2 - x + 3)}{1288408} - \frac{405 \log(5x^2 + 3x + 2)}{1288408} + \frac{6850x^3 - 9275x^2 + 11154x - 4342}{345092(10x^4 + x^3 + 16x^2 + 7x + 6)^2} + \frac{5(42842610x^3 - 57114690x^2 + 14085977x - 5711469)}{14886061058(10x^4 + x^3 + 16x^2 + 7x + 6)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((3 - x + 2\*x^2)^3\*(2 + 3\*x + 5\*x^2)^3), x]

[Out] (-4342 + 11154\*x - 9275\*x^2 + 6850\*x^3)/(345092\*(6 + 7\*x + 16\*x^2 + x^3 + 10\*x^4)^2) + (5\*(14085977 + 51156233\*x - 5711469\*x^2 + 42842610\*x^3))/(14886061058\*(10\*x^4 + x^3 + 16\*x^2 + 7\*x + 6))

061058\*(6 + 7\*x + 16\*x^2 + x^3 + 10\*x^4)) + (880575\*ArcTan[(-1 + 4\*x)/Sqrt[23]])/(340783916\*Sqrt[23]) + (2768835\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(6190800\*44\*Sqrt[31]) + (405\*Log[3 - x + 2\*x^2])/1288408 - (405\*Log[2 + 3\*x + 5\*x^2])/1288408

**fricas** [A] time = 0.73, size = 297, normalized size = 1.64

$67202918046000 x^7 - 2238718468800 x^6 + 186872434930060 x^5 + 62827256425340 x^4 + 173919793526820 x^3 + 67376830890 \sqrt{31} (100 x^8 + 20 x^7 + 321 x^6 + 172 x^5 + 390 x^4 + 236 x^3 + 241 x^2 + 84 x + 36) \arctan\left(\frac{1}{31} \sqrt{31} (10 x + 3)\right) + 52466419650 \sqrt{23} (100 x^8 + 20 x^7 + 321 x^6 + 172 x^5 + 390 x^4 + 236 x^3 + 241 x^2 + 84 x + 36) \arctan\left(\frac{1}{23} \sqrt{23} (4 x - 1)\right) + 73595926401690 x^2 - 146799174285 (100 x^8 + 20 x^7 + 321 x^6 + 172 x^5 + 390 x^4 + 236 x^3 + 241 x^2 + 84 x + 36) \log(5 x^2 + 3 x + 2) + 146799174285 (100 x^8 + 20 x^7 + 321 x^6 + 172 x^5 + 390 x^4 + 236 x^3 + 241 x^2 + 84 x + 36) \log(2 x^2 - x + 3) + 78707350628632 x + 7381223830244 / (100 x^8 + 20 x^7 + 321 x^6 + 172 x^5 + 390 x^4 + 236 x^3 + 241 x^2 + 84 x + 36)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^3/(5\*x^2+3\*x+2)^3,x, algorithm="fricas")

[Out] 1/467005507511576\*(67202918046000\*x^7 - 2238718468800\*x^6 + 186872434930060\*x^5 + 62827256425340\*x^4 + 173919793526820\*x^3 + 67376830890\*sqrt(31)\*(100\*x^8 + 20\*x^7 + 321\*x^6 + 172\*x^5 + 390\*x^4 + 236\*x^3 + 241\*x^2 + 84\*x + 36)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 52466419650\*sqrt(23)\*(100\*x^8 + 20\*x^7 + 321\*x^6 + 172\*x^5 + 390\*x^4 + 236\*x^3 + 241\*x^2 + 84\*x + 36)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 73595926401690\*x^2 - 146799174285\*(100\*x^8 + 20\*x^7 + 321\*x^6 + 172\*x^5 + 390\*x^4 + 236\*x^3 + 241\*x^2 + 84\*x + 36)\*log(5\*x^2 + 3\*x + 2) + 146799174285\*(100\*x^8 + 20\*x^7 + 321\*x^6 + 172\*x^5 + 390\*x^4 + 236\*x^3 + 241\*x^2 + 84\*x + 36)\*log(2\*x^2 - x + 3) + 78707350628632\*x + 7381223830244)/(100\*x^8 + 20\*x^7 + 321\*x^6 + 172\*x^5 + 390\*x^4 + 236\*x^3 + 241\*x^2 + 84\*x + 36)

**giac** [A] time = 0.22, size = 116, normalized size = 0.64

$\frac{2768835}{19191481364} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10 x + 3)\right) + \frac{880575}{7838030068} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4 x - 1)\right) + \frac{4284261000 x^7 - 142720800 x^6 + 11913326210 x^5 + 4005307690 x^4 + 11087580870 x^3 + 4691822415 x^2 + 5017681412 x + 470561254}{(10 x^4 + x^3 + 16 x^2 + 7 x + 6)^2} - \frac{405}{1288408} \log(5 x^2 + 3 x + 2) + \frac{405}{1288408} \log(2 x^2 - x + 3)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^3/(5\*x^2+3\*x+2)^3,x, algorithm="giac")

[Out] 2768835/19191481364\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 880575/7838030068\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 1/29772122116\*(4284261000\*x^7 - 142720800\*x^6 + 11913326210\*x^5 + 4005307690\*x^4 + 11087580870\*x^3 + 4691822415\*x^2 + 5017681412\*x + 470561254)/(10\*x^4 + x^3 + 16\*x^2 + 7\*x + 6)^2 - 405/1288408\*log(5\*x^2 + 3\*x + 2) + 405/1288408\*log(2\*x^2 - x + 3)

**maple** [A] time = 0.01, size = 118, normalized size = 0.65

$\frac{2768835 \sqrt{31} \arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{19191481364} + \frac{880575 \sqrt{23} \arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{7838030068} + \frac{405 \ln(2x^2 - x + 3)}{1288408} - \frac{405 \ln(5x^2 + 3x + 6)}{1288408}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x)`

[Out] 
$$-25/2576816*(-3013197/961*x^3-14516062/4805*x^2-51193868/24025*x-5423968/24025)/(5*x^2+3*x+2)^2-405/1288408*\ln(5*x^2+3*x+2)+2768835/19191481364*31^{(1/2)}*\arctan(1/31*(10*x+3)*31^{(1/2)})+1/644204*(302907/529*x^3-368291/529*x^2+501587/2116*x-665819/1058)/(2*x^2-x+3)^2+405/1288408*\ln(2*x^2-x+3)+880575/7838030068*23^{(1/2)}*\arctan(1/23*(4*x-1)*23^{(1/2)})$$

**maxima** [A] time = 0.97, size = 138, normalized size = 0.76

$$\frac{2768835}{19191481364} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x + 3)\right) + \frac{880575}{7838030068} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4x - 1)\right) + \frac{4284261000 x^7 - 142720800 x^6 + 11913326210 x^5 + 4005307690 x^4 + 11087580870 x^3 + 4691822415 x^2 + 5017681412 x + 470561254}{(100 x^8 + 20 x^7 + 321 x^6 + 172 x^5 + 390 x^4 + 236 x^3 + 241 x^2 + 84 x + 36)} - \frac{405}{1288408} \log(5 x^2 + 3 x + 2) + \frac{405}{1288408} \log(2 x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x, algorithm="maxima")`

[Out] 
$$2768835/19191481364*\sqrt{31}*\arctan(1/31*\sqrt{31}*(10*x + 3)) + 880575/7838030068*\sqrt{23}*\arctan(1/23*\sqrt{23}*(4*x - 1)) + 1/29772122116*(4284261000*x^7 - 142720800*x^6 + 11913326210*x^5 + 4005307690*x^4 + 11087580870*x^3 + 4691822415*x^2 + 5017681412*x + 470561254)/(100*x^8 + 20*x^7 + 321*x^6 + 172*x^5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36) - 405/1288408*\log(5*x^2 + 3*x + 2) + 405/1288408*\log(2*x^2 - x + 3)$$

**mupad** [B] time = 3.59, size = 155, normalized size = 0.86

$$\frac{21421305 x^7}{14886061058} - \frac{356802 x^6}{7443030529} + \frac{1191332621 x^5}{297721221160} + \frac{400530769 x^4}{297721221160} + \frac{1108758087 x^3}{297721221160} + \frac{938364483 x^2}{595442442320} + \frac{1254420353 x}{744303052900} + \frac{235280627}{1488606105800} + \frac{x^8 + \frac{x^7}{5} + \frac{321 x^6}{100} + \frac{43 x^5}{25} + \frac{39 x^4}{10} + \frac{59 x^3}{25} + \frac{241 x^2}{100} + \frac{21 x}{25} + \frac{9}{25}}{x^8 + \frac{x^7}{5} + \frac{321 x^6}{100} + \frac{43 x^5}{25} + \frac{39 x^4}{10} + \frac{59 x^3}{25} + \frac{241 x^2}{100} + \frac{21 x}{25} + \frac{9}{25}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((2*x^2 - x + 3)^3*(3*x + 5*x^2 + 2)^3),x)`

[Out] 
$$\log(x + (23^{(1/2)}*1i)/4 - 1/4)*((23^{(1/2)}*880575i)/15676060136 + 405/1288408) - \log(x - (23^{(1/2)}*1i)/4 - 1/4)*((23^{(1/2)}*880575i)/15676060136 - 405/1288408) - \log(x - (31^{(1/2)}*1i)/10 + 3/10)*((31^{(1/2)}*2768835i)/38382962728 + 405/1288408) + \log(x + (31^{(1/2)}*1i)/10 + 3/10)*((31^{(1/2)}*2768835i)/38382962728 - 405/1288408) + ((1254420353*x)/744303052900 + (938364483*x^2)/595442442320 + (1108758087*x^3)/297721221160 + (400530769*x^4)/297721221160 + (1191332621*x^5)/297721221160 - (356802*x^6)/7443030529 + (21421305*x^7)/14886061058 + 235280627/1488606105800)/((21*x)/25 + (241*x^2)/100 + (59*x^3)/25 + (39*x^4)/10 + (43*x^5)/25 + (321*x^6)/100 + x^7/5 + x^8 + 9/25)$$

sympy [A] time = 0.47, size = 163, normalized size = 0.90

$$\frac{4284261000x^7 - 142720800x^6 + 11913326210x^5 + 4005307690x^4 + 11087580870x^3 - 2977212211600x^8 + 595442442320x^7 + 9556851199236x^6 + 5120805003952x^5 + 11611127625240x^4 + 7026220819376x^3 + 7175081429956x^2 + 2500858257744x + 1071796396176}{2977212211600x^8 + 595442442320x^7 + 9556851199236x^6 + 5120805003952x^5 + 11611127625240x^4 + 7026220819376x^3 + 7175081429956x^2 + 2500858257744x + 1071796396176} + 405 \log(x^2 - x/2 + 3/2)/1288408 - 405 \log(x^2 + 3x/5 + 2/5)/1288408 + 880575 \sqrt{23} \operatorname{atan}(4\sqrt{23}x/23 - \sqrt{23}/23)/7838030068 + 2768835 \sqrt{31} \operatorname{atan}(10\sqrt{31}x/31 + 3\sqrt{31}/31)/19191481364$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x\*\*2-x+3)\*\*3/(5\*x\*\*2+3\*x+2)\*\*3,x)

[Out] (4284261000\*x\*\*7 - 142720800\*x\*\*6 + 11913326210\*x\*\*5 + 4005307690\*x\*\*4 + 11087580870\*x\*\*3 + 4691822415\*x\*\*2 + 5017681412\*x + 470561254)/(2977212211600\*x\*\*8 + 595442442320\*x\*\*7 + 9556851199236\*x\*\*6 + 5120805003952\*x\*\*5 + 11611127625240\*x\*\*4 + 7026220819376\*x\*\*3 + 7175081429956\*x\*\*2 + 2500858257744\*x + 1071796396176) + 405\*log(x\*\*2 - x/2 + 3/2)/1288408 - 405\*log(x\*\*2 + 3\*x/5 + 2/5)/1288408 + 880575\*sqrt(23)\*atan(4\*sqrt(23)\*x/23 - sqrt(23)/23)/7838030068 + 2768835\*sqrt(31)\*atan(10\*sqrt(31)\*x/31 + 3\*sqrt(31)/31)/19191481364

$$3.58 \quad \int \sqrt{3 - x + 2x^2} (2 + 3x + 5x^2)^4 dx$$

**Optimal.** Leaf size=208

$$\frac{83948353(2x^2 - x + 3)^{3/2} x^2}{2293760} + \frac{804243809(2x^2 - x + 3)^{3/2} x}{36700160} + \frac{27185733541(2x^2 - x + 3)^{3/2}}{440401920} - \frac{359471503(1 - 4x)}{67108864} \sqrt{2x^2 - x + 3}$$

[Out] 27185733541/440401920\*(2\*x^2-x+3)^(3/2)+804243809/36700160\*x\*(2\*x^2-x+3)^(3/2)-83948353/2293760\*x^2\*(2\*x^2-x+3)^(3/2)+8325631/1032192\*x^3\*(2\*x^2-x+3)^(3/2)+4796405/43008\*x^4\*(2\*x^2-x+3)^(3/2)+233225/1536\*x^5\*(2\*x^2-x+3)^(3/2)+14125/144\*x^6\*(2\*x^2-x+3)^(3/2)+125/4\*x^7\*(2\*x^2-x+3)^(3/2)-8267844569/268435456\*arcsinh(1/23\*(1-4\*x)\*23^(1/2))\*2^(1/2)-359471503/67108864\*(1-4\*x)\*(2\*x^2-x+3)^(1/2)

**Rubi [A]** time = 0.31, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {1661, 640, 612, 619, 215}

$$\frac{125}{4} (2x^2 - x + 3)^{3/2} x^7 + \frac{14125}{144} (2x^2 - x + 3)^{3/2} x^6 + \frac{233225(2x^2 - x + 3)^{3/2} x^5}{1536} + \frac{4796405(2x^2 - x + 3)^{3/2} x^4}{43008} + \frac{83948353(2x^2 - x + 3)^{3/2} x^2}{2293760} + \frac{804243809(2x^2 - x + 3)^{3/2} x}{36700160} + \frac{27185733541(2x^2 - x + 3)^{3/2}}{440401920} - \frac{359471503(1 - 4x)}{67108864} \sqrt{2x^2 - x + 3}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[3 - x + 2\*x^2]\*(2 + 3\*x + 5\*x^2)^4,x]

[Out] (-359471503\*(1 - 4\*x)\*Sqrt[3 - x + 2\*x^2])/67108864 + (27185733541\*(3 - x + 2\*x^2)^(3/2))/440401920 + (804243809\*x\*(3 - x + 2\*x^2)^(3/2))/36700160 - (83948353\*x^2\*(3 - x + 2\*x^2)^(3/2))/2293760 + (8325631\*x^3\*(3 - x + 2\*x^2)^(3/2))/1032192 + (4796405\*x^4\*(3 - x + 2\*x^2)^(3/2))/43008 + (233225\*x^5\*(3 - x + 2\*x^2)^(3/2))/1536 + (14125\*x^6\*(3 - x + 2\*x^2)^(3/2))/144 + (125\*x^7\*(3 - x + 2\*x^2)^(3/2))/4 - (8267844569\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(134217728\*Sqrt[2])

**Rule 215**

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

**Rule 612**

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N

$eQ[b^2 - 4ac, 0] \ \&\& \ GtQ[p, 0] \ \&\& \ IntegerQ[4p]$

### Rule 619

$Int[(a_.) + (b_.)(x_) + (c_.)(x_)^2]^{(p_)}, x\_Symbol] \ :> \ Dist[1/(2c*((-4*c)/(b^2 - 4*a*c))^{p}), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] \ ; \ FreeQ[\{a, b, c, p\}, x] \ \&\& \ GtQ[4*a - b^2/c, 0]$

### Rule 640

$Int[((d_.) + (e_.)(x_))*((a_.) + (b_.)(x_) + (c_.)(x_)^2)^{(p_)}, x\_Symbol] \ :> \ Simp[(e*(a + b*x + c*x^2)^{(p + 1)})/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] \ ; \ FreeQ[\{a, b, c, d, e, p\}, x] \ \&\& \ NeQ[2*c*d - b*e, 0] \ \&\& \ NeQ[p, -1]$

### Rule 1661

$Int[(Pq_)*((a_.) + (b_.)(x_) + (c_.)(x_)^2)^{(p_)}, x\_Symbol] \ :> \ With[\{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]\}, Simp[(e*x^{(q - 1)}*(a + b*x + c*x^2)^{(p + 1)})/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a + b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^{(q - 2)} - b*e*(q + p)*x^{(q - 1)} - c*e*(q + 2*p + 1)*x^q, x], x], x]] \ ; \ FreeQ[\{a, b, c, p\}, x] \ \&\& \ PolyQ[Pq, x] \ \&\& \ NeQ[b^2 - 4*a*c, 0] \ \&\& \ !LeQ[p, -1]$

### Rubi steps

$$\begin{aligned}
\int \sqrt{3-x+2x^2} (2+3x+5x^2)^4 dx &= \frac{125}{4}x^7(3-x+2x^2)^{3/2} + \frac{1}{20} \int \sqrt{3-x+2x^2} (320+1920x+7520x^2+11200x^3+6400x^4) dx \\
&= \frac{14125}{144}x^6(3-x+2x^2)^{3/2} + \frac{125}{4}x^7(3-x+2x^2)^{3/2} + \frac{1}{360} \int \sqrt{3-x+2x^2} (320+1920x+7520x^2+11200x^3+6400x^4) dx \\
&= \frac{233225x^5(3-x+2x^2)^{3/2}}{1536} + \frac{14125}{144}x^6(3-x+2x^2)^{3/2} + \frac{125}{4}x^7(3-x+2x^2)^{3/2} + \frac{1}{360} \int \sqrt{3-x+2x^2} (320+1920x+7520x^2+11200x^3+6400x^4) dx \\
&= \frac{4796405x^4(3-x+2x^2)^{3/2}}{43008} + \frac{233225x^5(3-x+2x^2)^{3/2}}{1536} + \frac{14125}{144}x^6(3-x+2x^2)^{3/2} + \frac{1}{360} \int \sqrt{3-x+2x^2} (320+1920x+7520x^2+11200x^3+6400x^4) dx \\
&= \frac{8325631x^3(3-x+2x^2)^{3/2}}{1032192} + \frac{4796405x^4(3-x+2x^2)^{3/2}}{43008} + \frac{233225x^5(3-x+2x^2)^{3/2}}{1536} + \frac{14125}{144}x^6(3-x+2x^2)^{3/2} + \frac{1}{360} \int \sqrt{3-x+2x^2} (320+1920x+7520x^2+11200x^3+6400x^4) dx \\
&= -\frac{83948353x^2(3-x+2x^2)^{3/2}}{2293760} + \frac{8325631x^3(3-x+2x^2)^{3/2}}{1032192} + \frac{4796405x^4(3-x+2x^2)^{3/2}}{43008} + \frac{233225x^5(3-x+2x^2)^{3/2}}{1536} + \frac{14125}{144}x^6(3-x+2x^2)^{3/2} + \frac{1}{360} \int \sqrt{3-x+2x^2} (320+1920x+7520x^2+11200x^3+6400x^4) dx \\
&= \frac{804243809x(3-x+2x^2)^{3/2}}{36700160} - \frac{83948353x^2(3-x+2x^2)^{3/2}}{2293760} + \frac{8325631x^3(3-x+2x^2)^{3/2}}{1032192} + \frac{4796405x^4(3-x+2x^2)^{3/2}}{43008} + \frac{233225x^5(3-x+2x^2)^{3/2}}{1536} + \frac{14125}{144}x^6(3-x+2x^2)^{3/2} + \frac{1}{360} \int \sqrt{3-x+2x^2} (320+1920x+7520x^2+11200x^3+6400x^4) dx \\
&= \frac{27185733541(3-x+2x^2)^{3/2}}{440401920} + \frac{804243809x(3-x+2x^2)^{3/2}}{36700160} - \frac{83948353x^2(3-x+2x^2)^{3/2}}{2293760} + \frac{8325631x^3(3-x+2x^2)^{3/2}}{1032192} + \frac{4796405x^4(3-x+2x^2)^{3/2}}{43008} + \frac{233225x^5(3-x+2x^2)^{3/2}}{1536} + \frac{14125}{144}x^6(3-x+2x^2)^{3/2} + \frac{1}{360} \int \sqrt{3-x+2x^2} (320+1920x+7520x^2+11200x^3+6400x^4) dx \\
&= -\frac{359471503(1-4x)\sqrt{3-x+2x^2}}{67108864} + \frac{27185733541(3-x+2x^2)^{3/2}}{440401920} + \frac{804243809x(3-x+2x^2)^{3/2}}{36700160} - \frac{83948353x^2(3-x+2x^2)^{3/2}}{2293760} + \frac{8325631x^3(3-x+2x^2)^{3/2}}{1032192} + \frac{4796405x^4(3-x+2x^2)^{3/2}}{43008} + \frac{233225x^5(3-x+2x^2)^{3/2}}{1536} + \frac{14125}{144}x^6(3-x+2x^2)^{3/2} + \frac{1}{360} \int \sqrt{3-x+2x^2} (320+1920x+7520x^2+11200x^3+6400x^4) dx \\
&= -\frac{359471503(1-4x)\sqrt{3-x+2x^2}}{67108864} + \frac{27185733541(3-x+2x^2)^{3/2}}{440401920} + \frac{804243809x(3-x+2x^2)^{3/2}}{36700160} - \frac{83948353x^2(3-x+2x^2)^{3/2}}{2293760} + \frac{8325631x^3(3-x+2x^2)^{3/2}}{1032192} + \frac{4796405x^4(3-x+2x^2)^{3/2}}{43008} + \frac{233225x^5(3-x+2x^2)^{3/2}}{1536} + \frac{14125}{144}x^6(3-x+2x^2)^{3/2} + \frac{1}{360} \int \sqrt{3-x+2x^2} (320+1920x+7520x^2+11200x^3+6400x^4) dx \\
&= -\frac{359471503(1-4x)\sqrt{3-x+2x^2}}{67108864} + \frac{27185733541(3-x+2x^2)^{3/2}}{440401920} + \frac{804243809x(3-x+2x^2)^{3/2}}{36700160} - \frac{83948353x^2(3-x+2x^2)^{3/2}}{2293760} + \frac{8325631x^3(3-x+2x^2)^{3/2}}{1032192} + \frac{4796405x^4(3-x+2x^2)^{3/2}}{43008} + \frac{233225x^5(3-x+2x^2)^{3/2}}{1536} + \frac{14125}{144}x^6(3-x+2x^2)^{3/2} + \frac{1}{360} \int \sqrt{3-x+2x^2} (320+1920x+7520x^2+11200x^3+6400x^4) dx
\end{aligned}$$

**Mathematica [A]** time = 0.30, size = 85, normalized size = 0.41

$$4\sqrt{2x^2-x+3} (1321205760000x^9 + 3486515200000x^8 + 6327795712000x^7 + 7725962035200x^6 + 7612808028000x^5 + 5441280000000x^4 + 3072000000000x^3 + 1280000000000x^2 + 256000000000x + 12800000000)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[3 - x + 2\*x^2]\*(2 + 3\*x + 5\*x^2)^4, x]



[Out]  $(4\sqrt{3-x+2x^2})(3801512106459 + 537752185764x - 174418077792x^2 + 2211683657856x^3 + 5354741991424x^4 + 7612808028160x^5 + 7725962035200x^6 + 6327795712000x^7 + 3486515200000x^8 + 1321205760000x^9) - 2604371039235\sqrt{2}\operatorname{ArcSinh}[(1-4x)/\sqrt{23}]/84557168640$

**fricas** [A] time = 0.66, size = 98, normalized size = 0.47

$$\frac{1}{21139292160} (1321205760000x^9 + 3486515200000x^8 + 6327795712000x^7 + 7725962035200x^6 + 7612808028160x^5 + 5354741991424x^4 + 2211683657856x^3 - 174418077792x^2 + 537752185764x + 3801512106459)\sqrt{2x^2 - x + 3} + 8267844569/536870912\sqrt{2}\log(-4\sqrt{2}\sqrt{2x^2 - x + 3})(4x - 1) - 32x^2 + 16x - 25)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x+2)^4*(2*x^2-x+3)^(1/2),x, algorithm="fricas")`

[Out]  $1/21139292160*(1321205760000x^9 + 3486515200000x^8 + 6327795712000x^7 + 7725962035200x^6 + 7612808028160x^5 + 5354741991424x^4 + 2211683657856x^3 - 174418077792x^2 + 537752185764x + 3801512106459)\sqrt{2x^2 - x + 3} + 8267844569/536870912\sqrt{2}\log(-4\sqrt{2}\sqrt{2x^2 - x + 3})(4x - 1) - 32x^2 + 16x - 25)$

**giac** [A] time = 0.25, size = 93, normalized size = 0.45

$$\frac{1}{21139292160} (4(8(4(16(20(40(140(160(36x+95)x+27587)x+4715553)x+185859571)x+2614620113)x+17278778577)x-5450564931)x+134438046441)x+3801512106459)\sqrt{2x^2-x+3}-8267844569/268435456\sqrt{2}\log(-2\sqrt{2}(\sqrt{2}x-\sqrt{2x^2-x+3}))+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x+2)^4*(2*x^2-x+3)^(1/2),x, algorithm="giac")`

[Out]  $1/21139292160*(4*(8*(4*(16*(20*(40*(140*(160*(36x+95)x+27587)x+4715553)x+185859571)x+2614620113)x+17278778577)x-5450564931)x+134438046441)x+3801512106459)\sqrt{2x^2-x+3}-8267844569/268435456\sqrt{2}\log(-2\sqrt{2}(\sqrt{2}x-\sqrt{2x^2-x+3}))+1)$

**maple** [A] time = 0.03, size = 166, normalized size = 0.80

$$\frac{125(2x^2-x+3)^{\frac{3}{2}}x^7}{4} + \frac{14125(2x^2-x+3)^{\frac{3}{2}}x^6}{144} + \frac{233225(2x^2-x+3)^{\frac{3}{2}}x^5}{1536} + \frac{4796405(2x^2-x+3)^{\frac{3}{2}}x^4}{43008} + \frac{8325631(2x^2-x+3)^{\frac{3}{2}}x^3}{1032192} + \frac{8325631(2x^2-x+3)^{\frac{3}{2}}x^2}{1032192} + \frac{8325631(2x^2-x+3)^{\frac{3}{2}}x}{1032192} + \frac{8325631(2x^2-x+3)^{\frac{3}{2}}}{1032192}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^2+3*x+2)^4*(2*x^2-x+3)^(1/2),x)`

[Out]  $125/4x^7(2x^2-x+3)^{3/2} + 27185733541/440401920(2x^2-x+3)^{3/2} + 14125/144x^6(2x^2-x+3)^{3/2} + 233225/1536x^5(2x^2-x+3)^{3/2} + 4796405/43008x^4(2x^2-x+3)^{3/2} + 8325631/1032192x^3(2x^2-x+3)^{3/2} - 83948353/2293760x^2(2x^2-x+3)^{3/2} - 83948353/2293760x(2x^2-x+3)^{3/2} - 83948353/2293760(2x^2-x+3)^{3/2}$

$x^2*(2*x^2-x+3)^{(3/2)}+804243809/36700160*x*(2*x^2-x+3)^{(3/2)}+8267844569/268435456*2^{(1/2)}*\operatorname{arcsinh}(4/23*23^{(1/2)}*(x-1/4))+359471503/67108864*(4*x-1)*(2*x^2-x+3)^{(1/2)}$

**maxima** [A] time = 1.01, size = 177, normalized size = 0.85

$$\frac{125}{4} (2x^2 - x + 3)^{\frac{3}{2}} x^7 + \frac{14125}{144} (2x^2 - x + 3)^{\frac{3}{2}} x^6 + \frac{233225}{1536} (2x^2 - x + 3)^{\frac{3}{2}} x^5 + \frac{4796405}{43008} (2x^2 - x + 3)^{\frac{3}{2}} x^4 + \frac{8325631}{1032192} (2x^2 - x + 3)^{\frac{3}{2}} x^3 - \frac{83948353}{2293760} (2x^2 - x + 3)^{\frac{3}{2}} x^2 + \frac{804243809}{36700160} (2x^2 - x + 3)^{\frac{3}{2}} x + \frac{27185733541}{440401920} (2x^2 - x + 3)^{\frac{3}{2}} + \frac{359471503}{16777216} \sqrt{2x^2 - x + 3} x + \frac{8267844569}{268435456} \sqrt{2} \operatorname{arcsinh}\left(\frac{1}{23} \sqrt{23} (4x - 1)\right) - \frac{359471503}{67108864} \sqrt{2x^2 - x + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^4\*(2\*x^2-x+3)^(1/2),x, algorithm="maxima")

[Out] 125/4\*(2\*x^2 - x + 3)^(3/2)\*x^7 + 14125/144\*(2\*x^2 - x + 3)^(3/2)\*x^6 + 233225/1536\*(2\*x^2 - x + 3)^(3/2)\*x^5 + 4796405/43008\*(2\*x^2 - x + 3)^(3/2)\*x^4 + 8325631/1032192\*(2\*x^2 - x + 3)^(3/2)\*x^3 - 83948353/2293760\*(2\*x^2 - x + 3)^(3/2)\*x^2 + 804243809/36700160\*(2\*x^2 - x + 3)^(3/2)\*x + 27185733541/440401920\*(2\*x^2 - x + 3)^(3/2) + 359471503/16777216\*sqrt(2\*x^2 - x + 3)\*x + 8267844569/268435456\*sqrt(2)\*arcsinh(1/23\*sqrt(23)\*(4\*x - 1)) - 359471503/67108864\*sqrt(2\*x^2 - x + 3)

**mupad** [B] time = 5.03, size = 221, normalized size = 1.06

$$\frac{8325631 x^3 (2x^2 - x + 3)^{3/2}}{1032192} - \frac{83948353 x^2 (2x^2 - x + 3)^{3/2}}{2293760} + \frac{4796405 x^4 (2x^2 - x + 3)^{3/2}}{43008} + \frac{233225 x^5 (2x^2 - x + 3)^{3/2}}{1536}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)^(1/2)\*(3\*x + 5\*x^2 + 2)^4,x)

[Out] (8325631\*x^3\*(2\*x^2 - x + 3)^(3/2))/1032192 - (83948353\*x^2\*(2\*x^2 - x + 3)^(3/2))/2293760 + (4796405\*x^4\*(2\*x^2 - x + 3)^(3/2))/43008 + (233225\*x^5\*(2\*x^2 - x + 3)^(3/2))/1536 + (14125\*x^6\*(2\*x^2 - x + 3)^(3/2))/144 + (125\*x^7\*(2\*x^2 - x + 3)^(3/2))/4 - (41987163941\*2^(1/2)\*log((2\*x^2 - x + 3)^(1/2) + (2^(1/2)\*(2\*x - 1/2))/2))/1174405120 - (1825528867\*(x/2 - 1/8)\*(2\*x^2 - x + 3)^(1/2))/36700160 + (27185733541\*(2\*x^2 - x + 3)^(1/2)\*(32\*x^2 - 4\*x + 45))/7046430720 + (804243809\*x\*(2\*x^2 - x + 3)^(3/2))/36700160 + (625271871443\*2^(1/2)\*log(2\*(2\*x^2 - x + 3)^(1/2) + (2^(1/2)\*(4\*x - 1))/2))/9395240960

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{2x^2 - x + 3} (5x^2 + 3x + 2)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**4*(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2)**4, x)
```

$$3.59 \quad \int \sqrt{3 - x + 2x^2} (2 + 3x + 5x^2)^3 dx$$

**Optimal.** Leaf size=166

$$\frac{531681(2x^2 - x + 3)^{3/2} x^2}{71680} - \frac{9627393(2x^2 - x + 3)^{3/2} x}{1146880} - \frac{22548119(2x^2 - x + 3)^{3/2}}{4587520} - \frac{6766097(1 - 4x)\sqrt{2x^2 - x + 3}}{2097152}$$

[Out] -22548119/4587520\*(2\*x^2-x+3)^(3/2)-9627393/1146880\*x\*(2\*x^2-x+3)^(3/2)+531681/71680\*x^2\*(2\*x^2-x+3)^(3/2)+247435/10752\*x^3\*(2\*x^2-x+3)^(3/2)+8825/448\*x^4\*(2\*x^2-x+3)^(3/2)+125/16\*x^5\*(2\*x^2-x+3)^(3/2)-155620231/8388608\*arcsinh(1/23\*(1-4\*x)\*23^(1/2))\*2^(1/2)-6766097/2097152\*(1-4\*x)\*(2\*x^2-x+3)^(1/2)

**Rubi [A]** time = 0.18, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {1661, 640, 612, 619, 215}

$$\frac{125}{16} (2x^2 - x + 3)^{3/2} x^5 + \frac{8825}{448} (2x^2 - x + 3)^{3/2} x^4 + \frac{247435 (2x^2 - x + 3)^{3/2} x^3}{10752} + \frac{531681 (2x^2 - x + 3)^{3/2} x^2}{71680} - \frac{9627393 (2x^2 - x + 3)^{3/2} x}{1146880} - \frac{22548119 (2x^2 - x + 3)^{3/2}}{4587520} - \frac{6766097 (1 - 4x) \sqrt{2x^2 - x + 3}}{2097152}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[3 - x + 2\*x^2]\*(2 + 3\*x + 5\*x^2)^3,x]

[Out] (-6766097\*(1 - 4\*x)\*Sqrt[3 - x + 2\*x^2])/2097152 - (22548119\*(3 - x + 2\*x^2)^(3/2))/4587520 - (9627393\*x\*(3 - x + 2\*x^2)^(3/2))/1146880 + (531681\*x^2\*(3 - x + 2\*x^2)^(3/2))/71680 + (247435\*x^3\*(3 - x + 2\*x^2)^(3/2))/10752 + (8825\*x^4\*(3 - x + 2\*x^2)^(3/2))/448 + (125\*x^5\*(3 - x + 2\*x^2)^(3/2))/16 - (155620231\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(4194304\*Sqrt[2])

Rule 215

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 612

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^(p)), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

### Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

### Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x]] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

### Rubi steps

$$\begin{aligned}
\int \sqrt{3-x+2x^2} (2+3x+5x^2)^3 dx &= \frac{125}{16} x^5 (3-x+2x^2)^{3/2} + \frac{1}{16} \int \sqrt{3-x+2x^2} (128+576x+1824x^2+33) \\
&= \frac{8825}{448} x^4 (3-x+2x^2)^{3/2} + \frac{125}{16} x^5 (3-x+2x^2)^{3/2} + \frac{1}{224} \int \sqrt{3-x+2x^2} \\
&= \frac{247435x^3 (3-x+2x^2)^{3/2}}{10752} + \frac{8825}{448} x^4 (3-x+2x^2)^{3/2} + \frac{125}{16} x^5 (3-x+2x^2)^{3/2} \\
&= \frac{531681x^2 (3-x+2x^2)^{3/2}}{71680} + \frac{247435x^3 (3-x+2x^2)^{3/2}}{10752} + \frac{8825}{448} x^4 (3-x+2x^2)^{3/2} \\
&= -\frac{9627393x (3-x+2x^2)^{3/2}}{1146880} + \frac{531681x^2 (3-x+2x^2)^{3/2}}{71680} + \frac{247435x^3 (3-x+2x^2)^{3/2}}{10752} \\
&= -\frac{22548119 (3-x+2x^2)^{3/2}}{4587520} - \frac{9627393x (3-x+2x^2)^{3/2}}{1146880} + \frac{531681x^2 (3-x+2x^2)^{3/2}}{71680} \\
&= -\frac{6766097(1-4x)\sqrt{3-x+2x^2}}{2097152} - \frac{22548119 (3-x+2x^2)^{3/2}}{4587520} - \frac{9627393x (3-x+2x^2)^{3/2}}{1146880} \\
&= -\frac{6766097(1-4x)\sqrt{3-x+2x^2}}{2097152} - \frac{22548119 (3-x+2x^2)^{3/2}}{4587520} - \frac{9627393x (3-x+2x^2)^{3/2}}{1146880} \\
&= -\frac{6766097(1-4x)\sqrt{3-x+2x^2}}{2097152} - \frac{22548119 (3-x+2x^2)^{3/2}}{4587520} - \frac{9627393x (3-x+2x^2)^{3/2}}{1146880}
\end{aligned}$$

**Mathematica [A]** time = 0.17, size = 75, normalized size = 0.45

$$\frac{4\sqrt{2x^2-x+3} (3440640000x^7 + 6955008000x^6 + 10958233600x^5 + 11212171264x^4 + 9872163456x^3 + 4583812128x^2 + 16340124255\sqrt{2}\operatorname{ArcSinh}[(1-4x)/\sqrt{23}])}{880803840}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[3 - x + 2\*x^2]\*(2 + 3\*x + 5\*x^2)^3, x]

[Out] (4\*Sqrt[3 - x + 2\*x^2]\*(-3957369321 - 1621307916\*x + 4583812128\*x^2 + 9872163456\*x^3 + 11212171264\*x^4 + 10958233600\*x^5 + 6955008000\*x^6 + 3440640000\*x^7) - 16340124255\*Sqrt[2]\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/880803840

**fricas [A]** time = 0.90, size = 88, normalized size = 0.53

$$\frac{1}{220200960} (3440640000 x^7 + 6955008000 x^6 + 10958233600 x^5 + 11212171264 x^4 + 9872163456 x^3 + 4583812128 x^2 + 16340124255 \sqrt{2} \operatorname{ArcSinh}[(1 - 4x)/\sqrt{23}])$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^3\*(2\*x^2-x+3)^(1/2),x, algorithm="fricas")

[Out] 1/220200960\*(3440640000\*x^7 + 6955008000\*x^6 + 10958233600\*x^5 + 11212171264\*x^4 + 9872163456\*x^3 + 4583812128\*x^2 - 1621307916\*x - 3957369321)\*sqrt(2\*x^2 - x + 3) + 155620231/16777216\*sqrt(2)\*log(-4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25)

**giac** [A] time = 0.25, size = 83, normalized size = 0.50

$$\frac{1}{220200960} (4 (8 (4 (16 (100 (120 (140 x + 283) x + 53507) x + 5474693) x + 77126277) x + 143244129) x - 405326979) x - 3957369321) \sqrt{2x^2 - x + 3} - 155620231/8388608 \sqrt{2} \log(-2\sqrt{2}(\sqrt{2}x - \sqrt{2x^2 - x + 3})) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^3\*(2\*x^2-x+3)^(1/2),x, algorithm="giac")

[Out] 1/220200960\*(4\*(8\*(4\*(16\*(100\*(120\*(140\*x + 283)\*x + 53507)\*x + 5474693)\*x + 77126277)\*x + 143244129)\*x - 405326979)\*x - 3957369321)\*sqrt(2\*x^2 - x + 3) - 155620231/8388608\*sqrt(2)\*log(-2\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 1)

**maple** [A] time = 0.01, size = 132, normalized size = 0.80

$$\frac{125}{16} (2x^2 - x + 3)^{\frac{3}{2}} x^5 + \frac{8825}{448} (2x^2 - x + 3)^{\frac{3}{2}} x^4 + \frac{247435}{10752} (2x^2 - x + 3)^{\frac{3}{2}} x^3 + \frac{531681}{71680} (2x^2 - x + 3)^{\frac{3}{2}} x^2 - \frac{9627393}{1146880} (2x^2 - x + 3)^{\frac{3}{2}} - \frac{155620231}{8388608} \sqrt{2} \log(-2\sqrt{2}(\sqrt{2}x - \sqrt{2x^2 - x + 3})) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+3\*x+2)^3\*(2\*x^2-x+3)^(1/2),x)

[Out] -22548119/4587520\*(2\*x^2-x+3)^(3/2)+125/16\*(2\*x^2-x+3)^(3/2)\*x^5+8825/448\*(2\*x^2-x+3)^(3/2)\*x^4+247435/10752\*(2\*x^2-x+3)^(3/2)\*x^3+531681/71680\*(2\*x^2-x+3)^(3/2)\*x^2-9627393/1146880\*(2\*x^2-x+3)^(3/2)\*x+155620231/8388608\*2^(1/2)\*arcsinh(4/23\*23^(1/2)\*(x-1/4))+6766097/2097152\*(4\*x-1)\*(2\*x^2-x+3)^(1/2)

**maxima** [A] time = 0.98, size = 143, normalized size = 0.86

$$\frac{125}{16} (2x^2 - x + 3)^{\frac{3}{2}} x^5 + \frac{8825}{448} (2x^2 - x + 3)^{\frac{3}{2}} x^4 + \frac{247435}{10752} (2x^2 - x + 3)^{\frac{3}{2}} x^3 + \frac{531681}{71680} (2x^2 - x + 3)^{\frac{3}{2}} x^2 - \frac{9627393}{1146880} (2x^2 - x + 3)^{\frac{3}{2}} - \frac{155620231}{8388608} \sqrt{2} \log(-2\sqrt{2}(\sqrt{2}x - \sqrt{2x^2 - x + 3})) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^3\*(2\*x^2-x+3)^(1/2),x, algorithm="maxima")

[Out]  $125/16*(2*x^2 - x + 3)^{(3/2)}*x^5 + 8825/448*(2*x^2 - x + 3)^{(3/2)}*x^4 + 247435/10752*(2*x^2 - x + 3)^{(3/2)}*x^3 + 531681/71680*(2*x^2 - x + 3)^{(3/2)}*x^2 - 9627393/1146880*(2*x^2 - x + 3)^{(3/2)}*x - 22548119/4587520*(2*x^2 - x + 3)^{(3/2)} + 6766097/524288*\text{sqrt}(2*x^2 - x + 3)*x + 155620231/8388608*\text{sqrt}(2*x^2 - x + 3)*\text{arcsinh}(1/23*\text{sqrt}(23)*(4*x - 1)) - 6766097/2097152*\text{sqrt}(2*x^2 - x + 3)$

**mupad [B]** time = 4.69, size = 187, normalized size = 1.13

$$\frac{531681 x^2 (2x^2 - x + 3)^{3/2}}{71680} + \frac{247435 x^3 (2x^2 - x + 3)^{3/2}}{10752} + \frac{8825 x^4 (2x^2 - x + 3)^{3/2}}{448} + \frac{125 x^5 (2x^2 - x + 3)^{3/2}}{16} + \frac{875}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((2*x^2 - x + 3)^{(1/2)}*(3*x + 5*x^2 + 2)^3, x)$

[Out]  $(531681*x^2*(2*x^2 - x + 3)^{(3/2)})/71680 + (247435*x^3*(2*x^2 - x + 3)^{(3/2)})/10752 + (8825*x^4*(2*x^2 - x + 3)^{(3/2)})/448 + (125*x^5*(2*x^2 - x + 3)^{(3/2)})/16 + (875316037*2^{(1/2)}*\log((2*x^2 - x + 3)^{(1/2)} + (2^{(1/2)}*(2*x - 1/2))/2))/36700160 + (38057219*(x/2 - 1/8)*(2*x^2 - x + 3)^{(1/2)})/1146880 - (22548119*(2*x^2 - x + 3)^{(1/2)}*(32*x^2 - 4*x + 45))/73400320 - (9627393*x*(2*x^2 - x + 3)^{(3/2)})/1146880 - (1555820211*2^{(1/2)}*\log(2*(2*x^2 - x + 3)^{(1/2)} + (2^{(1/2)}*(4*x - 1))/2))/293601280$

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{2x^2 - x + 3} (5x^2 + 3x + 2)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((5*x**2+3*x+2)**3*(2*x**2-x+3)**(1/2), x)$

[Out]  $\text{Integral}(\text{sqrt}(2*x**2 - x + 3)*(5*x**2 + 3*x + 2)**3, x)$



$$3.60 \quad \int \sqrt{3 - x + 2x^2} (2 + 3x + 5x^2)^2 dx$$

**Optimal.** Leaf size=124

$$\frac{63}{16} (2x^2 - x + 3)^{3/2} x^2 + \frac{769}{256} (2x^2 - x + 3)^{3/2} x - \frac{2107 (2x^2 - x + 3)^{3/2}}{3072} + \frac{12371(1 - 4x)\sqrt{2x^2 - x + 3}}{16384} + \frac{25}{12} (2x^2 - x + 3)^{1/2}$$

[Out]  $-2107/3072*(2*x^2-x+3)^{(3/2)}+769/256*x*(2*x^2-x+3)^{(3/2)}+63/16*x^2*(2*x^2-x+3)^{(3/2)}+25/12*x^3*(2*x^2-x+3)^{(3/2)}+284533/65536*\operatorname{arcsinh}(1/23*(1-4*x))*23^{(1/2)}*2^{(1/2)}+12371/16384*(1-4*x)*(2*x^2-x+3)^{(1/2)}$

**Rubi [A]** time = 0.10, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {1661, 640, 612, 619, 215}

$$\frac{25}{12} (2x^2 - x + 3)^{3/2} x^3 + \frac{63}{16} (2x^2 - x + 3)^{3/2} x^2 + \frac{769}{256} (2x^2 - x + 3)^{3/2} x - \frac{2107 (2x^2 - x + 3)^{3/2}}{3072} + \frac{12371(1 - 4x)\sqrt{2x^2 - x + 3}}{16384}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[3 - x + 2\*x^2]\*(2 + 3\*x + 5\*x^2)^2,x]

[Out]  $(12371*(1 - 4*x)*\operatorname{Sqrt}[3 - x + 2*x^2])/16384 - (2107*(3 - x + 2*x^2)^{(3/2)})/3072 + (769*x*(3 - x + 2*x^2)^{(3/2)})/256 + (63*x^2*(3 - x + 2*x^2)^{(3/2)})/16 + (25*x^3*(3 - x + 2*x^2)^{(3/2)})/12 + (284533*\operatorname{ArcSinh}[(1 - 4*x)/\operatorname{Sqrt}[23]])/(32768*\operatorname{Sqrt}[2])$

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[Rt[b, 2]\*x]/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 612

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

#### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b

+ 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

### Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

### Rule 1661

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(q + 2\*p + 1)), x] + Dist[1/(c\*(q + 2\*p + 1)), Int[(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*(q + 2\*p + 1)\*Pq - a\*e\*(q - 1)\*x^(q - 2) - b\*e\*(q + p)\*x^(q - 1) - c\*e\*(q + 2\*p + 1)\*x^q, x], x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

### Rubi steps

$$\begin{aligned}
 \int \sqrt{3-x+2x^2} (2+3x+5x^2)^2 dx &= \frac{25}{12}x^3(3-x+2x^2)^{3/2} + \frac{1}{12} \int \sqrt{3-x+2x^2} \left(48+144x+123x^2 + \frac{945x^3}{2}\right) dx \\
 &= \frac{63}{16}x^2(3-x+2x^2)^{3/2} + \frac{25}{12}x^3(3-x+2x^2)^{3/2} + \frac{1}{120} \int \sqrt{3-x+2x^2} \left(48+144x+123x^2 + \frac{945x^3}{2}\right) dx \\
 &= \frac{769}{256}x(3-x+2x^2)^{3/2} + \frac{63}{16}x^2(3-x+2x^2)^{3/2} + \frac{25}{12}x^3(3-x+2x^2)^{3/2} + \frac{1}{120} \int \sqrt{3-x+2x^2} \left(48+144x+123x^2 + \frac{945x^3}{2}\right) dx \\
 &= -\frac{2107(3-x+2x^2)^{3/2}}{3072} + \frac{769}{256}x(3-x+2x^2)^{3/2} + \frac{63}{16}x^2(3-x+2x^2)^{3/2} + \frac{1}{120} \int \sqrt{3-x+2x^2} \left(48+144x+123x^2 + \frac{945x^3}{2}\right) dx \\
 &= \frac{12371(1-4x)\sqrt{3-x+2x^2}}{16384} - \frac{2107(3-x+2x^2)^{3/2}}{3072} + \frac{769}{256}x(3-x+2x^2)^{3/2} + \frac{1}{120} \int \sqrt{3-x+2x^2} \left(48+144x+123x^2 + \frac{945x^3}{2}\right) dx \\
 &= \frac{12371(1-4x)\sqrt{3-x+2x^2}}{16384} - \frac{2107(3-x+2x^2)^{3/2}}{3072} + \frac{769}{256}x(3-x+2x^2)^{3/2} + \frac{1}{120} \int \sqrt{3-x+2x^2} \left(48+144x+123x^2 + \frac{945x^3}{2}\right) dx \\
 &= \frac{12371(1-4x)\sqrt{3-x+2x^2}}{16384} - \frac{2107(3-x+2x^2)^{3/2}}{3072} + \frac{769}{256}x(3-x+2x^2)^{3/2} + \frac{1}{120} \int \sqrt{3-x+2x^2} \left(48+144x+123x^2 + \frac{945x^3}{2}\right) dx
 \end{aligned}$$

**Mathematica** [A] time = 0.10, size = 65, normalized size = 0.52

$$\frac{4\sqrt{2x^2 - x + 3} (204800x^5 + 284672x^4 + 408960x^3 + 365536x^2 + 328204x - 64023) + 853599\sqrt{2} \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{196608}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[3 - x + 2\*x^2]\*(2 + 3\*x + 5\*x^2)^2,x]

[Out] (4\*Sqrt[3 - x + 2\*x^2]\*(-64023 + 328204\*x + 365536\*x^2 + 408960\*x^3 + 284672\*x^4 + 204800\*x^5) + 853599\*Sqrt[2]\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/196608

**fricas** [A] time = 0.85, size = 78, normalized size = 0.63

$$\frac{1}{49152} (204800x^5 + 284672x^4 + 408960x^3 + 365536x^2 + 328204x - 64023)\sqrt{2x^2 - x + 3} + \frac{284533}{131072}\sqrt{2} \log\left(\dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^2\*(2\*x^2-x+3)^(1/2),x, algorithm="fricas")

[Out] 1/49152\*(204800\*x^5 + 284672\*x^4 + 408960\*x^3 + 365536\*x^2 + 328204\*x - 64023)\*sqrt(2\*x^2 - x + 3) + 284533/131072\*sqrt(2)\*log(4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25)

**giac** [A] time = 0.22, size = 73, normalized size = 0.59

$$\frac{1}{49152} (4(8(4(16(100x + 139)x + 3195)x + 11423)x + 82051)x - 64023)\sqrt{2x^2 - x + 3} + \frac{284533}{65536}\sqrt{2} \log\left(-2\sqrt{2}\sqrt{2x^2 - x + 3} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^2\*(2\*x^2-x+3)^(1/2),x, algorithm="giac")

[Out] 1/49152\*(4\*(8\*(4\*(16\*(100\*x + 139)\*x + 3195)\*x + 11423)\*x + 82051)\*x - 64023)\*sqrt(2\*x^2 - x + 3) + 284533/65536\*sqrt(2)\*log(-2\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 1)

**maple** [A] time = 0.01, size = 98, normalized size = 0.79

$$\frac{25(2x^2 - x + 3)^{\frac{3}{2}}x^3}{12} + \frac{63(2x^2 - x + 3)^{\frac{3}{2}}x^2}{16} + \frac{769(2x^2 - x + 3)^{\frac{3}{2}}x}{256} - \frac{284533\sqrt{2} \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x - \frac{1}{4}\right)}{23}\right)}{65536} - \frac{2107(2x^2 - x + 3)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+3\*x+2)^2\*(2\*x^2-x+3)^(1/2),x)

[Out]  $25/12*(2*x^2-x+3)^{(3/2)}*x^3+63/16*(2*x^2-x+3)^{(3/2)}*x^2+769/256*(2*x^2-x+3)^{(3/2)}*x-2107/3072*(2*x^2-x+3)^{(3/2)}-12371/16384*(4*x-1)*(2*x^2-x+3)^{(1/2)}-284533/65536*2^{(1/2)}*\operatorname{arcsinh}(4/23*23^{(1/2)}*(x-1/4))$

**maxima** [A] time = 0.98, size = 109, normalized size = 0.88

$$\frac{25}{12} (2x^2 - x + 3)^{\frac{3}{2}} x^3 + \frac{63}{16} (2x^2 - x + 3)^{\frac{3}{2}} x^2 + \frac{769}{256} (2x^2 - x + 3)^{\frac{3}{2}} x - \frac{2107}{3072} (2x^2 - x + 3)^{\frac{3}{2}} - \frac{12371}{4096} \sqrt{2x^2 - x + 3} x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x+2)^2*(2*x^2-x+3)^(1/2),x, algorithm="maxima")`

[Out]  $25/12*(2*x^2 - x + 3)^{(3/2)}*x^3 + 63/16*(2*x^2 - x + 3)^{(3/2)}*x^2 + 769/256*(2*x^2 - x + 3)^{(3/2)}*x - 2107/3072*(2*x^2 - x + 3)^{(3/2)} - 12371/4096*\operatorname{sqrt}(2*x^2 - x + 3)*x - 284533/65536*\operatorname{sqrt}(2)*\operatorname{arcsinh}(1/23*\operatorname{sqrt}(23)*(4*x - 1)) + 12371/16384*\operatorname{sqrt}(2*x^2 - x + 3)$

**mupad** [B] time = 4.19, size = 153, normalized size = 1.23

$$\frac{63x^2(2x^2-x+3)^{3/2}}{16} + \frac{25x^3(2x^2-x+3)^{3/2}}{12} - \frac{29509\sqrt{2}\ln\left(\sqrt{2x^2-x+3} + \frac{\sqrt{2}\left(2x-\frac{1}{2}\right)}{2}\right)}{8192} - \frac{1283\left(\frac{x}{2} - \frac{1}{8}\right)\sqrt{2x^2}}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^2,x)`

[Out]  $(63*x^2*(2*x^2 - x + 3)^{(3/2)})/16 + (25*x^3*(2*x^2 - x + 3)^{(3/2)})/12 - (29509*2^{(1/2)}*\log((2*x^2 - x + 3)^{(1/2)} + (2^{(1/2)}*(2*x - 1/2))/2))/8192 - (1283*(x/2 - 1/8)*(2*x^2 - x + 3)^{(1/2)})/256 - (2107*(2*x^2 - x + 3)^{(1/2)}*(3*2*x^2 - 4*x + 45))/49152 + (769*x*(2*x^2 - x + 3)^{(3/2)})/256 - (48461*2^{(1/2)}*\log(2*(2*x^2 - x + 3)^{(1/2)} + (2^{(1/2)}*(4*x - 1))/2))/65536$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{2x^2 - x + 3} (5x^2 + 3x + 2)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**2+3*x+2)**2*(2*x**2-x+3)**(1/2),x)`

[Out] `Integral(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2)**2, x)`

### 3.61 $\int \sqrt{3 - x + 2x^2} (2 + 3x + 5x^2) dx$

Optimal. Leaf size=82

$$\frac{5}{8}x(2x^2 - x + 3)^{3/2} + \frac{73}{96}(2x^2 - x + 3)^{3/2} - \frac{81}{512}(1 - 4x)\sqrt{2x^2 - x + 3} - \frac{1863 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1024\sqrt{2}}$$

[Out] 73/96\*(2\*x^2-x+3)^(3/2)+5/8\*x\*(2\*x^2-x+3)^(3/2)-1863/2048\*arcsinh(1/23\*(1-4\*x)\*23^(1/2))\*2^(1/2)-81/512\*(1-4\*x)\*(2\*x^2-x+3)^(1/2)

Rubi [A] time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {1661, 640, 612, 619, 215}

$$\frac{5}{8}x(2x^2 - x + 3)^{3/2} + \frac{73}{96}(2x^2 - x + 3)^{3/2} - \frac{81}{512}(1 - 4x)\sqrt{2x^2 - x + 3} - \frac{1863 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1024\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[3 - x + 2\*x^2]\*(2 + 3\*x + 5\*x^2), x]

[Out] (-81\*(1 - 4\*x)\*Sqrt[3 - x + 2\*x^2])/512 + (73\*(3 - x + 2\*x^2)^(3/2))/96 + (5\*x\*(3 - x + 2\*x^2)^(3/2))/8 - (1863\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(1024\*Sqrt[2])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 612

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

#### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
  := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
  *e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
  && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q =
  Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
  c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
  b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
  e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x]] /; FreeQ[{a, b, c,
  p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
 \int \sqrt{3-x+2x^2} (2+3x+5x^2) dx &= \frac{5}{8}x(3-x+2x^2)^{3/2} + \frac{1}{8} \int \left(1 + \frac{73x}{2}\right) \sqrt{3-x+2x^2} dx \\
 &= \frac{73}{96} (3-x+2x^2)^{3/2} + \frac{5}{8}x(3-x+2x^2)^{3/2} + \frac{81}{64} \int \sqrt{3-x+2x^2} dx \\
 &= -\frac{81}{512}(1-4x)\sqrt{3-x+2x^2} + \frac{73}{96} (3-x+2x^2)^{3/2} + \frac{5}{8}x(3-x+2x^2)^{3/2} + \dots \\
 &= -\frac{81}{512}(1-4x)\sqrt{3-x+2x^2} + \frac{73}{96} (3-x+2x^2)^{3/2} + \frac{5}{8}x(3-x+2x^2)^{3/2} + \dots \\
 &= -\frac{81}{512}(1-4x)\sqrt{3-x+2x^2} + \frac{73}{96} (3-x+2x^2)^{3/2} + \frac{5}{8}x(3-x+2x^2)^{3/2} - \dots
 \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 55, normalized size = 0.67

$$\frac{4\sqrt{2x^2-x+3} (1920x^3 + 1376x^2 + 2684x + 3261) - 5589\sqrt{2} \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{6144}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[3 - x + 2\*x^2]\*(2 + 3\*x + 5\*x^2), x]

[Out]  $(4\sqrt{3-x+2x^2})(3261+2684x+1376x^2+1920x^3) - 5589\sqrt{2} \operatorname{ArcSinh}[(1-4x)/\sqrt{23}]/6144$

**fricas** [A] time = 1.01, size = 68, normalized size = 0.83

$$\frac{1}{1536} (1920x^3 + 1376x^2 + 2684x + 3261)\sqrt{2x^2 - x + 3} + \frac{1863}{4096} \sqrt{2} \log\left(-4\sqrt{2}\sqrt{2x^2 - x + 3}(4x - 1) - 32x^2 - 16x + 25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x+2)*(2*x^2-x+3)^(1/2),x, algorithm="fricas")`

[Out]  $1/1536*(1920*x^3 + 1376*x^2 + 2684*x + 3261)*\sqrt{2*x^2 - x + 3} + 1863/4096*\sqrt{2}*\log(-4*\sqrt{2}*\sqrt{2*x^2 - x + 3}*(4*x - 1) - 32*x^2 + 16*x - 25)$

**giac** [A] time = 0.24, size = 63, normalized size = 0.77

$$\frac{1}{1536} (4(8(60x+43)x+671)x+3261)\sqrt{2x^2-x+3} - \frac{1863}{2048} \sqrt{2} \log\left(-2\sqrt{2}\left(\sqrt{2}x - \sqrt{2x^2-x+3}\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x+2)*(2*x^2-x+3)^(1/2),x, algorithm="giac")`

[Out]  $1/1536*(4*(8*(60*x + 43)*x + 671)*x + 3261)*\sqrt{2*x^2 - x + 3} - 1863/2048*\sqrt{2}*\log(-2*\sqrt{2}*(\sqrt{2}*x - \sqrt{2*x^2 - x + 3})) + 1)$

**maple** [A] time = 0.01, size = 64, normalized size = 0.78

$$\frac{5(2x^2 - x + 3)^{\frac{3}{2}}x}{8} + \frac{1863\sqrt{2} \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{2048} + \frac{73(2x^2 - x + 3)^{\frac{3}{2}}}{96} + \frac{81(4x - 1)\sqrt{2x^2 - x + 3}}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^2+3*x+2)*(2*x^2-x+3)^(1/2),x)`

[Out]  $5/8*(2*x^2-x+3)^(3/2)*x+73/96*(2*x^2-x+3)^(3/2)+81/512*(4*x-1)*(2*x^2-x+3)^(1/2)+1863/2048*2^(1/2)*\operatorname{arcsinh}(4/23*23^(1/2)*(x-1/4))$

**maxima** [A] time = 0.96, size = 75, normalized size = 0.91

$$\frac{5}{8} (2x^2 - x + 3)^{\frac{3}{2}}x + \frac{73}{96} (2x^2 - x + 3)^{\frac{3}{2}} + \frac{81}{128} \sqrt{2x^2 - x + 3}x + \frac{1863}{2048} \sqrt{2} \operatorname{arsinh}\left(\frac{1}{23} \sqrt{23}(4x - 1)\right) - \frac{81}{512} \sqrt{2x^2 - x + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)\*(2\*x^2-x+3)^(1/2),x, algorithm="maxima")

[Out] 5/8\*(2\*x^2 - x + 3)^(3/2)\*x + 73/96\*(2\*x^2 - x + 3)^(3/2) + 81/128\*sqrt(2\*x^2 - x + 3)\*x + 1863/2048\*sqrt(2)\*arcsinh(1/23\*sqrt(23)\*(4\*x - 1)) - 81/512\*sqrt(2\*x^2 - x + 3)

mupad [B] time = 3.84, size = 119, normalized size = 1.45

$$\frac{23\sqrt{2}\ln\left(\sqrt{2x^2-x+3} + \frac{\sqrt{2}\left(2x-\frac{1}{2}\right)}{2}\right)}{256} + \frac{\left(\frac{x}{2} - \frac{1}{8}\right)\sqrt{2x^2-x+3}}{8} + \frac{73\sqrt{2x^2-x+3}(32x^2-4x+45)}{1536} + \frac{5x(2x^2-x+3)^{3/2}}{2048}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)^(1/2)\*(3\*x + 5\*x^2 + 2),x)

[Out] (23\*2^(1/2)\*log((2\*x^2 - x + 3)^(1/2) + (2^(1/2)\*(2\*x - 1/2))/2))/256 + ((x/2 - 1/8)\*(2\*x^2 - x + 3)^(1/2))/8 + (73\*(2\*x^2 - x + 3)^(1/2)\*(32\*x^2 - 4\*x + 45))/1536 + (5\*x\*(2\*x^2 - x + 3)^(3/2))/8 + (1679\*2^(1/2)\*log(2\*(2\*x^2 - x + 3)^(1/2) + (2^(1/2)\*(4\*x - 1))/2))/2048

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{2x^2 - x + 3} (5x^2 + 3x + 2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*2+3\*x+2)\*(2\*x\*\*2-x+3)\*\*(1/2),x)

[Out] Integral(sqrt(2\*x\*\*2 - x + 3)\*(5\*x\*\*2 + 3\*x + 2), x)



$$3.62 \quad \int \frac{\sqrt{3-x+2x^2}}{2+3x+5x^2} dx$$

Optimal. Leaf size=174

$$\frac{1}{5} \sqrt{\frac{11}{31} (13 + 10\sqrt{2})} \tan^{-1} \left( \frac{\sqrt{\frac{11}{62(13+10\sqrt{2})}} ((20 + 13\sqrt{2})x + 7\sqrt{2} + 6)}{\sqrt{2x^2 - x + 3}} \right) - \frac{1}{5} \sqrt{\frac{11}{31} (10\sqrt{2} - 13)} \tanh^{-1} \left( \frac{\sqrt{\frac{11}{62(13+10\sqrt{2})}} ((20 + 13\sqrt{2})x + 7\sqrt{2} + 6)}{\sqrt{2x^2 - x + 3}} \right)$$

[Out]  $-1/5*\operatorname{arcsinh}(1/23*(1-4*x)*23^{(1/2)})*2^{(1/2)}-1/155*\operatorname{arctanh}(1/62*(6+x*(20-13*2^{(1/2)}))-7*2^{(1/2)})*682^{(1/2)/(-13+10*2^{(1/2)})^{(1/2)/(2*x^2-x+3)^{(1/2)}}*(-4433+3410*2^{(1/2)})^{(1/2)}+1/155*\operatorname{arctan}(1/62*(6+7*2^{(1/2)}+x*(20+13*2^{(1/2)}))*682^{(1/2)/(13+10*2^{(1/2)})^{(1/2)/(2*x^2-x+3)^{(1/2)}}*(4433+3410*2^{(1/2)})^{(1/2)})$

**Rubi** [A] time = 0.44, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$ , Rules used = {989, 619, 215, 1035, 1029, 206, 204}

$$\frac{1}{5} \sqrt{\frac{11}{31} (13 + 10\sqrt{2})} \tan^{-1} \left( \frac{\sqrt{\frac{11}{62(13+10\sqrt{2})}} ((20 + 13\sqrt{2})x + 7\sqrt{2} + 6)}{\sqrt{2x^2 - x + 3}} \right) - \frac{1}{5} \sqrt{\frac{11}{31} (10\sqrt{2} - 13)} \tanh^{-1} \left( \frac{\sqrt{\frac{11}{62(13+10\sqrt{2})}} ((20 + 13\sqrt{2})x + 7\sqrt{2} + 6)}{\sqrt{2x^2 - x + 3}} \right)$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Sqrt}[3 - x + 2*x^2]/(2 + 3*x + 5*x^2), x]$

[Out]  $-(\operatorname{Sqrt}[2]*\operatorname{ArcSinh}[(1 - 4*x)/\operatorname{Sqrt}[23]])/5 + (\operatorname{Sqrt}[(11*(13 + 10*\operatorname{Sqrt}[2]))]/31)*\operatorname{ArcTan}[(\operatorname{Sqrt}[11/(62*(13 + 10*\operatorname{Sqrt}[2]))])*(6 + 7*\operatorname{Sqrt}[2] + (20 + 13*\operatorname{Sqrt}[2])*x)]/\operatorname{Sqrt}[3 - x + 2*x^2])/5 - (\operatorname{Sqrt}[(11*(-13 + 10*\operatorname{Sqrt}[2]))]/31)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[11/(62*(-13 + 10*\operatorname{Sqrt}[2]))])*(6 - 7*\operatorname{Sqrt}[2] + (20 - 13*\operatorname{Sqrt}[2])*x)]/\operatorname{Sqrt}[3 - x + 2*x^2])/5$

Rule 204

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x\_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTan}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[-a, 2]]/(\operatorname{Rt}[-a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x] \ \&\& \operatorname{PosQ}[a/b] \ \&\& (\operatorname{LtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 206

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x] \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 215

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[Rt[b, 2]\*x]/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 619

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

Rule 989

Int[Sqrt[(a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2]/((d\_) + (e\_)\*(x\_) + (f\_)\*(x\_)^2), x\_Symbol] := Dist[c/f, Int[1/Sqrt[a + b\*x + c\*x^2], x], x] - Dist[1/f, Int[(c\*d - a\*f + (c\*e - b\*f)\*x)/(Sqrt[a + b\*x + c\*x^2]\*(d + e\*x + f\*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0]

Rule 1029

Int[((g\_) + (h\_)\*(x\_))/(((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)\*Sqrt[(d\_) + (e\_)\*(x\_) + (f\_)\*(x\_)^2]), x\_Symbol] := Dist[-2\*g\*(g\*b - 2\*a\*h), Subst[Int[1/Simp[g\*(g\*b - 2\*a\*h)\*(b^2 - 4\*a\*c) - (b\*d - a\*e)\*x^2, x], x], x, Simp[g\*b - 2\*a\*h - (b\*h - 2\*g\*c)\*x, x]/Sqrt[d + e\*x + f\*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && NeQ[b\*d - a\*e, 0] && EqQ[h^2\*(b\*d - a\*e) - 2\*g\*h\*(c\*d - a\*f) + g^2\*(c\*e - b\*f), 0]

Rule 1035

Int[((g\_) + (h\_)\*(x\_))/(((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)\*Sqrt[(d\_) + (e\_)\*(x\_) + (f\_)\*(x\_)^2]), x\_Symbol] := With[{q = Rt[(c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f), 2]}, Dist[1/(2\*q), Int[Simp[h\*(b\*d - a\*e) - g\*(c\*d - a\*f - q) - (g\*(c\*e - b\*f) - h\*(c\*d - a\*f + q))\*x, x]/((a + b\*x + c\*x^2)\*Sqrt[d + e\*x + f\*x^2]), x], x] - Dist[1/(2\*q), Int[Simp[h\*(b\*d - a\*e) - g\*(c\*d - a\*f + q) - (g\*(c\*e - b\*f) - h\*(c\*d - a\*f - q))\*x, x]/((a + b\*x + c\*x^2)\*Sqrt[d + e\*x + f\*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && NeQ[b\*d - a\*e, 0] && NegQ[b^2 - 4\*a\*c]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{3-x+2x^2}}{2+3x+5x^2} dx &= -\left(\frac{1}{5} \int \frac{-11+11x}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx\right) + \frac{2}{5} \int \frac{1}{\sqrt{3-x+2x^2}} dx \\
&= \frac{1}{5} \sqrt{\frac{2}{23}} \operatorname{Subst} \left( \int \frac{1}{\sqrt{1+\frac{x^2}{23}}} dx, x, -1+4x \right) + \frac{\int \frac{121(2+\sqrt{2})-121\sqrt{2}x}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{110\sqrt{2}} - \frac{\int \frac{121(2-\sqrt{2})+121x}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{110\sqrt{2}} \\
&= -\frac{1}{5} \sqrt{2} \sinh^{-1} \left( \frac{1-4x}{\sqrt{23}} \right) - \frac{1}{5} (1331(20-13\sqrt{2})) \operatorname{Subst} \left( \int \frac{1}{-907742(13-10\sqrt{2})-11x} dx, x, -1+4x \right) \\
&= -\frac{1}{5} \sqrt{2} \sinh^{-1} \left( \frac{1-4x}{\sqrt{23}} \right) + \frac{1}{5} \sqrt{\frac{11}{31}(13+10\sqrt{2})} \tan^{-1} \left( \frac{\sqrt{\frac{11}{62(13+10\sqrt{2})}} (6+7\sqrt{2}+(20+11x))}{\sqrt{3-x+2x^2}} \right)
\end{aligned}$$

**Mathematica [C]** time = 0.46, size = 185, normalized size = 1.06

$$-\frac{1}{5} \sqrt{2} \sinh^{-1} \left( \frac{1-4x}{\sqrt{23}} \right) - \frac{1}{5} i \sqrt{\frac{11}{62}} \left( \sqrt{13+i\sqrt{31}} \tanh^{-1} \left( \frac{-4i\sqrt{31}x-22x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}} \right) - \sqrt{13-i\sqrt{31}} \tanh^{-1} \left( \frac{-4i\sqrt{31}x-22x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[3 - x + 2\*x^2]/(2 + 3\*x + 5\*x^2), x]

[Out] -1/5\*(Sqrt[2]\*ArcSinh[(1 - 4\*x)/Sqrt[23]]) - (I/5)\*Sqrt[11/62]\*(Sqrt[13 + I\*Sqrt[31]]\*ArcTanh[(63 + I\*Sqrt[31] - 22\*x - (4\*I)\*Sqrt[31]\*x)/(2\*Sqrt[286 + (22\*I)\*Sqrt[31]]\*Sqrt[3 - x + 2\*x^2])] - Sqrt[13 - I\*Sqrt[31]]\*ArcTanh[(63 - I\*Sqrt[31] - 22\*x + (4\*I)\*Sqrt[31]\*x)/(2\*Sqrt[286 - (22\*I)\*Sqrt[31]]\*Sqrt[3 - x + 2\*x^2])])

**fricas [B]** time = 2.11, size = 2016, normalized size = 11.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(1/2)/(5\*x^2+3\*x+2), x, algorithm="fricas")

[Out] 1/1550\*6050^(1/4)\*sqrt(31)\*sqrt(5)\*sqrt(2)\*sqrt(13\*sqrt(2) + 20)\*arctan(1/89125\*(460\*sqrt(5)\*(4\*6050^(3/4)\*sqrt(31)\*(4702\*x^7 - 19541\*x^6 + 40352\*x^5 - 68777\*x^4 + 35480\*x^3 - 19080\*x^2 - sqrt(2)\*(4028\*x^7 - 14488\*x^6 + 30919

$$\begin{aligned}
& *x^5 - 46671*x^4 + 22688*x^3 - 9144*x^2 - 27648*x + 17280) - 34560*x + 2764 \\
& 8) + 5*6050^{(1/4)}*\sqrt{31}*(22836*x^7 - 355266*x^6 + 1914360*x^5 - 4475096* \\
& x^4 + 5840640*x^3 - 4011840*x^2 - \sqrt{2}*(18463*x^7 - 280047*x^6 + 1453472 \\
& *x^5 - 3238500*x^4 + 4140576*x^3 - 2378592*x^2 - 3068928*x + 1990656) - 398 \\
& 1312*x + 3068928))*\sqrt{2*x^2 - x + 3}*\sqrt{13*\sqrt{2} + 20) + 253000*\sqrt{(} \\
& 31)*\sqrt{2}*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^ \\
& 4 - 642048*x^3 - 98496*x^2 - \sqrt{2}*(8746*x^8 - 102335*x^7 + 396104*x^6 - \\
& 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1 \\
& 154304*x - 456192) - 2*\sqrt{10}*(\sqrt{5}*(4*6050^{(3/4)}*\sqrt{31}*(15454*x^7 \\
& - 22399*x^6 + 73509*x^5 - 37360*x^4 + 52200*x^3 + 13824*x^2 - \sqrt{2}*(1543 \\
& 8*x^7 - 22007*x^6 + 69837*x^5 - 21232*x^4 + 19368*x^3 + 44928*x^2 - 44928*x \\
& ) - 13824*x) + 5*6050^{(1/4)}*\sqrt{31}*(77254*x^7 - 1000024*x^6 + 3868360*x^5 \\
& - 5120640*x^4 + 7012800*x^3 + 2405376*x^2 - \sqrt{2}*(69479*x^7 - 898236*x^ \\
& 6 + 3454740*x^5 - 4394304*x^4 + 5347296*x^3 + 4478976*x^2 - 4478976*x) - 24 \\
& 05376*x))*\sqrt{2*x^2 - x + 3}*\sqrt{13*\sqrt{2} + 20) + 550*\sqrt{31}*\sqrt{2}* \\
& (123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336* \\
& x^3 - 3822336*x^2 - \sqrt{2}*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x \\
& ^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) + 25 \\
& *\sqrt{31}*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 1087819 \\
& 20*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*\sqrt{2}*(4*x^8 - 76*x^7 + 517 \\
& *x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))* \\
& \sqrt{-(6050^{(1/4)}*\sqrt{5}*\sqrt{2*x^2 - x + 3}*(\sqrt{2}*(3*x + 5) - 8*x + 2) \\
& *\sqrt{13*\sqrt{2} + 20) - 245*x^2 - 220*\sqrt{2}*(2*x^2 - x + 3) + 755*x - 10 \\
& 00)/x^2) + 2875*\sqrt{31}*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835 \\
& 344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*\sqrt{2}*(1348 \\
& *x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + \\
& 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 1419 \\
& 1920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 247726 \\
& 08*x + 18579456)) + 1/1550*6050^{(1/4)}*\sqrt{31}*\sqrt{5}*\sqrt{2}*\sqrt{13*\sqrt{2} \\
& (2) + 20)*\arctan(1/89125*(460*\sqrt{5}*(4*6050^{(3/4)}*\sqrt{31}*(4702*x^7 - 19 \\
& 541*x^6 + 40352*x^5 - 68777*x^4 + 35480*x^3 - 19080*x^2 - \sqrt{2}*(4028*x^7 \\
& - 14488*x^6 + 30919*x^5 - 46671*x^4 + 22688*x^3 - 9144*x^2 - 27648*x + 172 \\
& 80) - 34560*x + 27648) + 5*6050^{(1/4)}*\sqrt{31}*(22836*x^7 - 355266*x^6 + 19 \\
& 14360*x^5 - 4475096*x^4 + 5840640*x^3 - 4011840*x^2 - \sqrt{2}*(18463*x^7 - \\
& 280047*x^6 + 1453472*x^5 - 3238500*x^4 + 4140576*x^3 - 2378592*x^2 - 306892 \\
& 8*x + 1990656) - 3981312*x + 3068928))*\sqrt{2*x^2 - x + 3}*\sqrt{13*\sqrt{2} \\
& + 20) - 253000*\sqrt{31}*\sqrt{2}*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385 \\
& 256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - \sqrt{2}*(8746*x^8 - 102335 \\
& *x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 54 \\
& 6048*x - 539136) + 1154304*x - 456192) - 2*\sqrt{10}*(\sqrt{5}*(4*6050^{(3/4)}* \\
& \sqrt{31}*(15454*x^7 - 22399*x^6 + 73509*x^5 - 37360*x^4 + 52200*x^3 + 13824 \\
& *x^2 - \sqrt{2}*(15438*x^7 - 22007*x^6 + 69837*x^5 - 21232*x^4 + 19368*x^3 + \\
& 44928*x^2 - 44928*x) - 13824*x) + 5*6050^{(1/4)}*\sqrt{31}*(77254*x^7 - 10000 \\
& 24*x^6 + 3868360*x^5 - 5120640*x^4 + 7012800*x^3 + 2405376*x^2 - \sqrt{2}*(6 \\
& 9479*x^7 - 898236*x^6 + 3454740*x^5 - 4394304*x^4 + 5347296*x^3 + 4478976*x
\end{aligned}$$

```

^2 - 4478976*x) - 2405376*x))*sqrt(2*x^2 - x + 3)*sqrt(13*sqrt(2) + 20) - 5
50*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 +
396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 2
44047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*
x) + 3276288*x) - 25*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90
866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*
(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944
*x) + 144820224*x))*sqrt((6050^(1/4)*sqrt(5)*sqrt(2*x^2 - x + 3)*(sqrt(2)*
(3*x + 5) - 8*x + 2)*sqrt(13*sqrt(2) + 20) + 245*x^2 + 220*sqrt(2)*(2*x^2 -
x + 3) - 755*x + 1000)/x^2) - 2875*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53
385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 -
7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 556
8*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 -
4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34
615296*x^2 - 24772608*x + 18579456)) - 1/6200*6050^(1/4)*sqrt(5)*sqrt(13*sq
rt(2) + 20)*(13*sqrt(2) - 20)*log(40*(6050^(1/4)*sqrt(5)*sqrt(2*x^2 - x + 3
)*(sqrt(2)*(3*x + 5) - 8*x + 2)*sqrt(13*sqrt(2) + 20) + 245*x^2 + 220*sqrt(
2)*(2*x^2 - x + 3) - 755*x + 1000)/x^2) + 1/6200*6050^(1/4)*sqrt(5)*sqrt(13
*sqrt(2) + 20)*(13*sqrt(2) - 20)*log(-40*(6050^(1/4)*sqrt(5)*sqrt(2*x^2 - x
+ 3)*(sqrt(2)*(3*x + 5) - 8*x + 2)*sqrt(13*sqrt(2) + 20) - 245*x^2 - 220*s
qrt(2)*(2*x^2 - x + 3) + 755*x - 1000)/x^2) + 1/10*sqrt(2)*log(-4*sqrt(2)*s
qrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)

```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(1/2)/(5\*x^2+3\*x+2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP  
UT:sage2:=int(sage0,x);OUTPUT:Francis algorithm failure for[-1.0,infinity,  
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,inf  
inity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity  
]proot error [1.0,infinity,infinity,infinity,infinity]Evaluation time: 5.57  
Done

**maple** [B] time = 0.15, size = 2065, normalized size = 11.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^(1/2)/(5\*x^2+3\*x+2),x)



$$\frac{/(2^{(1/2)+1-x})^2+23)*(2^{(1/2)-1+x})/(2^{(1/2)+1-x}*(8+3*2^{(1/2)})))*(-775687+549362*2^{(1/2)})^{(1/2)+520*(-8866+6820*2^{(1/2)})^{(1/2)}*\arctan(1/11692487*(-775687+549362*2^{(1/2)})^{(1/2)}*(-23*(8+3*2^{(1/2)}))*(-23*(2^{(1/2)-1+x})^2/(2^{(1/2)+1-x})^2+24*2^{(1/2)-41}))^{(1/2)}*(6485*2^{(1/2)}*(2^{(1/2)-1+x})^2/(2^{(1/2)+1-x})^2+10368*(2^{(1/2)-1+x})^2/(2^{(1/2)+1-x})^2+22379*2^{(1/2)+32016)/(23*(2^{(1/2)-1+x})^4/(2^{(1/2)+1-x})^4+82*(2^{(1/2)-1+x})^2/(2^{(1/2)+1-x})^2+23)*(2^{(1/2)-1+x})/(2^{(1/2)+1-x}*(8+3*2^{(1/2)})))*(-775687+549362*2^{(1/2)})^{(1/2)+465124*\operatorname{arctanh}(31/2*(8*(2^{(1/2)-1+x})^2/(2^{(1/2)+1-x})^2+3*2^{(1/2)}*(2^{(1/2)-1+x})^2/(2^{(1/2)+1-x})^2+8-3*2^{(1/2)})^{(1/2)})/(-8866+6820*2^{(1/2)})^{(1/2)})*2^{(1/2)-866822*\operatorname{arctanh}(31/2*(8*(2^{(1/2)-1+x})^2/(2^{(1/2)+1-x})^2+3*2^{(1/2)}*(2^{(1/2)-1+x})^2/(2^{(1/2)+1-x})^2+8-3*2^{(1/2)})^{(1/2)})/(-8866+6820*2^{(1/2)})^{(1/2)})/((8*(2^{(1/2)-1+x})^2/(2^{(1/2)+1-x})^2+3*2^{(1/2)}*(2^{(1/2)-1+x})^2/(2^{(1/2)+1-x})^2+8-3*2^{(1/2)})/(1+(2^{(1/2)-1+x})/(2^{(1/2)+1-x}))^2)^{(1/2)}/(1+(2^{(1/2)-1+x})/(2^{(1/2)+1-x}))/((8+3*2^{(1/2)})/(-8866+6820*2^{(1/2)})^{(1/2)})$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2 - x + 3}}{5x^2 + 3x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(1/2)/(5\*x^2+3\*x+2),x, algorithm="maxima")

[Out] integrate(sqrt(2\*x^2 - x + 3)/(5\*x^2 + 3\*x + 2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2x^2 - x + 3}}{5x^2 + 3x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)^(1/2)/(3\*x + 5\*x^2 + 2),x)

[Out] int((2\*x^2 - x + 3)^(1/2)/(3\*x + 5\*x^2 + 2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2 - x + 3}}{5x^2 + 3x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2-x+3)\*\*(1/2)/(5\*x\*\*2+3\*x+2),x)

[Out] Integral(sqrt(2\*x\*\*2 - x + 3)/(5\*x\*\*2 + 3\*x + 2), x)

$$3.63 \quad \int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^2} dx$$

Optimal. Leaf size=188

$$\frac{\sqrt{2x^2-x+3}(10x+3)}{31(5x^2+3x+2)} + \frac{1}{62} \sqrt{\frac{1}{682}(70517+49942\sqrt{2})} \tan^{-1} \left( \frac{\sqrt{\frac{11}{31(70517+49942\sqrt{2})}}((973+696\sqrt{2})x+277\sqrt{2})}{\sqrt{2x^2-x+3}} \right)$$

[Out] 1/31\*(3+10\*x)\*(2\*x^2-x+3)^(1/2)/(5\*x^2+3\*x+2)-1/42284\*arctanh(1/31\*(419+x\*(973-696\*2^(1/2))-277\*2^(1/2))\*341^(1/2)/(-70517+49942\*2^(1/2))^(1/2)/(2\*x^2-x+3)^(1/2))\*(-48092594+34060444\*2^(1/2))^(1/2)+1/42284\*arctan(1/31\*(419+277\*2^(1/2)+x\*(973+696\*2^(1/2)))\*341^(1/2)/(70517+49942\*2^(1/2))^(1/2)/(2\*x^2-x+3)^(1/2))\*(48092594+34060444\*2^(1/2))^(1/2)

**Rubi [A]** time = 0.39, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {971, 1035, 1029, 206, 204}

$$\frac{\sqrt{2x^2-x+3}(10x+3)}{31(5x^2+3x+2)} + \frac{1}{62} \sqrt{\frac{1}{682}(70517+49942\sqrt{2})} \tan^{-1} \left( \frac{\sqrt{\frac{11}{31(70517+49942\sqrt{2})}}((973+696\sqrt{2})x+277\sqrt{2})}{\sqrt{2x^2-x+3}} \right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[3 - x + 2\*x^2]/(2 + 3\*x + 5\*x^2)^2,x]

[Out] ((3 + 10\*x)\*Sqrt[3 - x + 2\*x^2])/(31\*(2 + 3\*x + 5\*x^2)) + (Sqrt[(70517 + 49942\*Sqrt[2])/682]\*ArcTan[(Sqrt[11/(31\*(70517 + 49942\*Sqrt[2]))])\*(419 + 277\*Sqrt[2] + (973 + 696\*Sqrt[2])\*x)]/Sqrt[3 - x + 2\*x^2])]/62 - (Sqrt[(-70517 + 49942\*Sqrt[2])/682]\*ArcTanh[(Sqrt[11/(31\*(-70517 + 49942\*Sqrt[2]))])\*(419 - 277\*Sqrt[2] + (973 - 696\*Sqrt[2])\*x)]/Sqrt[3 - x + 2\*x^2])]/62

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]]/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt



Q[a, 0] || LtQ[b, 0])

### Rule 971

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p + 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] && !IGtQ[q, 0]
```

### Rule 1029

```
Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*g*(g*b - 2*a*h), Subst[Int[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f), 0]
```

### Rule 1035

```
Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 - 4*a*c]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^2} dx &= \frac{(3+10x)\sqrt{3-x+2x^2}}{31(2+3x+5x^2)} - \frac{1}{31} \int \frac{-\frac{63}{2} + 11x}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx \\
&= \frac{(3+10x)\sqrt{3-x+2x^2}}{31(2+3x+5x^2)} - \frac{\int \frac{\frac{11}{2}(85-63\sqrt{2}) - \frac{11}{2}(41-22\sqrt{2})x}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{682\sqrt{2}} + \frac{\int \frac{\frac{11}{2}(85+63\sqrt{2}) - \frac{11}{2}(41+22\sqrt{2})x}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{682\sqrt{2}} \\
&= \frac{(3+10x)\sqrt{3-x+2x^2}}{31(2+3x+5x^2)} - \frac{1}{248} \left( 11 \left( 99884 - 70517\sqrt{2} \right) \right) \text{Subst} \left( \int \frac{1}{-\frac{3751}{4} (70517 - 49942\sqrt{2}) + \frac{11}{4} (70517 + 49942\sqrt{2})x} dx \right) \\
&= \frac{(3+10x)\sqrt{3-x+2x^2}}{31(2+3x+5x^2)} + \frac{1}{62} \sqrt{\frac{1}{682} (70517 + 49942\sqrt{2})} \tan^{-1} \left( \frac{\sqrt{\frac{11}{31(70517+49942\sqrt{2})}} (4x + 3)}{\sqrt{\frac{11}{31(70517-49942\sqrt{2})}} (4x + 3)} \right)
\end{aligned}$$

**Mathematica [C]** time = 1.04, size = 214, normalized size = 1.14

$$\frac{27280\sqrt{2x^2-x+3}(10x+3)}{5x^2+3x+2} + i\sqrt{286-22i\sqrt{31}}(973\sqrt{31}+1271i)\tanh^{-1}\left(\frac{4i\sqrt{31}x-22x-i\sqrt{31}+63}{2\sqrt{286-22i\sqrt{31}}\sqrt{2x^2-x+3}}\right) - i\sqrt{286+22i\sqrt{31}}\left(\frac{4x+3}{\sqrt{286+22i\sqrt{31}}}\right)$$


---

845680

Antiderivative was successfully verified.

[In] Integrate[Sqrt[3 - x + 2\*x^2]/(2 + 3\*x + 5\*x^2)^2, x]

[Out] ((27280\*(3 + 10\*x)\*Sqrt[3 - x + 2\*x^2])/(2 + 3\*x + 5\*x^2) + I\*Sqrt[286 - (2\*I)\*Sqrt[31]]\*(1271\*I + 973\*Sqrt[31])\*ArcTanh[(63 - I\*Sqrt[31] - 22\*x + (4\*I)\*Sqrt[31]\*x)/(2\*Sqrt[286 - (22\*I)\*Sqrt[31]]\*Sqrt[3 - x + 2\*x^2])] - I\*Sqrt[286 + (22\*I)\*Sqrt[31]]\*(-1271\*I + 973\*Sqrt[31])\*ArcTanh[(63 + I\*Sqrt[31] + (-22 - (4\*I)\*Sqrt[31])\*x)/(2\*Sqrt[286 + (22\*I)\*Sqrt[31]]\*Sqrt[3 - x + 2\*x^2])])/845680

**fricas [B]** time = 2.37, size = 2102, normalized size = 11.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(1/2)/(5\*x^2+3\*x+2)^2,x, algorithm="fricas")

[Out] -1/186703822445536\*(88412\*4988406728^(1/4)\*sqrt(24971)\*sqrt(341)\*sqrt(2)\*(5\*x^2 + 3\*x + 2)\*sqrt(70517\*sqrt(2) + 99884)\*arctan(1/10668926457462302923\*(

$$\begin{aligned}
& 3096404*\sqrt{24971}*(11*4988406728^{(3/4)}*\sqrt{341}*(537184*x^7 - 2047820*x^6 \\
& + 4310846*x^5 - 6853210*x^4 + 3421536*x^3 - 1589328*x^2 - \sqrt{2}*(370014 \\
& *x^7 - 1438653*x^6 + 3014868*x^5 - 4873381*x^4 + 2452952*x^3 - 1184616*x^2 \\
& - 2633472*x + 1893888) - 3787776*x + 2633472) + 774101*4988406728^{(1/4)}*\sqrt{341} \\
& *(40625*x^7 - 622509*x^6 + 3280912*x^5 - 7459052*x^4 + 9621216*x^3 - 5992992*x^2 \\
& - \sqrt{2}*(28204*x^7 - 433677*x^6 + 2297444*x^5 - 5257628*x^4 + 6800832*x^3 \\
& - 4341024*x^2 - 4810752*x + 3442176) - 6884352*x + 4810752))*\sqrt{2*x^2 - x + 3} \\
& *\sqrt{70517*\sqrt{2} + 99884) + 30285984782473634104*\sqrt{31}*\sqrt{2}*(28180*x^8 \\
& - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 \\
& - \sqrt{2}*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 \\
& - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*\sqrt{49942} \\
& *( \sqrt{24971}*(11*4988406728^{(3/4)}*\sqrt{341}*(84604*x^7 - 121310*x^6 + 389610*x^5 \\
& - 147168*x^4 + 168912*x^3 + 186624*x^2 - \sqrt{2}*(57082*x^7 - 82029*x^6 \\
& + 264639*x^5 - 107216*x^4 + 130104*x^3 + 110592*x^2 - 110592*x) - 186624*x) \\
& + 774101*4988406728^{(1/4)}*\sqrt{341}*(6379*x^7 - 82508*x^6 + 318020*x^5 \\
& - 410688*x^4 + 523872*x^3 + 331776*x^2 - \sqrt{2}*(4365*x^7 - 56468*x^6 \\
& + 217820*x^5 - 282816*x^4 + 366624*x^3 + 207360*x^2 - 207360*x) - 331776*x) \\
& )*\sqrt{2*x^2 - x + 3}*\sqrt{70517*\sqrt{2} + 99884) + 425261673562*\sqrt{31} \\
& *\sqrt{2}*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 \\
& + 798336*x^3 - 3822336*x^2 - \sqrt{2}*(15550*x^8 - 118051*x^7 + 244047*x^6 \\
& - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) \\
& + 3276288*x) + 19330076071*\sqrt{31}*(254591*x^8 - 4815126*x^7 + 32303580*x^6 \\
& - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*\sqrt{2} \\
& *(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) \\
& + 144820224*x))*\sqrt{-(4988406728^{(1/4)}*\sqrt{24971}*\sqrt{341}*\sqrt{31}*\sqrt{2*x^2 - x + 3} \\
& *( \sqrt{2}*(10*x + 3) - 13*x - 7)*\sqrt{70517*\sqrt{2} + 99884) - 1175859419*x^2 - 1055873764*\sqrt{2} \\
& *(2*x^2 - x + 3) + 3623566781*x - 4799426200)/x^2) + 344158917982654933*\sqrt{31} \\
& *(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096 \\
& *x^3 + 37981440*x^2 - 7744*\sqrt{2}*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 \\
& + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936) \\
& )/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 \\
& + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 88412*4988406728^{(1/4)} \\
& *\sqrt{24971}*\sqrt{341}*\sqrt{2}*(5*x^2 + 3*x + 2)*\sqrt{70517*\sqrt{2} + 99884} \\
& *\arctan(1/10668926457462302923*(3096404*\sqrt{24971}*(11*4988406728^{(3/4)} \\
& *\sqrt{341}*(537184*x^7 - 2047820*x^6 + 4310846*x^5 - 6853210*x^4 + 3421536*x^3 \\
& - 1589328*x^2 - \sqrt{2}*(370014*x^7 - 1438653*x^6 + 3014868*x^5 - 4873381*x^4 \\
& + 2452952*x^3 - 1184616*x^2 - 2633472*x + 1893888) - 3787776*x + 2633472) \\
& + 774101*4988406728^{(1/4)}*\sqrt{341}*(40625*x^7 - 622509*x^6 + 3280912*x^5 \\
& - 7459052*x^4 + 9621216*x^3 - 5992992*x^2 - \sqrt{2}*(28204*x^7 - 433677*x^6 \\
& + 2297444*x^5 - 5257628*x^4 + 6800832*x^3 - 4341024*x^2 - 4810752*x + 3442176) \\
& - 6884352*x + 4810752))*\sqrt{2*x^2 - x + 3}*\sqrt{70517*\sqrt{2} + 99884) - 30285984782473634104 \\
& *\sqrt{31}*\sqrt{2}*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 \\
& - 642048*x^3 - 98496*x^2 - \sqrt{2}*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 \\
& + 1320710*x^4
\end{aligned}$$

```

- 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*sq
rt(49942)*(sqrt(24971)*(11*4988406728^(3/4)*sqrt(341)*(84604*x^7 - 121310*x
^6 + 389610*x^5 - 147168*x^4 + 168912*x^3 + 186624*x^2 - sqrt(2)*(57082*x^7
- 82029*x^6 + 264639*x^5 - 107216*x^4 + 130104*x^3 + 110592*x^2 - 110592*x
) - 186624*x) + 774101*4988406728^(1/4)*sqrt(341)*(6379*x^7 - 82508*x^6 + 3
18020*x^5 - 410688*x^4 + 523872*x^3 + 331776*x^2 - sqrt(2)*(4365*x^7 - 5646
8*x^6 + 217820*x^5 - 282816*x^4 + 366624*x^3 + 207360*x^2 - 207360*x) - 331
776*x))*sqrt(2*x^2 - x + 3)*sqrt(70517*sqrt(2) + 99884) - 425261673562*sqrt
(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*
x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x
^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 32
76288*x) - 19330076071*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 -
90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)
*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 19
44*x) + 144820224*x))*sqrt((4988406728^(1/4)*sqrt(24971)*sqrt(341)*sqrt(31)
*sqrt(2*x^2 - x + 3)*(sqrt(2)*(10*x + 3) - 13*x - 7)*sqrt(70517*sqrt(2) + 9
9884) + 1175859419*x^2 + 1055873764*sqrt(2)*(2*x^2 - x + 3) - 3623566781*x
+ 4799426200)/x^2) - 344158917982654933*sqrt(31)*(2828123*x^8 - 9696916*x^7
+ 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*
x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4
- 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*
x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3
- 34615296*x^2 - 24772608*x + 18579456)) - 4988406728^(1/4)*sqrt(24971)*sq
rt(341)*sqrt(31)*(499420*x^2 - 70517*sqrt(2)*(5*x^2 + 3*x + 2) + 299652*x +
199768)*sqrt(70517*sqrt(2) + 99884)*log(199768*(4988406728^(1/4)*sqrt(2497
1)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(10*x + 3) - 13*x - 7)*s
qrt(70517*sqrt(2) + 99884) + 1175859419*x^2 + 1055873764*sqrt(2)*(2*x^2 - x
+ 3) - 3623566781*x + 4799426200)/x^2) + 4988406728^(1/4)*sqrt(24971)*sqrt
(341)*sqrt(31)*(499420*x^2 - 70517*sqrt(2)*(5*x^2 + 3*x + 2) + 299652*x + 1
99768)*sqrt(70517*sqrt(2) + 99884)*log(-199768*(4988406728^(1/4)*sqrt(24971
)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(10*x + 3) - 13*x - 7)*sq
rt(70517*sqrt(2) + 99884) - 1175859419*x^2 - 1055873764*sqrt(2)*(2*x^2 - x
+ 3) + 3623566781*x - 4799426200)/x^2) - 6022703949856*sqrt(2*x^2 - x + 3)*
(10*x + 3))/(5*x^2 + 3*x + 2)

```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,inf
```

nity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity  
 ]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai  
 lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity  
 ,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf  
 inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinit  
 y]Evaluation time: 17.67Done

**maple [B]** time = 0.23, size = 16357, normalized size = 87.01

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^(1/2)/(5\*x^2+3\*x+2)^2,x)

[Out] result too large to display

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2 - x + 3}}{(5x^2 + 3x + 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(1/2)/(5\*x^2+3\*x+2)^2,x, algorithm="maxima")

[Out] integrate(sqrt(2\*x^2 - x + 3)/(5\*x^2 + 3\*x + 2)^2, x)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2x^2 - x + 3}}{(5x^2 + 3x + 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)^(1/2)/(3\*x + 5\*x^2 + 2)^2,x)

[Out] int((2\*x^2 - x + 3)^(1/2)/(3\*x + 5\*x^2 + 2)^2, x)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2 - x + 3}}{(5x^2 + 3x + 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2-x+3)\*\*(1/2)/(5\*x\*\*2+3\*x+2)\*\*2,x)

[Out] Integral(sqrt(2\*x\*\*2 - x + 3)/(5\*x\*\*2 + 3\*x + 2)\*\*2, x)



a, 0] || LtQ[b, 0])

### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 971

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_)\*((d\_) + (e\_)\*(x\_) + (f\_)\*(x\_)^2)^(q\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^q)/((b^2 - 4\*a\*c)\*(p + 1)), x] - Dist[1/((b^2 - 4\*a\*c)\*(p + 1)), Int[(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^(q - 1)\*Simp[2\*c\*d\*(2\*p + 3) + b\*e\*q + (2\*b\*f\*q + 2\*c\*e\*(2\*p + q + 3))\*x + 2\*c\*f\*(2\*p + 2\*q + 3)\*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && LtQ[p, -1] && GtQ[q, 0] && !IGtQ[q, 0]

### Rule 1029

Int[((g\_) + (h\_)\*(x\_))/(((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)\*Sqrt[(d\_) + (e\_)\*(x\_) + (f\_)\*(x\_)^2]), x\_Symbol] := Dist[-2\*g\*(g\*b - 2\*a\*h), Subst[Int[1/Simp[g\*(g\*b - 2\*a\*h)\*(b^2 - 4\*a\*c) - (b\*d - a\*e)\*x^2, x], x], x, Simp[g\*b - 2\*a\*h - (b\*h - 2\*g\*c)\*x, x]/Sqrt[d + e\*x + f\*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && NeQ[b\*d - a\*e, 0] && EqQ[h^2\*(b\*d - a\*e) - 2\*g\*h\*(c\*d - a\*f) + g^2\*(c\*e - b\*f), 0]

### Rule 1035

Int[((g\_) + (h\_)\*(x\_))/(((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)\*Sqrt[(d\_) + (e\_)\*(x\_) + (f\_)\*(x\_)^2]), x\_Symbol] := With[{q = Rt[(c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f), 2]}, Dist[1/(2\*q), Int[Simp[h\*(b\*d - a\*e) - g\*(c\*d - a\*f - q) - (g\*(c\*e - b\*f) - h\*(c\*d - a\*f + q))\*x, x]/((a + b\*x + c\*x^2)\*Sqrt[d + e\*x + f\*x^2]), x], x] - Dist[1/(2\*q), Int[Simp[h\*(b\*d - a\*e) - g\*(c\*d - a\*f + q) - (g\*(c\*e - b\*f) - h\*(c\*d - a\*f - q))\*x, x]/((a + b\*x + c\*x^2)\*Sqrt[d + e\*x + f\*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && NeQ[b\*d - a\*e, 0] && NegQ[b^2 - 4\*a\*c]

### Rule 1060

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_)\*((A\_) + (B\_)\*(x\_) + (C\_)\*(x\_)^2)\*((d\_) + (e\_)\*(x\_) + (f\_)\*(x\_)^2)^(q\_), x\_Symbol] := Simp[((a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^(q + 1)\*((A\*c - a\*C)\*(2\*a\*c\*e - b\*(c\*d +

$a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] + \text{Dist}[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), \text{Int}[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*\text{Simp}[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, C, q\}, x\} \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[e^2 - 4*d*f, 0] \&\& \text{LtQ}[p, -1] \&\& \text{NeQ}[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] \&\& !( \text{IntegerQ}[p] \&\& \text{ILtQ}[q, -1] ) \&\& ! \text{IGtQ}[q, 0]$

### Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^3} dx &= \frac{(3+10x)\sqrt{3-x+2x^2}}{62(2+3x+5x^2)^2} - \frac{1}{62} \int \frac{-\frac{183}{2} + 31x - 40x^2}{\sqrt{3-x+2x^2}(2+3x+5x^2)^2} dx \\
 &= \frac{(3+10x)\sqrt{3-x+2x^2}}{62(2+3x+5x^2)^2} + \frac{(3464+13665x)\sqrt{3-x+2x^2}}{84568(2+3x+5x^2)} - \frac{\int \frac{-213004 + \frac{358655x}{4}}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{465124} \\
 &= \frac{(3+10x)\sqrt{3-x+2x^2}}{62(2+3x+5x^2)^2} + \frac{(3464+13665x)\sqrt{3-x+2x^2}}{84568(2+3x+5x^2)} - \frac{\int \frac{\frac{121}{4}(110061-77456\sqrt{2}) - \frac{121}{4}(448)}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{10232728\sqrt{2}} \\
 &= \frac{(3+10x)\sqrt{3-x+2x^2}}{62(2+3x+5x^2)^2} + \frac{(3464+13665x)\sqrt{3-x+2x^2}}{84568(2+3x+5x^2)} - \frac{(11(158798761480 - 11228}}{10232728\sqrt{2}} \\
 &= \frac{(3+10x)\sqrt{3-x+2x^2}}{62(2+3x+5x^2)^2} + \frac{(3464+13665x)\sqrt{3-x+2x^2}}{84568(2+3x+5x^2)} + \frac{\sqrt{\frac{1}{682}(112285869463 + 79}}{10232728\sqrt{2}}
 \end{aligned}$$



**Mathematica [C]** time = 2.08, size = 299, normalized size = 1.34

$$5 \frac{i\sqrt{286+22i\sqrt{31}} (258253\sqrt{31}+1004586i) \tanh^{-1}\left(\frac{(-22-4i\sqrt{31})x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)}{(\sqrt{31}-13i)^2} + \frac{2000\left(1364(\sqrt{31}+13i)\sqrt{2x^2-x+3}(68325x^3+58315x^2+51362x+1004586)\right)}{(\sqrt{31}-13i)^2}$$

14418844

Antiderivative was successfully verified.

[In] Integrate[Sqrt[3 - x + 2\*x^2]/(2 + 3\*x + 5\*x^2)^3,x]

[Out] (5\*((I\*Sqrt[286 + (22\*I)\*Sqrt[31]]\*(1004586\*I + 258253\*Sqrt[31])\*ArcTanh[(63 + I\*Sqrt[31] + (-22 - (4\*I)\*Sqrt[31])\*x]/(2\*Sqrt[286 + (22\*I)\*Sqrt[31]]\*Sqrt[3 - x + 2\*x^2])]/(-13\*I + Sqrt[31])^2 + (2000\*(1364\*(13\*I + Sqrt[31])\*Sqrt[3 - x + 2\*x^2]\*(11020 + 51362\*x + 58315\*x^2 + 68325\*x^3) - 5\*Sqrt[286 - (22\*I)\*Sqrt[31]]\*(-202151\*I + 174475\*Sqrt[31]))\*(2 + 3\*x + 5\*x^2)^2\*ArcTanh[(63 - I\*Sqrt[31] + (-22 + (4\*I)\*Sqrt[31])\*x]/(2\*Sqrt[286 - (22\*I)\*Sqrt[31]]\*Sqrt[3 - x + 2\*x^2])]))/((-13\*I + Sqrt[31])\*(13\*I + Sqrt[31])^2\*(-3\*I + Sqrt[31] - (10\*I)\*x)^2\*(3\*I + Sqrt[31] + (10\*I)\*x^2)))/14418844

**fricas [B]** time = 2.82, size = 2182, normalized size = 9.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(1/2)/(5\*x^2+3\*x+2)^3,x, algorithm="fricas")

[Out] 1/65052151896952926425996714240\*(14205421276\*788032707736935368450^(1/4)\*sqrt(39699690370)\*sqrt(341)\*sqrt(2)\*(25\*x^4 + 30\*x^3 + 29\*x^2 + 12\*x + 4)\*sqrt(112285869463\*sqrt(2) + 158798761480)\*arctan(1/861047662213971287591057659551879544939625\*(2461380802940\*sqrt(39699690370)\*(22\*788032707736935368450^(3/4)\*sqrt(341)\*(667937076\*x^7 - 2573871186\*x^6 + 5404850058\*x^5 - 8671430212\*x^4 + 4348809776\*x^3 - 2064441888\*x^2 - sqrt(2)\*(473555282\*x^7 - 1821195871\*x^6 + 3826055542\*x^5 - 6128133137\*x^4 + 3070797960\*x^3 - 1452037320\*x^2 - 3352976640\*x + 2366869248) - 4733738496\*x + 3352976640) + 615345200735\*788032707736935368450^(1/4)\*sqrt(341)\*(50730703\*x^7 - 778833417\*x^6 + 4116367112\*x^5 - 9392273180\*x^4 + 12133646496\*x^3 - 7660912032\*x^2 - sqrt(2)\*(35938543\*x^7 - 551546778\*x^6 + 2913578540\*x^5 - 6643469608\*x^4 + 8580088800\*x^3 - 5403919680\*x^2 - 6107913216\*x + 4313793024) - 8627586048\*x + 6107913216))\*sqrt(2\*x^2 - x + 3)\*sqrt(112285869463\*sqrt(2) + 158798761480) + 2444264331446112042193970130340819353377000\*sqrt(31)\*sqrt(2)\*(28180\*x^8 - 254666\*x^7 + 704270\*x^6 - 1385256\*x^5 + 1549144\*x^4 - 642048\*x^3 - 98496\*x^2 - sqrt(2)\*(8746\*x^8 - 102335\*x^7 + 396104\*x^6 - 783113\*x^5 + 1320710\*x^4 - 752088\*x^3 + 396144\*x^2 + 546048\*x - 539136) + 1154304\*x - 456192) - 2\*sqrt(396996

$$\begin{aligned}
& 90370/160673) * (\text{sqrt}(39699690370) * (22 * 788032707736935368450^{(3/4)} * \text{sqrt}(341) * \\
& (104024992 * x^7 - 149335248 * x^6 + 480784368 * x^5 - 188730368 * x^4 + 223535232 * \\
& x^3 + 214417152 * x^2 - \text{sqrt}(2) * (73906058 * x^7 - 106073653 * x^6 + 341348823 * x^5 \\
& - 133050960 * x^4 + 156704760 * x^3 + 154338048 * x^2 - 154338048 * x) - 214417152 \\
& * x) + 615345200735 * 788032707736935368450^{(1/4)} * \text{sqrt}(341) * (7903323 * x^7 - 102 \\
& 233612 * x^6 + 394216580 * x^5 - 510585408 * x^4 + 657060192 * x^3 + 391744512 * x^2 \\
& - 4 * \text{sqrt}(2) * (1401761 * x^7 - 18132196 * x^6 + 69912940 * x^5 - 90501120 * x^4 + 116 \\
& 274240 * x^3 + 70118784 * x^2 - 70118784 * x) - 391744512 * x) * \text{sqrt}(2 * x^2 - x + 3) \\
& * \text{sqrt}(112285869463 * \text{sqrt}(2) + 158798761480) + 43175912524323866211143695850 * \\
& \text{sqrt}(31) * \text{sqrt}(2) * (123408 * x^8 - 914152 * x^7 + 1578888 * x^6 - 3293072 * x^5 + 396 \\
& 480 * x^4 + 798336 * x^3 - 3822336 * x^2 - \text{sqrt}(2) * (15550 * x^8 - 118051 * x^7 + 2440 \\
& 47 * x^6 - 707374 * x^5 + 1053960 * x^4 - 1667952 * x^3 + 1209600 * x^2 - 1036800 * x) \\
& + 3276288 * x) + 1962541478378357555051986175 * \text{sqrt}(31) * (254591 * x^8 - 4815126 * \\
& x^7 + 32303580 * x^6 - 90866808 * x^5 + 108781920 * x^4 - 74219328 * x^3 - 16895692 \\
& 8 * x^2 - 15488 * \text{sqrt}(2) * (4 * x^8 - 76 * x^7 + 517 * x^6 - 1536 * x^5 + 2385 * x^4 - 361 \\
& 8 * x^3 + 2268 * x^2 - 1944 * x) + 144820224 * x) * \text{sqrt}(-(788032707736935368450^{(1/4)} \\
& * \text{sqrt}(39699690370) * \text{sqrt}(341) * \text{sqrt}(31) * \text{sqrt}(2 * x^2 - x + 3) * (\text{sqrt}(2) * (12053 \\
& * x + 5138) - 17191 * x - 6915) * \text{sqrt}(112285869463 * \text{sqrt}(2) + 158798761480) - 15 \\
& 0182556985858180945 * x^2 - 134857806273015509420 * \text{sqrt}(2) * (2 * x^2 - x + 3) + 4 \\
& 62807471527848680055 * x - 612990028513706861000) / x^2) + 27775731039160364115 \\
& 840569662963856288375 * \text{sqrt}(31) * (2828123 * x^8 - 9696916 * x^7 + 53385560 * x^6 - \\
& 142835344 * x^5 + 254146592 * x^4 - 249300096 * x^3 + 37981440 * x^2 - 7744 * \text{sqrt}(2) \\
& * (1348 * x^8 - 2692 * x^7 + 9789 * x^6 - 10070 * x^5 + 15569 * x^4 - 5568 * x^3 + 1080 * \\
& x^2 + 4320 * x - 5184) + 223064064 * x - 94887936) / (2585191 * x^8 - 4661200 * x^7 \\
& + 14191920 * x^6 + 490880 * x^5 - 13562944 * x^4 + 44249088 * x^3 - 34615296 * x^2 - \\
& 24772608 * x + 18579456) + 14205421276 * 788032707736935368450^{(1/4)} * \text{sqrt}(3969 \\
& 9690370) * \text{sqrt}(341) * \text{sqrt}(2) * (25 * x^4 + 30 * x^3 + 29 * x^2 + 12 * x + 4) * \text{sqrt}(11228 \\
& 5869463 * \text{sqrt}(2) + 158798761480) * \arctan(1/8610476622139712875910576595518795 \\
& 44939625 * (2461380802940 * \text{sqrt}(39699690370) * (22 * 788032707736935368450^{(3/4)} * \text{sq} \\
& \text{qrt}(341) * (667937076 * x^7 - 2573871186 * x^6 + 5404850058 * x^5 - 8671430212 * x^4 \\
& + 4348809776 * x^3 - 2064441888 * x^2 - \text{sqrt}(2) * (473555282 * x^7 - 1821195871 * x^6 \\
& + 3826055542 * x^5 - 6128133137 * x^4 + 3070797960 * x^3 - 1452037320 * x^2 - 3352 \\
& 976640 * x + 2366869248) - 4733738496 * x + 3352976640) + 615345200735 * 78803270 \\
& 7736935368450^{(1/4)} * \text{sqrt}(341) * (50730703 * x^7 - 778833417 * x^6 + 4116367112 * x^ \\
& 5 - 9392273180 * x^4 + 12133646496 * x^3 - 7660912032 * x^2 - \text{sqrt}(2) * (35938543 * x \\
& ^7 - 551546778 * x^6 + 2913578540 * x^5 - 6643469608 * x^4 + 8580088800 * x^3 - 540 \\
& 3919680 * x^2 - 6107913216 * x + 4313793024) - 8627586048 * x + 6107913216) * \text{sqrt} \\
& (2 * x^2 - x + 3) * \text{sqrt}(112285869463 * \text{sqrt}(2) + 158798761480) - 244426433144611 \\
& 2042193970130340819353377000 * \text{sqrt}(31) * \text{sqrt}(2) * (28180 * x^8 - 254666 * x^7 + 704 \\
& 270 * x^6 - 1385256 * x^5 + 1549144 * x^4 - 642048 * x^3 - 98496 * x^2 - \text{sqrt}(2) * (874 \\
& 6 * x^8 - 102335 * x^7 + 396104 * x^6 - 783113 * x^5 + 1320710 * x^4 - 752088 * x^3 + 3 \\
& 96144 * x^2 + 546048 * x - 539136) + 1154304 * x - 456192) - 2 * \text{sqrt}(39699690370/1 \\
& 60673) * (\text{sqrt}(39699690370) * (22 * 788032707736935368450^{(3/4)} * \text{sqrt}(341) * (104024 \\
& 992 * x^7 - 149335248 * x^6 + 480784368 * x^5 - 188730368 * x^4 + 223535232 * x^3 + 2 \\
& 14417152 * x^2 - \text{sqrt}(2) * (73906058 * x^7 - 106073653 * x^6 + 341348823 * x^5 - 1330
\end{aligned}$$

$$\begin{aligned}
& 50960x^4 + 156704760x^3 + 154338048x^2 - 154338048x) - 214417152x) + 6 \\
& 15345200735*788032707736935368450^{(1/4)}*\sqrt{341}*(7903323x^7 - 102233612x^6 \\
& + 394216580x^5 - 510585408x^4 + 657060192x^3 + 391744512x^2 - 4*\sqrt{2} \\
& * (1401761x^7 - 18132196x^6 + 69912940x^5 - 90501120x^4 + 116274240x^3 \\
& + 70118784x^2 - 70118784x) - 391744512x))*\sqrt{2x^2 - x + 3}*\sqrt{112285869463*\sqrt{2} \\
& + 158798761480) - 43175912524323866211143695850*\sqrt{31})*\sqrt{2}*(123408x^8 \\
& - 914152x^7 + 1578888x^6 - 3293072x^5 + 396480x^4 + 798336x^3 - 3822336x^2 \\
& - \sqrt{2}*(15550x^8 - 118051x^7 + 244047x^6 - 707374x^5 + 1053960x^4 - 1667952x^3 \\
& + 1209600x^2 - 1036800x) + 3276288x) - 1962541478378357555051986175*\sqrt{31}*(254591x^8 - 4815126x^7 + 3 \\
& 2303580x^6 - 90866808x^5 + 108781920x^4 - 74219328x^3 - 168956928x^2 - 15488*\sqrt{2} \\
& *(4x^8 - 76x^7 + 517x^6 - 1536x^5 + 2385x^4 - 3618x^3 + 2268x^2 - 1944x) + 144820224x) \\
& )*\sqrt{(788032707736935368450^{(1/4)}*\sqrt{39699690370})*\sqrt{341})*\sqrt{31})*\sqrt{2x^2 - x + 3} \\
& *( \sqrt{2}*(12053x + 5138) - 17191x - 6915)*\sqrt{112285869463*\sqrt{2} + 158798761480} \\
& + 150182556985858180945x^2 + 134857806273015509420*\sqrt{2}*(2x^2 - x + 3) - 462807471 \\
& 527848680055x + 612990028513706861000)/x^2) - 2777573103916036411584056966 \\
& 2963856288375*\sqrt{31}*(2828123x^8 - 9696916x^7 + 53385560x^6 - 142835344x^5 \\
& + 254146592x^4 - 249300096x^3 + 37981440x^2 - 7744*\sqrt{2}*(1348x^8 - 2692x^7 \\
& + 9789x^6 - 10070x^5 + 15569x^4 - 5568x^3 + 1080x^2 + 4320x - 5184) + 223064064x \\
& - 94887936))/(2585191x^8 - 4661200x^7 + 14191920x^6 + 490880x^5 - 13562944x^4 \\
& + 44249088x^3 - 34615296x^2 - 24772608x + 18579456) + 788032707736935368450^{(1/4)} \\
& *\sqrt{39699690370})*\sqrt{341})*\sqrt{31}*(3969969037000x^4 + 4763962844400x^3 + 4605164082920x^2 \\
& - 112285869463*\sqrt{2}*(25x^4 + 30x^3 + 29x^2 + 12x + 4) + 1905585137760x + 63 \\
& 5195045920)*\sqrt{112285869463*\sqrt{2} + 158798761480}*\log(635195045920/160673 \\
& *(788032707736935368450^{(1/4)}*\sqrt{39699690370})*\sqrt{341})*\sqrt{31})*\sqrt{2x^2 - x + 3} \\
& *( \sqrt{2}*(12053x + 5138) - 17191x - 6915)*\sqrt{112285869463*\sqrt{2} + 158798761480} \\
& + 150182556985858180945x^2 + 134857806273015509420*\sqrt{2}*(2x^2 - x + 3) - 462807471 \\
& 527848680055x + 612990028513706861000)/x^2) - 788032707736935368450^{(1/4)} \\
& *\sqrt{39699690370})*\sqrt{341})*\sqrt{31}*(3969969037000x^4 + 4763962844400x^3 + 4605164082920x^2 \\
& - 112285869463*\sqrt{2}*(25x^4 + 30x^3 + 29x^2 + 12x + 4) + 1905585137760x + 635195045920) \\
& *\sqrt{112285869463*\sqrt{2} + 158798761480}*\log(-635195045920/160673*(788032707736935368450^{(1/4)} \\
& *\sqrt{39699690370})*\sqrt{341})*\sqrt{31})*\sqrt{2x^2 - x + 3}*( \sqrt{2}*(12053x + 5138) - 17191x \\
& - 6915)*\sqrt{112285869463*\sqrt{2} + 158798761480) - 150182556985858180945x^2 - 134857806273015509420 \\
& *\sqrt{2}*(2x^2 - x + 3) + 462807471527848680055x - 612990028513706861000)/x^2) + 769228926981280465731680 \\
& *(68325x^3 + 58315x^2 + 51362x + 11020)*\sqrt{2x^2 - x + 3}))/ (25x^4 + 30x^3 + 29x^2 + 12x + 4)
\end{aligned}$$

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(1/2)/(5\*x^2+3\*x+2)^3,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP  
 UT:sage2:=int(sage0,x):;OUTPUT:Francis algorithm failure for[-1.0,infinity,  
 infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infi  
 nity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity  
 ]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai  
 lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity  
 ,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf  
 inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infiniti  
 y]Evaluation time: 48.06Done

**maple** [B] time = 0.37, size = 44343, normalized size = 198.85

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^(1/2)/(5\*x^2+3\*x+2)^3,x)

[Out] result too large to display

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2 - x + 3}}{(5x^2 + 3x + 2)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(1/2)/(5\*x^2+3\*x+2)^3,x, algorithm="maxima")

[Out] integrate(sqrt(2\*x^2 - x + 3)/(5\*x^2 + 3\*x + 2)^3, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2x^2 - x + 3}}{(5x^2 + 3x + 2)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)^(1/2)/(3\*x + 5\*x^2 + 2)^3,x)

[Out] int((2\*x^2 - x + 3)^(1/2)/(3\*x + 5\*x^2 + 2)^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2 - x + 3}}{(5x^2 + 3x + 2)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(1/2)/(5*x**2+3*x+2)**3,x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)/(5*x**2 + 3*x + 2)**3, x)
```

$$3.65 \quad \int (3 - x + 2x^2)^{3/2} (2 + 3x + 5x^2)^4 dx$$

**Optimal.** Leaf size=231

$$\frac{56422489(2x^2 - x + 3)^{5/2} x^2}{8257536} + \frac{48669967(2x^2 - x + 3)^{5/2} x}{22020096} + \frac{2124689283(2x^2 - x + 3)^{5/2}}{146800640} - \frac{382121949(1 - 4x)}{134217728}$$

[Out]  $-382121949/134217728*(1-4*x)*(2*x^2-x+3)^{(3/2)}+2124689283/146800640*(2*x^2-x+3)^{(5/2)}+48669967/22020096*x*(2*x^2-x+3)^{(5/2)}-56422489/8257536*x^2*(2*x^2-x+3)^{(5/2)}+10444117/294912*x^3*(2*x^2-x+3)^{(5/2)}+941905/9216*x^4*(2*x^2-x+3)^{(5/2)}+95165/768*x^5*(2*x^2-x+3)^{(5/2)}+7625/96*x^6*(2*x^2-x+3)^{(5/2)}+625/24*x^7*(2*x^2-x+3)^{(5/2)}-606427533063/8589934592*\operatorname{arcsinh}(1/23*(1-4*x)*23^{(1/2)})*2^{(1/2)}-26366414481/2147483648*(1-4*x)*(2*x^2-x+3)^{(1/2)}$

**Rubi [A]** time = 0.34, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {1661, 640, 612, 619, 215}

$$\frac{625}{24} (2x^2 - x + 3)^{5/2} x^7 + \frac{7625}{96} (2x^2 - x + 3)^{5/2} x^6 + \frac{95165}{768} (2x^2 - x + 3)^{5/2} x^5 + \frac{941905(2x^2 - x + 3)^{5/2} x^4}{9216} + \frac{10444}{134217728} (2x^2 - x + 3)^{5/2}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^(3/2)\*(2 + 3\*x + 5\*x^2)^4,x]

[Out]  $(-26366414481*(1 - 4*x)*\operatorname{Sqrt}[3 - x + 2*x^2])/2147483648 - (382121949*(1 - 4*x)*(3 - x + 2*x^2)^{(3/2)})/134217728 + (2124689283*(3 - x + 2*x^2)^{(5/2)})/146800640 + (48669967*x*(3 - x + 2*x^2)^{(5/2)})/22020096 - (56422489*x^2*(3 - x + 2*x^2)^{(5/2)})/8257536 + (10444117*x^3*(3 - x + 2*x^2)^{(5/2)})/294912 + (941905*x^4*(3 - x + 2*x^2)^{(5/2)})/9216 + (95165*x^5*(3 - x + 2*x^2)^{(5/2)})/768 + (7625*x^6*(3 - x + 2*x^2)^{(5/2)})/96 + (625*x^7*(3 - x + 2*x^2)^{(5/2)})/24 - (606427533063*\operatorname{ArcSinh}[(1 - 4*x)/\operatorname{Sqrt}[23]])/(4294967296*\operatorname{Sqrt}[2])$

### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

### Rule 612

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N

$eQ[b^2 - 4ac, 0] \ \&\& \ GtQ[p, 0] \ \&\& \ IntegerQ[4p]$

### Rule 619

$Int[(a_.) + (b_.)x + (c_.)x^2]^{(p_.)}, x\_Symbol] \ :> \ Dist[1/(2c*((-4c)/(b^2 - 4ac))^{(p)}), Subst[Int[Simp[1 - x^2/(b^2 - 4ac)], x]^p, x], x, b + 2cx], x] \ ; \ FreeQ[\{a, b, c, p\}, x] \ \&\& \ GtQ[4a - b^2/c, 0]$

### Rule 640

$Int[((d_.) + (e_.)x)((a_.) + (b_.)x + (c_.)x^2)^{(p_.)}, x\_Symbol] \ :> \ Simp[(e(a + bx + cx^2)^{(p+1)})/(2c(p+1)), x] + Dist[(2cd - be)/(2c), Int[(a + bx + cx^2)^p, x], x] \ ; \ FreeQ[\{a, b, c, d, e, p\}, x] \ \&\& \ NeQ[2cd - be, 0] \ \&\& \ NeQ[p, -1]$

### Rule 1661

$Int[(Pq_.)((a_.) + (b_.)x + (c_.)x^2)^{(p_.)}, x\_Symbol] \ :> \ With[\{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]\}, Simp[(e*x^{(q-1)}*(a + bx + cx^2)^{(p+1)})/(c*(q + 2p + 1)), x] + Dist[1/(c*(q + 2p + 1)), Int[(a + bx + cx^2)^p*ExpandToSum[c*(q + 2p + 1)*Pq - a*e*(q-1)*x^{(q-2)} - b*e*(q+p)*x^{(q-1)} - c*e*(q + 2p + 1)*x^q, x], x], x]] \ ; \ FreeQ[\{a, b, c, p\}, x] \ \&\& \ PolyQ[Pq, x] \ \&\& \ NeQ[b^2 - 4ac, 0] \ \&\& \ !LeQ[p, -1]$

### Rubi steps

$$\begin{aligned}
\int (3-x+2x^2)^{3/2} (2+3x+5x^2)^4 dx &= \frac{625}{24}x^7 (3-x+2x^2)^{5/2} + \frac{1}{24} \int (3-x+2x^2)^{3/2} (384+2304x+9024x^2 \\
&= \frac{7625}{96}x^6 (3-x+2x^2)^{5/2} + \frac{625}{24}x^7 (3-x+2x^2)^{5/2} + \frac{1}{528} \int (3-x+2x^2)^{3/2} \\
&= \frac{95165}{768}x^5 (3-x+2x^2)^{5/2} + \frac{7625}{96}x^6 (3-x+2x^2)^{5/2} + \frac{625}{24}x^7 (3-x+2x^2)^{5/2} \\
&= \frac{941905x^4 (3-x+2x^2)^{5/2}}{9216} + \frac{95165}{768}x^5 (3-x+2x^2)^{5/2} + \frac{7625}{96}x^6 (3-x+2x^2)^{5/2} \\
&= \frac{10444117x^3 (3-x+2x^2)^{5/2}}{294912} + \frac{941905x^4 (3-x+2x^2)^{5/2}}{9216} + \frac{95165}{768}x^5 (3-x+2x^2)^{5/2} \\
&= -\frac{56422489x^2 (3-x+2x^2)^{5/2}}{8257536} + \frac{10444117x^3 (3-x+2x^2)^{5/2}}{294912} + \frac{941905x^4 (3-x+2x^2)^{5/2}}{9216} \\
&= \frac{48669967x (3-x+2x^2)^{5/2}}{22020096} - \frac{56422489x^2 (3-x+2x^2)^{5/2}}{8257536} + \frac{10444117x^3 (3-x+2x^2)^{5/2}}{294912} \\
&= \frac{2124689283 (3-x+2x^2)^{5/2}}{146800640} + \frac{48669967x (3-x+2x^2)^{5/2}}{22020096} - \frac{56422489x^2 (3-x+2x^2)^{5/2}}{8257536} \\
&= -\frac{382121949(1-4x) (3-x+2x^2)^{3/2}}{134217728} + \frac{2124689283 (3-x+2x^2)^{5/2}}{146800640} \\
&= -\frac{26366414481(1-4x)\sqrt{3-x+2x^2}}{2147483648} - \frac{382121949(1-4x) (3-x+2x^2)^{3/2}}{134217728} \\
&= -\frac{26366414481(1-4x)\sqrt{3-x+2x^2}}{2147483648} - \frac{382121949(1-4x) (3-x+2x^2)^{3/2}}{134217728} \\
&= -\frac{26366414481(1-4x)\sqrt{3-x+2x^2}}{2147483648} - \frac{382121949(1-4x) (3-x+2x^2)^{3/2}}{134217728}
\end{aligned}$$

**Mathematica [A]** time = 0.37, size = 95, normalized size = 0.41

$$4\sqrt{2x^2-x+3} (70464307200000x^{11} + 144451829760000x^{10} + 349379651174400x^9 + 534038708224000x^8 + 70464307200000x^7 + 144451829760000x^6 + 349379651174400x^5 + 534038708224000x^4 + 70464307200000x^3 + 144451829760000x^2 + 349379651174400x + 534038708224000)$$

Antiderivative was successfully verified.



[In] Integrate[(3 - x + 2\*x^2)^(3/2)\*(2 + 3\*x + 5\*x^2)^4,x]

[Out] (4\*sqrt[3 - x + 2\*x^2]\*(74032009514181 + 12971175524316\*x + 65151998063712\*x^2 + 239021184223104\*x^3 + 451581382260736\*x^4 + 675479464714240\*x^5 + 765087080448000\*x^6 + 745133229998080\*x^7 + 534038708224000\*x^8 + 349379651174400\*x^9 + 144451829760000\*x^10 + 70464307200000\*x^11) - 191024672914845\*sqrt[2]\*ArcSinh[(1 - 4\*x)/sqrt[23]])/2705829396480

**fricas** [A] time = 0.67, size = 108, normalized size = 0.47

$$\frac{1}{676457349120} (70464307200000 x^{11} + 144451829760000 x^{10} + 349379651174400 x^9 + 534038708224000 x^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)\*(5\*x^2+3\*x+2)^4,x, algorithm="fricas")

[Out] 1/676457349120\*(70464307200000\*x^11 + 144451829760000\*x^10 + 349379651174400\*x^9 + 534038708224000\*x^8 + 745133229998080\*x^7 + 765087080448000\*x^6 + 675479464714240\*x^5 + 451581382260736\*x^4 + 239021184223104\*x^3 + 65151998063712\*x^2 + 12971175524316\*x + 74032009514181)\*sqrt(2\*x^2 - x + 3) + 606427533063/17179869184\*sqrt(2)\*log(-4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25)

**giac** [A] time = 0.25, size = 103, normalized size = 0.45

$$\frac{1}{676457349120} (4 (8 (4 (16 (20 (8 (28 (160 (12 (200 (20 x + 41) x + 19833) x + 363785) x + 81213077) x + 2334860$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)\*(5\*x^2+3\*x+2)^4,x, algorithm="giac")

[Out] 1/676457349120\*(4\*(8\*(4\*(16\*(20\*(8\*(28\*(160\*(12\*(200\*(20\*x + 41)\*x + 19833)\*x + 363785)\*x + 81213077)\*x + 2334860475)\*x + 16491197869)\*x + 220498721807)\*x + 1867353001743)\*x + 2035999939491)\*x + 3242793881079)\*x + 74032009514181)\*sqrt(2\*x^2 - x + 3) - 606427533063/8589934592\*sqrt(2)\*log(-2\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 1)

**maple** [A] time = 0.04, size = 185, normalized size = 0.80

$$\frac{625 (2x^2 - x + 3)^{\frac{5}{2}} x^7}{24} + \frac{7625 (2x^2 - x + 3)^{\frac{5}{2}} x^6}{96} + \frac{95165 (2x^2 - x + 3)^{\frac{5}{2}} x^5}{768} + \frac{941905 (2x^2 - x + 3)^{\frac{5}{2}} x^4}{9216} + \frac{10444117}{9216}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^(3/2)\*(5\*x^2+3\*x+2)^4,x)

[Out]  $625/24*x^7*(2*x^2-x+3)^{(5/2)}+7625/96*x^6*(2*x^2-x+3)^{(5/2)}+2124689283/146800640*(2*x^2-x+3)^{(5/2)}+95165/768*x^5*(2*x^2-x+3)^{(5/2)}+941905/9216*x^4*(2*x^2-x+3)^{(5/2)}+10444117/294912*x^3*(2*x^2-x+3)^{(5/2)}-56422489/8257536*x^2*(2*x^2-x+3)^{(5/2)}+48669967/22020096*x*(2*x^2-x+3)^{(5/2)}+606427533063/8589934592*2^{(1/2)}*\operatorname{arcsinh}(4/23*23^{(1/2)}*(x-1/4))+26366414481/2147483648*(4*x-1)*(2*x^2-x+3)^{(1/2)}+382121949/134217728*(4*x-1)*(2*x^2-x+3)^{(3/2)}$

**maxima** [A] time = 1.02, size = 206, normalized size = 0.89

$$\frac{625}{24} (2x^2 - x + 3)^{\frac{5}{2}} x^7 + \frac{7625}{96} (2x^2 - x + 3)^{\frac{5}{2}} x^6 + \frac{95165}{768} (2x^2 - x + 3)^{\frac{5}{2}} x^5 + \frac{941905}{9216} (2x^2 - x + 3)^{\frac{5}{2}} x^4 + \frac{10444117}{294912}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^4,x, algorithm="maxima")`

[Out]  $625/24*(2*x^2 - x + 3)^{(5/2)}*x^7 + 7625/96*(2*x^2 - x + 3)^{(5/2)}*x^6 + 95165/768*(2*x^2 - x + 3)^{(5/2)}*x^5 + 941905/9216*(2*x^2 - x + 3)^{(5/2)}*x^4 + 10444117/294912*(2*x^2 - x + 3)^{(5/2)}*x^3 - 56422489/8257536*(2*x^2 - x + 3)^{(5/2)}*x^2 + 48669967/22020096*(2*x^2 - x + 3)^{(5/2)}*x + 2124689283/146800640*(2*x^2 - x + 3)^{(5/2)} + 382121949/33554432*(2*x^2 - x + 3)^{(3/2)}*x - 382121949/134217728*(2*x^2 - x + 3)^{(3/2)} + 26366414481/536870912*\operatorname{sqrt}(2*x^2 - x + 3)*x + 606427533063/8589934592*\operatorname{sqrt}(2)*\operatorname{arcsinh}(1/23*\operatorname{sqrt}(23))*(4*x - 1) - 26366414481/2147483648*\operatorname{sqrt}(2*x^2 - x + 3)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (2x^2 - x + 3)^{3/2} (5x^2 + 3x + 2)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^4,x)`

[Out] `int((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^4, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2x^2 - x + 3)^{\frac{3}{2}} (5x^2 + 3x + 2)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**2-x+3)**(3/2)*(5*x**2+3*x+2)**4,x)`

[Out] `Integral((2*x**2 - x + 3)**(3/2)*(5*x**2 + 3*x + 2)**4, x)`

$$3.66 \quad \int (3 - x + 2x^2)^{3/2} (2 + 3x + 5x^2)^3 dx$$

**Optimal.** Leaf size=189

$$\frac{384739(2x^2 - x + 3)^{5/2} x^2}{43008} - \frac{81685(2x^2 - x + 3)^{5/2} x}{114688} - \frac{4625907(2x^2 - x + 3)^{5/2}}{2293760} - \frac{667795(1 - 4x)(2x^2 - x + 3)^{5/2}}{2097152}$$

[Out]  $-667795/2097152*(1-4*x)*(2*x^2-x+3)^{(3/2)}-4625907/2293760*(2*x^2-x+3)^{(5/2)}$   
 $-81685/114688*x*(2*x^2-x+3)^{(5/2)}+384739/43008*x^2*(2*x^2-x+3)^{(5/2)}+27785/$   
 $1536*x^3*(2*x^2-x+3)^{(5/2)}+725/48*x^4*(2*x^2-x+3)^{(5/2)}+25/4*x^5*(2*x^2-x+3)$   
 $)^{(5/2)}-1059790665/134217728*\operatorname{arcsinh}(1/23*(1-4*x)*23^{(1/2)})*2^{(1/2)}-4607785$   
 $5/33554432*(1-4*x)*(2*x^2-x+3)^{(1/2)}$

**Rubi [A]** time = 0.19, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {1661, 640, 612, 619, 215}

$$\frac{25}{4} (2x^2 - x + 3)^{5/2} x^5 + \frac{725}{48} (2x^2 - x + 3)^{5/2} x^4 + \frac{27785(2x^2 - x + 3)^{5/2} x^3}{1536} + \frac{384739(2x^2 - x + 3)^{5/2} x^2}{43008} - \frac{81685(2x^2 - x + 3)^{5/2} x}{114688} - \frac{4625907(2x^2 - x + 3)^{5/2}}{2293760} - \frac{667795(1 - 4x)(2x^2 - x + 3)^{5/2}}{2097152}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^(3/2)\*(2 + 3\*x + 5\*x^2)^3,x]

[Out]  $(-46077855*(1 - 4*x)*\operatorname{Sqrt}[3 - x + 2*x^2])/33554432 - (667795*(1 - 4*x)*(3 - x + 2*x^2)^{(3/2)})/2097152 - (4625907*(3 - x + 2*x^2)^{(5/2)})/2293760 - (81685*x*(3 - x + 2*x^2)^{(5/2)})/114688 + (384739*x^2*(3 - x + 2*x^2)^{(5/2)})/43008 + (27785*x^3*(3 - x + 2*x^2)^{(5/2)})/1536 + (725*x^4*(3 - x + 2*x^2)^{(5/2)})/48 + (25*x^5*(3 - x + 2*x^2)^{(5/2)})/4 - (1059790665*\operatorname{ArcSinh}[(1 - 4*x)/\operatorname{Sqrt}[23]])/(67108864*\operatorname{Sqrt}[2])$

**Rule 215**

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

**Rule 612**

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^(p)), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int (3-x+2x^2)^{3/2} (2+3x+5x^2)^3 dx &= \frac{25}{4}x^5(3-x+2x^2)^{5/2} + \frac{1}{20} \int (3-x+2x^2)^{3/2} (160+720x+2280x^2 \\
&= \frac{725}{48}x^4(3-x+2x^2)^{5/2} + \frac{25}{4}x^5(3-x+2x^2)^{5/2} + \frac{1}{360} \int (3-x+2x^2)^{3/2} \\
&= \frac{27785x^3(3-x+2x^2)^{5/2}}{1536} + \frac{725}{48}x^4(3-x+2x^2)^{5/2} + \frac{25}{4}x^5(3-x+2x^2)^{5/2} \\
&= \frac{384739x^2(3-x+2x^2)^{5/2}}{43008} + \frac{27785x^3(3-x+2x^2)^{5/2}}{1536} + \frac{725}{48}x^4(3-x+2x^2)^{5/2} \\
&= -\frac{81685x(3-x+2x^2)^{5/2}}{114688} + \frac{384739x^2(3-x+2x^2)^{5/2}}{43008} + \frac{27785x^3(3-x+2x^2)^{5/2}}{1536} \\
&= -\frac{4625907(3-x+2x^2)^{5/2}}{2293760} - \frac{81685x(3-x+2x^2)^{5/2}}{114688} + \frac{384739x^2(3-x+2x^2)^{5/2}}{43008} \\
&= -\frac{667795(1-4x)(3-x+2x^2)^{3/2}}{2097152} - \frac{4625907(3-x+2x^2)^{5/2}}{2293760} - \frac{81685x(3-x+2x^2)^{5/2}}{114688} \\
&= -\frac{46077855(1-4x)\sqrt{3-x+2x^2}}{33554432} - \frac{667795(1-4x)(3-x+2x^2)^{3/2}}{2097152} \\
&= -\frac{46077855(1-4x)\sqrt{3-x+2x^2}}{33554432} - \frac{667795(1-4x)(3-x+2x^2)^{3/2}}{2097152} \\
&= -\frac{46077855(1-4x)\sqrt{3-x+2x^2}}{33554432} - \frac{667795(1-4x)(3-x+2x^2)^{3/2}}{2097152}
\end{aligned}$$

**Mathematica [A]** time = 0.23, size = 85, normalized size = 0.45

$$4\sqrt{2x^2-x+3} (88080384000x^9 + 124780544000x^8 + 328328806400x^7 + 430820229120x^6 + 571298324480x^5 + 430820229120x^4 + 124780544000x^3 + 88080384000x^2 + 124780544000x + 571298324480)$$

Antiderivative was successfully verified.

[In] Integrate[(3-x+2\*x^2)^(3/2)\*(2+3\*x+5\*x^2)^3,x]

[Out] (4\*Sqrt[3-x+2\*x^2]\*(-72152399943+53985432012\*x+199615064544\*x^2+389257196928\*x^3+487891884032\*x^4+571298324480\*x^5+430820229120\*x^6+124780544000\*x^7+88080384000\*x^8+571298324480\*x^9))

$328328806400x^7 + 124780544000x^8 + 88080384000x^9) - 111278019825\sqrt{2} \operatorname{ArcSinh}[(1 - 4x)/\sqrt{23}]/14092861440$

**fricas** [A] time = 0.99, size = 98, normalized size = 0.52

$$\frac{1}{3523215360} (88080384000x^9 + 124780544000x^8 + 328328806400x^7 + 430820229120x^6 + 571298324480x^5 -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)\*(5\*x^2+3\*x+2)^3,x, algorithm="fricas")

[Out]  $\frac{1}{3523215360} (88080384000x^9 + 124780544000x^8 + 328328806400x^7 + 430820229120x^6 + 571298324480x^5 + 487891884032x^4 + 389257196928x^3 + 199615064544x^2 + 53985432012x - 72152399943) \sqrt{2x^2 - x + 3} + 1059790665/268435456 \sqrt{2} \log(-4\sqrt{2} \sqrt{2x^2 - x + 3} (4x - 1) - 32x^2 + 16x - 25)$

**giac** [A] time = 0.26, size = 93, normalized size = 0.49

$$\frac{1}{3523215360} (4(8(4(16(20(8(140(160(12x + 17)x + 7157)x + 1314759)x + 13947713)x + 238228459)x + 3041071851)x + 6237970767)x + 13496358003)x - 72152399943) \sqrt{2x^2 - x + 3} - 1059790665/134217728 \sqrt{2} \log(-2\sqrt{2} (\sqrt{2}x - \sqrt{2x^2 - x + 3}) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)\*(5\*x^2+3\*x+2)^3,x, algorithm="giac")

[Out]  $\frac{1}{3523215360} (4(8(4(16(20(8(140(160(12x + 17)x + 7157)x + 1314759)x + 13947713)x + 238228459)x + 3041071851)x + 6237970767)x + 13496358003)x - 72152399943) \sqrt{2x^2 - x + 3} - 1059790665/134217728 \sqrt{2} \log(-2\sqrt{2} (\sqrt{2}x - \sqrt{2x^2 - x + 3}) + 1)$

**maple** [A] time = 0.01, size = 151, normalized size = 0.80

$$\frac{25(2x^2 - x + 3)^{\frac{5}{2}}x^5}{4} + \frac{725(2x^2 - x + 3)^{\frac{5}{2}}x^4}{48} + \frac{27785(2x^2 - x + 3)^{\frac{5}{2}}x^3}{1536} + \frac{384739(2x^2 - x + 3)^{\frac{5}{2}}x^2}{43008} - \frac{81685(2x^2 - x + 3)^{\frac{5}{2}}x}{114688} + \frac{1059790665\sqrt{2}\operatorname{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{134217728\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^(3/2)\*(5\*x^2+3\*x+2)^3,x)

[Out]  $-4625907/2293760(2x^2-x+3)^{5/2} + 25/4(2x^2-x+3)^{5/2}x^5 + 725/48(2x^2-x+3)^{5/2}x^4 + 27785/1536(2x^2-x+3)^{5/2}x^3 + 384739/43008(2x^2-x+3)^{5/2}x^2 - 81685/114688(2x^2-x+3)^{5/2}x + 1059790665/134217728\sqrt{2}\operatorname{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right) + 46077855/33554432(4x-1)(2x^2-x+3)^{1/2} + 667795/2097152(4x-1)(2x^2-x+3)^{3/2}$

**maxima** [A] time = 0.99, size = 172, normalized size = 0.91

$$\frac{25}{4} (2x^2 - x + 3)^{\frac{5}{2}} x^5 + \frac{725}{48} (2x^2 - x + 3)^{\frac{5}{2}} x^4 + \frac{27785}{1536} (2x^2 - x + 3)^{\frac{5}{2}} x^3 + \frac{384739}{43008} (2x^2 - x + 3)^{\frac{5}{2}} x^2 - \frac{81685}{114688} (2x^2 - x + 3)^{\frac{5}{2}} x - \frac{4625907}{2293760} (2x^2 - x + 3)^{\frac{5}{2}} - \frac{667795}{524288} (2x^2 - x + 3)^{\frac{3}{2}} x - \frac{667795}{2097152} (2x^2 - x + 3)^{\frac{3}{2}} + \frac{46077855}{8388608} \sqrt{2x^2 - x + 3} x + \frac{1059790665}{134217728} \sqrt{2} \operatorname{arcsinh}\left(\frac{1}{23} \sqrt{23} (4x - 1)\right) - \frac{46077855}{33554432} \sqrt{2x^2 - x + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)\*(5\*x^2+3\*x+2)^3,x, algorithm="maxima")

[Out] 25/4\*(2\*x^2 - x + 3)^(5/2)\*x^5 + 725/48\*(2\*x^2 - x + 3)^(5/2)\*x^4 + 27785/1536\*(2\*x^2 - x + 3)^(5/2)\*x^3 + 384739/43008\*(2\*x^2 - x + 3)^(5/2)\*x^2 - 81685/114688\*(2\*x^2 - x + 3)^(5/2)\*x - 4625907/2293760\*(2\*x^2 - x + 3)^(5/2) - 667795/524288\*(2\*x^2 - x + 3)^(3/2)\*x - 667795/2097152\*(2\*x^2 - x + 3)^(3/2) + 46077855/8388608\*sqrt(2\*x^2 - x + 3)\*x + 1059790665/134217728\*sqrt(2)\*arcsinh(1/23\*sqrt(23)\*(4\*x - 1)) - 46077855/33554432\*sqrt(2\*x^2 - x + 3)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (2x^2 - x + 3)^{3/2} (5x^2 + 3x + 2)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)^(3/2)\*(3\*x + 5\*x^2 + 2)^3,x)

[Out] int((2\*x^2 - x + 3)^(3/2)\*(3\*x + 5\*x^2 + 2)^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2x^2 - x + 3)^{\frac{3}{2}} (5x^2 + 3x + 2)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2-x+3)\*\*(3/2)\*(5\*x\*\*2+3\*x+2)\*\*3,x)

[Out] Integral((2\*x\*\*2 - x + 3)\*\*(3/2)\*(5\*x\*\*2 + 3\*x + 2)\*\*3, x)

$$3.67 \quad \int (3 - x + 2x^2)^{3/2} (2 + 3x + 5x^2)^2 dx$$

**Optimal.** Leaf size=147

$$\frac{1235}{448} (2x^2 - x + 3)^{5/2} x^2 + \frac{24499 (2x^2 - x + 3)^{5/2} x}{10752} + \frac{73861 (2x^2 - x + 3)^{5/2}}{215040} + \frac{24293(1 - 4x) (2x^2 - x + 3)^{3/2}}{196608} + \frac{558739}{1048576} (1 - 4x) (2x^2 - x + 3)^{1/2}$$

[Out] 24293/196608\*(1-4\*x)\*(2\*x^2-x+3)^(3/2)+73861/215040\*(2\*x^2-x+3)^(5/2)+24499/10752\*x\*(2\*x^2-x+3)^(5/2)+1235/448\*x^2\*(2\*x^2-x+3)^(5/2)+25/16\*x^3\*(2\*x^2-x+3)^(5/2)+12850997/4194304\*arcsinh(1/23\*(1-4\*x)\*23^(1/2))\*2^(1/2)+558739/1048576\*(1-4\*x)\*(2\*x^2-x+3)^(1/2)

**Rubi [A]** time = 0.12, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {1661, 640, 612, 619, 215}

$$\frac{25}{16} (2x^2 - x + 3)^{5/2} x^3 + \frac{1235}{448} (2x^2 - x + 3)^{5/2} x^2 + \frac{24499 (2x^2 - x + 3)^{5/2} x}{10752} + \frac{73861 (2x^2 - x + 3)^{5/2}}{215040} + \frac{24293(1 - 4x) (2x^2 - x + 3)^{3/2}}{196608} + \frac{558739}{1048576} (1 - 4x) (2x^2 - x + 3)^{1/2}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^(3/2)\*(2 + 3\*x + 5\*x^2)^2,x]

[Out] (558739\*(1 - 4\*x)\*Sqrt[3 - x + 2\*x^2])/1048576 + (24293\*(1 - 4\*x)\*(3 - x + 2\*x^2)^(3/2))/196608 + (73861\*(3 - x + 2\*x^2)^(5/2))/215040 + (24499\*x\*(3 - x + 2\*x^2)^(5/2))/10752 + (1235\*x^2\*(3 - x + 2\*x^2)^(5/2))/448 + (25\*x^3\*(3 - x + 2\*x^2)^(5/2))/16 + (12850997\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(2097152\*Sqrt[2])

**Rule 215**

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

**Rule 612**

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

**Rule 619**



```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^(p)), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

### Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

### Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x]] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

### Rubi steps

$$\begin{aligned}
\int (3-x+2x^2)^{3/2} (2+3x+5x^2)^2 dx &= \frac{25}{16}x^3(3-x+2x^2)^{5/2} + \frac{1}{16} \int (3-x+2x^2)^{3/2} (64+192x+239x^2+1235x^3) dx \\
&= \frac{1235}{448}x^2(3-x+2x^2)^{5/2} + \frac{25}{16}x^3(3-x+2x^2)^{5/2} + \frac{1}{224} \int (3-x+2x^2)^{3/2} (64+192x+239x^2+1235x^3) dx \\
&= \frac{24499x(3-x+2x^2)^{5/2}}{10752} + \frac{1235}{448}x^2(3-x+2x^2)^{5/2} + \frac{25}{16}x^3(3-x+2x^2)^{5/2} + \frac{1}{224} \int (3-x+2x^2)^{3/2} (64+192x+239x^2+1235x^3) dx \\
&= \frac{73861(3-x+2x^2)^{5/2}}{215040} + \frac{24499x(3-x+2x^2)^{5/2}}{10752} + \frac{1235}{448}x^2(3-x+2x^2)^{5/2} + \frac{1}{224} \int (3-x+2x^2)^{3/2} (64+192x+239x^2+1235x^3) dx \\
&= \frac{24293(1-4x)(3-x+2x^2)^{3/2}}{196608} + \frac{73861(3-x+2x^2)^{5/2}}{215040} + \frac{24499x(3-x+2x^2)^{5/2}}{10752} + \frac{1}{224} \int (3-x+2x^2)^{3/2} (64+192x+239x^2+1235x^3) dx \\
&= \frac{558739(1-4x)\sqrt{3-x+2x^2}}{1048576} + \frac{24293(1-4x)(3-x+2x^2)^{3/2}}{196608} + \frac{73861(3-x+2x^2)^{5/2}}{215040} + \frac{24499x(3-x+2x^2)^{5/2}}{10752} + \frac{1}{224} \int (3-x+2x^2)^{3/2} (64+192x+239x^2+1235x^3) dx \\
&= \frac{558739(1-4x)\sqrt{3-x+2x^2}}{1048576} + \frac{24293(1-4x)(3-x+2x^2)^{3/2}}{196608} + \frac{73861(3-x+2x^2)^{5/2}}{215040} + \frac{24499x(3-x+2x^2)^{5/2}}{10752} + \frac{1}{224} \int (3-x+2x^2)^{3/2} (64+192x+239x^2+1235x^3) dx \\
&= \frac{558739(1-4x)\sqrt{3-x+2x^2}}{1048576} + \frac{24293(1-4x)(3-x+2x^2)^{3/2}}{196608} + \frac{73861(3-x+2x^2)^{5/2}}{215040} + \frac{24499x(3-x+2x^2)^{5/2}}{10752} + \frac{1}{224} \int (3-x+2x^2)^{3/2} (64+192x+239x^2+1235x^3) dx
\end{aligned}$$

**Mathematica [A]** time = 0.14, size = 75, normalized size = 0.51

$$\frac{4\sqrt{2x^2-x+3} (688128000x^7 + 525926400x^6 + 2025840640x^5 + 2061273088x^4 + 2728413312x^3 + 1799647136x^2 + 2728413312x + 1349354685) \operatorname{ArcSinh}\left[\frac{1-4x}{\sqrt{23}}\right]}{440401920}$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)^(3/2)\*(2 + 3\*x + 5\*x^2)^2,x]

[Out] (4\*Sqrt[3 - x + 2\*x^2]\*(439831323 + 1619403428\*x + 1799647136\*x^2 + 2728413312\*x^3 + 2061273088\*x^4 + 2025840640\*x^5 + 525926400\*x^6 + 688128000\*x^7) + 1349354685\*Sqrt[2]\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/440401920

**fricas [A]** time = 0.80, size = 88, normalized size = 0.60

$$\frac{1}{110100480} (688128000x^7 + 525926400x^6 + 2025840640x^5 + 2061273088x^4 + 2728413312x^3 + 1799647136x^2 + 2728413312x + 1349354685) \operatorname{ArcSinh}\left[\frac{1-4x}{\sqrt{23}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)\*(5\*x^2+3\*x+2)^2,x, algorithm="fricas")

[Out] 1/110100480\*(688128000\*x^7 + 525926400\*x^6 + 2025840640\*x^5 + 2061273088\*x^4 + 2728413312\*x^3 + 1799647136\*x^2 + 1619403428\*x + 439831323)\*sqrt(2\*x^2 - x + 3) + 12850997/8388608\*sqrt(2)\*log(4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25)

**giac** [A] time = 0.26, size = 83, normalized size = 0.56

$$\frac{1}{110100480} (4 (8 (4 (16 (20 (120 (140 x + 107)x + 49459)x + 1006481)x + 21315729)x + 56238973)x + 4048508$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)\*(5\*x^2+3\*x+2)^2,x, algorithm="giac")

[Out] 1/110100480\*(4\*(8\*(4\*(16\*(20\*(120\*(140\*x + 107)\*x + 49459)\*x + 1006481)\*x + 21315729)\*x + 56238973)\*x + 404850857)\*x + 439831323)\*sqrt(2\*x^2 - x + 3) + 12850997/4194304\*sqrt(2)\*log(-2\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 1)

**maple** [A] time = 0.01, size = 117, normalized size = 0.80

$$\frac{25 (2x^2 - x + 3)^{\frac{5}{2}} x^3}{16} + \frac{1235 (2x^2 - x + 3)^{\frac{5}{2}} x^2}{448} + \frac{24499 (2x^2 - x + 3)^{\frac{5}{2}} x}{10752} - \frac{12850997\sqrt{2} \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{4194304} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^(3/2)\*(5\*x^2+3\*x+2)^2,x)

[Out] 73861/215040\*(2\*x^2-x+3)^(5/2)+25/16\*(2\*x^2-x+3)^(5/2)\*x^3+1235/448\*(2\*x^2-x+3)^(5/2)\*x^2+24499/10752\*(2\*x^2-x+3)^(5/2)\*x-12850997/4194304\*2^(1/2)\*arc sinh(4/23\*23^(1/2)\*(x-1/4))-558739/1048576\*(4\*x-1)\*(2\*x^2-x+3)^(1/2)-24293/196608\*(4\*x-1)\*(2\*x^2-x+3)^(3/2)

**maxima** [A] time = 0.98, size = 138, normalized size = 0.94

$$\frac{25}{16} (2x^2 - x + 3)^{\frac{5}{2}} x^3 + \frac{1235}{448} (2x^2 - x + 3)^{\frac{5}{2}} x^2 + \frac{24499}{10752} (2x^2 - x + 3)^{\frac{5}{2}} x + \frac{73861}{215040} (2x^2 - x + 3)^{\frac{5}{2}} - \frac{24293}{49152} (2x^2 -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)\*(5\*x^2+3\*x+2)^2,x, algorithm="maxima")

[Out] 25/16\*(2\*x^2 - x + 3)^(5/2)\*x^3 + 1235/448\*(2\*x^2 - x + 3)^(5/2)\*x^2 + 24499/10752\*(2\*x^2 - x + 3)^(5/2)\*x + 73861/215040\*(2\*x^2 - x + 3)^(5/2) - 2429

$\frac{3}{49152}*(2*x^2 - x + 3)^{(3/2)}*x + \frac{24293}{196608}*(2*x^2 - x + 3)^{(3/2)} - \frac{558739}{262144}*\sqrt{2*x^2 - x + 3}*x - \frac{12850997}{4194304}*\sqrt{2}*\operatorname{arcsinh}(1/23*\sqrt{23}*(4*x - 1)) + \frac{558739}{1048576}*\sqrt{2*x^2 - x + 3}$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (2x^2 - x + 3)^{3/2} (5x^2 + 3x + 2)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^2, x)`

[Out] `int((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^2, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2x^2 - x + 3)^{\frac{3}{2}} (5x^2 + 3x + 2)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**2-x+3)**(3/2)*(5*x**2+3*x+2)**2, x)`

[Out] `Integral((2*x**2 - x + 3)**(3/2)*(5*x**2 + 3*x + 2)**2, x)`

$$3.68 \quad \int (3 - x + 2x^2)^{3/2} (2 + 3x + 5x^2) dx$$

**Optimal.** Leaf size=105

$$\frac{5}{12}x(2x^2 - x + 3)^{5/2} + \frac{107}{240}(2x^2 - x + 3)^{5/2} - \frac{179(1 - 4x)(2x^2 - x + 3)^{3/2}}{1536} - \frac{4117(1 - 4x)\sqrt{2x^2 - x + 3}}{8192} - \frac{94691 \operatorname{arcsinh}\left(\frac{1 - 4x}{\sqrt{23}}\right)}{16384}$$

[Out]  $-179/1536*(1-4*x)*(2*x^2-x+3)^{(3/2)}+107/240*(2*x^2-x+3)^{(5/2)}+5/12*x*(2*x^2-x+3)^{(5/2)}-94691/32768*\operatorname{arcsinh}(1/23*(1-4*x)*23^{(1/2)})*2^{(1/2)}-4117/8192*(1-4*x)*(2*x^2-x+3)^{(1/2)}$

**Rubi [A]** time = 0.05, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {1661, 640, 612, 619, 215}

$$\frac{5}{12}x(2x^2 - x + 3)^{5/2} + \frac{107}{240}(2x^2 - x + 3)^{5/2} - \frac{179(1 - 4x)(2x^2 - x + 3)^{3/2}}{1536} - \frac{4117(1 - 4x)\sqrt{2x^2 - x + 3}}{8192} - \frac{94691 \operatorname{arcsinh}\left(\frac{1 - 4x}{\sqrt{23}}\right)}{16384}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(3 - x + 2*x^2)^{(3/2)}*(2 + 3*x + 5*x^2), x]$

[Out]  $(-4117*(1 - 4*x)*\operatorname{Sqrt}[3 - x + 2*x^2])/8192 - (179*(1 - 4*x)*(3 - x + 2*x^2)^{(3/2)})/1536 + (107*(3 - x + 2*x^2)^{(5/2)})/240 + (5*x*(3 - x + 2*x^2)^{(5/2)})/12 - (94691*\operatorname{ArcSinh}[(1 - 4*x)/\operatorname{Sqrt}[23]])/(16384*\operatorname{Sqrt}[2])$

### Rule 215

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_) + (b_)*(x_)^2], x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{ArcSinh}[(\operatorname{Rt}[b, 2]*x)/\operatorname{Sqrt}[a]]/\operatorname{Rt}[b, 2], x] /;$  FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

### Rule 612

$\operatorname{Int}[((a_) + (b_)*(x_) + (c_)*(x_)^2)^{(p_)}, x\_Symbol] \rightarrow \operatorname{Simp}[(b + 2*c*x)*(a + b*x + c*x^2)^p/(2*c*(2*p + 1)), x] - \operatorname{Dist}[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), \operatorname{Int}[(a + b*x + c*x^2)^{(p - 1)}, x], x] /;$  FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

### Rule 619

$\operatorname{Int}[((a_) + (b_)*(x_) + (c_)*(x_)^2)^{(p_)}, x\_Symbol] \rightarrow \operatorname{Dist}[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), \operatorname{Subst}[\operatorname{Int}[\operatorname{Simp}[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /;$  FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x]] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int (3 - x + 2x^2)^{3/2} (2 + 3x + 5x^2) dx &= \frac{5}{12}x(3 - x + 2x^2)^{5/2} + \frac{1}{12} \int \left(9 + \frac{107x}{2}\right) (3 - x + 2x^2)^{3/2} dx \\
&= \frac{107}{240} (3 - x + 2x^2)^{5/2} + \frac{5}{12}x(3 - x + 2x^2)^{5/2} + \frac{179}{96} \int (3 - x + 2x^2)^{3/2} dx \\
&= -\frac{179(1 - 4x)(3 - x + 2x^2)^{3/2}}{1536} + \frac{107}{240} (3 - x + 2x^2)^{5/2} + \frac{5}{12}x(3 - x + 2x^2)^{5/2} \\
&= -\frac{4117(1 - 4x)\sqrt{3 - x + 2x^2}}{8192} - \frac{179(1 - 4x)(3 - x + 2x^2)^{3/2}}{1536} + \frac{107}{240} (3 - x + 2x^2)^{5/2} \\
&= -\frac{4117(1 - 4x)\sqrt{3 - x + 2x^2}}{8192} - \frac{179(1 - 4x)(3 - x + 2x^2)^{3/2}}{1536} + \frac{107}{240} (3 - x + 2x^2)^{5/2} \\
&= -\frac{4117(1 - 4x)\sqrt{3 - x + 2x^2}}{8192} - \frac{179(1 - 4x)(3 - x + 2x^2)^{3/2}}{1536} + \frac{107}{240} (3 - x + 2x^2)^{5/2}
\end{aligned}$$

**Mathematica** [A] time = 0.08, size = 65, normalized size = 0.62

$$\frac{4\sqrt{2x^2 - x + 3} (204800x^5 + 14336x^4 + 561024x^3 + 319072x^2 + 565276x + 388341) - 1420365\sqrt{2} \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{491520}$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)^(3/2)\*(2 + 3\*x + 5\*x^2), x]

[Out] (4\*sqrt(3 - x + 2\*x^2)\*(388341 + 565276\*x + 319072\*x^2 + 561024\*x^3 + 14336\*x^4 + 204800\*x^5) - 1420365\*sqrt(2)\*ArcSinh[(1 - 4\*x)/sqrt(23)]) / 491520

**fricas** [A] time = 0.74, size = 78, normalized size = 0.74

$$\frac{1}{122880} (204800x^5 + 14336x^4 + 561024x^3 + 319072x^2 + 565276x + 388341) \sqrt{2x^2 - x + 3} + \frac{94691}{65536} \sqrt{2} \log\left(\dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)\*(5\*x^2+3\*x+2), x, algorithm="fricas")

[Out] 1/122880\*(204800\*x^5 + 14336\*x^4 + 561024\*x^3 + 319072\*x^2 + 565276\*x + 388341)\*sqrt(2\*x^2 - x + 3) + 94691/65536\*sqrt(2)\*log(-4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25)

**giac** [A] time = 0.23, size = 73, normalized size = 0.70

$$\frac{1}{122880} (4(8(4(16(100x+7)x+4383)x+9971)x+141319)x+388341) \sqrt{2x^2 - x + 3} - \frac{94691}{32768} \sqrt{2} \log\left(-2\sqrt{2}\dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)\*(5\*x^2+3\*x+2), x, algorithm="giac")

[Out] 1/122880\*(4\*(8\*(4\*(16\*(100\*x + 7)\*x + 4383)\*x + 9971)\*x + 141319)\*x + 388341)\*sqrt(2\*x^2 - x + 3) - 94691/32768\*sqrt(2)\*log(-2\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 1)

**maple** [A] time = 0.01, size = 83, normalized size = 0.79

$$\frac{5(2x^2 - x + 3)^{\frac{5}{2}}x}{12} + \frac{94691\sqrt{2} \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{32768} + \frac{107(2x^2 - x + 3)^{\frac{5}{2}}}{240} + \frac{179(4x - 1)(2x^2 - x + 3)^{\frac{3}{2}}}{1536} + \frac{4117(4\dots)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^(3/2)\*(5\*x^2+3\*x+2), x)

[Out] 5/12\*(2\*x^2-x+3)^(5/2)\*x+107/240\*(2\*x^2-x+3)^(5/2)+179/1536\*(4\*x-1)\*(2\*x^2-x+3)^(3/2)+4117/8192\*(4\*x-1)\*(2\*x^2-x+3)^(1/2)+94691/32768\*2^(1/2)\*arcsinh(4/23\*23^(1/2)\*(x-1/4))

**maxima** [A] time = 0.96, size = 104, normalized size = 0.99

$$\frac{5}{12} (2x^2 - x + 3)^{\frac{5}{2}}x + \frac{107}{240} (2x^2 - x + 3)^{\frac{5}{2}} + \frac{179}{384} (2x^2 - x + 3)^{\frac{3}{2}}x - \frac{179}{1536} (2x^2 - x + 3)^{\frac{3}{2}} + \frac{4117}{2048} \sqrt{2x^2 - x + 3}x + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)\*(5\*x^2+3\*x+2),x, algorithm="maxima")

[Out] 5/12\*(2\*x^2 - x + 3)^(5/2)\*x + 107/240\*(2\*x^2 - x + 3)^(5/2) + 179/384\*(2\*x^2 - x + 3)^(3/2)\*x - 179/1536\*(2\*x^2 - x + 3)^(3/2) + 4117/2048\*sqrt(2\*x^2 - x + 3)\*x + 94691/32768\*sqrt(2)\*arcsinh(1/23\*sqrt(23)\*(4\*x - 1)) - 4117/8192\*sqrt(2\*x^2 - x + 3)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (2x^2 - x + 3)^{3/2} (5x^2 + 3x + 2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)^(3/2)\*(3\*x + 5\*x^2 + 2),x)

[Out] int((2\*x^2 - x + 3)^(3/2)\*(3\*x + 5\*x^2 + 2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2x^2 - x + 3)^{3/2} (5x^2 + 3x + 2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2-x+3)\*\*(3/2)\*(5\*x\*\*2+3\*x+2),x)

[Out] Integral((2\*x\*\*2 - x + 3)\*\*(3/2)\*(5\*x\*\*2 + 3\*x + 2), x)



$$3.69 \quad \int \frac{(3-x+2x^2)^{3/2}}{2+3x+5x^2} dx$$

Optimal. Leaf size=197

$$-\frac{1}{100} \sqrt{2x^2 - x + 3} (49 - 20x) + \frac{11}{125} \sqrt{\frac{11}{31} (247 + 500\sqrt{2})} \tan^{-1} \left( \frac{\sqrt{\frac{11}{62(247+500\sqrt{2})}} ((130 + 69\sqrt{2})x + 61\sqrt{2} + 8)}{\sqrt{2x^2 - x + 3}} \right)$$

[Out] -2203/2000\*arcsinh(1/23\*(1-4\*x)\*23^(1/2))\*2^(1/2)-1/100\*(49-20\*x)\*(2\*x^2-x+3)^(1/2)-11/3875\*arctanh(1/62\*(8+x\*(130-69\*2^(1/2))-61\*2^(1/2))\*682^(1/2)/(-247+500\*2^(1/2))^(1/2)/(2\*x^2-x+3)^(1/2))\*(-84227+170500\*2^(1/2))^(1/2)+11/3875\*arctan(1/62\*(8+61\*2^(1/2)+x\*(130+69\*2^(1/2)))\*682^(1/2)/(247+500\*2^(1/2))^(1/2)/(2\*x^2-x+3)^(1/2))\*(84227+170500\*2^(1/2))^(1/2)

**Rubi [A]** time = 0.49, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$ , Rules used = {977, 1076, 619, 215, 1035, 1029, 206, 204}

$$-\frac{1}{100} \sqrt{2x^2 - x + 3} (49 - 20x) + \frac{11}{125} \sqrt{\frac{11}{31} (247 + 500\sqrt{2})} \tan^{-1} \left( \frac{\sqrt{\frac{11}{62(247+500\sqrt{2})}} ((130 + 69\sqrt{2})x + 61\sqrt{2} + 8)}{\sqrt{2x^2 - x + 3}} \right)$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^(3/2)/(2 + 3\*x + 5\*x^2), x]

[Out] -((49 - 20\*x)\*Sqrt[3 - x + 2\*x^2])/100 - (2203\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(1000\*Sqrt[2]) + (11\*Sqrt[(11\*(247 + 500\*Sqrt[2]))/31]\*ArcTan[(Sqrt[11/(62\*(247 + 500\*Sqrt[2]))])\*(8 + 61\*Sqrt[2] + (130 + 69\*Sqrt[2])\*x)]/Sqrt[3 - x + 2\*x^2])/125 - (11\*Sqrt[(11\*(-247 + 500\*Sqrt[2]))/31]\*ArcTanh[(Sqrt[11/(62\*(-247 + 500\*Sqrt[2]))])\*(8 - 61\*Sqrt[2] + (130 - 69\*Sqrt[2])\*x)]/Sqrt[3 - x + 2\*x^2])/125

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

$Q[a, 0] \parallel LtQ[b, 0]$

### Rule 215

$Int[1/Sqrt[(a_) + (b_)*(x_)^2], x\_Symbol] := Simp[ArcSinh[Rt[b, 2]*x]/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[\{a, b\}, x] \&\& GtQ[a, 0] \&\& PosQ[b]$

### Rule 619

$Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x\_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^(p)), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[\{a, b, c, p\}, x] \&\& GtQ[4*a - b^2/c, 0]$

### Rule 977

$Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x\_Symbol] := Simp[((b*f*(3*p + 2*q) - c*e*(2*p + q) + 2*c*f*(p + q)*x)*(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^(q + 1))/(2*f^2*(p + q)*(2*p + 2*q + 1)), x] - Dist[1/(2*f^2*(p + q)*(2*p + 2*q + 1)), Int[(a + b*x + c*x^2)^(p - 2)*(d + e*x + f*x^2)^q*Simp[(b*d - a*e)*(c*e - b*f)*(1 - p)*(2*p + q) - (p + q)*(b^2*d*f*(1 - p) - a*(f*(b*e - 2*a*f)*(2*p + 2*q + 1) + c*(2*d*f - e^2*(2*p + q)))] + (2*(c*d - a*f)*(c*e - b*f)*(1 - p)*(2*p + q) - (p + q)*((b^2 - 4*a*c)*e*f*(1 - p) + b*(c*(e^2 - 4*d*f)*(2*p + q) + f*(2*c*d - b*e + 2*a*f)*(2*p + 2*q + 1)))]*x + ((c*e - b*f)^2*(1 - p)*p + c*(p + q)*(f*(b*e - 2*a*f)*(4*p + 2*q - 1) - c*(2*d*f*(1 - 2*p) + e^2*(3*p + q - 1)))]*x^2, x], x] /; FreeQ[\{a, b, c, d, e, f, q\}, x] \&\& NeQ[b^2 - 4*a*c, 0] \&\& NeQ[e^2 - 4*d*f, 0] \&\& GtQ[p, 1] \&\& NeQ[p + q, 0] \&\& NeQ[2*p + 2*q + 1, 0] \&\& !IGtQ[p, 0] \&\& !IGtQ[q, 0]$

### Rule 1029

$Int[((g_) + (h_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2], x\_Symbol] := Dist[-2*g*(g*b - 2*a*h), Subst[Int[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[\{a, b, c, d, e, f, g, h\}, x] \&\& NeQ[b^2 - 4*a*c, 0] \&\& NeQ[e^2 - 4*d*f, 0] \&\& NeQ[b*d - a*e, 0] \&\& EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f), 0]$

### Rule 1035

$Int[((g_) + (h_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2], x\_Symbol] := With[\{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)], 2\}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt$

$[d + e*x + f*x^2]), x], x] - \text{Dist}[1/(2*q), \text{Int}[\text{Simp}[h*(b*d - a*e) - g*(c*d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*\text{Sqrt}[d + e*x + f*x^2]), x], x]] /; \text{FreeQ}[\{a, b, c, d, e, f, g, h\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[e^2 - 4*d*f, 0] \&\& \text{NeQ}[b*d - a*e, 0] \&\& \text{NegQ}[b^2 - 4*a*c]$

### Rule 1076

$\text{Int}[(A_.) + (B_.)*(x_) + (C_.)*(x_)^2]/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*\text{Sqrt}[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x\_Symbol] :> \text{Dist}[C/c, \text{Int}[1/\text{Sqrt}[d + e*x + f*x^2], x], x] + \text{Dist}[1/c, \text{Int}[(A*c - a*C + (B*c - b*C)*x]/((a + b*x + c*x^2)*\text{Sqrt}[d + e*x + f*x^2]), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, C\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[e^2 - 4*d*f, 0]$

### Rubi steps

$$\begin{aligned} \int \frac{(3-x+2x^2)^{3/2}}{2+3x+5x^2} dx &= -\frac{1}{100}(49-20x)\sqrt{3-x+2x^2} - \frac{1}{50} \int \frac{-\frac{731}{2} + \frac{1195x}{4} - \frac{2203x^2}{4}}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx \\ &= -\frac{1}{100}(49-20x)\sqrt{3-x+2x^2} - \frac{1}{250} \int \frac{-726+3146x}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx + \frac{2203}{100} \int \frac{1}{\sqrt{3-x+2x^2}} dx \\ &= -\frac{1}{100}(49-20x)\sqrt{3-x+2x^2} + \frac{\int \frac{2662(16+3\sqrt{2})+2662(10-13\sqrt{2})x}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{5500\sqrt{2}} - \frac{\int \frac{2662(16-3\sqrt{2})+2662(10-13\sqrt{2})x}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{5500\sqrt{2}} \\ &= -\frac{1}{100}(49-20x)\sqrt{3-x+2x^2} - \frac{2203 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1000\sqrt{2}} - \frac{1}{125} \left(322102 \left(1000 - 247\sqrt{2}\right)\right) \\ &= -\frac{1}{100}(49-20x)\sqrt{3-x+2x^2} - \frac{2203 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1000\sqrt{2}} + \frac{11}{125} \sqrt{\frac{11}{31}} \left(247 + 500\sqrt{2}\right) \tan^{-1}\left(\frac{\sqrt{31}\sqrt{2x^2-x+3}}{\sqrt{2x^2-x+3}}\right) \end{aligned}$$

**Mathematica** [C] time = 0.69, size = 310, normalized size = 1.57

$$400\sqrt{31} \sqrt{2x^2-x+3} x - 980\sqrt{31} \sqrt{2x^2-x+3} + 44\sqrt{286+22i\sqrt{31}} \left(\sqrt{31}-13i\right) \tanh^{-1}\left(\frac{-4i\sqrt{31}x-22x+i\sqrt{31}+2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x}}{\sqrt{2x^2-x+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2), x]
```

```
[Out] (-980*Sqrt[31]*Sqrt[3 - x + 2*x^2] + 400*Sqrt[31]*x*Sqrt[3 - x + 2*x^2] - 2
203*Sqrt[62]*ArcSinh[(1 - 4*x)/Sqrt[23]] + 44*Sqrt[286 + (22*I)*Sqrt[31]]*(
-13*I + Sqrt[31])*ArcTanh[(63 + I*Sqrt[31] - 22*x - (4*I)*Sqrt[31]*x)/(2*Sqr
rt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] + 44*Sqrt[682*(13 - I*Sqrt[
31])]*ArcTanh[(63 - I*Sqrt[31] - 22*x + (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22
*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] + (572*I)*Sqrt[286 - (22*I)*Sqrt[31]]*A
rcTanh[(63 - I*Sqrt[31] - 22*x + (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqr
t[31]]*Sqrt[3 - x + 2*x^2])]/(2000*Sqrt[31])
```

**fricas** [B] time = 1.97, size = 2027, normalized size = 10.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2), x, algorithm="fricas")
```

```
[Out] 11/77500*24200^(1/4)*sqrt(31)*sqrt(10)*sqrt(2)*sqrt(247*sqrt(2) + 1000)*arc
tan(1/10605875*(230*sqrt(10)*(2*24200^(3/4)*sqrt(31)*(20846*x^7 - 109153*x^
6 + 215386*x^5 - 427391*x^4 + 234360*x^3 - 156600*x^2 - sqrt(2)*(28854*x^7
- 90639*x^6 + 200187*x^5 - 262838*x^4 + 117544*x^3 - 23472*x^2 - 186624*x +
86400) - 172800*x + 186624) + 5*24200^(1/4)*sqrt(31)*(112238*x^7 - 1817988
*x^6 + 10351960*x^5 - 25791248*x^4 + 34522560*x^3 - 28368000*x^2 - sqrt(2)*
(125839*x^7 - 1864281*x^6 + 9323336*x^5 - 19725020*x^4 + 24624288*x^3 - 108
62496*x^2 - 19989504*x + 10533888) - 21067776*x + 19989504))*sqrt(2*x^2 - x
+ 3)*sqrt(247*sqrt(2) + 1000) + 30107000*sqrt(31)*sqrt(2)*(28180*x^8 - 254
666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 -
sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 7
52088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - sqrt(5/
119)*(sqrt(10)*(2*24200^(3/4)*sqrt(31)*(46522*x^7 - 71117*x^6 + 257247*x^5
- 273360*x^4 + 484920*x^3 - 269568*x^2 - 16*sqrt(2)*(7714*x^7 - 10881*x^6 +
33771*x^5 - 5576*x^4 - 576*x^3 + 32184*x^2 - 32184*x) + 269568*x) + 5*2420
0^(1/4)*sqrt(31)*(309512*x^7 - 4017952*x^6 + 15741280*x^5 - 22625280*x^4 +
37693440*x^3 - 13519872*x^2 - sqrt(2)*(516957*x^7 - 6676948*x^6 + 25569820*
x^5 - 31522752*x^4 + 34450848*x^3 + 46199808*x^2 - 46199808*x) + 13519872*x
))*sqrt(2*x^2 - x + 3)*sqrt(247*sqrt(2) + 1000) + 130900*sqrt(31)*sqrt(2)*(
123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x
^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^
5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) + 595
0*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781
920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 51
7*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))
```

$$\begin{aligned}
& * \sqrt{(24200^{(1/4)} * \sqrt{10} * \sqrt{2x^2 - x + 3}) * (\sqrt{2} * (x - 75) + 74x - 76) * \sqrt{247 * \sqrt{2} + 1000} + 58310x^2 + 52360 * \sqrt{2} * (2x^2 - x + 3) - 179690x + 238000) / x^2} + 342125 * \sqrt{31} * (2828123x^8 - 9696916x^7 + 53385560x^6 - 142835344x^5 + 254146592x^4 - 249300096x^3 + 37981440x^2 - 7744 * \sqrt{2} * (1348x^8 - 2692x^7 + 9789x^6 - 10070x^5 + 15569x^4 - 5568x^3 + 1080x^2 + 4320x - 5184) + 223064064x - 94887936)) / (2585191x^8 - 4661200x^7 + 14191920x^6 + 490880x^5 - 13562944x^4 + 44249088x^3 - 34615296x^2 - 24772608x + 18579456) + 11/77500 * 24200^{(1/4)} * \sqrt{31} * \sqrt{10} * \sqrt{2} * \sqrt{247 * \sqrt{2} + 1000} * \arctan(1/10605875 * (230 * \sqrt{10} * (2 * 24200^{(3/4)} * \sqrt{31} * (20846x^7 - 109153x^6 + 215386x^5 - 427391x^4 + 234360x^3 - 156600x^2 - \sqrt{2} * (28854x^7 - 90639x^6 + 200187x^5 - 262838x^4 + 117544x^3 - 23472x^2 - 186624x + 86400) - 172800x + 186624) + 5 * 24200^{(1/4)} * \sqrt{31} * (112238x^7 - 1817988x^6 + 10351960x^5 - 25791248x^4 + 34522560x^3 - 28368000x^2 - \sqrt{2} * (125839x^7 - 1864281x^6 + 9323336x^5 - 19725020x^4 + 24624288x^3 - 10862496x^2 - 19989504x + 10533888) - 21067776x + 19989504)) * \sqrt{2x^2 - x + 3} * \sqrt{247 * \sqrt{2} + 1000} - 30107000 * \sqrt{31} * \sqrt{2} * (28180x^8 - 254666x^7 + 704270x^6 - 1385256x^5 + 1549144x^4 - 642048x^3 - 98496x^2 - \sqrt{2} * (8746x^8 - 102335x^7 + 396104x^6 - 783113x^5 + 1320710x^4 - 752088x^3 + 396144x^2 + 546048x - 539136) + 1154304x - 456192) - \sqrt{5/119} * (\sqrt{10} * (2 * 24200^{(3/4)} * \sqrt{31} * (46522x^7 - 71117x^6 + 257247x^5 - 273360x^4 + 484920x^3 - 269568x^2 - 16 * \sqrt{2} * (7714x^7 - 10881x^6 + 33771x^5 - 5576x^4 - 576x^3 + 32184x^2 - 32184x) + 269568x) + 5 * 24200^{(1/4)} * \sqrt{31} * (309512x^7 - 4017952x^6 + 15741280x^5 - 22625280x^4 + 37693440x^3 - 13519872x^2 - \sqrt{2} * (516957x^7 - 6676948x^6 + 25569820x^5 - 31522752x^4 + 34450848x^3 + 46199808x^2 - 46199808x) + 13519872x)) * \sqrt{2x^2 - x + 3} * \sqrt{247 * \sqrt{2} + 1000} - 130900 * \sqrt{31} * \sqrt{2} * (123408x^8 - 914152x^7 + 1578888x^6 - 3293072x^5 + 396480x^4 + 798336x^3 - 3822336x^2 - \sqrt{2} * (15550x^8 - 118051x^7 + 244047x^6 - 707374x^5 + 1053960x^4 - 1667952x^3 + 1209600x^2 - 1036800x) + 3276288x) - 5950 * \sqrt{31} * (254591x^8 - 4815126x^7 + 32303580x^6 - 90866808x^5 + 108781920x^4 - 74219328x^3 - 168956928x^2 - 15488 * \sqrt{2} * (4x^8 - 76x^7 + 517x^6 - 1536x^5 + 2385x^4 - 3618x^3 + 2268x^2 - 1944x) + 144820224x)) * \sqrt{-(24200^{(1/4)} * \sqrt{10} * \sqrt{2x^2 - x + 3}) * (\sqrt{2} * (x - 75) + 74x - 76) * \sqrt{247 * \sqrt{2} + 1000} - 58310x^2 - 52360 * \sqrt{2} * (2x^2 - x + 3) + 179690x - 238000) / x^2} - 342125 * \sqrt{31} * (2828123x^8 - 9696916x^7 + 53385560x^6 - 142835344x^5 + 254146592x^4 - 249300096x^3 + 37981440x^2 - 7744 * \sqrt{2} * (1348x^8 - 2692x^7 + 9789x^6 - 10070x^5 + 15569x^4 - 5568x^3 + 1080x^2 + 4320x - 5184) + 223064064x - 94887936)) / (2585191x^8 - 4661200x^7 + 14191920x^6 + 490880x^5 - 13562944x^4 + 44249088x^3 - 34615296x^2 - 24772608x + 18579456) + 11/36890000 * 24200^{(1/4)} * \sqrt{10} * \sqrt{247 * \sqrt{2} + 1000} * (247 * \sqrt{2} - 1000) * \log(1512500/119 * (24200^{(1/4)} * \sqrt{10} * \sqrt{2x^2 - x + 3}) * (\sqrt{2} * (x - 75) + 74x - 76) * \sqrt{247 * \sqrt{2} + 1000} + 58310x^2 + 52360 * \sqrt{2} * (2x^2 - x + 3) - 179690x + 238000) / x^2) - 11/36890000 * 24200^{(1/4)} * \sqrt{10} * \sqrt{247 * \sqrt{2} + 1000} * (247 * \sqrt{2} - 1000) * \log(-1512500/119 * (24200^{(1/4)} * \sqrt{10} * \sqrt{2x^2 - x + 3}) * (\sqrt{2} * (x - 75) + 74x - 76) * \sqrt{247 * \sqrt{2} + 1000} - 58310x^2 - 52360 * \sqrt{2} * (2x^2 - x + 3) + 179690x - 238000) / x^2)
\end{aligned}$$

```

qrt(10)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(x - 75) + 74*x - 76)*sqrt(247*sqrt(2)
+ 1000) - 58310*x^2 - 52360*sqrt(2)*(2*x^2 - x + 3) + 179690*x - 238000)/x
^2) + 1/100*sqrt(2*x^2 - x + 3)*(20*x - 49) + 2203/4000*sqrt(2)*log(-4*sqrt
(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)

```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infini
ty]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity
]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai
lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity
,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf
inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinit
y]Evaluation time: 10.3Done
```

**maple** [B] time = 0.05, size = 3460, normalized size = 17.56

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x)
```

```
[Out] 1/5*x*(2*x^2-x+3)^(1/2)-49/100*(2*x^2-x+3)^(1/2)+2203/2000*2^(1/2)*arcsinh(
4/23*23^(1/2)*(x-1/4))-2/1321375*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1
/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^^(1/2)*2^(1/2)*(4245*2^(1/
2)*(-8866+6820*2^(1/2))^^(1/2)*(-775687+549362*2^(1/2))^^(1/2)*arctan(1/11692
487*(-775687+549362*2^(1/2))^^(1/2)*(-23*(8+3*2^(1/2))*(-23*(x+2^(1/2)-1)^2/
(-x+2^(1/2)+1)^2+24*2^(1/2)-41))^^(1/2)*(6485*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^
(1/2)+1)^2+10368*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+22379*2^(1/2)+32016)/(23*
(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)^4+82*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23)*(x
+2^(1/2)-1)/(-x+2^(1/2)+1)*(8+3*2^(1/2)))+6154*(-8866+6820*2^(1/2))^^(1/2)*(-
775687+549362*2^(1/2))^^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^^(1
/2)*(-23*(8+3*2^(1/2))*(-23*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24*2^(1/2)-41)
)^^(1/2)*(6485*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+10368*(x+2^(1/2)-1)^
2/(-x+2^(1/2)+1)^2+22379*2^(1/2)+32016)/(23*(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)^
4+82*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23)*(x+2^(1/2)-1)/(-x+2^(1/2)+1)*(8+3
*2^(1/2)))+12325786*arctanh(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1
```



$820*2^{(1/2)})^{(1/2)}+3/10571*(8*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+3*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+8-3*2^{(1/2)})^{(1/2)}*2^{(1/2)}*(151*2^{(1/2)}*(-8866+6820*2^{(1/2)})^{(1/2)}*(-775687+549362*2^{(1/2)})^{(1/2)}*\arctan(1/11692487*(-775687+549362*2^{(1/2)})^{(1/2)}*(-23*(8+3*2^{(1/2)}))*(-23*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+24*2^{(1/2)}-41))^{(1/2)}*(6485*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+10368*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+22379*2^{(1/2)}+32016)/(23*(x+2^{(1/2)}-1)^4/(-x+2^{(1/2)}+1)^4+82*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+23)*(x+2^{(1/2)}-1)/(-x+2^{(1/2)}+1)*(8+3*2^{(1/2)})))+218*(-8866+6820*2^{(1/2)})^{(1/2)}*(-775687+549362*2^{(1/2)})^{(1/2)}*\arctan(1/11692487*(-775687+549362*2^{(1/2)})^{(1/2)}*(-23*(8+3*2^{(1/2)}))*(-23*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+24*2^{(1/2)}-41))^{(1/2)}*(6485*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+10368*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+22379*2^{(1/2)}+32016)/(23*(x+2^{(1/2)}-1)^4/(-x+2^{(1/2)}+1)^4+82*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+23)*(x+2^{(1/2)}-1)/(-x+2^{(1/2)}+1)*(8+3*2^{(1/2)})))+401698*\operatorname{arctanh}(31/2*(8*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+3*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+8-3*2^{(1/2)})^{(1/2)})/(-8866+6820*2^{(1/2)})^{(1/2)}*2^{(1/2)}-63426*\operatorname{arctanh}(31/2*(8*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+3*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+8-3*2^{(1/2)})^{(1/2)})/(-8866+6820*2^{(1/2)})^{(1/2)}*2^{(1/2)}*((8*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+3*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+8-3*2^{(1/2)})/(1+(x+2^{(1/2)}-1)/(-x+2^{(1/2)}+1)))^2)^{(1/2)})/(1+(x+2^{(1/2)}-1)/(-x+2^{(1/2)}+1)))/(8+3*2^{(1/2)})/(-8866+6820*2^{(1/2)})^{(1/2)}+9/21142*(8*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+3*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+8-3*2^{(1/2)})^{(1/2)}*2^{(1/2)}*(369*2^{(1/2)}*(-8866+6820*2^{(1/2)})^{(1/2)}*(-775687+549362*2^{(1/2)})^{(1/2)}*\arctan(1/11692487*(-775687+549362*2^{(1/2)})^{(1/2)}*(-23*(8+3*2^{(1/2)}))*(-23*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+24*2^{(1/2)}-41))^{(1/2)}*(6485*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+10368*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+22379*2^{(1/2)}+32016)/(23*(x+2^{(1/2)}-1)^4/(-x+2^{(1/2)}+1)^4+82*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+23)*(x+2^{(1/2)}-1)/(-x+2^{(1/2)}+1)*(8+3*2^{(1/2)})))+520*(-8866+6820*2^{(1/2)})^{(1/2)}*(-775687+549362*2^{(1/2)})^{(1/2)}*\arctan(1/11692487*(-775687+549362*2^{(1/2)})^{(1/2)}*(-23*(8+3*2^{(1/2)}))*(-23*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+24*2^{(1/2)}-41))^{(1/2)}*(6485*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+10368*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+22379*2^{(1/2)}+32016)/(23*(x+2^{(1/2)}-1)^4/(-x+2^{(1/2)}+1)^4+82*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+23)*(x+2^{(1/2)}-1)/(-x+2^{(1/2)}+1)*(8+3*2^{(1/2)})))+465124*\operatorname{arctanh}(31/2*(8*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+3*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+8-3*2^{(1/2)})^{(1/2)})/(-8866+6820*2^{(1/2)})^{(1/2)}*2^{(1/2)}-866822*\operatorname{arctanh}(31/2*(8*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+3*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+8-3*2^{(1/2)})^{(1/2)})/(-8866+6820*2^{(1/2)})^{(1/2)}*2^{(1/2)}*((8*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+3*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+8-3*2^{(1/2)})/(1+(x+2^{(1/2)}-1)/(-x+2^{(1/2)}+1)))^2)^{(1/2)})/(1+(x+2^{(1/2)}-1)/(-x+2^{(1/2)}+1)))/(8+3*2^{(1/2)})/(-8866+6820*2^{(1/2)})^{(1/2)}$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - x + 3)^{\frac{3}{2}}}{5x^2 + 3x + 2} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)/(5\*x^2+3\*x+2),x, algorithm="maxima")

[Out] integrate((2\*x^2 - x + 3)^(3/2)/(5\*x^2 + 3\*x + 2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2x^2 - x + 3)^{3/2}}{5x^2 + 3x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)^(3/2)/(3\*x + 5\*x^2 + 2),x)

[Out] int((2\*x^2 - x + 3)^(3/2)/(3\*x + 5\*x^2 + 2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - x + 3)^{3/2}}{5x^2 + 3x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2-x+3)\*\*(3/2)/(5\*x\*\*2+3\*x+2),x)

[Out] Integral((2\*x\*\*2 - x + 3)\*\*(3/2)/(5\*x\*\*2 + 3\*x + 2), x)

$$3.70 \quad \int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^2} dx$$

**Optimal.** Leaf size=232

$$\frac{(10x+3)(2x^2-x+3)^{3/2}}{31(5x^2+3x+2)} + \frac{4}{155}(4-5x)\sqrt{2x^2-x+3} + \frac{\sqrt{\frac{11}{31}(3169333+2265350\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{\frac{11}{62(3169333+2265350\sqrt{2})}}}{\dots}\right)}{1550}$$

[Out] 1/31\*(3+10\*x)\*(2\*x^2-x+3)^(3/2)/(5\*x^2+3\*x+2)-2/25\*arcsinh(1/23\*(1-4\*x)\*23^(1/2))\*2^(1/2)+4/155\*(4-5\*x)\*(2\*x^2-x+3)^(1/2)-1/48050\*arctanh(1/62\*(3514+x\*(9440-6477\*2^(1/2))-2963\*2^(1/2))\*682^(1/2)/(-3169333+2265350\*2^(1/2)))^(1/2)/(2\*x^2-x+3)^(1/2)\*(-1080742553+772484350\*2^(1/2))^(1/2)+1/48050\*arctan(1/62\*(3514+2963\*2^(1/2)+x\*(9440+6477\*2^(1/2)))\*682^(1/2)/(3169333+2265350\*2^(1/2)))^(1/2)/(2\*x^2-x+3)^(1/2)\*(1080742553+772484350\*2^(1/2))^(1/2)

**Rubi [A]** time = 0.58, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {971, 1066, 1076, 619, 215, 1035, 1029, 206, 204}

$$\frac{(10x+3)(2x^2-x+3)^{3/2}}{31(5x^2+3x+2)} + \frac{4}{155}(4-5x)\sqrt{2x^2-x+3} + \frac{\sqrt{\frac{11}{31}(3169333+2265350\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{\frac{11}{62(3169333+2265350\sqrt{2})}}}{\dots}\right)}{1550}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^(3/2)/(2 + 3\*x + 5\*x^2)^2, x]

[Out] (4\*(4 - 5\*x)\*Sqrt[3 - x + 2\*x^2])/155 + ((3 + 10\*x)\*(3 - x + 2\*x^2)^(3/2))/(31\*(2 + 3\*x + 5\*x^2)) - (2\*Sqrt[2]\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/25 + (Sqrt[(11\*(3169333 + 2265350\*Sqrt[2]))/31]\*ArcTan[(Sqrt[11/(62\*(3169333 + 2265350\*Sqrt[2]))])\*(3514 + 2963\*Sqrt[2] + (9440 + 6477\*Sqrt[2])\*x)]/Sqrt[3 - x + 2\*x^2])/1550 - (Sqrt[(11\*(-3169333 + 2265350\*Sqrt[2]))/31]\*ArcTanh[(Sqrt[11/(62\*(-3169333 + 2265350\*Sqrt[2]))])\*(3514 - 2963\*Sqrt[2] + (9440 - 6477\*Sqrt[2])\*x)]/Sqrt[3 - x + 2\*x^2])/1550

**Rule 204**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 215

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 619

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

Rule 971

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_)\*((d\_) + (e\_)\*(x\_) + (f\_)\*(x\_)^2)^(q\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^q)/((b^2 - 4\*a\*c)\*(p + 1)), x] - Dist[1/((b^2 - 4\*a\*c)\*(p + 1)), Int[(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^(q - 1)\*Simp[2\*c\*d\*(2\*p + 3) + b\*e\*q + (2\*b\*f\*q + 2\*c\*e\*(2\*p + q + 3))\*x + 2\*c\*f\*(2\*p + 2\*q + 3)\*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && LtQ[p, -1] && GtQ[q, 0] && !IGtQ[q, 0]

Rule 1029

Int[((g\_) + (h\_)\*(x\_))/(((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)\*Sqrt[(d\_) + (e\_)\*(x\_) + (f\_)\*(x\_)^2]), x\_Symbol] := Dist[-2\*g\*(g\*b - 2\*a\*h), Subst[Int[1/Simp[g\*(g\*b - 2\*a\*h)\*(b^2 - 4\*a\*c) - (b\*d - a\*e)\*x^2, x], x], x, Simp[g\*b - 2\*a\*h - (b\*h - 2\*g\*c)\*x, x]/Sqrt[d + e\*x + f\*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && NeQ[b\*d - a\*e, 0] && EqQ[h^2\*(b\*d - a\*e) - 2\*g\*h\*(c\*d - a\*f) + g^2\*(c\*e - b\*f), 0]

Rule 1035

Int[((g\_) + (h\_)\*(x\_))/(((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)\*Sqrt[(d\_) + (e\_)\*(x\_) + (f\_)\*(x\_)^2]), x\_Symbol] := With[{q = Rt[(c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f), 2]}, Dist[1/(2\*q), Int[Simp[h\*(b\*d - a\*e) - g\*(c\*d - a\*f - q) - (g\*(c\*e - b\*f) - h\*(c\*d - a\*f + q))\*x, x]/((a + b\*x + c\*x^2)\*Sqrt[d + e\*x + f\*x^2]), x], x] - Dist[1/(2\*q), Int[Simp[h\*(b\*d - a\*e) - g\*(c\*d - a\*f + q) - (g\*(c\*e - b\*f) - h\*(c\*d - a\*f - q))\*x, x]/((a + b\*x + c\*x^2)\*S

```

qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]

```

### Rule 1066

```

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((A_) + (B_)*(x_) + (C_)*(x_)
^2)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[((B*c*f*(2*
p + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a +
b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x
^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) -
c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f
- e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3)))] + (2*p*(c*d - a*f)
*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e
*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e +
2*A*f)*(2*p + 2*q + 3)))]*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C
e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^
2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3)))]*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]

```

### Rule 1076

```

Int[((A_) + (B_)*(x_) + (C_)*(x_)^2)/(((a_) + (b_)*(x_) + (c_)*(x_)^2)
*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]

```

### Rubi steps

$$\begin{aligned}
\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^2} dx &= \frac{(3+10x)(3-x+2x^2)^{3/2}}{31(2+3x+5x^2)} - \frac{1}{31} \int \frac{\sqrt{3-x+2x^2} \left(-\frac{69}{2} + 13x + 40x^2\right)}{2+3x+5x^2} dx \\
&= \frac{4}{155}(4-5x)\sqrt{3-x+2x^2} + \frac{(3+10x)(3-x+2x^2)^{3/2}}{31(2+3x+5x^2)} + \frac{\int \frac{13070-5750x+2480x^2}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{3100} \\
&= \frac{4}{155}(4-5x)\sqrt{3-x+2x^2} + \frac{(3+10x)(3-x+2x^2)^{3/2}}{31(2+3x+5x^2)} + \frac{\int \frac{60390-36190x}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{15500} + \frac{4}{25} \\
&= \frac{4}{155}(4-5x)\sqrt{3-x+2x^2} + \frac{(3+10x)(3-x+2x^2)^{3/2}}{31(2+3x+5x^2)} + \frac{1}{25} \left(2\sqrt{\frac{2}{23}}\right) \text{Subst} \left( \int \frac{1}{\sqrt{1+}} \right) \\
&= \frac{4}{155}(4-5x)\sqrt{3-x+2x^2} + \frac{(3+10x)(3-x+2x^2)^{3/2}}{31(2+3x+5x^2)} - \frac{2}{25}\sqrt{2} \sinh^{-1} \left( \frac{1-4x}{\sqrt{23}} \right) + \frac{1}{15} \\
&= \frac{4}{155}(4-5x)\sqrt{3-x+2x^2} + \frac{(3+10x)(3-x+2x^2)^{3/2}}{31(2+3x+5x^2)} - \frac{2}{25}\sqrt{2} \sinh^{-1} \left( \frac{1-4x}{\sqrt{23}} \right) + \frac{1}{15}
\end{aligned}$$

**Mathematica [C]** time = 2.56, size = 530, normalized size = 2.28

$$\frac{62000\sqrt{2x^2-x+3}x^2}{10x-i\sqrt{31}+3} + \frac{62000\sqrt{2x^2-x+3}x^2}{10x+i\sqrt{31}+3} - \frac{31000\sqrt{2x^2-x+3}x}{10x-i\sqrt{31}+3} - \frac{31000\sqrt{2x^2-x+3}x}{10x+i\sqrt{31}+3} - 12400\sqrt{2x^2-x+3}x + \frac{93000\sqrt{2x^2-x+3}}{10x-i\sqrt{31}+3} + \frac{93000\sqrt{2x^2-x+3}}{10x+i\sqrt{31}+3}$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)^(3/2)/(2 + 3\*x + 5\*x^2)^2, x]

[Out] (9920\*Sqrt[3 - x + 2\*x^2] - 12400\*x\*Sqrt[3 - x + 2\*x^2] + (93000\*Sqrt[3 - x + 2\*x^2]))/(3 - I\*Sqrt[31] + 10\*x) - (31000\*x\*Sqrt[3 - x + 2\*x^2])/(3 - I\*Sqrt[31] + 10\*x) + (62000\*x^2\*Sqrt[3 - x + 2\*x^2])/(3 - I\*Sqrt[31] + 10\*x) + (93000\*Sqrt[3 - x + 2\*x^2])/(3 + I\*Sqrt[31] + 10\*x) - (31000\*x\*Sqrt[3 - x + 2\*x^2])/(3 + I\*Sqrt[31] + 10\*x) + (62000\*x^2\*Sqrt[3 - x + 2\*x^2])/(3 + I\*Sqrt[31] + 10\*x) - 7688\*Sqrt[2]\*ArcSinh[(1 - 4\*x)/Sqrt[23]] - (Sqrt[286 + (22\*I)\*Sqrt[31]]\*(10199\*I + 6477\*Sqrt[31]))\*ArcTanh[(63 + I\*Sqrt[31] - 22\*x -

$$\frac{(4*I)*\text{Sqrt}[31]*x)/(2*\text{Sqrt}[286 + (22*I)*\text{Sqrt}[31]]*\text{Sqrt}[3 - x + 2*x^2])]}{(-13*I + \text{Sqrt}[31]) - (6477*\text{Sqrt}[682*(13 - I*\text{Sqrt}[31])]*\text{ArcTanh}[(63 - I*\text{Sqrt}[31] - 22*x + (4*I)*\text{Sqrt}[31]*x)/(2*\text{Sqrt}[286 - (22*I)*\text{Sqrt}[31]]*\text{Sqrt}[3 - x + 2*x^2])])}/(13*I + \text{Sqrt}[31]) + ((10199*I)*\text{Sqrt}[286 - (22*I)*\text{Sqrt}[31]]*\text{ArcTanh}[(63 - I*\text{Sqrt}[31] - 22*x + (4*I)*\text{Sqrt}[31]*x)/(2*\text{Sqrt}[286 - (22*I)*\text{Sqrt}[31]]*\text{Sqrt}[3 - x + 2*x^2])])}/(13*I + \text{Sqrt}[31])}]/96100$$

**fricas [B]** time = 2.64, size = 2150, normalized size = 9.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)/(5\*x^2+3\*x+2)^2,x, algorithm="fricas")

[Out] 1/90746855745853600\*(10421084\*1987037073032^(1/4)\*sqrt(45307)\*sqrt(62)\*sqrt(2)\*(5\*x^2 + 3\*x + 2)\*sqrt(3169333\*sqrt(2) + 4530700)\*arctan(1/172758074198807633719789\*(64607782\*sqrt(45307)\*(2\*1987037073032^(3/4)\*sqrt(62)\*(2433118\*x^7 - 9616349\*x^6 + 20077988\*x^5 - 32895253\*x^4 + 16664280\*x^3 - 8289000\*x^2 - sqrt(2)\*(1842432\*x^7 - 6916062\*x^6 + 14611071\*x^5 - 22920229\*x^4 + 11367152\*x^3 - 5107176\*x^2 - 12897792\*x + 8726400) - 17452800\*x + 12897792) + 1404517\*1987037073032^(1/4)\*sqrt(62)\*(373384\*x^7 - 5757834\*x^6 + 30631880\*x^5 - 70476664\*x^4 + 91370880\*x^3 - 59457600\*x^2 - sqrt(2)\*(276977\*x^7 - 4232733\*x^6 + 22218448\*x^5 - 50249260\*x^4 + 64668384\*x^3 - 39479328\*x^2 - 46697472\*x + 32016384) - 64032768\*x + 46697472))\*sqrt(2\*x^2 - x + 3)\*sqrt(3169333\*sqrt(2) + 4530700) + 490410017080486186043272\*sqrt(31)\*sqrt(2)\*(28180\*x^8 - 254666\*x^7 + 704270\*x^6 - 1385256\*x^5 + 1549144\*x^4 - 642048\*x^3 - 98496\*x^2 - sqrt(2)\*(8746\*x^8 - 102335\*x^7 + 396104\*x^6 - 783113\*x^5 + 1320710\*x^4 - 752088\*x^3 + 396144\*x^2 + 546048\*x - 539136) + 1154304\*x - 456192) - sqrt(45307/2711)\*(sqrt(45307)\*(2\*1987037073032^(3/4)\*sqrt(62)\*(8480726\*x^7 - 12210811\*x^6 + 39548601\*x^5 - 16962480\*x^4 + 21434760\*x^3 + 14432256\*x^2 - sqrt(2)\*(6779042\*x^7 - 9704193\*x^6 + 31062363\*x^5 - 11094928\*x^4 + 12114072\*x^3 + 16301952\*x^2 - 16301952\*x) - 14432256\*x) + 1404517\*1987037073032^(1/4)\*sqrt(62)\*(1312966\*x^7 - 16987736\*x^6 + 65572040\*x^5 - 85530240\*x^4 + 112374720\*x^3 + 57314304\*x^2 - sqrt(2)\*(1011501\*x^7 - 13081364\*x^6 + 50391260\*x^5 - 64806336\*x^4 + 81634464\*x^3 + 56070144\*x^2 - 56070144\*x) - 57314304\*x))\*sqrt(2\*x^2 - x + 3)\*sqrt(3169333\*sqrt(2) + 4530700) + 7590571938849196\*sqrt(31)\*sqrt(2)\*(123408\*x^8 - 914152\*x^7 + 1578888\*x^6 - 3293072\*x^5 + 396480\*x^4 + 798336\*x^3 - 3822336\*x^2 - sqrt(2)\*(15550\*x^8 - 118051\*x^7 + 244047\*x^6 - 707374\*x^5 + 1053960\*x^4 - 1667952\*x^3 + 1209600\*x^2 - 1036800\*x) + 3276288\*x) + 345025997220418\*sqrt(31)\*(254591\*x^8 - 4815126\*x^7 + 32303580\*x^6 - 90866808\*x^5 + 108781920\*x^4 - 74219328\*x^3 - 168956928\*x^2 - 15488\*sqrt(2)\*(4\*x^8 - 76\*x^7 + 517\*x^6 - 1536\*x^5 + 2385\*x^4 - 3618\*x^3 + 2268\*x^2 - 1944\*x) + 144820224\*x))\*sqrt(-(1987037073032^(1/4)\*sqrt(45307)\*sqrt(62)\*sqrt(31)\*sqrt(2\*x^2 - x + 3)\*(sqrt(2)\*(1867\*x + 1425) - 3292\*x - 442)\*sqrt(3169333\*sqrt(2) + 4530700) - 11567627293306\*x^2 - 10387257161336\*sqrt(2)

$$\begin{aligned}
& )*(2*x^2 - x + 3) + 35647177985494*x - 47214805278800)/x^2) + 5572841103187 \\
& 343023219*\sqrt{31}*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 \\
& + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*\sqrt{2}*(1348*x^8 - \\
& 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x \\
& - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x \\
& ^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + \\
& 18579456) + 10421084*1987037073032^{(1/4)}*\sqrt{45307}*\sqrt{62}*\sqrt{2}*(5* \\
& x^2 + 3*x + 2)*\sqrt{3169333*\sqrt{2} + 4530700}*\arctan(1/1727580741988076337 \\
& 19789*(64607782*\sqrt{45307}*(2*1987037073032^{(3/4)}*\sqrt{62}*(2433118*x^7 - \\
& 9616349*x^6 + 20077988*x^5 - 32895253*x^4 + 16664280*x^3 - 8289000*x^2 - sq \\
& rt(2)*(1842432*x^7 - 6916062*x^6 + 14611071*x^5 - 22920229*x^4 + 11367152*x \\
& ^3 - 5107176*x^2 - 12897792*x + 8726400) - 17452800*x + 12897792) + 1404517 \\
& *1987037073032^{(1/4)}*\sqrt{62}*(373384*x^7 - 5757834*x^6 + 30631880*x^5 - 70 \\
& 476664*x^4 + 91370880*x^3 - 59457600*x^2 - \sqrt{2}*(276977*x^7 - 4232733*x^ \\
& 6 + 22218448*x^5 - 50249260*x^4 + 64668384*x^3 - 39479328*x^2 - 46697472*x \\
& + 32016384) - 64032768*x + 46697472))*\sqrt{2*x^2 - x + 3}*\sqrt{3169333*\sqrt{2} \\
& (2) + 4530700) - 490410017080486186043272*\sqrt{31}*\sqrt{2}*(28180*x^8 - 254 \\
& 666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - \\
& \sqrt{2}*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 7 \\
& 52088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - \sqrt{45 \\
& 307/2711}*(\sqrt{45307}*(2*1987037073032^{(3/4)}*\sqrt{62}*(8480726*x^7 - 12210 \\
& 811*x^6 + 39548601*x^5 - 16962480*x^4 + 21434760*x^3 + 14432256*x^2 - \sqrt{2} \\
& (2)*(6779042*x^7 - 9704193*x^6 + 31062363*x^5 - 11094928*x^4 + 12114072*x^3 \\
& + 16301952*x^2 - 16301952*x) - 14432256*x) + 1404517*1987037073032^{(1/4)}*sq \\
& rt(62)*(1312966*x^7 - 16987736*x^6 + 65572040*x^5 - 85530240*x^4 + 11237472 \\
& 0*x^3 + 57314304*x^2 - \sqrt{2}*(1011501*x^7 - 13081364*x^6 + 50391260*x^5 - \\
& 64806336*x^4 + 81634464*x^3 + 56070144*x^2 - 56070144*x) - 57314304*x))*sq \\
& rt(2*x^2 - x + 3)*\sqrt{3169333*\sqrt{2} + 4530700) - 7590571938849196*\sqrt{3} \\
& 1)*\sqrt{2}*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^ \\
& 4 + 798336*x^3 - 3822336*x^2 - \sqrt{2}*(15550*x^8 - 118051*x^7 + 244047*x^6 \\
& - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276 \\
& 288*x) - 345025997220418*\sqrt{31}*(254591*x^8 - 4815126*x^7 + 32303580*x^6 \\
& - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*\sqrt{2} \\
& (2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - \\
& 1944*x) + 144820224*x))*\sqrt{((1987037073032^{(1/4)}*\sqrt{45307}*\sqrt{62}*\sqrt{2} \\
& (31)*\sqrt{2*x^2 - x + 3}*(\sqrt{2}*(1867*x + 1425) - 3292*x - 442)*\sqrt{3169 \\
& 333*\sqrt{2} + 4530700) + 11567627293306*x^2 + 10387257161336*\sqrt{2}*(2*x^2 \\
& - x + 3) - 35647177985494*x + 47214805278800)/x^2) - 557284110318734302321 \\
& 9*\sqrt{31}*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 2541 \\
& 46592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*\sqrt{2}*(1348*x^8 - 2692*x^ \\
& 7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) \\
& + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490 \\
& 880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 1857945 \\
& 6)) + 1987037073032^{(1/4)}*\sqrt{45307}*\sqrt{62}*\sqrt{31}*(22653500*x^2 - 316 \\
& 9333*\sqrt{2}*(5*x^2 + 3*x + 2) + 13592100*x + 9061400)*\sqrt{3169333*\sqrt{2}}
\end{aligned}$$

```

+ 4530700)*log(113267500/2711*(1987037073032^(1/4)*sqrt(45307)*sqrt(62)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(1867*x + 1425) - 3292*x - 442)*sqrt(3169333*sqrt(2) + 4530700) + 11567627293306*x^2 + 10387257161336*sqrt(2)*(2*x^2 - x + 3) - 35647177985494*x + 47214805278800)/x^2) - 1987037073032^(1/4)*sqrt(45307)*sqrt(62)*sqrt(31)*(22653500*x^2 - 3169333*sqrt(2)*(5*x^2 + 3*x + 2) + 13592100*x + 9061400)*sqrt(3169333*sqrt(2) + 4530700)*log(-113267500/2711*(1987037073032^(1/4)*sqrt(45307)*sqrt(62)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(1867*x + 1425) - 3292*x - 442)*sqrt(3169333*sqrt(2) + 4530700) - 11567627293306*x^2 - 10387257161336*sqrt(2)*(2*x^2 - x + 3) + 35647177985494*x - 47214805278800)/x^2) + 3629874229834144*sqrt(2)*(5*x^2 + 3*x + 2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 6440099440028320*sqrt(2*x^2 - x + 3)*(13*x + 7))/(5*x^2 + 3*x + 2)

```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Unable to divide, perhaps due to rounding error
%%{174900625, [8]%%}+%%{%%{-419761500,0}:[1,0,-2]%%}, [7]%%}+%%{-685610450, [6]%%}+%%{%%{3246155600,0}:[1,0,-2]%%}, [5]%%}+%%{1574105625, [4]%%}+%%{%%{-10885814900,0}:[1,0,-2]%%}, [3]%%}+%%{-3861805800, [2]%%}+%%{%%{20372424800,0}:[1,0,-2]%%}, [1]%%}+%%{21939534400, [0]%%} / %%{50, [8]%%}+%%{%%{poly1[-120,0]:[1,0,-2]%%}, [7]%%}+%%{-196, [6]%%}+%%{%%{poly1[928,0]:[1,0,-2]%%}, [5]%%}+%%{450, [4]%%}+%%{%%{poly1[-3112,0]:[1,0,-2]%%}, [3]%%}+%%{-1104, [2]%%}+%%{%%{poly1[5824,0]:[1,0,-2]%%}, [1]%%}+%%{6272, [0]%%} Error: Bad Argument Value
```

**maple** [B] time = 0.16, size = 28185, normalized size = 121.49

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x)
```

```
[Out] result too large to display
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - x + 3)^{\frac{3}{2}}}{(5x^2 + 3x + 2)^2} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)/(5\*x^2+3\*x+2)^2,x, algorithm="maxima")

[Out] integrate((2\*x^2 - x + 3)^(3/2)/(5\*x^2 + 3\*x + 2)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(2x^2 - x + 3)^{3/2}}{(5x^2 + 3x + 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)^(3/2)/(3\*x + 5\*x^2 + 2)^2,x)

[Out] int((2\*x^2 - x + 3)^(3/2)/(3\*x + 5\*x^2 + 2)^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - x + 3)^{\frac{3}{2}}}{(5x^2 + 3x + 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2-x+3)\*\*(3/2)/(5\*x\*\*2+3\*x+2)\*\*2,x)

[Out] Integral((2\*x\*\*2 - x + 3)\*\*(3/2)/(5\*x\*\*2 + 3\*x + 2)\*\*2, x)

$$3.71 \quad \int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^3} dx$$

**Optimal.** Leaf size=223

$$\frac{(10x+3)(2x^2-x+3)^{3/2}}{62(5x^2+3x+2)^2} + \frac{3(696x+277)\sqrt{2x^2-x+3}}{3844(5x^2+3x+2)} + \frac{3\sqrt{\frac{1}{682}(366990269+259509026\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{31(366990269+259509026\sqrt{2})}}{7688}\right)}{7688}$$

[Out] 1/62\*(3+10\*x)\*(2\*x^2-x+3)^(3/2)/(5\*x^2+3\*x+2)^2+3/3844\*(277+696\*x)\*(2\*x^2-x+3)^(1/2)/(5\*x^2+3\*x+2)-3/5243216\*arctanh(1/31\*(29367+x\*(70517-49942\*2^(1/2)))-20575\*2^(1/2))\*341^(1/2)/(-366990269+259509026\*2^(1/2))^(1/2)/(2\*x^2-x+3)^(1/2))\*(-250287363458+176985155732\*2^(1/2))^(1/2)+3/5243216\*arctan(1/31\*(29367+20575\*2^(1/2)+x\*(70517+49942\*2^(1/2)))\*341^(1/2)/(366990269+259509026\*2^(1/2))^(1/2)/(2\*x^2-x+3)^(1/2))\*(250287363458+176985155732\*2^(1/2))^(1/2))

**Rubi [A]** time = 0.43, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {971, 1013, 1035, 1029, 206, 204}

$$\frac{(10x+3)(2x^2-x+3)^{3/2}}{62(5x^2+3x+2)^2} + \frac{3(696x+277)\sqrt{2x^2-x+3}}{3844(5x^2+3x+2)} + \frac{3\sqrt{\frac{1}{682}(366990269+259509026\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{31(366990269+259509026\sqrt{2})}}{7688}\right)}{7688}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^(3/2)/(2 + 3\*x + 5\*x^2)^3, x]

[Out] ((3 + 10\*x)\*(3 - x + 2\*x^2)^(3/2))/(62\*(2 + 3\*x + 5\*x^2)^2) + (3\*(277 + 696\*x)\*Sqrt[3 - x + 2\*x^2])/(3844\*(2 + 3\*x + 5\*x^2)) + (3\*Sqrt[(366990269 + 259509026\*Sqrt[2])/682]\*ArcTan[(Sqrt[11/(31\*(366990269 + 259509026\*Sqrt[2]))])\*(29367 + 20575\*Sqrt[2] + (70517 + 49942\*Sqrt[2])\*x)]/Sqrt[3 - x + 2\*x^2]])/7688 - (3\*Sqrt[(-366990269 + 259509026\*Sqrt[2])/682]\*ArcTanh[(Sqrt[11/(31\*(-366990269 + 259509026\*Sqrt[2]))])\*(29367 - 20575\*Sqrt[2] + (70517 - 49942\*Sqrt[2])\*x)]/Sqrt[3 - x + 2\*x^2]])/7688

**Rule 204**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[

a, 0] || LtQ[b, 0])

### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 971

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_)\*((d\_) + (e\_)\*(x\_) + (f\_)\*(x\_)^2)^(q\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^q)/((b^2 - 4\*a\*c)\*(p + 1)), x] - Dist[1/((b^2 - 4\*a\*c)\*(p + 1)), Int[(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^(q - 1)\*Simp[2\*c\*d\*(2\*p + 3) + b\*e\*q + (2\*b\*f\*q + 2\*c\*e\*(2\*p + q + 3))\*x + 2\*c\*f\*(2\*p + 2\*q + 3)\*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && LtQ[p, -1] && GtQ[q, 0] && !IGtQ[q, 0]

### Rule 1013

Int[((g\_) + (h\_)\*(x\_))\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_)\*((d\_) + (e\_)\*(x\_) + (f\_)\*(x\_)^2)^(q\_), x\_Symbol] := Simp[((g\*b - 2\*a\*h - (b\*h - 2\*g\*c)\*x)\*(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^q)/((b^2 - 4\*a\*c)\*(p + 1)), x] - Dist[1/((b^2 - 4\*a\*c)\*(p + 1)), Int[(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^(q - 1)\*Simp[e\*q\*(g\*b - 2\*a\*h) - d\*(b\*h - 2\*g\*c)\*(2\*p + 3) + (2\*f\*q\*(g\*b - 2\*a\*h) - e\*(b\*h - 2\*g\*c)\*(2\*p + q + 3))\*x - f\*(b\*h - 2\*g\*c)\*(2\*p + 2\*q + 3)\*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && LtQ[p, -1] && GtQ[q, 0]

### Rule 1029

Int[((g\_) + (h\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)\*Sqrt[(d\_) + (e\_)\*(x\_) + (f\_)\*(x\_)^2]), x\_Symbol] := Dist[-2\*g\*(g\*b - 2\*a\*h), Subst[Int[1/Simp[g\*(g\*b - 2\*a\*h)\*(b^2 - 4\*a\*c) - (b\*d - a\*e)\*x^2, x], x], x, Simp[g\*b - 2\*a\*h - (b\*h - 2\*g\*c)\*x, x]/Sqrt[d + e\*x + f\*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && NeQ[b\*d - a\*e, 0] && EqQ[h^2\*(b\*d - a\*e) - 2\*g\*h\*(c\*d - a\*f) + g^2\*(c\*e - b\*f), 0]

### Rule 1035

Int[((g\_) + (h\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)\*Sqrt[(d\_) + (e\_)\*(x\_) + (f\_)\*(x\_)^2]), x\_Symbol] := With[{q = Rt[(c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f), 2]}, Dist[1/(2\*q), Int[Simp[h\*(b\*d - a\*e) - g\*(c\*d - a\*f - q) - (g\*(c\*e - b\*f) - h\*(c\*d - a\*f + q))\*x, x]/((a + b\*x + c\*x^2)\*Sqrt

$[d + e*x + f*x^2]), x], x] - \text{Dist}[1/(2*q), \text{Int}[\text{Simp}[h*(b*d - a*e) - g*(c*d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*\text{Sqrt}[d + e*x + f*x^2]), x], x]] /;$  FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && NeQ[b\*d - a\*e, 0] && NegQ[b^2 - 4\*a\*c]

### Rubi steps

$$\begin{aligned} \int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^3} dx &= \frac{(3+10x)(3-x+2x^2)^{3/2}}{62(2+3x+5x^2)^2} - \frac{1}{62} \int \frac{\left(-\frac{189}{2} + 33x\right) \sqrt{3-x+2x^2}}{(2+3x+5x^2)^2} dx \\ &= \frac{(3+10x)(3-x+2x^2)^{3/2}}{62(2+3x+5x^2)^2} + \frac{3(277+696x)\sqrt{3-x+2x^2}}{3844(2+3x+5x^2)} + \frac{\int \frac{\frac{13359}{4} - 1353x}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{1922} \\ &= \frac{(3+10x)(3-x+2x^2)^{3/2}}{62(2+3x+5x^2)^2} + \frac{3(277+696x)\sqrt{3-x+2x^2}}{3844(2+3x+5x^2)} + \frac{\int \frac{-\frac{33}{4}(6257-4453\sqrt{2}) + \frac{33}{4}(2649-1353\sqrt{2})}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{42284\sqrt{2}} \\ &= \frac{(3+10x)(3-x+2x^2)^{3/2}}{62(2+3x+5x^2)^2} + \frac{3(277+696x)\sqrt{3-x+2x^2}}{3844(2+3x+5x^2)} + \frac{(99(519018052 - 366990269\sqrt{2}) + 366990269\sqrt{2})}{42284\sqrt{2}} \\ &= \frac{(3+10x)(3-x+2x^2)^{3/2}}{62(2+3x+5x^2)^2} + \frac{3(277+696x)\sqrt{3-x+2x^2}}{3844(2+3x+5x^2)} + \frac{3\sqrt{\frac{1}{682}(366990269 + 259519018052\sqrt{2})}}{42284\sqrt{2}} \end{aligned}$$

**Mathematica [C]** time = 5.34, size = 1262, normalized size = 5.66

result too large to display

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)^(3/2)/(2 + 3\*x + 5\*x^2)^3, x]

[Out] (((248000\*I)\*Sqrt[31]\*(3 - x + 2\*x^2)^(3/2))/(3\*I + Sqrt[31] + (10\*I)\*x)^2 + (744000\*(3 - x + 2\*x^2)^(3/2))/(3 - I\*Sqrt[31] + 10\*x) + ((248000\*I)\*Sqrt[31]\*(3 - x + 2\*x^2)^(3/2))/(3 + I\*Sqrt[31] + 10\*x)^2 + (744000\*(3 - x + 2\*x^2)^(3/2))/(3 + I\*Sqrt[31] + 10\*x) + (3\*I)\*Sqrt[31]\*(20\*(1199 + (98\*I)\*Sqr

$$\begin{aligned}
& t[31] - 20*(11 + (2*I)*\text{Sqrt}[31])*x)*\text{Sqrt}[3 - x + 2*x^2] + \text{Sqrt}[2]*(13453 + \\
& (4406*I)*\text{Sqrt}[31])* \text{ArcSinh}[(1 - 4*x)/\text{Sqrt}[23]] - (352*\text{Sqrt}[286 + (22*I)*\text{Sqr} \\
& t[31]]*(-69*I + 13*\text{Sqrt}[31])* \text{ArcTanh}[(63 + I*\text{Sqrt}[31] + (-22 - (4*I)*\text{Sqrt}[3 \\
& 1])*x)/(2*\text{Sqrt}[286 + (22*I)*\text{Sqrt}[31]]*\text{Sqrt}[3 - x + 2*x^2])))/(-13*I + \text{Sqrt}[ \\
& 31])) + 558*(20*(27 + (4*I)*\text{Sqrt}[31] - 20*x)*\text{Sqrt}[3 - x + 2*x^2] + \text{Sqrt}[2]* \\
& (569 + (88*I)*\text{Sqrt}[31])* \text{ArcSinh}[(1 - 4*x)/\text{Sqrt}[23]] - (4*\text{Sqrt}[286 + (22*I)* \\
& \text{Sqrt}[31]]*(-81*I + 37*\text{Sqrt}[31])* \text{ArcTanh}[(63 + I*\text{Sqrt}[31] + (-22 - (4*I)*\text{Sqr} \\
& t[31])*x)/(2*\text{Sqrt}[286 + (22*I)*\text{Sqrt}[31]]*\text{Sqrt}[3 - x + 2*x^2])))/(-13*I + \text{Sq} \\
& rt[31])) + (744*\text{Sqrt}[31]*(220*(-439 + (497*I)*\text{Sqrt}[31] + 20*(69 + (13*I)*\text{Sq} \\
& rt[31])*x)*\text{Sqrt}[3 - x + 2*x^2] + 88*\text{Sqrt}[2]*(4426 - (398*I)*\text{Sqrt}[31] + 5*(4 \\
& 7 - (281*I)*\text{Sqrt}[31])*x)* \text{ArcSinh}[(-1 + 4*x)/\text{Sqrt}[23]] + \text{Sqrt}[286 + (22*I)*\text{S} \\
& qrt[31]]*(19548 - (4904*I)*\text{Sqrt}[31] + (-23345 - (8565*I)*\text{Sqrt}[31])*x)* \text{ArcTa} \\
& nh[(63 + I*\text{Sqrt}[31] + (-22 - (4*I)*\text{Sqrt}[31])*x)/(2*\text{Sqrt}[286 + (22*I)*\text{Sqrt}[3 \\
& 1]]*\text{Sqrt}[3 - x + 2*x^2]))))/(11*(-13*I + \text{Sqrt}[31])^2*(-3*I + \text{Sqrt}[31] - (10 \\
& *I)*x)) + (744*\text{Sqrt}[31]*(220*(-439 - (497*I)*\text{Sqrt}[31] + 20*(69 - (13*I)*\text{Sqr} \\
& t[31])*x)*\text{Sqrt}[3 - x + 2*x^2] + 88*\text{Sqrt}[2]*(4426 + (398*I)*\text{Sqrt}[31] + 5*(47 \\
& + (281*I)*\text{Sqrt}[31])*x)* \text{ArcSinh}[(-1 + 4*x)/\text{Sqrt}[23]] + \text{Sqrt}[286 - (22*I)*\text{Sq} \\
& rt[31]]*(-19548 - (4904*I)*\text{Sqrt}[31] + 5*(4669 - (1713*I)*\text{Sqrt}[31])*x)* \text{ArcTa} \\
& nh[(-63 + I*\text{Sqrt}[31] + (22 - (4*I)*\text{Sqrt}[31])*x)/(2*\text{Sqrt}[286 - (22*I)*\text{Sqrt}[3 \\
& 1]]*\text{Sqrt}[3 - x + 2*x^2]))))/(11*(13*I + \text{Sqrt}[31])^2*(3*I + \text{Sqrt}[31] + (10*I \\
& )*x)) + (3*\text{Sqrt}[31]*(20*(12549 - (2473*I)*\text{Sqrt}[31] + (20*I)*(81*I + 37*\text{Sqrt} \\
& [31])*x)*\text{Sqrt}[3 - x + 2*x^2] + \text{Sqrt}[2]*(38303 - (70731*I)*\text{Sqrt}[31])* \text{ArcSinh} \\
& [(1 - 4*x)/\text{Sqrt}[23]] + (352*I)*\text{Sqrt}[286 - (22*I)*\text{Sqrt}[31]]*(69*I + 13*\text{Sqrt}[ \\
& 31])* \text{ArcTanh}[(63 - I*\text{Sqrt}[31] + (-22 + (4*I)*\text{Sqrt}[31])*x)/(2*\text{Sqrt}[286 - (22 \\
& *I)*\text{Sqrt}[31]]*\text{Sqrt}[3 - x + 2*x^2])))/(13*I + \text{Sqrt}[31]) + 558*(-20*(-27 + ( \\
& 4*I)*\text{Sqrt}[31] + 20*x)*\text{Sqrt}[3 - x + 2*x^2] + \text{Sqrt}[2]*(569 - (88*I)*\text{Sqrt}[31]) \\
& * \text{ArcSinh}[(1 - 4*x)/\text{Sqrt}[23]] - (4*\text{Sqrt}[286 - (22*I)*\text{Sqrt}[31]]*(81*I + 37*\text{Sq} \\
& rt[31])* \text{ArcTanh}[(63 - I*\text{Sqrt}[31] + (-22 + (4*I)*\text{Sqrt}[31])*x)/(2*\text{Sqrt}[286 - \\
& (22*I)*\text{Sqrt}[31]]*\text{Sqrt}[3 - x + 2*x^2])))/(13*I + \text{Sqrt}[31])))/4766560
\end{aligned}$$

**fricas** [B] time = 2.61, size = 2183, normalized size = 9.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)/(5\*x^2+3\*x+2)^3,x, algorithm="fricas")

[Out]  $-1/85773071417697924109696*(189113268*134689869150937352^{(1/4)}*\text{sqrt}(1297545$   
 $13)*\text{sqrt}(341)*\text{sqrt}(2)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*\text{sqrt}(366990269*$   
 $\text{sqrt}(2) + 519018052)*\text{arctan}(1/1067259092343193675559267622545473*(160895596$   
 $12*\text{sqrt}(129754513)*(11*134689869150937352^{(3/4)}*\text{sqrt}(341)*(38305160*x^7 - 1$   
 $47261352*x^6 + 309398878*x^5 - 495410374*x^4 + 248212864*x^3 - 117285552*x^$   
 $2 - \text{sqrt}(2)*(26988622*x^7 - 104036813*x^6 + 218448200*x^5 - 350579241*x^4 +$   
 $175844824*x^3 - 83534472*x^2 - 191303424*x + 135585792) - 271171584*x + 19$   
 $1303424) + 4022389903*134689869150937352^{(1/4)}*\text{sqrt}(341)*(2906601*x^7 - 446$

$$\begin{aligned}
& 04657*x^6 + 235604928*x^5 - 537156764*x^4 + 693706464*x^3 - 436717728*x^2 - \\
& \text{sqrt}(2)*(2050114*x^7 - 31475955*x^6 + 166375268*x^5 - 379661892*x^4 + 4905 \\
& 00864*x^3 - 309827808*x^2 - 348696576*x + 246965760) - 493931520*x + 348696 \\
& 576))*\text{sqrt}(2*x^2 - x + 3)*\text{sqrt}(366990269*\text{sqrt}(2) + 519018052) + 30296387137 \\
& 48420756426308089806504*\text{sqrt}(31)*\text{sqrt}(2)*(28180*x^8 - 254666*x^7 + 704270*x \\
& ^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - \text{sqrt}(2)*(8746*x^8 \\
& - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144 \\
& *x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*\text{sqrt}(259509026/713)*(\text{sq} \\
& \text{rt}(129754513)*(11*134689869150937352^{(3/4)}*\text{sqrt}(341)*(5980372*x^7 - 8582986 \\
& *x^6 + 27618126*x^5 - 10751392*x^4 + 12649968*x^3 + 12517632*x^2 - \text{sqrt}(2)* \\
& (4201650*x^7 - 6032009*x^6 + 19421619*x^5 - 7633552*x^4 + 9050328*x^3 + 864 \\
& 0000*x^2 - 8640000*x) - 12517632*x) + 4022389903*134689869150937352^{(1/4)}*\text{s} \\
& \text{qrt}(341)*(453599*x^7 - 5867420*x^6 + 22622900*x^5 - 29282112*x^4 + 37610208 \\
& *x^3 + 22726656*x^2 - \text{sqrt}(2)*(319303*x^7 - 4130364*x^6 + 15927060*x^5 - 20 \\
& 630592*x^4 + 26556768*x^3 + 15800832*x^2 - 15800832*x) - 22726656*x))*\text{sqrt}( \\
& 2*x^2 - x + 3)*\text{sqrt}(366990269*\text{sqrt}(2) + 519018052) + 8186887989068712800954 \\
& *\text{sqrt}(31)*\text{sqrt}(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 39 \\
& 6480*x^4 + 798336*x^3 - 3822336*x^2 - \text{sqrt}(2)*(15550*x^8 - 118051*x^7 + 244 \\
& 047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) \\
& + 3276288*x) + 372131272230396036407*\text{sqrt}(31)*(254591*x^8 - 4815126*x^7 + \\
& 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 \\
& - 15488*\text{sqrt}(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 \\
& + 2268*x^2 - 1944*x) + 144820224*x))*\text{sqrt}(-(134689869150937352^{(1/4)}*\text{sqrt}(1 \\
& 29754513)*\text{sqrt}(341)*\text{sqrt}(31)*\text{sqrt}(2*x^2 - x + 3)*(\text{sqrt}(2)*(696*x + 277) - 9 \\
& 73*x - 419)*\text{sqrt}(366990269*\text{sqrt}(2) + 519018052) - 4356437317274441*x^2 - 39 \\
& 11902897144396*\text{sqrt}(2)*(2*x^2 - x + 3) + 13424939487927359*x - 177813768052 \\
& 01800)/x^2) + 34427712656232054050298955565983*\text{sqrt}(31)*(2828123*x^8 - 9696 \\
& 916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37 \\
& 981440*x^2 - 7744*\text{sqrt}(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 155 \\
& 69*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2 \\
& 585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249 \\
& 088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 189113268*13468986915093 \\
& 7352^{(1/4)}*\text{sqrt}(129754513)*\text{sqrt}(341)*\text{sqrt}(2)*(25*x^4 + 30*x^3 + 29*x^2 + 12 \\
& *x + 4)*\text{sqrt}(366990269*\text{sqrt}(2) + 519018052)*\text{arctan}(1/1067259092343193675559 \\
& 267622545473*(16089559612*\text{sqrt}(129754513)*(11*134689869150937352^{(3/4)}*\text{sqrt} \\
& (341)*(38305160*x^7 - 147261352*x^6 + 309398878*x^5 - 495410374*x^4 + 24821 \\
& 2864*x^3 - 117285552*x^2 - \text{sqrt}(2)*(26988622*x^7 - 104036813*x^6 + 21844820 \\
& 0*x^5 - 350579241*x^4 + 175844824*x^3 - 83534472*x^2 - 191303424*x + 135585 \\
& 792) - 271171584*x + 191303424) + 4022389903*134689869150937352^{(1/4)}*\text{sqrt}( \\
& 341)*(2906601*x^7 - 44604657*x^6 + 235604928*x^5 - 537156764*x^4 + 69370646 \\
& 4*x^3 - 436717728*x^2 - \text{sqrt}(2)*(2050114*x^7 - 31475955*x^6 + 166375268*x^5 \\
& - 379661892*x^4 + 490500864*x^3 - 309827808*x^2 - 348696576*x + 246965760) \\
& - 493931520*x + 348696576))*\text{sqrt}(2*x^2 - x + 3)*\text{sqrt}(366990269*\text{sqrt}(2) + 5 \\
& 19018052) - 3029638713748420756426308089806504*\text{sqrt}(31)*\text{sqrt}(2)*(28180*x^8 \\
& - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*
\end{aligned}$$

$$\begin{aligned}
& x^2 - \sqrt{2} \cdot (8746x^8 - 102335x^7 + 396104x^6 - 783113x^5 + 1320710x^4 - 752088x^3 + 396144x^2 + 546048x - 539136) + 1154304x - 456192) - 2 \cdot \\
& \sqrt{259509026/713} \cdot (\sqrt{129754513} \cdot (11 \cdot 134689869150937352^{3/4} \cdot \sqrt{341}) \\
& \cdot (5980372x^7 - 8582986x^6 + 27618126x^5 - 10751392x^4 + 12649968x^3 + \\
& 12517632x^2 - \sqrt{2} \cdot (4201650x^7 - 6032009x^6 + 19421619x^5 - 7633552x^4 \\
& + 9050328x^3 + 8640000x^2 - 8640000x) - 12517632x) + 4022389903 \cdot 134 \\
& 689869150937352^{1/4} \cdot \sqrt{341} \cdot (453599x^7 - 5867420x^6 + 22622900x^5 - \\
& 29282112x^4 + 37610208x^3 + 22726656x^2 - \sqrt{2} \cdot (319303x^7 - 4130364x^6 \\
& + 15927060x^5 - 20630592x^4 + 26556768x^3 + 15800832x^2 - 15800832x) \\
& - 22726656x) \cdot \sqrt{2x^2 - x + 3} \cdot \sqrt{366990269 \cdot \sqrt{2} + 519018052} - \\
& 8186887989068712800954 \cdot \sqrt{31} \cdot \sqrt{2} \cdot (123408x^8 - 914152x^7 + 1578888 \\
& x^6 - 3293072x^5 + 396480x^4 + 798336x^3 - 3822336x^2 - \sqrt{2} \cdot (15550 \\
& x^8 - 118051x^7 + 244047x^6 - 707374x^5 + 1053960x^4 - 1667952x^3 + 1 \\
& 209600x^2 - 1036800x) + 3276288x) - 372131272230396036407 \cdot \sqrt{31} \cdot (2545 \\
& 91x^8 - 4815126x^7 + 32303580x^6 - 90866808x^5 + 108781920x^4 - 742193 \\
& 28x^3 - 168956928x^2 - 15488 \cdot \sqrt{2} \cdot (4x^8 - 76x^7 + 517x^6 - 1536x^5 \\
& + 2385x^4 - 3618x^3 + 2268x^2 - 1944x) + 144820224x) \cdot \sqrt{(134689869 \\
& 150937352^{1/4} \cdot \sqrt{129754513} \cdot \sqrt{341} \cdot \sqrt{31} \cdot \sqrt{2x^2 - x + 3} \cdot (\sqrt{2} \cdot (696x + 277) - 973x - 419) \cdot \sqrt{366990269 \cdot \sqrt{2} + 519018052} + 435 \\
& 6437317274441x^2 + 3911902897144396 \cdot \sqrt{2} \cdot (2x^2 - x + 3) - 134249394879 \\
& 27359x + 17781376805201800) / x^2) - 34427712656232054050298955565983 \cdot \sqrt{3} \\
& 1) \cdot (2828123x^8 - 9696916x^7 + 53385560x^6 - 142835344x^5 + 254146592x^4 \\
& - 249300096x^3 + 37981440x^2 - 7744 \cdot \sqrt{2} \cdot (1348x^8 - 2692x^7 + 9789 \\
& x^6 - 10070x^5 + 15569x^4 - 5568x^3 + 1080x^2 + 4320x - 5184) + 22306 \\
& 4064x - 94887936) / (2585191x^8 - 4661200x^7 + 14191920x^6 + 490880x^5 \\
& - 13562944x^4 + 44249088x^3 - 34615296x^2 - 24772608x + 18579456) - 3 \cdot \\
& 134689869150937352^{1/4} \cdot \sqrt{129754513} \cdot \sqrt{341} \cdot \sqrt{31} \cdot (12975451300x^4 \\
& + 15570541560x^3 + 15051523508x^2 - 366990269 \cdot \sqrt{2} \cdot (25x^4 + 30x^3 \\
& + 29x^2 + 12x + 4) + 6228216624x + 2076072208) \cdot \sqrt{366990269 \cdot \sqrt{2} + \\
& 519018052} \cdot \log(9342324936/713 \cdot (134689869150937352^{1/4} \cdot \sqrt{129754513} \cdot \sqrt{341} \cdot \sqrt{31} \cdot \sqrt{2x^2 - x + 3} \cdot (\sqrt{2} \cdot (696x + 277) - 973x - 419) \cdot \sqrt{366990269 \cdot \sqrt{2} + 519018052} + 4356437317274441x^2 + 391190289714439 \\
& 6 \cdot \sqrt{2} \cdot (2x^2 - x + 3) - 13424939487927359x + 17781376805201800) / x^2) + \\
& 3 \cdot 134689869150937352^{1/4} \cdot \sqrt{129754513} \cdot \sqrt{341} \cdot \sqrt{31} \cdot (12975451300 \\
& x^4 + 15570541560x^3 + 15051523508x^2 - 366990269 \cdot \sqrt{2} \cdot (25x^4 + 30x^3 \\
& + 29x^2 + 12x + 4) + 6228216624x + 2076072208) \cdot \sqrt{366990269 \cdot \sqrt{2} + \\
& 519018052} \cdot \log(-9342324936/713 \cdot (134689869150937352^{1/4} \cdot \sqrt{129754513} \cdot \sqrt{341} \cdot \sqrt{31} \cdot \sqrt{2x^2 - x + 3} \cdot (\sqrt{2} \cdot (696x + 277) - 973x - 419) \cdot \sqrt{366990269 \cdot \sqrt{2} + 519018052} - 4356437317274441x^2 - 39119028971 \\
& 44396 \cdot \sqrt{2} \cdot (2x^2 - x + 3) + 13424939487927359x - 17781376805201800) / x^2) - \\
& 22313494125311634784 \cdot (11680x^3 + 10171x^2 + 8343x + 2220) \cdot \sqrt{2x^2 - x + 3} / (25x^4 + 30x^3 + 29x^2 + 12x + 4)
\end{aligned}$$

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infi
nity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity
]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai
lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity
,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf
inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinity]
Evaluation time: 36.71Done
```

**maple [B]** time = 0.35, size = 81552, normalized size = 365.70

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x)
```

```
[Out] result too large to display
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - x + 3)^{\frac{3}{2}}}{(5x^2 + 3x + 2)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 - x + 3)^(3/2)/(5*x^2 + 3*x + 2)^3, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(2x^2 - x + 3)^{3/2}}{(5x^2 + 3x + 2)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(3/2)/(3*x + 5*x^2 + 2)^3,x)
```

```
[Out] int((2*x^2 - x + 3)^(3/2)/(3*x + 5*x^2 + 2)^3, x)
```



sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - x + 3)^{\frac{3}{2}}}{(5x^2 + 3x + 2)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2-x+3)\*\*(3/2)/(5\*x\*\*2+3\*x+2)\*\*3,x)

[Out] Integral((2\*x\*\*2 - x + 3)\*\*(3/2)/(5\*x\*\*2 + 3\*x + 2)\*\*3, x)

$$3.72 \quad \int (3 - x + 2x^2)^{5/2} (2 + 3x + 5x^2)^4 dx$$

**Optimal.** Leaf size=254

$$\frac{122595067(2x^2 - x + 3)^{7/2} x^2}{19169280} + \frac{112244125(2x^2 - x + 3)^{7/2} x}{122683392} + \frac{25250178739(2x^2 - x + 3)^{7/2}}{5725224960} - \frac{401135647(1 - 4x)}{335544320}$$

[Out] -9226119881/2147483648\*(1-4\*x)\*(2\*x^2-x+3)^(3/2)-401135647/335544320\*(1-4\*x)\*(2\*x^2-x+3)^(5/2)+25250178739/5725224960\*(2\*x^2-x+3)^(7/2)+112244125/122683392\*x\*(2\*x^2-x+3)^(7/2)+122595067/19169280\*x^2\*(2\*x^2-x+3)^(7/2)+23460839/532480\*x^3\*(2\*x^2-x+3)^(7/2)+3684995/39936\*x^4\*(2\*x^2-x+3)^(7/2)+1046225/9984\*x^5\*(2\*x^2-x+3)^(7/2)+13875/208\*x^6\*(2\*x^2-x+3)^(7/2)+625/28\*x^7\*(2\*x^2-x+3)^(7/2)-14641852251147/137438953472\*arcsinh(1/23\*(1-4\*x)\*23^(1/2))\*2^(1/2)-636602271789/34359738368\*(1-4\*x)\*(2\*x^2-x+3)^(1/2)

**Rubi [A]** time = 0.37, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {1661, 640, 612, 619, 215}

$$\frac{625}{28} (2x^2 - x + 3)^{7/2} x^7 + \frac{13875}{208} (2x^2 - x + 3)^{7/2} x^6 + \frac{1046225 (2x^2 - x + 3)^{7/2} x^5}{9984} + \frac{3684995 (2x^2 - x + 3)^{7/2} x^4}{39936} + \dots$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^(5/2)\*(2 + 3\*x + 5\*x^2)^4,x]

[Out] (-636602271789\*(1 - 4\*x)\*Sqrt[3 - x + 2\*x^2])/34359738368 - (9226119881\*(1 - 4\*x)\*(3 - x + 2\*x^2)^(3/2))/2147483648 - (401135647\*(1 - 4\*x)\*(3 - x + 2\*x^2)^(5/2))/335544320 + (25250178739\*(3 - x + 2\*x^2)^(7/2))/5725224960 + (112244125\*x\*(3 - x + 2\*x^2)^(7/2))/122683392 + (122595067\*x^2\*(3 - x + 2\*x^2)^(7/2))/19169280 + (23460839\*x^3\*(3 - x + 2\*x^2)^(7/2))/532480 + (3684995\*x^4\*(3 - x + 2\*x^2)^(7/2))/39936 + (1046225\*x^5\*(3 - x + 2\*x^2)^(7/2))/9984 + (13875\*x^6\*(3 - x + 2\*x^2)^(7/2))/208 + (625\*x^7\*(3 - x + 2\*x^2)^(7/2))/28 - (14641852251147\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(68719476736\*Sqrt[2])

**Rule 215**

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[Rt[b, 2]\*x]/Sqrt[a]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

**Rule 612**

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2

$\ast p + 1)), \text{Int}[(a + b*x + c*x^2)^{(p - 1)}, x], x] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ \text{IntegerQ}[4*p]$

### Rule 619

$\text{Int}[(a_. + (b_.)*(x_) + (c_.)*(x_)^2)^{(p_)}, x\_Symbol] \text{:>} \text{Dist}[1/(2*c*((-4*c)/(b^2 - 4*a*c))^{p}), \text{Subst}[\text{Int}[\text{Simp}[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; \text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{GtQ}[4*a - b^2/c, 0]$

### Rule 640

$\text{Int}[(d_. + (e_.)*(x_))*((a_. + (b_.)*(x_) + (c_.)*(x_)^2)^{(p_)}, x\_Symbol] \text{:>} \text{Simp}[(e*(a + b*x + c*x^2)^{(p + 1)})/(2*c*(p + 1)), x] + \text{Dist}[(2*c*d - b*e)/(2*c), \text{Int}[(a + b*x + c*x^2)^p, x], x] /; \text{FreeQ}[\{a, b, c, d, e, p\}, x] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0] \ \&\& \ \text{NeQ}[p, -1]$

### Rule 1661

$\text{Int}[(Pq_)*((a_. + (b_.)*(x_) + (c_.)*(x_)^2)^{(p_)}, x\_Symbol] \text{:>} \text{With}[\{q = \text{Expon}[Pq, x], e = \text{Coeff}[Pq, x, \text{Expon}[Pq, x]]\}, \text{Simp}[(e*x^{(q - 1)}*(a + b*x + c*x^2)^{(p + 1)})/(c*(q + 2*p + 1)), x] + \text{Dist}[1/(c*(q + 2*p + 1)), \text{Int}[(a + b*x + c*x^2)^p * \text{ExpandToSum}[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^{(q - 2)} - b*e*(q + p)*x^{(q - 1)} - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; \text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{PolyQ}[Pq, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ !\text{LeQ}[p, -1]$

### Rubi steps

$$\begin{aligned}
\int (3-x+2x^2)^{5/2} (2+3x+5x^2)^4 dx &= \frac{625}{28} x^7 (3-x+2x^2)^{7/2} + \frac{1}{28} \int (3-x+2x^2)^{5/2} (448+2688x+10528x^2) dx \\
&= \frac{13875}{208} x^6 (3-x+2x^2)^{7/2} + \frac{625}{28} x^7 (3-x+2x^2)^{7/2} + \frac{1}{728} \int (3-x+2x^2)^{5/2} (448+2688x+10528x^2) dx \\
&= \frac{1046225x^5 (3-x+2x^2)^{7/2}}{9984} + \frac{13875}{208} x^6 (3-x+2x^2)^{7/2} + \frac{625}{28} x^7 (3-x+2x^2)^{7/2} \\
&= \frac{3684995x^4 (3-x+2x^2)^{7/2}}{39936} + \frac{1046225x^5 (3-x+2x^2)^{7/2}}{9984} + \frac{13875}{208} x^6 (3-x+2x^2)^{7/2} \\
&= \frac{23460839x^3 (3-x+2x^2)^{7/2}}{532480} + \frac{3684995x^4 (3-x+2x^2)^{7/2}}{39936} + \frac{1046225x^5 (3-x+2x^2)^{7/2}}{9984} \\
&= \frac{122595067x^2 (3-x+2x^2)^{7/2}}{19169280} + \frac{23460839x^3 (3-x+2x^2)^{7/2}}{532480} + \frac{3684995x^4 (3-x+2x^2)^{7/2}}{39936} \\
&= \frac{112244125x (3-x+2x^2)^{7/2}}{122683392} + \frac{122595067x^2 (3-x+2x^2)^{7/2}}{19169280} + \frac{23460839x^3 (3-x+2x^2)^{7/2}}{532480} \\
&= \frac{25250178739 (3-x+2x^2)^{7/2}}{5725224960} + \frac{112244125x (3-x+2x^2)^{7/2}}{122683392} + \frac{122595067x^2 (3-x+2x^2)^{7/2}}{19169280} \\
&= -\frac{401135647(1-4x)(3-x+2x^2)^{5/2}}{335544320} + \frac{25250178739(3-x+2x^2)^{7/2}}{5725224960} \\
&= -\frac{9226119881(1-4x)(3-x+2x^2)^{3/2}}{2147483648} - \frac{401135647(1-4x)(3-x+2x^2)^{5/2}}{335544320} \\
&= -\frac{636602271789(1-4x)\sqrt{3-x+2x^2}}{34359738368} - \frac{9226119881(1-4x)(3-x+2x^2)^{3/2}}{2147483648} \\
&= -\frac{636602271789(1-4x)\sqrt{3-x+2x^2}}{34359738368} - \frac{9226119881(1-4x)(3-x+2x^2)^{3/2}}{2147483648} \\
&= -\frac{636602271789(1-4x)\sqrt{3-x+2x^2}}{34359738368} - \frac{9226119881(1-4x)(3-x+2x^2)^{3/2}}{2147483648}
\end{aligned}$$

**Mathematica** [A] time = 0.45, size = 105, normalized size = 0.41

$$4\sqrt{2x^2-x+3} (25125558681600000x^{13} + 37398427729920000x^{12} + 137233466130432000x^{11} + 20493241166000000x^{10} + 137233466130432000x^9 + 37398427729920000x^8 + 25125558681600000x^7 + 137233466130432000x^6 + 37398427729920000x^5 + 25125558681600000x^4 + 137233466130432000x^3 + 37398427729920000x^2 + 25125558681600000x + 137233466130432000)$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)^(5/2)\*(2 + 3\*x + 5\*x^2)^4,x]

[Out] (4\*Sqrt[3 - x + 2\*x^2]\*(10820567498568669 + 12071614275862524\*x + 50064174038215008\*x^2 + 142490931553577856\*x^3 + 257786732552566784\*x^4 + 405468382284161024\*x^5 + 485091164642279424\*x^6 + 530502956133122048\*x^7 + 439064558846345216\*x^8 + 363646430503501824\*x^9 + 204932411660697600\*x^10 + 137233466130432000\*x^11 + 37398427729920000\*x^12 + 25125558681600000\*x^13) - 59958384968446965\*Sqrt[2]\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/562812514467840

**fricas** [A] time = 0.81, size = 118, normalized size = 0.46

$$\frac{1}{140703128616960} (25125558681600000 x^{13} + 37398427729920000 x^{12} + 137233466130432000 x^{11} + 204932411660697600 x^{10} + 439064558846345216 x^8 + 530502956133122048 x^7 + 485091164642279424 x^6 + 405468382284161024 x^5 + 257786732552566784 x^4 + 142490931553577856 x^3 + 50064174038215008 x^2 + 12071614275862524 x + 10820567498568669) \sqrt{2x^2 - x + 3} + \frac{14641852251147}{274877906944} \sqrt{2} \log(-4\sqrt{2} \sqrt{2x^2 - x + 3}) (4x - 1) - 32x^2 + 16x - 25$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(5/2)\*(5\*x^2+3\*x+2)^4,x, algorithm="fricas")

[Out] 1/140703128616960\*(25125558681600000\*x^13 + 37398427729920000\*x^12 + 137233466130432000\*x^11 + 204932411660697600\*x^10 + 363646430503501824\*x^9 + 439064558846345216\*x^8 + 530502956133122048\*x^7 + 485091164642279424\*x^6 + 405468382284161024\*x^5 + 257786732552566784\*x^4 + 142490931553577856\*x^3 + 50064174038215008\*x^2 + 12071614275862524\*x + 10820567498568669)\*sqrt(2\*x^2 - x + 3) + 14641852251147/274877906944\*sqrt(2)\*log(-4\*sqrt(2)\*sqrt(2\*x^2 - x + 3))\*(4\*x - 1) - 32\*x^2 + 16\*x - 25)

**giac** [A] time = 0.54, size = 113, normalized size = 0.44

$$\frac{1}{140703128616960} (4 (8 (4 (16 (4 (8 (4 (32 (12 (200 (20 (240 (260 x + 387) x + 340823) x + 10179103) x + 3612502719) x + 52340574127) x + 2023708176167) x + 7401903757359) x + 49495652134297) x + 125872428004183) x + 1113210402762327) x + 1564505438694219) x + 3017903568965631) x + 10820567498568669) \sqrt{2x^2 - x + 3} - \frac{14641852251147}{137438953472} \sqrt{2} \log(-2\sqrt{2} (\sqrt{2} x - \sqrt{2x^2 - x + 3}) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(5/2)\*(5\*x^2+3\*x+2)^4,x, algorithm="giac")

[Out] 1/140703128616960\*(4\*(8\*(4\*(16\*(4\*(8\*(4\*(32\*(12\*(200\*(20\*(240\*(260\*x + 387)\*x + 340823)\*x + 10179103)\*x + 3612502719)\*x + 52340574127)\*x + 2023708176167)\*x + 7401903757359)\*x + 49495652134297)\*x + 125872428004183)\*x + 1113210402762327)\*x + 1564505438694219)\*x + 3017903568965631)\*x + 10820567498568669)\*sqrt(2\*x^2 - x + 3) - 14641852251147/137438953472\*sqrt(2)\*log(-2\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 1)

**maple** [A] time = 0.04, size = 204, normalized size = 0.80

$$\frac{625 (2x^2 - x + 3)^{\frac{7}{2}} x^7}{28} + \frac{13875 (2x^2 - x + 3)^{\frac{7}{2}} x^6}{208} + \frac{1046225 (2x^2 - x + 3)^{\frac{7}{2}} x^5}{9984} + \frac{3684995 (2x^2 - x + 3)^{\frac{7}{2}} x^4}{39936} + \frac{234125 (2x^2 - x + 3)^{\frac{7}{2}} x^3}{159744} + \frac{1170625 (2x^2 - x + 3)^{\frac{7}{2}} x^2}{79872} + \frac{5853125 (2x^2 - x + 3)^{\frac{7}{2}} x}{39936} + \frac{29265625 (2x^2 - x + 3)^{\frac{7}{2}}}{159744} - \frac{14641852251147 \sqrt{2} \log(-2\sqrt{2} (\sqrt{2} x - \sqrt{2x^2 - x + 3}) + 1)}{137438953472}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^4,x)`

[Out]  $625/28*x^7*(2*x^2-x+3)^{(7/2)}+13875/208*x^6*(2*x^2-x+3)^{(7/2)}+25250178739/5725224960*(2*x^2-x+3)^{(7/2)}+1046225/9984*x^5*(2*x^2-x+3)^{(7/2)}+3684995/39936*x^4*(2*x^2-x+3)^{(7/2)}+23460839/532480*x^3*(2*x^2-x+3)^{(7/2)}+122595067/19169280*x^2*(2*x^2-x+3)^{(7/2)}+112244125/122683392*x*(2*x^2-x+3)^{(7/2)}+14641852251147/137438953472*2^{(1/2)}*\operatorname{arcsinh}(4/23*23^{(1/2)}*(x-1/4))+636602271789/34359738368*(4*x-1)*(2*x^2-x+3)^{(1/2)}+401135647/335544320*(4*x-1)*(2*x^2-x+3)^{(5/2)}+9226119881/2147483648*(4*x-1)*(2*x^2-x+3)^{(3/2)}$

**maxima** [A] time = 1.02, size = 235, normalized size = 0.93

$$\frac{625}{28} (2x^2 - x + 3)^{\frac{7}{2}} x^7 + \frac{13875}{208} (2x^2 - x + 3)^{\frac{7}{2}} x^6 + \frac{1046225}{9984} (2x^2 - x + 3)^{\frac{7}{2}} x^5 + \frac{3684995}{39936} (2x^2 - x + 3)^{\frac{7}{2}} x^4 + \frac{23460839}{532480} (2x^2 - x + 3)^{\frac{7}{2}} x^3 + \frac{122595067}{19169280} (2x^2 - x + 3)^{\frac{7}{2}} x^2 + \frac{112244125}{122683392} (2x^2 - x + 3)^{\frac{7}{2}} x + \frac{14641852251147}{137438953472} \sqrt{2} \operatorname{arcsinh}\left(\frac{4}{23} \sqrt{23} (x - \frac{1}{4})\right) + \frac{636602271789}{34359738368} (4x - 1) \sqrt{2x^2 - x + 3} + \frac{401135647}{335544320} (4x - 1) (2x^2 - x + 3)^{\frac{5}{2}} + \frac{9226119881}{2147483648} (4x - 1) (2x^2 - x + 3)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^4,x, algorithm="maxima")`

[Out]  $625/28*(2*x^2 - x + 3)^{(7/2)}*x^7 + 13875/208*(2*x^2 - x + 3)^{(7/2)}*x^6 + 1046225/9984*(2*x^2 - x + 3)^{(7/2)}*x^5 + 3684995/39936*(2*x^2 - x + 3)^{(7/2)}*x^4 + 23460839/532480*(2*x^2 - x + 3)^{(7/2)}*x^3 + 122595067/19169280*(2*x^2 - x + 3)^{(7/2)}*x^2 + 112244125/122683392*(2*x^2 - x + 3)^{(7/2)}*x + 25250178739/5725224960*(2*x^2 - x + 3)^{(7/2)} + 401135647/83886080*(2*x^2 - x + 3)^{(5/2)}*x - 401135647/335544320*(2*x^2 - x + 3)^{(5/2)} + 9226119881/536870912*(2*x^2 - x + 3)^{(3/2)}*x - 9226119881/2147483648*(2*x^2 - x + 3)^{(3/2)} + 636602271789/8589934592*\operatorname{sqrt}(2*x^2 - x + 3)*x + 14641852251147/137438953472*\operatorname{sqrt}(2)*\operatorname{arcsinh}(1/23*\operatorname{sqrt}(23)*(4*x - 1)) - 636602271789/34359738368*\operatorname{sqrt}(2*x^2 - x + 3)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (2x^2 - x + 3)^{5/2} (5x^2 + 3x + 2)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^4,x)`

[Out] `int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^4, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2x^2 - x + 3)^{\frac{5}{2}} (5x^2 + 3x + 2)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(5/2)*(5*x**2+3*x+2)**4,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2)**4, x)
```

$$3.73 \quad \int (3 - x + 2x^2)^{5/2} (2 + 3x + 5x^2)^3 dx$$

**Optimal.** Leaf size=212

$$\frac{80483(2x^2 - x + 3)^{7/2} x^2}{9216} + \frac{509257(2x^2 - x + 3)^{7/2} x}{294912} - \frac{1696165(2x^2 - x + 3)^{7/2}}{2752512} - \frac{57915(1 - 4x)(2x^2 - x + 3)^{5/2}}{2097152}$$

[Out] -6660225/67108864\*(1-4\*x)\*(2\*x^2-x+3)^(3/2)-57915/2097152\*(1-4\*x)\*(2\*x^2-x+3)^(5/2)-1696165/2752512\*(2\*x^2-x+3)^(7/2)+509257/294912\*x\*(2\*x^2-x+3)^(7/2)+80483/9216\*x^2\*(2\*x^2-x+3)^(7/2)+3823/256\*x^3\*(2\*x^2-x+3)^(7/2)+1175/96\*x^4\*(2\*x^2-x+3)^(7/2)+125/24\*x^5\*(2\*x^2-x+3)^(7/2)-10569777075/4294967296\*arcsinh(1/23\*(1-4\*x)\*23^(1/2))\*2^(1/2)-459555525/1073741824\*(1-4\*x)\*(2\*x^2-x+3)^(1/2)

**Rubi [A]** time = 0.22, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {1661, 640, 612, 619, 215}

$$\frac{125}{24} (2x^2 - x + 3)^{7/2} x^5 + \frac{1175}{96} (2x^2 - x + 3)^{7/2} x^4 + \frac{3823}{256} (2x^2 - x + 3)^{7/2} x^3 + \frac{80483(2x^2 - x + 3)^{7/2} x^2}{9216} + \frac{509257(2x^2 - x + 3)^{7/2} x}{294912} - \frac{1696165(2x^2 - x + 3)^{7/2}}{2752512} - \frac{57915(1 - 4x)(2x^2 - x + 3)^{5/2}}{2097152}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^(5/2)\*(2 + 3\*x + 5\*x^2)^3,x]

[Out] (-459555525\*(1 - 4\*x)\*Sqrt[3 - x + 2\*x^2])/1073741824 - (6660225\*(1 - 4\*x)\*(3 - x + 2\*x^2)^(3/2))/67108864 - (57915\*(1 - 4\*x)\*(3 - x + 2\*x^2)^(5/2))/2097152 - (1696165\*(3 - x + 2\*x^2)^(7/2))/2752512 + (509257\*x\*(3 - x + 2\*x^2)^(7/2))/294912 + (80483\*x^2\*(3 - x + 2\*x^2)^(7/2))/9216 + (3823\*x^3\*(3 - x + 2\*x^2)^(7/2))/256 + (1175\*x^4\*(3 - x + 2\*x^2)^(7/2))/96 + (125\*x^5\*(3 - x + 2\*x^2)^(7/2))/24 - (10569777075\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(2147483648\*Sqrt[2])

**Rule 215**

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

**Rule 612**

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N



$eQ[b^2 - 4ac, 0] \ \&\& \ GtQ[p, 0] \ \&\& \ IntegerQ[4p]$

### Rule 619

$Int[(a_.) + (b_.)(x_) + (c_.)(x_)^2]^{(p_)}, x\_Symbol] \ :> \ Dist[1/(2c*((-4*c)/(b^2 - 4ac))^{(p)}, Subst[Int[Simp[1 - x^2/(b^2 - 4ac), x]^p, x], x, b + 2cx], x] \ ; \ FreeQ[\{a, b, c, p\}, x] \ \&\& \ GtQ[4a - b^2/c, 0]$

### Rule 640

$Int[((d_.) + (e_.)(x_))*((a_.) + (b_.)(x_) + (c_.)(x_)^2)^{(p_)}, x\_Symbol] \ :> \ Simp[(e*(a + bx + cx^2)^{(p+1)})/(2c*(p+1)), x] + Dist[(2cd - b*e)/(2c), Int[(a + bx + cx^2)^p, x], x] \ ; \ FreeQ[\{a, b, c, d, e, p\}, x] \ \&\& \ NeQ[2cd - b*e, 0] \ \&\& \ NeQ[p, -1]$

### Rule 1661

$Int[(Pq_)*((a_.) + (b_.)(x_) + (c_.)(x_)^2)^{(p_)}, x\_Symbol] \ :> \ With[\{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]\}, Simp[(e*x^{(q-1)}*(a + bx + cx^2)^{(p+1)})/(c*(q + 2p + 1)), x] + Dist[1/(c*(q + 2p + 1)), Int[(a + bx + cx^2)^p*ExpandToSum[c*(q + 2p + 1)*Pq - a*e*(q-1)*x^{(q-2)} - b*e*(q+p)*x^{(q-1)} - c*e*(q + 2p + 1)*x^q, x], x], x]] \ ; \ FreeQ[\{a, b, c, p\}, x] \ \&\& \ PolyQ[Pq, x] \ \&\& \ NeQ[b^2 - 4ac, 0] \ \&\& \ !LeQ[p, -1]$

### Rubi steps

$$\begin{aligned}
\int (3-x+2x^2)^{5/2} (2+3x+5x^2)^3 dx &= \frac{125}{24} x^5 (3-x+2x^2)^{7/2} + \frac{1}{24} \int (3-x+2x^2)^{5/2} (192+864x+2736x^2- \\
&= \frac{1175}{96} x^4 (3-x+2x^2)^{7/2} + \frac{125}{24} x^5 (3-x+2x^2)^{7/2} + \frac{1}{528} \int (3-x+2x^2)^{5/2} \\
&= \frac{3823}{256} x^3 (3-x+2x^2)^{7/2} + \frac{1175}{96} x^4 (3-x+2x^2)^{7/2} + \frac{125}{24} x^5 (3-x+2x^2)^{7/2} \\
&= \frac{80483x^2 (3-x+2x^2)^{7/2}}{9216} + \frac{3823}{256} x^3 (3-x+2x^2)^{7/2} + \frac{1175}{96} x^4 (3-x+2x^2)^{7/2} \\
&= \frac{509257x (3-x+2x^2)^{7/2}}{294912} + \frac{80483x^2 (3-x+2x^2)^{7/2}}{9216} + \frac{3823}{256} x^3 (3-x+2x^2)^{7/2} \\
&= -\frac{1696165 (3-x+2x^2)^{7/2}}{2752512} + \frac{509257x (3-x+2x^2)^{7/2}}{294912} + \frac{80483x^2 (3-x+2x^2)^{7/2}}{9216} \\
&= -\frac{57915(1-4x)(3-x+2x^2)^{5/2}}{2097152} - \frac{1696165 (3-x+2x^2)^{7/2}}{2752512} + \frac{509257x (3-x+2x^2)^{7/2}}{294912} \\
&= -\frac{6660225(1-4x)(3-x+2x^2)^{3/2}}{67108864} - \frac{57915(1-4x)(3-x+2x^2)^{5/2}}{2097152} + \frac{509257x (3-x+2x^2)^{7/2}}{294912} \\
&= -\frac{459555525(1-4x)\sqrt{3-x+2x^2}}{1073741824} - \frac{6660225(1-4x)(3-x+2x^2)^{3/2}}{67108864} + \frac{509257x (3-x+2x^2)^{7/2}}{294912} \\
&= -\frac{459555525(1-4x)\sqrt{3-x+2x^2}}{1073741824} - \frac{6660225(1-4x)(3-x+2x^2)^{3/2}}{67108864} + \frac{509257x (3-x+2x^2)^{7/2}}{294912} \\
&= -\frac{459555525(1-4x)\sqrt{3-x+2x^2}}{1073741824} - \frac{6660225(1-4x)(3-x+2x^2)^{3/2}}{67108864} + \frac{509257x (3-x+2x^2)^{7/2}}{294912}
\end{aligned}$$

**Mathematica [A]** time = 0.30, size = 95, normalized size = 0.45

$$4\sqrt{2x^2-x+3} (2818572288000x^{11} + 2395786444800x^{10} + 12943588589568x^9 + 14341894045696x^8 + 278355$$

Antiderivative was successfully verified.

[In] Integrate[(3-x+2\*x^2)^(5/2)\*(2+3\*x+5\*x^2)^3,x]

[Out]  $(4\sqrt{3-x+2x^2}) \cdot (-1191399152715 + 4560943728924x + 10060731582048x^2 + 20384824684416x^3 + 26186527209472x^4 + 34378613923840x^5 + 28347538538496x^6 + 27835561148416x^7 + 14341894045696x^8 + 12943588589568x^9 + 2395786444800x^{10} + 2818572288000x^{11}) - 665895955725\sqrt{2} \cdot \text{ArcSinh}\left(\frac{1-4x}{\sqrt{23}}\right) / 270582939648$

**fricas** [A] time = 1.00, size = 108, normalized size = 0.51

$$\frac{1}{67645734912} (2818572288000x^{11} + 2395786444800x^{10} + 12943588589568x^9 + 14341894045696x^8 + 27835561148416x^7 + 28347538538496x^6 + 34378613923840x^5 + 26186527209472x^4 + 20384824684416x^3 + 10060731582048x^2 + 4560943728924x - 1191399152715) \sqrt{2x^2 - x + 3} + 10569777075/85899345 \cdot 92 \sqrt{2} \log(-4\sqrt{2} \sqrt{2x^2 - x + 3} (4x - 1) - 32x^2 + 16x - 25)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^3,x, algorithm="fricas")`

[Out]  $1/67645734912 \cdot (2818572288000x^{11} + 2395786444800x^{10} + 12943588589568x^9 + 14341894045696x^8 + 27835561148416x^7 + 28347538538496x^6 + 34378613923840x^5 + 26186527209472x^4 + 20384824684416x^3 + 10060731582048x^2 + 4560943728924x - 1191399152715) \sqrt{2x^2 - x + 3} + 10569777075/85899345 \cdot 92 \sqrt{2} \log(-4\sqrt{2} \sqrt{2x^2 - x + 3} (4x - 1) - 32x^2 + 16x - 25)$

**giac** [A] time = 0.55, size = 103, normalized size = 0.49

$$\frac{1}{67645734912} (4(8(4(16(4(8(28(32(12(200(20x+17)x+18369)x+244241)x+15169177)x+432549111)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^3,x, algorithm="giac")`

[Out]  $1/67645734912 \cdot (4 \cdot (8 \cdot (4 \cdot (16 \cdot (4 \cdot (8 \cdot (28 \cdot (32 \cdot (12 \cdot (200 \cdot (20x + 17)x + 18369)x + 244241)x + 15169177)x + 432549111)x + 4196608145)x + 12786390239)x + 159256442847)x + 314397861939)x + 1140235932231)x - 1191399152715) \sqrt{2x^2 - x + 3} - 10569777075/4294967296 \sqrt{2} \log(-2\sqrt{2} (\sqrt{2} \sqrt{2x^2 - x + 3}) + 1)$

**maple** [A] time = 0.01, size = 170, normalized size = 0.80

$$\frac{125(2x^2-x+3)^{\frac{7}{2}}x^5}{24} + \frac{1175(2x^2-x+3)^{\frac{7}{2}}x^4}{96} + \frac{3823(2x^2-x+3)^{\frac{7}{2}}x^3}{256} + \frac{80483(2x^2-x+3)^{\frac{7}{2}}x^2}{9216} + \frac{509257(2x^2-x+3)^{\frac{7}{2}}x}{294912}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^3,x)`

[Out]  $-1696165/2752512*(2*x^2-x+3)^{(7/2)}+125/24*(2*x^2-x+3)^{(7/2)}*x^5+1175/96*(2*x^2-x+3)^{(7/2)}*x^4+3823/256*(2*x^2-x+3)^{(7/2)}*x^3+80483/9216*(2*x^2-x+3)^{(7/2)}*x^2+509257/294912*(2*x^2-x+3)^{(7/2)}*x+10569777075/4294967296*2^{(1/2)}*\arcsinh(4/23*23^{(1/2)}*(x-1/4))+459555525/1073741824*(4*x-1)*(2*x^2-x+3)^{(1/2)}+57915/2097152*(4*x-1)*(2*x^2-x+3)^{(5/2)}+6660225/67108864*(4*x-1)*(2*x^2-x+3)^{(3/2)}$

**maxima** [A] time = 1.00, size = 201, normalized size = 0.95

$$\frac{125}{24} (2x^2 - x + 3)^{\frac{7}{2}} x^5 + \frac{1175}{96} (2x^2 - x + 3)^{\frac{7}{2}} x^4 + \frac{3823}{256} (2x^2 - x + 3)^{\frac{7}{2}} x^3 + \frac{80483}{9216} (2x^2 - x + 3)^{\frac{7}{2}} x^2 + \frac{509257}{294912} (2x^2 - x + 3)^{\frac{7}{2}} x + \frac{10569777075}{4294967296} \sqrt{2} \operatorname{arcsinh}\left(\frac{4}{23} \sqrt{23} (x - \frac{1}{4})\right) + \frac{459555525}{1073741824} (4x - 1) (2x^2 - x + 3)^{\frac{1}{2}} + \frac{57915}{2097152} (4x - 1) (2x^2 - x + 3)^{\frac{5}{2}} + \frac{6660225}{67108864} (4x - 1) (2x^2 - x + 3)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^3,x, algorithm="maxima")`

[Out]  $125/24*(2*x^2 - x + 3)^{(7/2)}*x^5 + 1175/96*(2*x^2 - x + 3)^{(7/2)}*x^4 + 3823/256*(2*x^2 - x + 3)^{(7/2)}*x^3 + 80483/9216*(2*x^2 - x + 3)^{(7/2)}*x^2 + 509257/294912*(2*x^2 - x + 3)^{(7/2)}*x - 1696165/2752512*(2*x^2 - x + 3)^{(7/2)} + 57915/524288*(2*x^2 - x + 3)^{(5/2)}*x - 57915/2097152*(2*x^2 - x + 3)^{(5/2)} + 6660225/16777216*(2*x^2 - x + 3)^{(3/2)}*x - 6660225/67108864*(2*x^2 - x + 3)^{(3/2)} + 459555525/268435456*\sqrt{2*x^2 - x + 3}*x + 10569777075/4294967296*\sqrt{2}*\operatorname{arcsinh}(1/23*\sqrt{23}*(4*x - 1)) - 459555525/1073741824*\sqrt{2}*x^2 - x + 3)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (2x^2 - x + 3)^{5/2} (5x^2 + 3x + 2)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^3,x)`

[Out] `int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^3, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2x^2 - x + 3)^{\frac{5}{2}} (5x^2 + 3x + 2)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**2-x+3)**(5/2)*(5*x**2+3*x+2)**3,x)`

[Out] `Integral((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2)**3, x)`

$$3.74 \quad \int (3 - x + 2x^2)^{5/2} (2 + 3x + 5x^2)^2 dx$$

**Optimal.** Leaf size=170

$$\frac{305}{144}x^2(2x^2 - x + 3)^{7/2} + \frac{8467x(2x^2 - x + 3)^{7/2}}{4608} + \frac{23225(2x^2 - x + 3)^{7/2}}{43008} - \frac{1547(1 - 4x)(2x^2 - x + 3)^{5/2}}{98304} - \frac{177905}{98304}(2x^2 - x + 3)^{3/2}$$

[Out]  $-177905/3145728*(1-4*x)*(2*x^2-x+3)^{(3/2)}-1547/98304*(1-4*x)*(2*x^2-x+3)^{(5/2)}+23225/43008*(2*x^2-x+3)^{(7/2)}+8467/4608*x*(2*x^2-x+3)^{(7/2)}+305/144*x^2*(2*x^2-x+3)^{(7/2)}+5/4*x^3*(2*x^2-x+3)^{(7/2)}-94111745/67108864*\operatorname{arcsinh}(1/23*(1-4*x)*23^{(1/2)})*2^{(1/2)}-4091815/16777216*(1-4*x)*(2*x^2-x+3)^{(1/2)}$

**Rubi [A]** time = 0.13, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {1661, 640, 612, 619, 215}

$$\frac{5}{4}x^3(2x^2 - x + 3)^{7/2} + \frac{305}{144}x^2(2x^2 - x + 3)^{7/2} + \frac{8467x(2x^2 - x + 3)^{7/2}}{4608} + \frac{23225(2x^2 - x + 3)^{7/2}}{43008} - \frac{1547(1 - 4x)(2x^2 - x + 3)^{5/2}}{98304} - \frac{177905}{98304}(2x^2 - x + 3)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^(5/2)\*(2 + 3\*x + 5\*x^2)^2, x]

[Out]  $(-4091815*(1 - 4*x)*\operatorname{Sqrt}[3 - x + 2*x^2])/16777216 - (177905*(1 - 4*x)*(3 - x + 2*x^2)^{(3/2)})/3145728 - (1547*(1 - 4*x)*(3 - x + 2*x^2)^{(5/2)})/98304 + (23225*(3 - x + 2*x^2)^{(7/2)})/43008 + (8467*x*(3 - x + 2*x^2)^{(7/2)})/4608 + (305*x^2*(3 - x + 2*x^2)^{(7/2)})/144 + (5*x^3*(3 - x + 2*x^2)^{(7/2)})/4 - (94111745*\operatorname{ArcSinh}[(1 - 4*x)/\operatorname{Sqrt}[23]])/(33554432*\operatorname{Sqrt}[2])$

**Rule 215**

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] :> Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

**Rule 612**

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

**Rule 619**

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

### Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

### Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

### Rubi steps

$$\begin{aligned}
\int (3-x+2x^2)^{5/2} (2+3x+5x^2)^2 dx &= \frac{5}{4}x^3(3-x+2x^2)^{7/2} + \frac{1}{20} \int (3-x+2x^2)^{5/2} (80+240x+355x^2+1) \\
&= \frac{305}{144}x^2(3-x+2x^2)^{7/2} + \frac{5}{4}x^3(3-x+2x^2)^{7/2} + \frac{1}{360} \int (3-x+2x^2)^5 \\
&= \frac{8467x(3-x+2x^2)^{7/2}}{4608} + \frac{305}{144}x^2(3-x+2x^2)^{7/2} + \frac{5}{4}x^3(3-x+2x^2)^{7/2} \\
&= \frac{23225(3-x+2x^2)^{7/2}}{43008} + \frac{8467x(3-x+2x^2)^{7/2}}{4608} + \frac{305}{144}x^2(3-x+2x^2)^{7/2} \\
&= -\frac{1547(1-4x)(3-x+2x^2)^{5/2}}{98304} + \frac{23225(3-x+2x^2)^{7/2}}{43008} + \frac{8467x(3-x+2x^2)^{7/2}}{4608} \\
&= -\frac{177905(1-4x)(3-x+2x^2)^{3/2}}{3145728} - \frac{1547(1-4x)(3-x+2x^2)^{5/2}}{98304} + \frac{23225(3-x+2x^2)^{7/2}}{43008} \\
&= -\frac{4091815(1-4x)\sqrt{3-x+2x^2}}{16777216} - \frac{177905(1-4x)(3-x+2x^2)^{3/2}}{3145728} - \frac{23225(3-x+2x^2)^{7/2}}{43008} \\
&= -\frac{4091815(1-4x)\sqrt{3-x+2x^2}}{16777216} - \frac{177905(1-4x)(3-x+2x^2)^{3/2}}{3145728} - \frac{23225(3-x+2x^2)^{7/2}}{43008} \\
&= -\frac{4091815(1-4x)\sqrt{3-x+2x^2}}{16777216} - \frac{177905(1-4x)(3-x+2x^2)^{3/2}}{3145728} - \frac{23225(3-x+2x^2)^{7/2}}{43008}
\end{aligned}$$

**Mathematica** [A] time = 0.18, size = 85, normalized size = 0.50

$$4\sqrt{2x^2-x+3} (10569646080x^9 + 2055208960x^8 + 44163137536x^7 + 26401898496x^6 + 75389820928x^5 + 5727272000x^4 + 57147467776x^3 + 75389820928x^2 + 26401898496x + 44163137536) - 5929039935\sqrt{2}\operatorname{ArcSinh}\left[\frac{1-4x}{\sqrt{23}}\right]/4227858432$$

Antiderivative was successfully verified.

[In] Integrate[(3-x+2\*x^2)^(5/2)\*(2+3\*x+5\*x^2)^2,x]

[Out] (4\*Sqrt[3-x+2\*x^2]\*(14824182519+39533249652\*x+42992644128\*x^2+77872272000\*x^3+57147467776\*x^4+75389820928\*x^5+26401898496\*x^6+44163137536\*x^7+2055208960\*x^8+10569646080\*x^9)-5929039935\*Sqrt[2]\*ArcSinh[(1-4\*x)/Sqrt[23]])/4227858432

**fricas** [A] time = 0.79, size = 98, normalized size = 0.58

$$\frac{1}{1056964608} (10569646080 x^9 + 2055208960 x^8 + 44163137536 x^7 + 26401898496 x^6 + 75389820928 x^5 + 57147467776 x^4 + 77872272000 x^3 + 42992644128 x^2 + 39533249652 x + 14824182519) \sqrt{2x^2 - x + 3} + 94111745/13421772 \cdot 8 \sqrt{2} \log(-4 \sqrt{2} \sqrt{2x^2 - x + 3} (4x - 1) - 32x^2 + 16x - 25)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(5/2)\*(5\*x^2+3\*x+2)^2,x, algorithm="fricas")

[Out] 1/1056964608\*(10569646080\*x^9 + 2055208960\*x^8 + 44163137536\*x^7 + 26401898496\*x^6 + 75389820928\*x^5 + 57147467776\*x^4 + 77872272000\*x^3 + 42992644128\*x^2 + 39533249652\*x + 14824182519)\*sqrt(2\*x^2 - x + 3) + 94111745/13421772\*8\*sqrt(2)\*log(-4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25)

**giac** [A] time = 0.52, size = 93, normalized size = 0.55

$$\frac{1}{1056964608} (4 (8 (4 (16 (4 (8 (28 (160 (36x + 7)x + 24067)x + 402861)x + 9202859)x + 27904037)x + 608377125)x + 1343520129)x + 9883312413)x + 14824182519) \sqrt{2x^2 - x + 3} - 94111745/67108864 \sqrt{2} \log(-2 \sqrt{2} (\sqrt{2} x - \sqrt{2x^2 - x + 3}) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(5/2)\*(5\*x^2+3\*x+2)^2,x, algorithm="giac")

[Out] 1/1056964608\*(4\*(8\*(4\*(16\*(4\*(8\*(28\*(160\*(36\*x + 7)\*x + 24067)\*x + 402861)\*x + 9202859)\*x + 27904037)\*x + 608377125)\*x + 1343520129)\*x + 9883312413)\*x + 14824182519)\*sqrt(2\*x^2 - x + 3) - 94111745/67108864\*sqrt(2)\*log(-2\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 1)

**maple** [A] time = 0.01, size = 136, normalized size = 0.80

$$\frac{5(2x^2 - x + 3)^{\frac{7}{2}} x^3}{4} + \frac{305(2x^2 - x + 3)^{\frac{7}{2}} x^2}{144} + \frac{8467(2x^2 - x + 3)^{\frac{7}{2}} x}{4608} + \frac{94111745\sqrt{2} \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{67108864} + \frac{23225}{43008}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^(5/2)\*(5\*x^2+3\*x+2)^2,x)

[Out] 23225/43008\*(2\*x^2-x+3)^(7/2)+5/4\*(2\*x^2-x+3)^(7/2)\*x^3+305/144\*(2\*x^2-x+3)^(7/2)\*x^2+8467/4608\*(2\*x^2-x+3)^(7/2)\*x+94111745/67108864\*2^(1/2)\*arcsinh(4/23\*23^(1/2)\*(x-1/4))+4091815/16777216\*(4\*x-1)\*(2\*x^2-x+3)^(1/2)+1547/9830\*4\*(4\*x-1)\*(2\*x^2-x+3)^(5/2)+177905/3145728\*(4\*x-1)\*(2\*x^2-x+3)^(3/2)

**maxima** [A] time = 1.03, size = 167, normalized size = 0.98

$$\frac{5}{4} (2x^2 - x + 3)^{\frac{7}{2}} x^3 + \frac{305}{144} (2x^2 - x + 3)^{\frac{7}{2}} x^2 + \frac{8467}{4608} (2x^2 - x + 3)^{\frac{7}{2}} x + \frac{23225}{43008} (2x^2 - x + 3)^{\frac{7}{2}} + \frac{1547}{24576} (2x^2 - x + 3)^{\frac{5}{2}} + \frac{1547}{24576} (2x^2 - x + 3)^{\frac{3}{2}}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(5/2)\*(5\*x^2+3\*x+2)^2,x, algorithm="maxima")

[Out] 5/4\*(2\*x^2 - x + 3)^(7/2)\*x^3 + 305/144\*(2\*x^2 - x + 3)^(7/2)\*x^2 + 8467/4608\*(2\*x^2 - x + 3)^(7/2)\*x + 23225/43008\*(2\*x^2 - x + 3)^(7/2) + 1547/24576\*(2\*x^2 - x + 3)^(5/2)\*x - 1547/98304\*(2\*x^2 - x + 3)^(5/2) + 177905/786432\*(2\*x^2 - x + 3)^(3/2)\*x - 177905/3145728\*(2\*x^2 - x + 3)^(3/2) + 4091815/4194304\*sqrt(2\*x^2 - x + 3)\*x + 94111745/67108864\*sqrt(2)\*arcsinh(1/23\*sqrt(23)\*(4\*x - 1)) - 4091815/16777216\*sqrt(2\*x^2 - x + 3)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (2x^2 - x + 3)^{5/2} (5x^2 + 3x + 2)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)^(5/2)\*(3\*x + 5\*x^2 + 2)^2,x)

[Out] int((2\*x^2 - x + 3)^(5/2)\*(3\*x + 5\*x^2 + 2)^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2x^2 - x + 3)^{5/2} (5x^2 + 3x + 2)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2-x+3)\*\*(5/2)\*(5\*x\*\*2+3\*x+2)\*\*2,x)

[Out] Integral((2\*x\*\*2 - x + 3)\*\*(5/2)\*(5\*x\*\*2 + 3\*x + 2)\*\*2, x)

$$3.75 \quad \int (3 - x + 2x^2)^{5/2} (2 + 3x + 5x^2) dx$$

**Optimal.** Leaf size=128

$$\frac{5}{16}x(2x^2 - x + 3)^{7/2} + \frac{141}{448}(2x^2 - x + 3)^{7/2} - \frac{277(1 - 4x)(2x^2 - x + 3)^{5/2}}{3072} - \frac{31855(1 - 4x)(2x^2 - x + 3)^{3/2}}{98304} - \frac{732665}{1048576}$$

[Out] -31855/98304\*(1-4\*x)\*(2\*x^2-x+3)^(3/2)-277/3072\*(1-4\*x)\*(2\*x^2-x+3)^(5/2)+141/448\*(2\*x^2-x+3)^(7/2)+5/16\*x\*(2\*x^2-x+3)^(7/2)-16851295/2097152\*arcsinh(1/23\*(1-4\*x)\*23^(1/2))\*2^(1/2)-732665/524288\*(1-4\*x)\*(2\*x^2-x+3)^(1/2)

**Rubi [A]** time = 0.06, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {1661, 640, 612, 619, 215}

$$\frac{5}{16}x(2x^2 - x + 3)^{7/2} + \frac{141}{448}(2x^2 - x + 3)^{7/2} - \frac{277(1 - 4x)(2x^2 - x + 3)^{5/2}}{3072} - \frac{31855(1 - 4x)(2x^2 - x + 3)^{3/2}}{98304} - \frac{732665}{1048576}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^(5/2)\*(2 + 3\*x + 5\*x^2), x]

[Out] (-732665\*(1 - 4\*x)\*Sqrt[3 - x + 2\*x^2])/524288 - (31855\*(1 - 4\*x)\*(3 - x + 2\*x^2)^(3/2))/98304 - (277\*(1 - 4\*x)\*(3 - x + 2\*x^2)^(5/2))/3072 + (141\*(3 - x + 2\*x^2)^(7/2))/448 + (5\*x\*(3 - x + 2\*x^2)^(7/2))/16 - (16851295\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(1048576\*Sqrt[2])

Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[Rt[b, 2]\*x]/Sqrt[a]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 612

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b

+ 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

### Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

### Rule 1661

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(q + 2\*p + 1)), x] + Dist[1/(c\*(q + 2\*p + 1)), Int[(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*(q + 2\*p + 1)\*Pq - a\*e\*(q - 1)\*x^(q - 2) - b\*e\*(q + p)\*x^(q - 1) - c\*e\*(q + 2\*p + 1)\*x^q, x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

### Rubi steps

$$\begin{aligned}
 \int (3 - x + 2x^2)^{5/2} (2 + 3x + 5x^2) dx &= \frac{5}{16}x(3 - x + 2x^2)^{7/2} + \frac{1}{16} \int \left(17 + \frac{141x}{2}\right) (3 - x + 2x^2)^{5/2} dx \\
 &= \frac{141}{448} (3 - x + 2x^2)^{7/2} + \frac{5}{16}x(3 - x + 2x^2)^{7/2} + \frac{277}{128} \int (3 - x + 2x^2)^{5/2} dx \\
 &= -\frac{277(1 - 4x)(3 - x + 2x^2)^{5/2}}{3072} + \frac{141}{448} (3 - x + 2x^2)^{7/2} + \frac{5}{16}x(3 - x + 2x^2)^{5/2} \\
 &= -\frac{31855(1 - 4x)(3 - x + 2x^2)^{3/2}}{98304} - \frac{277(1 - 4x)(3 - x + 2x^2)^{5/2}}{3072} + \frac{141}{448} (3 - x + 2x^2)^{7/2} \\
 &= -\frac{732665(1 - 4x)\sqrt{3 - x + 2x^2}}{524288} - \frac{31855(1 - 4x)(3 - x + 2x^2)^{3/2}}{98304} - \frac{277}{128} (3 - x + 2x^2)^{5/2} \\
 &= -\frac{732665(1 - 4x)\sqrt{3 - x + 2x^2}}{524288} - \frac{31855(1 - 4x)(3 - x + 2x^2)^{3/2}}{98304} - \frac{277}{128} (3 - x + 2x^2)^{5/2} \\
 &= -\frac{732665(1 - 4x)\sqrt{3 - x + 2x^2}}{524288} - \frac{31855(1 - 4x)(3 - x + 2x^2)^{3/2}}{98304} - \frac{277}{128} (3 - x + 2x^2)^{5/2}
 \end{aligned}$$

**Mathematica** [A] time = 0.11, size = 75, normalized size = 0.59

$$4\sqrt{2x^2 - x + 3} (27525120x^7 - 13565952x^6 + 118808576x^5 - 1619968x^4 + 172684416x^3 + 67272352x^2 + 14895744x - 353877195)$$

---

44040192

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)^(5/2)\*(2 + 3\*x + 5\*x^2), x]

[Out] (4\*Sqrt[3 - x + 2\*x^2]\*(58536675 + 148957444\*x + 67272352\*x^2 + 172684416\*x^3 - 1619968\*x^4 + 118808576\*x^5 - 13565952\*x^6 + 27525120\*x^7) - 353877195\*Sqrt[2]\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/44040192

**fricas** [A] time = 0.70, size = 88, normalized size = 0.69

$$\frac{1}{11010048} (27525120x^7 - 13565952x^6 + 118808576x^5 - 1619968x^4 + 172684416x^3 + 67272352x^2 + 14895744x - 353877195)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(5/2)\*(5\*x^2+3\*x+2), x, algorithm="fricas")

[Out] 1/11010048\*(27525120\*x^7 - 13565952\*x^6 + 118808576\*x^5 - 1619968\*x^4 + 172684416\*x^3 + 67272352\*x^2 + 148957444\*x + 58536675)\*sqrt(2\*x^2 - x + 3) + 16851295/4194304\*sqrt(2)\*log(-4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25)

**giac** [A] time = 0.39, size = 83, normalized size = 0.65

$$\frac{1}{11010048} (4(8(4(16(4(24(140x - 69)x + 14503)x - 791)x + 1349097)x + 2102261)x + 37239361)x + 58536675)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(5/2)\*(5\*x^2+3\*x+2), x, algorithm="giac")

[Out] 1/11010048\*(4\*(8\*(4\*(16\*(4\*(24\*(140\*x - 69)\*x + 14503)\*x - 791)\*x + 1349097)\*x + 2102261)\*x + 37239361)\*x + 58536675)\*sqrt(2\*x^2 - x + 3) - 16851295/2097152\*sqrt(2)\*log(-2\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 1)

**maple** [A] time = 0.01, size = 102, normalized size = 0.80

$$\frac{5(2x^2 - x + 3)^{\frac{7}{2}} x}{16} + \frac{16851295\sqrt{2} \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x - \frac{1}{4}\right)}{23}\right)}{2097152} + \frac{141(2x^2 - x + 3)^{\frac{7}{2}}}{448} + \frac{277(4x - 1)(2x^2 - x + 3)^{\frac{5}{2}}}{3072} + \frac{31851295}{11010048}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2),x)`

[Out]  $5/16*(2*x^2-x+3)^{(7/2)}*x+141/448*(2*x^2-x+3)^{(7/2)}+277/3072*(4*x-1)*(2*x^2-x+3)^{(5/2)}+31855/98304*(4*x-1)*(2*x^2-x+3)^{(3/2)}+732665/524288*(4*x-1)*(2*x^2-x+3)^{(1/2)}+16851295/2097152*2^{(1/2)}*\operatorname{arcsinh}(4/23*23^{(1/2)}*(x-1/4))$

**maxima** [A] time = 0.97, size = 133, normalized size = 1.04

$$\frac{5}{16} (2x^2 - x + 3)^{\frac{7}{2}} x + \frac{141}{448} (2x^2 - x + 3)^{\frac{7}{2}} + \frac{277}{768} (2x^2 - x + 3)^{\frac{5}{2}} x - \frac{277}{3072} (2x^2 - x + 3)^{\frac{5}{2}} + \frac{31855}{24576} (2x^2 - x + 3)^{\frac{3}{2}} x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2),x, algorithm="maxima")`

[Out]  $5/16*(2*x^2 - x + 3)^{(7/2)}*x + 141/448*(2*x^2 - x + 3)^{(7/2)} + 277/768*(2*x^2 - x + 3)^{(5/2)}*x - 277/3072*(2*x^2 - x + 3)^{(5/2)} + 31855/24576*(2*x^2 - x + 3)^{(3/2)}*x - 31855/98304*(2*x^2 - x + 3)^{(3/2)} + 732665/131072*\operatorname{sqrt}(2*x^2 - x + 3)*x + 16851295/2097152*\operatorname{sqrt}(2)*\operatorname{arcsinh}(1/23*\operatorname{sqrt}(23)*(4*x - 1)) - 732665/524288*\operatorname{sqrt}(2*x^2 - x + 3)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (2x^2 - x + 3)^{5/2} (5x^2 + 3x + 2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2),x)`

[Out] `int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2x^2 - x + 3)^{\frac{5}{2}} (5x^2 + 3x + 2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**2-x+3)**(5/2)*(5*x**2+3*x+2),x)`

[Out] `Integral((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2), x)`

$$3.76 \quad \int \frac{(3-x+2x^2)^{5/2}}{2+3x+5x^2} dx$$

Optimal. Leaf size=222

$$-\frac{1}{600}(103-60x)(2x^2-x+3)^{3/2} - \frac{(226249-99620x)\sqrt{2x^2-x+3}}{80000} - \frac{121\sqrt{\frac{11}{31}(25000\sqrt{2}-15457)} \tan^{-1}\left(\frac{\sqrt{62(25000\sqrt{2}-15457)}}{3125}\right)}{3125}$$

[Out]  $-1/600*(103-60*x)*(2*x^2-x+3)^{(3/2)}-7216203/1600000*\operatorname{arcsinh}(1/23*(1-4*x)*23^{(1/2)})*2^{(1/2)}-1/80000*(226249-99620*x)*(2*x^2-x+3)^{(1/2)}-121/96875*\arctan(1/62*(196-443*2^{(1/2)}-x*(690+247*2^{(1/2)}))*682^{(1/2)}/(-15457+25000*2^{(1/2)})^{(1/2)})/(2*x^2-x+3)^{(1/2)}*(-5270837+8525000*2^{(1/2)})^{(1/2)}+121/96875*\operatorname{arctanh}(1/62*(196-x*(690-247*2^{(1/2)})+443*2^{(1/2)})*682^{(1/2)}/(15457+25000*2^{(1/2)})^{(1/2)})/(2*x^2-x+3)^{(1/2)}*(5270837+8525000*2^{(1/2)})^{(1/2)}$

**Rubi [A]** time = 0.54, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {977, 1066, 1076, 619, 215, 1035, 1029, 206, 204}

$$-\frac{1}{600}(103-60x)(2x^2-x+3)^{3/2} - \frac{(226249-99620x)\sqrt{2x^2-x+3}}{80000} - \frac{121\sqrt{\frac{11}{31}(25000\sqrt{2}-15457)} \tan^{-1}\left(\frac{\sqrt{62(25000\sqrt{2}-15457)}}{3125}\right)}{3125}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^(5/2)/(2 + 3\*x + 5\*x^2), x]

[Out]  $-((226249-99620*x)*\operatorname{Sqrt}[3-x+2*x^2])/80000 - ((103-60*x)*(3-x+2*x^2)^{(3/2)})/600 - (7216203*\operatorname{ArcSinh}[(1-4*x)/\operatorname{Sqrt}[23]])/(800000*\operatorname{Sqrt}[2]) - (121*\operatorname{Sqrt}[(11*(-15457+25000*\operatorname{Sqrt}[2]))/31]*\operatorname{ArcTan}[(\operatorname{Sqrt}[11/(62*(-15457+25000*\operatorname{Sqrt}[2])))]*(196-443*\operatorname{Sqrt}[2]-(690+247*\operatorname{Sqrt}[2])*x))/\operatorname{Sqrt}[3-x+2*x^2])/3125 + (121*\operatorname{Sqrt}[(11*(15457+25000*\operatorname{Sqrt}[2]))/31]*\operatorname{ArcTanh}[(\operatorname{Sqrt}[11/(62*(15457+25000*\operatorname{Sqrt}[2])))]*(196+443*\operatorname{Sqrt}[2]-(690-247*\operatorname{Sqrt}[2])*x))/\operatorname{Sqrt}[3-x+2*x^2])/3125$

Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 215

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 619

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^(p)), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

Rule 977

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_)\*((d\_) + (e\_)\*(x\_) + (f\_)\*(x\_)^2)^(q\_), x\_Symbol] := Simp[((b\*f\*(3\*p + 2\*q) - c\*e\*(2\*p + q) + 2\*c\*f\*(p + q)\*x)\*(a + b\*x + c\*x^2)^(p - 1)\*(d + e\*x + f\*x^2)^(q + 1))/(2\*f^2\*(p + q)\*(2\*p + 2\*q + 1)), x] - Dist[1/(2\*f^2\*(p + q)\*(2\*p + 2\*q + 1)), Int[(a + b\*x + c\*x^2)^(p - 2)\*(d + e\*x + f\*x^2)^q\*Simp[(b\*d - a\*e)\*(c\*e - b\*f)\*(1 - p)\*(2\*p + q) - (p + q)\*(b^2\*d\*f\*(1 - p) - a\*(f\*(b\*e - 2\*a\*f)\*(2\*p + 2\*q + 1) + c\*(2\*d\*f - e^2\*(2\*p + q)))] + (2\*(c\*d - a\*f)\*(c\*e - b\*f)\*(1 - p)\*(2\*p + q) - (p + q)\*((b^2 - 4\*a\*c)\*e\*f\*(1 - p) + b\*(c\*(e^2 - 4\*d\*f)\*(2\*p + q) + f\*(2\*c\*d - b\*e + 2\*a\*f)\*(2\*p + 2\*q + 1)))]\*x + ((c\*e - b\*f)^2\*(1 - p)\*p + c\*(p + q)\*(f\*(b\*e - 2\*a\*f)\*(4\*p + 2\*q - 1) - c\*(2\*d\*f\*(1 - 2\*p) + e^2\*(3\*p + q - 1)))]\*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2\*p + 2\*q + 1, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]

Rule 1029

Int[((g\_) + (h\_)\*(x\_))/(((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)\*Sqrt[(d\_) + (e\_)\*(x\_) + (f\_)\*(x\_)^2]), x\_Symbol] := Dist[-2\*g\*(g\*b - 2\*a\*h), Subst[Int[1/Simp[g\*(g\*b - 2\*a\*h)\*(b^2 - 4\*a\*c) - (b\*d - a\*e)\*x^2, x], x], x, Simp[g\*b - 2\*a\*h - (b\*h - 2\*g\*c)\*x, x]/Sqrt[d + e\*x + f\*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && NeQ[b\*d - a\*e, 0] && EqQ[h^2\*(b\*d - a\*e) - 2\*g\*h\*(c\*d - a\*f) + g^2\*(c\*e - b\*f), 0]

Rule 1035

```

Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d
- a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]

```

### Rule 1066

```

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((B*c*f*(2*
p + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a +
b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x
^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) -
c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f
- e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)
*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e
*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e +
2*A*f)*(2*p + 2*q + 3)))]*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C
e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^
2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3)))]*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]

```

### Rule 1076

```

Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]

```

### Rubi steps



$$\begin{aligned}
\int \frac{(3-x+2x^2)^{5/2}}{2+3x+5x^2} dx &= -\frac{1}{600}(103-60x)(3-x+2x^2)^{3/2} - \frac{1}{300} \int \frac{\left(-\frac{4731}{2} + \frac{6135x}{4} - \frac{14943x^2}{4}\right) \sqrt{3-x+2x^2}}{2+3x+5x^2} dx \\
&= -\frac{(226249-99620x)\sqrt{3-x+2x^2}}{80000} - \frac{1}{600}(103-60x)(3-x+2x^2)^{3/2} + \frac{\int \frac{\frac{3205293}{8} - \frac{11339}{16}}{\sqrt{3-x+2x^2}}}{30} \\
&= -\frac{(226249-99620x)\sqrt{3-x+2x^2}}{80000} - \frac{1}{600}(103-60x)(3-x+2x^2)^{3/2} + \frac{\int \frac{-702768-76}{\sqrt{3-x+2x^2}(2+3x+5x^2)}}{15000} \\
&= -\frac{(226249-99620x)\sqrt{3-x+2x^2}}{80000} - \frac{1}{600}(103-60x)(3-x+2x^2)^{3/2} - \frac{\int \frac{-702768(108-)}{\sqrt{3-x+2x^2}}}{15000} \\
&= -\frac{(226249-99620x)\sqrt{3-x+2x^2}}{80000} - \frac{1}{600}(103-60x)(3-x+2x^2)^{3/2} - \frac{7216203 \sinh^{-1}\left(\frac{3-x+2x^2}{\sqrt{3-x+2x^2}}\right)}{800000} \\
&= -\frac{(226249-99620x)\sqrt{3-x+2x^2}}{80000} - \frac{1}{600}(103-60x)(3-x+2x^2)^{3/2} - \frac{7216203 \sinh^{-1}\left(\frac{3-x+2x^2}{\sqrt{3-x+2x^2}}\right)}{800000}
\end{aligned}$$

**Mathematica [C]** time = 1.05, size = 229, normalized size = 1.03

$$46464\sqrt{286+22i\sqrt{31}}(403-69i\sqrt{31})\tanh^{-1}\left(\frac{(-22-4i\sqrt{31})x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)-46464i\sqrt{286-22i\sqrt{31}}(69\sqrt{31}-403)$$

Antiderivative was successfully verified.

[In] Integrate[(3-x+2\*x^2)^(5/2)/(2+3\*x+5\*x^2),x]

[Out] (620\*Sqrt[3-x+2\*x^2]\*(-802347+412060\*x-106400\*x^2+48000\*x^3)+671106879\*Sqrt[2]\*ArcSinh[(-1+4\*x)/Sqrt[23]]+46464\*Sqrt[286+(22\*I)\*Sqrt[31]]\*(403-(69\*I)\*Sqrt[31])\*ArcTanh[(63+I\*Sqrt[31]+(-22-(4\*I)\*Sqrt[31])\*x)/(2\*Sqrt[286+(22\*I)\*Sqrt[31]]\*Sqrt[3-x+2\*x^2])]-46464\*I\*Sqrt[286-(22\*I)\*Sqrt[31]]\*(-403\*I+69\*Sqrt[31])\*ArcTanh[(-63+I\*Sqrt[31]+(22-(4\*I)\*Sqrt[31])\*x)/(2\*Sqrt[286-(22\*I)\*Sqrt[31]]\*Sqrt[3-x+2\*x^2])])/148800000

fricas [B] time = 2.25, size = 2010, normalized size = 9.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(5/2)/(5\*x^2+3\*x+2),x, algorithm="fricas")

[Out] 121/96875000\*6050^(1/4)\*sqrt(31)\*sqrt(2)\*sqrt(-772850000\*sqrt(2) + 2500000000)\*arctan(1/254496437500\*(722441500000\*sqrt(31)\*sqrt(2)\*(28180\*x^8 - 254666\*x^7 + 704270\*x^6 - 1385256\*x^5 + 1549144\*x^4 - 642048\*x^3 - 98496\*x^2 - sqrt(2)\*(8746\*x^8 - 102335\*x^7 + 396104\*x^6 - 783113\*x^5 + 1320710\*x^4 - 752088\*x^3 + 396144\*x^2 + 546048\*x - 539136) + 1154304\*x - 456192) + 2300\*(4\*6050^(3/4)\*sqrt(31)\*(35898\*x^7 - 441939\*x^6 + 782418\*x^5 - 2117233\*x^4 + 1272680\*x^3 - 1081800\*x^2 - sqrt(2)\*(173702\*x^7 - 453907\*x^6 + 1056481\*x^5 - 1083344\*x^4 + 393672\*x^3 + 152064\*x^2 - 1043712\*x + 259200) - 518400\*x + 1043712) + 5\*6050^(1/4)\*sqrt(31)\*(317294\*x^7 - 5870544\*x^6 + 38857480\*x^5 - 111531424\*x^4 + 156761280\*x^3 - 168192000\*x^2 - sqrt(2)\*(712757\*x^7 - 10233303\*x^6 + 48529768\*x^5 - 94500260\*x^4 + 113086944\*x^3 - 22282848\*x^2 - 106417152\*x + 37407744) - 74815488\*x + 106417152))\*sqrt(2\*x^2 - x + 3)\*sqrt(-772850000\*sqrt(2) + 2500000000) - sqrt(10/5711)\*(314105000\*sqrt(31)\*sqrt(2)\*(123408\*x^8 - 914152\*x^7 + 1578888\*x^6 - 3293072\*x^5 + 396480\*x^4 + 798336\*x^3 - 3822336\*x^2 - sqrt(2)\*(15550\*x^8 - 118051\*x^7 + 244047\*x^6 - 707374\*x^5 + 1053960\*x^4 - 1667952\*x^3 + 1209600\*x^2 - 1036800\*x) + 3276288\*x) - (4\*6050^(3/4)\*sqrt(31)\*(167914\*x^7 - 195429\*x^6 + 331239\*x^5 + 1685680\*x^4 - 3693960\*x^3 + 4195584\*x^2 + 22\*sqrt(2)\*(37846\*x^7 - 52859\*x^6 + 160569\*x^5 - 4464\*x^4 - 49464\*x^3 + 202176\*x^2 - 202176\*x) - 4195584\*x) - 5\*6050^(1/4)\*sqrt(31)\*(160956\*x^7 - 2232176\*x^6 + 11218640\*x^5 - 38096640\*x^4 + 139374720\*x^3 - 296027136\*x^2 - sqrt(2)\*(3246491\*x^7 - 41888524\*x^6 + 159670660\*x^5 - 190080576\*x^4 + 180496224\*x^3 + 376648704\*x^2 - 376648704\*x) + 296027136\*x))\*sqrt(2\*x^2 - x + 3)\*sqrt(-772850000\*sqrt(2) + 2500000000) + 14277500\*sqrt(31)\*(254591\*x^8 - 4815126\*x^7 + 32303580\*x^6 - 90866808\*x^5 + 108781920\*x^4 - 74219328\*x^3 - 168956928\*x^2 - 15488\*sqrt(2)\*(4\*x^8 - 76\*x^7 + 517\*x^6 - 1536\*x^5 + 2385\*x^4 - 3618\*x^3 + 2268\*x^2 - 1944\*x) + 144820224\*x))\*sqrt((6050^(1/4)\*sqrt(2\*x^2 - x + 3)\*(sqrt(2)\*(163\*x - 725) + 562\*x - 888)\*sqrt(-772850000\*sqrt(2) + 2500000000) + 139919500\*x^2 + 125642000\*sqrt(2)\*(2\*x^2 - x + 3) - 431180500\*x + 571100000)/x^2) + 8209562500\*sqrt(31)\*(2828123\*x^8 - 9696916\*x^7 + 53385560\*x^6 - 142835344\*x^5 + 254146592\*x^4 - 249300096\*x^3 + 37981440\*x^2 - 7744\*sqrt(2)\*(1348\*x^8 - 2692\*x^7 + 9789\*x^6 - 10070\*x^5 + 15569\*x^4 - 5568\*x^3 + 1080\*x^2 + 4320\*x - 5184) + 223064064\*x - 94887936))/(2585191\*x^8 - 4661200\*x^7 + 14191920\*x^6 + 490880\*x^5 - 13562944\*x^4 + 44249088\*x^3 - 34615296\*x^2 - 24772608\*x + 18579456)) + 121/96875000\*6050^(1/4)\*sqrt(31)\*sqrt(2)\*sqrt(-772850000\*sqrt(2) + 2500000000)\*arctan(-1/254496437500\*(722441500000\*sqrt(31)\*sqrt(2)\*(28180\*x^8 - 254666\*x^7 + 704270\*x^6 - 1385256\*x^5 + 1549144\*x^4 - 642048\*x^3 - 98496\*x^2 - sqrt(2)\*(8746\*x

$$\begin{aligned}
&^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 3961 \\
&44*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2300*(4*6050^{(3/4)}*\sqrt{ \\
&(31)*(35898*x^7 - 441939*x^6 + 782418*x^5 - 2117233*x^4 + 1272680*x^3 - 108 \\
&1800*x^2 - \sqrt{2}*(173702*x^7 - 453907*x^6 + 1056481*x^5 - 1083344*x^4 + 3 \\
&93672*x^3 + 152064*x^2 - 1043712*x + 259200) - 518400*x + 1043712) + 5*6050 \\
&^{(1/4)}*\sqrt{31}*(317294*x^7 - 5870544*x^6 + 38857480*x^5 - 111531424*x^4 + \\
&156761280*x^3 - 168192000*x^2 - \sqrt{2}*(712757*x^7 - 10233303*x^6 + 485297 \\
&68*x^5 - 94500260*x^4 + 113086944*x^3 - 22282848*x^2 - 106417152*x + 374077 \\
&44) - 74815488*x + 106417152))*\sqrt{2*x^2 - x + 3}*\sqrt{-772850000*\sqrt{2} \\
&+ 2500000000) - \sqrt{10/5711}*(314105000*\sqrt{31}*\sqrt{2}*(123408*x^8 - 914 \\
&152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 \\
&- \sqrt{2}*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 \\
&- 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) + (4*6050^{(3/4)}*\sqrt{ \\
&31}*(167914*x^7 - 195429*x^6 + 331239*x^5 + 1685680*x^4 - 3693960*x^3 + 419 \\
&5584*x^2 + 22*\sqrt{2}*(37846*x^7 - 52859*x^6 + 160569*x^5 - 4464*x^4 - 4946 \\
&4*x^3 + 202176*x^2 - 202176*x) - 4195584*x) - 5*6050^{(1/4)}*\sqrt{31}*(160956 \\
&*x^7 - 2232176*x^6 + 11218640*x^5 - 38096640*x^4 + 139374720*x^3 - 29602713 \\
&6*x^2 - \sqrt{2}*(3246491*x^7 - 41888524*x^6 + 159670660*x^5 - 190080576*x^4 \\
&+ 180496224*x^3 + 376648704*x^2 - 376648704*x) + 296027136*x))*\sqrt{2*x^2 \\
&- x + 3}*\sqrt{-772850000*\sqrt{2} + 2500000000) + 14277500*\sqrt{31}*(254591* \\
&x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328* \\
&x^3 - 168956928*x^2 - 15488*\sqrt{2}*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + \\
&2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*\sqrt{-(6050^{(1/4)}* \\
&\sqrt{2*x^2 - x + 3}*(\sqrt{2}*(163*x - 725) + 562*x - 888))*\sqrt{-772850000*s \\
&qrt{2} + 2500000000) - 139919500*x^2 - 125642000*\sqrt{2}*(2*x^2 - x + 3) + \\
&431180500*x - 571100000)/x^2) + 8209562500*\sqrt{31}*(2828123*x^8 - 9696916* \\
&x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 379814 \\
&40*x^2 - 7744*\sqrt{2}*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x \\
&^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(25851 \\
&91*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088* \\
&x^3 - 34615296*x^2 - 24772608*x + 18579456)) - 121/2213012500000*6050^{(1/4)} \\
&*(15457*\sqrt{2} + 50000)*\sqrt{-772850000*\sqrt{2} + 2500000000)*\log(91506250 \\
&00/5711*(6050^{(1/4)}*\sqrt{2*x^2 - x + 3}*(\sqrt{2}*(163*x - 725) + 562*x - 88 \\
&8))*\sqrt{-772850000*\sqrt{2} + 2500000000) + 139919500*x^2 + 125642000*\sqrt{2} \\
&)*(2*x^2 - x + 3) - 431180500*x + 571100000)/x^2) + 121/2213012500000*6050^{ \\
&(1/4)}*(15457*\sqrt{2} + 50000)*\sqrt{-772850000*\sqrt{2} + 2500000000)*\log(-91 \\
&50625000/5711*(6050^{(1/4)}*\sqrt{2*x^2 - x + 3}*(\sqrt{2}*(163*x - 725) + 562* \\
&x - 888))*\sqrt{-772850000*\sqrt{2} + 2500000000) - 139919500*x^2 - 125642000* \\
&\sqrt{2}*(2*x^2 - x + 3) + 431180500*x - 571100000)/x^2) + 1/240000*(48000*x \\
&^3 - 106400*x^2 + 412060*x - 802347)*\sqrt{2*x^2 - x + 3) + 7216203/3200000* \\
&\sqrt{2}*\log(-4*\sqrt{2}*\sqrt{2*x^2 - x + 3}*(4*x - 1) - 32*x^2 + 16*x - 25)
\end{aligned}$$

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]root error [1.0,infinity,infinity,infinity,infi
nity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity
]root error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai
lure for[-1.0,infinity,infinity,infinity,infinity]root error [1.0,infinity
,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf
inity,infinity,infinity]root error [1.0,infinity,infinity,infinity,infinity]
Evaluation time: 15.78Done
```

**maple [B]** time = 0.05, size = 4860, normalized size = 21.89

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x)
```

```
[Out] 1/5*x^3*(2*x^2-x+3)^(1/2)-133/300*x^2*(2*x^2-x+3)^(1/2)+20603/12000*x*(2*x^
2-x+3)^(1/2)-267449/80000*(2*x^2-x+3)^(1/2)+7216203/1600000*2^(1/2)*arcsinh
(4/23*23^(1/2)*(x-1/4))+4/33034375*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^
(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^^(1/2)*2^(1/2)*(75195*2^
(1/2)*(-8866+6820*2^(1/2))^^(1/2)*(-775687+549362*2^(1/2))^^(1/2)*arctan(1/11
692487*(-775687+549362*2^(1/2))^^(1/2)*(-23*(8+3*2^(1/2)))*(-23*(x+2^(1/2)-1)
^2/(-x+2^(1/2)+1)^2+24*2^(1/2)-41))^^(1/2)*(6485*2^(1/2)*(x+2^(1/2)-1)^2/(-x
+2^(1/2)+1)^2+10368*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+22379*2^(1/2)+32016)/(
23*(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)^4+82*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23)
*(x+2^(1/2)-1)/(-x+2^(1/2)+1)*(8+3*2^(1/2)))+106294*(-8866+6820*2^(1/2))^^(1
/2)*(-775687+549362*2^(1/2))^^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2)
))^^(1/2)*(-23*(8+3*2^(1/2)))*(-23*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24*2^(1/2)
-41))^^(1/2)*(6485*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+10368*(x+2^(1/2)
-1)^2/(-x+2^(1/2)+1)^2+22379*2^(1/2)+32016)/(23*(x+2^(1/2)-1)^4/(-x+2^(1/2)
+1)^4+82*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23)*(x+2^(1/2)-1)/(-x+2^(1/2)+1)
*(8+3*2^(1/2)))+108099046*arctanh(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+
3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^^(1/2)/(-8866+6820*2
^(1/2))^^(1/2))*2^(1/2)-158290154*arctanh(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)
+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^^(1/2)/(-8866
+6820*2^(1/2))^^(1/2))/((8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^
(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))/(1+(x+2^(1/2)-1)/(-x+2^(1/2)+1))^2
)^^(1/2)/(1+(x+2^(1/2)-1)/(-x+2^(1/2)+1))/(8+3*2^(1/2))/(-8866+6820*2^(1/2)
)^^(1/2)+6/6606875*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)
)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^^(1/2)*2^(1/2)*(10915*2^(1/2)*(-8866+6820*
```





$$\begin{aligned}
 & -1)^2/(-x+2^{(1/2)}+1)^2+3*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+8-3*2^{(1/2)} \\
 & ^{(1/2)}/(-8866+6820*2^{(1/2)})^{(1/2)})*2^{(1/2)}-63426*\operatorname{arctanh}(31/2*(8*(x+2^{(1/2)} \\
 & -1)^2/(-x+2^{(1/2)}+1)^2+3*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+8-3*2^{(1/2)} \\
 & ^{(1/2)})^{(1/2)}/(-8866+6820*2^{(1/2)})^{(1/2)}))/((8*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+3*2^{(1/2)} \\
 & *(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+8-3*2^{(1/2)})/(1+(x+2^{(1/2)}-1)/(-x+2^{(1/2)}+1))^{(1/2)}/(1+(x+2^{(1/2)}-1)/(-x+2^{(1/2)}+1))^{(1/2)}/(8+3*2^{(1/2)})/ \\
 & (-8866+6820*2^{(1/2)})^{(1/2)}+27/21142*(8*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+3*2^{(1/2)} \\
 & *(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+8-3*2^{(1/2)})^{(1/2)}*2^{(1/2)}*(369*2^{(1/2)} \\
 & *(-8866+6820*2^{(1/2)})^{(1/2)}*(-775687+549362*2^{(1/2)})^{(1/2)}*\operatorname{arctan}(1/116 \\
 & 92487*(-775687+549362*2^{(1/2)})^{(1/2)}*(-23*(8+3*2^{(1/2)})*(-23*(x+2^{(1/2)}-1)^2/ \\
 & (-x+2^{(1/2)}+1)^2+24*2^{(1/2)}-41))^{(1/2)}*(6485*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+10368*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+22379*2^{(1/2)}+32016)/(2 \\
 & 3*(x+2^{(1/2)}-1)^4/(-x+2^{(1/2)}+1)^4+82*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+23)* \\
 & (x+2^{(1/2)}-1)/(-x+2^{(1/2)}+1)*(8+3*2^{(1/2)})))+520*(-8866+6820*2^{(1/2)})^{(1/2)}* \\
 & (-775687+549362*2^{(1/2)})^{(1/2)}*\operatorname{arctan}(1/11692487*(-775687+549362*2^{(1/2)})^{(1/2)} \\
 & *(-23*(8+3*2^{(1/2)})*(-23*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+24*2^{(1/2)}-41))^{(1/2)} \\
 & *(6485*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+10368*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+22379*2^{(1/2)}+32016)/(23*(x+2^{(1/2)}-1)^4/(-x+2^{(1/2)}+1)^4+82*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+23)*(x+2^{(1/2)}-1)/(-x+2^{(1/2)}+1)*(8+3*2^{(1/2)})))+465124*\operatorname{arctanh}(31/2*(8*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+3*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+8-3*2^{(1/2)})^{(1/2)}/(-8866+6820*2^{(1/2)})^{(1/2)})*2^{(1/2)}-866822*\operatorname{arctanh}(31/2*(8*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+3*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+8-3*2^{(1/2)})^{(1/2)}/(-8866+6820*2^{(1/2)})^{(1/2)}))/((8*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+3*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+8-3*2^{(1/2)})/(1+(x+2^{(1/2)}-1)/(-x+2^{(1/2)}+1))^{(1/2)}/(1+(x+2^{(1/2)}-1)/(-x+2^{(1/2)}+1))^{(1/2)}/(8+3*2^{(1/2)})/(-8866+6820*2^{(1/2)})^{(1/2)}
 \end{aligned}$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - x + 3)^{5/2}}{5x^2 + 3x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(5/2)/(5\*x^2+3\*x+2),x, algorithm="maxima")

[Out] integrate((2\*x^2 - x + 3)^(5/2)/(5\*x^2 + 3\*x + 2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(2x^2 - x + 3)^{5/2}}{5x^2 + 3x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(5/2)/(3*x + 5*x^2 + 2), x)
```

```
[Out] int((2*x^2 - x + 3)^(5/2)/(3*x + 5*x^2 + 2), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - x + 3)^{\frac{5}{2}}}{5x^2 + 3x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(5/2)/(5*x**2+3*x+2), x)
```

```
[Out] Integral((2*x**2 - x + 3)**(5/2)/(5*x**2 + 3*x + 2), x)
```



$$3.77 \quad \int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^2} dx$$

**Optimal.** Leaf size=255

$$\frac{(10x+3)(2x^2-x+3)^{5/2}}{31(5x^2+3x+2)} + \frac{4}{155}(4-5x)(2x^2-x+3)^{3/2} - \frac{(2240x+1277)\sqrt{2x^2-x+3}}{7750} + \frac{11\sqrt{\frac{11}{31}(224510383+1)}}{31}$$

[Out] 4/155\*(4-5\*x)\*(2\*x^2-x+3)^(3/2)+1/31\*(3+10\*x)\*(2\*x^2-x+3)^(5/2)/(5\*x^2+3\*x+2)-4799/5000\*arcsinh(1/23\*(1-4\*x)\*23^(1/2))\*2^(1/2)-1/7750\*(1277+2240\*x)\*(2\*x^2-x+3)^(1/2)-11/1201250\*arctanh(1/62\*(21136+x\*(87710-54423\*2^(1/2)))-33287\*2^(1/2))\*682^(1/2)/(-224510383+194487500\*2^(1/2))^(1/2)/(2\*x^2-x+3)^(1/2))\*(-76558040603+66320237500\*2^(1/2))^(1/2)+11/1201250\*arctan(1/62\*(21136+33287\*2^(1/2)+x\*(87710+54423\*2^(1/2)))\*682^(1/2)/(224510383+194487500\*2^(1/2))^(1/2)/(2\*x^2-x+3)^(1/2))\*(76558040603+66320237500\*2^(1/2))^(1/2)

**Rubi [A]** time = 0.66, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {971, 1066, 1076, 619, 215, 1035, 1029, 206, 204}

$$\frac{(10x+3)(2x^2-x+3)^{5/2}}{31(5x^2+3x+2)} + \frac{4}{155}(4-5x)(2x^2-x+3)^{3/2} - \frac{(2240x+1277)\sqrt{2x^2-x+3}}{7750} + \frac{11\sqrt{\frac{11}{31}(224510383+1)}}{31}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^(5/2)/(2 + 3\*x + 5\*x^2)^2, x]

[Out] -((1277 + 2240\*x)\*Sqrt[3 - x + 2\*x^2])/7750 + (4\*(4 - 5\*x)\*(3 - x + 2\*x^2)^(3/2))/155 + ((3 + 10\*x)\*(3 - x + 2\*x^2)^(5/2))/(31\*(2 + 3\*x + 5\*x^2)) - (4799\*ArcSinh[(1 - 4\*x)/Sqrt[23]]/(2500\*Sqrt[2]) + (11\*Sqrt[(11\*(224510383 + 194487500\*Sqrt[2]))/31]\*ArcTan[(Sqrt[11/(62\*(224510383 + 194487500\*Sqrt[2]))])\*(21136 + 33287\*Sqrt[2] + (87710 + 54423\*Sqrt[2])\*x)]/Sqrt[3 - x + 2\*x^2]))/38750 - (11\*Sqrt[(11\*(-224510383 + 194487500\*Sqrt[2]))/31]\*ArcTanh[(Sqrt[11/(62\*(-224510383 + 194487500\*Sqrt[2]))])\*(21136 - 33287\*Sqrt[2] + (87710 - 54423\*Sqrt[2])\*x)]/Sqrt[3 - x + 2\*x^2]))/38750

**Rule 204**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[

a, 0] || LtQ[b, 0])

### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 215

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

### Rule 619

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

### Rule 971

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_)\*((d\_) + (e\_)\*(x\_) + (f\_)\*(x\_)^2)^(q\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^q)/((b^2 - 4\*a\*c)\*(p + 1)), x] - Dist[1/((b^2 - 4\*a\*c)\*(p + 1)), Int[(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^(q - 1)\*Simp[2\*c\*d\*(2\*p + 3) + b\*e\*q + (2\*b\*f\*q + 2\*c\*e\*(2\*p + q + 3))\*x + 2\*c\*f\*(2\*p + 2\*q + 3)\*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && LtQ[p, -1] && GtQ[q, 0] && !IGtQ[q, 0]

### Rule 1029

Int[((g\_) + (h\_)\*(x\_))/(((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)\*Sqrt[(d\_) + (e\_)\*(x\_) + (f\_)\*(x\_)^2]), x\_Symbol] := Dist[-2\*g\*(g\*b - 2\*a\*h), Subst[Int[1/Simp[g\*(g\*b - 2\*a\*h)\*(b^2 - 4\*a\*c) - (b\*d - a\*e)\*x^2, x], x], x, Simp[g\*b - 2\*a\*h - (b\*h - 2\*g\*c)\*x, x]/Sqrt[d + e\*x + f\*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && NeQ[b\*d - a\*e, 0] && EqQ[h^2\*(b\*d - a\*e) - 2\*g\*h\*(c\*d - a\*f) + g^2\*(c\*e - b\*f), 0]

### Rule 1035

Int[((g\_) + (h\_)\*(x\_))/(((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)\*Sqrt[(d\_) + (e\_)\*(x\_) + (f\_)\*(x\_)^2]), x\_Symbol] := With[{q = Rt[(c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f), 2]}, Dist[1/(2\*q), Int[Simp[h\*(b\*d - a\*e) - g\*(c\*d - a\*f - q) - (g\*(c\*e - b\*f) - h\*(c\*d - a\*f + q))\*x, x]/((a + b\*x + c\*x^2)\*Sqrt

```
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 - 4*a*c]
```

### Rule 1066

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((A_) + (B_)*(x_) + (C_)*(x_)^2)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*p + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3)))] + (2*p*(c*d - a*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3)))]*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3)))]*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]
```

### Rule 1076

```
Int[((A_) + (B_)*(x_) + (C_)*(x_)^2)/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^2} dx &= \frac{(3+10x)(3-x+2x^2)^{5/2}}{31(2+3x+5x^2)} - \frac{1}{31} \int \frac{(3-x+2x^2)^{3/2} \left(-\frac{75}{2} + 15x + 80x^2\right)}{2+3x+5x^2} dx \\
&= \frac{4}{155}(4-5x)(3-x+2x^2)^{3/2} + \frac{(3+10x)(3-x+2x^2)^{5/2}}{31(2+3x+5x^2)} + \frac{\int \frac{(87660-54300x-53760x^2)\sqrt{3-x+2x^2}}{2+3x+5x^2}}{18600} \\
&= -\frac{(1277+2240x)\sqrt{3-x+2x^2}}{7750} + \frac{4}{155}(4-5x)(3-x+2x^2)^{3/2} + \frac{(3+10x)(3-x+2x^2)^{5/2}}{31(2+3x+5x^2)} \\
&= -\frac{(1277+2240x)\sqrt{3-x+2x^2}}{7750} + \frac{4}{155}(4-5x)(3-x+2x^2)^{3/2} + \frac{(3+10x)(3-x+2x^2)^{5/2}}{31(2+3x+5x^2)} \\
&= -\frac{(1277+2240x)\sqrt{3-x+2x^2}}{7750} + \frac{4}{155}(4-5x)(3-x+2x^2)^{3/2} + \frac{(3+10x)(3-x+2x^2)^{5/2}}{31(2+3x+5x^2)} \\
&= -\frac{(1277+2240x)\sqrt{3-x+2x^2}}{7750} + \frac{4}{155}(4-5x)(3-x+2x^2)^{3/2} + \frac{(3+10x)(3-x+2x^2)^{5/2}}{31(2+3x+5x^2)} \\
&= -\frac{(1277+2240x)\sqrt{3-x+2x^2}}{7750} + \frac{4}{155}(4-5x)(3-x+2x^2)^{3/2} + \frac{(3+10x)(3-x+2x^2)^{5/2}}{31(2+3x+5x^2)}
\end{aligned}$$

**Mathematica [C]** time = 1.65, size = 685, normalized size = 2.69

$$7784100\sqrt{2x^2 - x + 3}x^2 - 5759180\sqrt{2x^2 - x + 3}x - 5577520\sqrt{2x^2 - x + 3} - 4611839\sqrt{2} (5x^2 + 3x + 2) \sinh$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)^(5/2)/(2 + 3\*x + 5\*x^2)^2,x]

[Out] -1/4805000\*(-5577520\*sqrt[3 - x + 2\*x^2] - 5759180\*x\*sqrt[3 - x + 2\*x^2] + 7784100\*x^2\*sqrt[3 - x + 2\*x^2] - 1922000\*x^3\*sqrt[3 - x + 2\*x^2] - 4611839

$$\begin{aligned} & * \text{Sqrt}[2] * (2 + 3*x + 5*x^2) * \text{ArcSinh}[(-1 + 4*x) / \text{Sqrt}[23]] + (11*I) * \text{Sqrt}[286 + \\ & (22*I) * \text{Sqrt}[31]] * (5177*I + 8771 * \text{Sqrt}[31]) * (2 + 3*x + 5*x^2) * \text{ArcTanh}[(63 + \\ & I * \text{Sqrt}[31] - 22*x - (4*I) * \text{Sqrt}[31] * x) / (2 * \text{Sqrt}[286 + (22*I) * \text{Sqrt}[31]] * \text{Sqrt}[3 \\ & - x + 2*x^2])] + (192962*I) * \text{Sqrt}[682 * (13 - I * \text{Sqrt}[31])] * \text{ArcTanh}[(-63 + I * \text{S} \\ & \text{qrt}[31] + 22*x - (4*I) * \text{Sqrt}[31] * x) / (2 * \text{Sqrt}[286 - (22*I) * \text{Sqrt}[31]] * \text{Sqrt}[3 - \\ & x + 2*x^2])] + 113894 * \text{Sqrt}[286 - (22*I) * \text{Sqrt}[31]] * \text{ArcTanh}[(-63 + I * \text{Sqrt}[31] \\ & + 22*x - (4*I) * \text{Sqrt}[31] * x) / (2 * \text{Sqrt}[286 - (22*I) * \text{Sqrt}[31]] * \text{Sqrt}[3 - x + 2*x \\ & ^2])] + (289443*I) * \text{Sqrt}[682 * (13 - I * \text{Sqrt}[31])] * x * \text{ArcTanh}[(-63 + I * \text{Sqrt}[31] \\ & + 22*x - (4*I) * \text{Sqrt}[31] * x) / (2 * \text{Sqrt}[286 - (22*I) * \text{Sqrt}[31]] * \text{Sqrt}[3 - x + 2*x^ \\ & 2])] + 170841 * \text{Sqrt}[286 - (22*I) * \text{Sqrt}[31]] * x * \text{ArcTanh}[(-63 + I * \text{Sqrt}[31] + 22* \\ & x - (4*I) * \text{Sqrt}[31] * x) / (2 * \text{Sqrt}[286 - (22*I) * \text{Sqrt}[31]] * \text{Sqrt}[3 - x + 2*x^2])] \\ & + (482405*I) * \text{Sqrt}[682 * (13 - I * \text{Sqrt}[31])] * x^2 * \text{ArcTanh}[(-63 + I * \text{Sqrt}[31] + 22 \\ & * x - (4*I) * \text{Sqrt}[31] * x) / (2 * \text{Sqrt}[286 - (22*I) * \text{Sqrt}[31]] * \text{Sqrt}[3 - x + 2*x^2])] \\ & + 284735 * \text{Sqrt}[286 - (22*I) * \text{Sqrt}[31]] * x^2 * \text{ArcTanh}[(-63 + I * \text{Sqrt}[31] + 22*x \\ & - (4*I) * \text{Sqrt}[31] * x) / (2 * \text{Sqrt}[286 - (22*I) * \text{Sqrt}[31]] * \text{Sqrt}[3 - x + 2*x^2])]) / ( \\ & 2 + 3*x + 5*x^2) \end{aligned}$$

**fricas [B]** time = 2.74, size = 2161, normalized size = 8.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(5/2)/(5\*x^2+3\*x+2)^2,x, algorithm="fricas")

[Out] 1/1322759922435707900000\*(38925001324\*1464599010050^(1/4)\*sqrt(155590)\*sqrt(62)\*sqrt(2)\*(5\*x^2 + 3\*x + 2)\*sqrt(224510383\*sqrt(2) + 388975000)\*arctan(1/296975447063866363819995875\*(110935670\*sqrt(155590)\*(4\*1464599010050^(3/4)\*sqrt(62)\*(18997882\*x^7 - 82713851\*x^6 + 169131062\*x^5 - 298338397\*x^4 + 156222120\*x^3 - 89116200\*x^2 - sqrt(2)\*(18111018\*x^7 - 62947113\*x^6 + 135463929\*x^5 - 197908246\*x^4 + 94500248\*x^3 - 34095024\*x^2 - 122404608\*x + 71452800) - 142905600\*x + 122404608) + 2411645\*1464599010050^(1/4)\*sqrt(62)\*(3035566\*x^7 - 47612316\*x^6 + 259553720\*x^5 - 615321136\*x^4 + 807721920\*x^3 - 579888000\*x^2 - sqrt(2)\*(2643323\*x^7 - 39854517\*x^6 + 204950152\*x^5 - 451004140\*x^4 + 573424416\*x^3 - 311722272\*x^2 - 434377728\*x + 268655616) - 537311232\*x + 434377728))\*sqrt(2\*x^2 - x + 3)\*sqrt(224510383\*sqrt(2) + 388975000) + 843027075536136774714827000\*sqrt(31)\*sqrt(2)\*(28180\*x^8 - 254666\*x^7 + 704270\*x^6 - 1385256\*x^5 + 1549144\*x^4 - 642048\*x^3 - 98496\*x^2 - sqrt(2)\*(8746\*x^8 - 102335\*x^7 + 396104\*x^6 - 783113\*x^5 + 1320710\*x^4 - 752088\*x^3 + 396144\*x^2 + 546048\*x - 539136) + 1154304\*x - 456192) - sqrt(77795/920561)\*(sqrt(155590)\*(4\*1464599010050^(3/4)\*sqrt(62)\*(58767374\*x^7 - 85793239\*x^6 + 285539949\*x^5 - 168939120\*x^4 + 253241640\*x^3 + 601344\*x^2 - 4\*sqrt(2)\*(17889302\*x^7 - 25424283\*x^6 + 80174553\*x^5 - 21241168\*x^4 + 15593832\*x^3 + 58564512\*x^2 - 58564512\*x) - 601344\*x) + 2411645\*1464599010050^(1/4)\*sqrt(62)\*(9891184\*x^7 - 128099264\*x^6 + 496592960\*x^5 - 666984960\*x^4 + 949582080\*x^3 + 183223296\*x^2 - sqrt(2)\*(10181049\*x^7 - 131588036\*x^6 + 505509740\*x^5

$$\begin{aligned}
& - 637596864*x^4 + 754818336*x^3 + 725677056*x^2 - 725677056*x) - 183223296 \\
& *x)) * \text{sqrt}(2*x^2 - x + 3) * \text{sqrt}(224510383 * \text{sqrt}(2) + 388975000) + 759924265600 \\
& 1778100 * \text{sqrt}(31) * \text{sqrt}(2) * (123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x \\
& ^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - \text{sqrt}(2) * (15550*x^8 - 118051*x^ \\
& 7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 103 \\
& 6800*x) + 3276288*x) + 345420120727353550 * \text{sqrt}(31) * (254591*x^8 - 4815126*x^ \\
& 7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x \\
& ^2 - 15488 * \text{sqrt}(2) * (4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x \\
& ^3 + 2268*x^2 - 1944*x) + 144820224*x) * \text{sqrt}(- (1464599010050^{1/4} * \text{sqrt}(15 \\
& 5590) * \text{sqrt}(62) * \text{sqrt}(31) * \text{sqrt}(2*x^2 - x + 3) * (\text{sqrt}(2) * (9733*x + 29025) - 387 \\
& 58*x + 19292) * \text{sqrt}(224510383 * \text{sqrt}(2) + 388975000) - 6744561519183110*x^2 - \\
& 6056340956001160 * \text{sqrt}(2) * (2*x^2 - x + 3) + 20784261008094890*x - 2752882252 \\
& 7278000) / x^2) + 9579853131092463349032125 * \text{sqrt}(31) * (2828123*x^8 - 9696916*x \\
& ^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 3798144 \\
& 0*x^2 - 7744 * \text{sqrt}(2) * (1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^ \\
& 4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936) / (258519 \\
& 1*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x \\
& ^3 - 34615296*x^2 - 24772608*x + 18579456) + 38925001324 * 1464599010050^{1/4} \\
& * \text{sqrt}(155590) * \text{sqrt}(62) * \text{sqrt}(2) * (5*x^2 + 3*x + 2) * \text{sqrt}(224510383 * \text{sqrt}(2) + \\
& 388975000) * \arctan(1/296975447063866363819995875 * (110935670 * \text{sqrt}(155590) * (4 \\
& * 1464599010050^{3/4} * \text{sqrt}(62) * (18997882*x^7 - 82713851*x^6 + 169131062*x^5 \\
& - 298338397*x^4 + 156222120*x^3 - 89116200*x^2 - \text{sqrt}(2) * (18111018*x^7 - 62 \\
& 947113*x^6 + 135463929*x^5 - 197908246*x^4 + 94500248*x^3 - 34095024*x^2 - \\
& 122404608*x + 71452800) - 142905600*x + 122404608) + 2411645 * 1464599010050^{ \\
& (1/4) * \text{sqrt}(62) * (3035566*x^7 - 47612316*x^6 + 259553720*x^5 - 615321136*x^4 \\
& + 807721920*x^3 - 579888000*x^2 - \text{sqrt}(2) * (2643323*x^7 - 39854517*x^6 + 204 \\
& 950152*x^5 - 451004140*x^4 + 573424416*x^3 - 311722272*x^2 - 434377728*x + \\
& 268655616) - 537311232*x + 434377728) * \text{sqrt}(2*x^2 - x + 3) * \text{sqrt}(224510383 * \text{sqrt}( \\
& 2) + 388975000) - 843027075536136774714827000 * \text{sqrt}(31) * \text{sqrt}(2) * (28180*x \\
& ^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 984 \\
& 96*x^2 - \text{sqrt}(2) * (8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710 \\
& *x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - \\
& \text{sqrt}(77795/920561) * (\text{sqrt}(155590) * (4 * 1464599010050^{3/4} * \text{sqrt}(62) * (58767374 \\
& *x^7 - 85793239*x^6 + 285539949*x^5 - 168939120*x^4 + 253241640*x^3 + 60134 \\
& 4*x^2 - 4 * \text{sqrt}(2) * (17889302*x^7 - 25424283*x^6 + 80174553*x^5 - 21241168*x^ \\
& 4 + 15593832*x^3 + 58564512*x^2 - 58564512*x) - 601344*x) + 2411645 * 1464599 \\
& 010050^{1/4} * \text{sqrt}(62) * (9891184*x^7 - 128099264*x^6 + 496592960*x^5 - 666984 \\
& 960*x^4 + 949582080*x^3 + 183223296*x^2 - \text{sqrt}(2) * (10181049*x^7 - 131588036 \\
& *x^6 + 505509740*x^5 - 637596864*x^4 + 754818336*x^3 + 725677056*x^2 - 7256 \\
& 77056*x) - 183223296*x) * \text{sqrt}(2*x^2 - x + 3) * \text{sqrt}(224510383 * \text{sqrt}(2) + 38897 \\
& 5000) - 7599242656001778100 * \text{sqrt}(31) * \text{sqrt}(2) * (123408*x^8 - 914152*x^7 + 157 \\
& 8888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - \text{sqrt}(2) * (1 \\
& 5550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 \\
& + 1209600*x^2 - 1036800*x) + 3276288*x) - 345420120727353550 * \text{sqrt}(31) * (254 \\
& 591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219
\end{aligned}$$

$$\begin{aligned}
& 328x^3 - 168956928x^2 - 15488\sqrt{2}(4x^8 - 76x^7 + 517x^6 - 1536x^5 \\
& + 2385x^4 - 3618x^3 + 2268x^2 - 1944x) + 144820224x) \sqrt{(14645990 \\
& 10050^{1/4} \sqrt{155590} \sqrt{62} \sqrt{31} \sqrt{2x^2 - x + 3} (\sqrt{2}(97 \\
& 33x + 29025) - 38758x + 19292) \sqrt{224510383 \sqrt{2} + 388975000}) + 6744 \\
& 561519183110x^2 + 6056340956001160 \sqrt{2} (2x^2 - x + 3) - 2078426100809 \\
& 4890x + 27528822527278000) / x^2) - 9579853131092463349032125 \sqrt{31} (2828 \\
& 123x^8 - 9696916x^7 + 53385560x^6 - 142835344x^5 + 254146592x^4 - 2493 \\
& 00096x^3 + 37981440x^2 - 7744 \sqrt{2} (1348x^8 - 2692x^7 + 9789x^6 - 1 \\
& 0070x^5 + 15569x^4 - 5568x^3 + 1080x^2 + 4320x - 5184) + 223064064x - \\
& 94887936) / (2585191x^8 - 4661200x^7 + 14191920x^6 + 490880x^5 - 135629 \\
& 44x^4 + 44249088x^3 - 34615296x^2 - 24772608x + 18579456) + 11 \cdot 1464599 \\
& 010050^{1/4} \sqrt{155590} \sqrt{62} \sqrt{31} (1944875000x^2 - 224510383 \sqrt{2} \\
& (2x^2 - x + 3) + 1166925000x + 777950000) \sqrt{224510383 \sqrt{2} + \\
& 388975000} \log(14708117187500/920561 (1464599010050^{1/4} \sqrt{155590} \sqrt{62} \\
& \sqrt{31} \sqrt{2x^2 - x + 3} (\sqrt{2}(9733x + 29025) - 38758x + 19 \\
& 292) \sqrt{224510383 \sqrt{2} + 388975000}) + 6744561519183110x^2 + 605634095 \\
& 6001160 \sqrt{2} (2x^2 - x + 3) - 20784261008094890x + 27528822527278000) / \\
& x^2) - 11 \cdot 1464599010050^{1/4} \sqrt{155590} \sqrt{62} \sqrt{31} (1944875000x^2 \\
& - 224510383 \sqrt{2} (5x^2 + 3x + 2) + 1166925000x + 777950000) \sqrt{22 \\
& 4510383 \sqrt{2} + 388975000} \log(-14708117187500/920561 (1464599010050^{1/4} \\
& ) \sqrt{155590} \sqrt{62} \sqrt{31} \sqrt{2x^2 - x + 3} (\sqrt{2}(9733x + 290 \\
& 25) - 38758x + 19292) \sqrt{224510383 \sqrt{2} + 388975000}) - 67445615191831 \\
& 10x^2 - 6056340956001160 \sqrt{2} (2x^2 - x + 3) + 20784261008094890x - 2 \\
& 7528822527278000) / x^2) + 634792486776896221210 \sqrt{2} (5x^2 + 3x + 2) \log(-4 \sqrt{2} \\
& \sqrt{2x^2 - x + 3} (4x - 1) - 32x^2 + 16x - 25) + 17067869 \\
& 9669123600 (3100x^3 - 12555x^2 + 9289x + 8996) \sqrt{2x^2 - x + 3} / (5x \\
& ^2 + 3x + 2)
\end{aligned}$$

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(5/2)/(5\*x^2+3\*x+2)^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP  
UT:sage2:=int(sage0,x);OUTPUT:Unable to divide, perhaps due to rounding er  
ror%%{15625, [8]%%}+%%{%%{[-37500, 0] : [1, 0, -2]%%}, [7]%%}+%%{-61250, [6]%%  
%%}+%%{%%{[290000, 0] : [1, 0, -2]%%}, [5]%%}+%%{140625, [4]%%}+%%{%%{[-972500  
, 0] : [1, 0, -2]%%}, [3]%%}+%%{-345000, [2]%%}+%%{%%{[1820000, 0] : [1, 0, -2]%%},  
[1]%%}+%%{1960000, [0]%%} / %%{50, [8]%%}+%%{%%{poly1[-120, 0] : [1, 0, -2]%%  
%%}, [7]%%}+%%{-196, [6]%%}+%%{%%{poly1[928, 0] : [1, 0, -2]%%}, [5]%%}+%%{450  
, [4]%%}+%%{%%{poly1[-3112, 0] : [1, 0, -2]%%}, [3]%%}+%%{-1104, [2]%%}+%%{%%  
{poly1[5824, 0] : [1, 0, -2]%%}, [1]%%}+%%{6272, [0]%%} Error: Bad Argument Val  
ue

maple [B] time = 0.16, size = 40028, normalized size = 156.97

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x)`

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - x + 3)^{\frac{5}{2}}}{(5x^2 + 3x + 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")`

[Out] `integrate((2*x^2 - x + 3)^(5/2)/(5*x^2 + 3*x + 2)^2, x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(2x^2 - x + 3)^{5/2}}{(5x^2 + 3x + 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2 - x + 3)^(5/2)/(3*x + 5*x^2 + 2)^2,x)`

[Out] `int((2*x^2 - x + 3)^(5/2)/(3*x + 5*x^2 + 2)^2, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - x + 3)^{\frac{5}{2}}}{(5x^2 + 3x + 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**2-x+3)**(5/2)/(5*x**2+3*x+2)**2,x)`

[Out] `Integral((2*x**2 - x + 3)**(5/2)/(5*x**2 + 3*x + 2)**2, x)`



$$3.78 \quad \int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^3} dx$$

Optimal. Leaf size=281

$$\frac{(10x+3)(2x^2-x+3)^{5/2}}{62(5x^2+3x+2)^2} + \frac{(2336x+769)(2x^2-x+3)^{3/2}}{3844(5x^2+3x+2)} + \frac{(11359-12920x)\sqrt{2x^2-x+3}}{48050} + \frac{\sqrt{11(1+4\sqrt{2})}}{\dots}$$

[Out] 1/62\*(3+10\*x)\*(2\*x^2-x+3)^(5/2)/(5\*x^2+3\*x+2)^2+1/3844\*(769+2336\*x)\*(2\*x^2-x+3)^(3/2)/(5\*x^2+3\*x+2)-4/125\*arcsinh(1/23\*(1-4\*x)\*23^(1/2))\*2^(1/2)+1/48050\*(11359-12920\*x)\*(2\*x^2-x+3)^(1/2)-1/29791000\*arctanh(1/62\*(3957722+x\*(9832420-6895071\*2^(1/2))-2937349\*2^(1/2))\*682^(1/2)/(-3531015707557+2498852071250\*2^(1/2)))^(1/2)/(2\*x^2-x+3)^(1/2))\*(2937349-1978861\*2^(1/2))\*(-11+44\*2^(1/2))^(1/2)+1/29791000\*arctan(1/62\*(3957722+2937349\*2^(1/2)+x\*(9832420+6895071\*2^(1/2)))\*682^(1/2)/(3531015707557+2498852071250\*2^(1/2)))^(1/2)/(2\*x^2-x+3)^(1/2))\*(2937349+1978861\*2^(1/2))\*(11+44\*2^(1/2))^(1/2)

**Rubi [A]** time = 0.65, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$ , Rules used = {971, 1054, 1066, 1076, 619, 215, 1035, 1029, 206, 204}

$$\frac{(10x+3)(2x^2-x+3)^{5/2}}{62(5x^2+3x+2)^2} + \frac{(2336x+769)(2x^2-x+3)^{3/2}}{3844(5x^2+3x+2)} + \frac{(11359-12920x)\sqrt{2x^2-x+3}}{48050} + \frac{\sqrt{11(1+4\sqrt{2})}}{\dots}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^(5/2)/(2 + 3\*x + 5\*x^2)^3,x]

[Out] ((11359 - 12920\*x)\*Sqrt[3 - x + 2\*x^2])/48050 + ((3 + 10\*x)\*(3 - x + 2\*x^2)^(5/2))/(62\*(2 + 3\*x + 5\*x^2)^2) + ((769 + 2336\*x)\*(3 - x + 2\*x^2)^(3/2))/(3844\*(2 + 3\*x + 5\*x^2)) - (4\*Sqrt[2]\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/125 + (Sqrt[11\*(1 + 4\*Sqrt[2])]\*(2937349 + 1978861\*Sqrt[2])\*ArcTan[(Sqrt[11/(62\*(3531015707557 + 2498852071250\*Sqrt[2]))\*(3957722 + 2937349\*Sqrt[2] + (9832420 + 6895071\*Sqrt[2])\*x)]/Sqrt[3 - x + 2\*x^2])]/29791000 - ((2937349 - 1978861\*Sqrt[2])\*Sqrt[11\*(-1 + 4\*Sqrt[2])]\*ArcTanh[(Sqrt[11/(62\*(-3531015707557 + 2498852071250\*Sqrt[2]))\*(3957722 - 2937349\*Sqrt[2] + (9832420 - 6895071\*Sqrt[2])\*x)]/Sqrt[3 - x + 2\*x^2])]/29791000

Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

### Rule 971

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_)\*((d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2)^(q\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^q)/((b^2 - 4\*a\*c)\*(p + 1)), x] - Dist[1/((b^2 - 4\*a\*c)\*(p + 1)), Int[(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^(q - 1)\*Simp[2\*c\*d\*(2\*p + 3) + b\*e\*q + (2\*b\*f\*q + 2\*c\*e\*(2\*p + q + 3))\*x + 2\*c\*f\*(2\*p + 2\*q + 3)\*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && LtQ[p, -1] && GtQ[q, 0] && !IGtQ[q, 0]

### Rule 1029

Int[((g\_.) + (h\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*Sqrt[(d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2], x\_Symbol] := Dist[-2\*g\*(g\*b - 2\*a\*h), Subst[Int[1/Simp[g\*(g\*b - 2\*a\*h)\*(b^2 - 4\*a\*c) - (b\*d - a\*e)\*x^2, x], x], x, Simp[g\*b - 2\*a\*h - (b\*h - 2\*g\*c)\*x, x]/Sqrt[d + e\*x + f\*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && NeQ[b\*d - a\*e, 0] && EqQ[h^2\*(b\*d - a\*e) - 2\*g\*h\*(c\*d - a\*f) + g^2\*(c\*e - b\*f), 0]

### Rule 1035

Int[((g\_.) + (h\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*Sqrt[(d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2], x\_Symbol] := With[{q = Rt[(c\*d - a\*f)^2 - (b\*d

```

- a*e)*(c*e - b*f), 2]], Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]

```

### Rule 1054

```

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((A_) + (B_)*(x_) + (C_)*(x_
)^2)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[((A*b*c - 2
*a*B*c + a*b*C - (c*(b*B - 2*A*c) - C*(b^2 - 2*a*c))*x)*(a + b*x + c*x^2)^(
p + 1)*(d + e*x + f*x^2)^q)/(c*(b^2 - 4*a*c)*(p + 1)), x] - Dist[1/(c*(b^2
- 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*
Simp[e*q*(A*b*c - 2*a*B*c + a*b*C) - d*(c*(b*B - 2*A*c)*(2*p + 3) + C*(2*a*
c - b^2*(p + 2))) + (2*f*q*(A*b*c - 2*a*B*c + a*b*C) - e*(c*(b*B - 2*A*c)*(
2*p + q + 3) + C*(2*a*c*(q + 1) - b^2*(p + q + 2)))]*x - f*(c*(b*B - 2*A*c)
*(2*p + 2*q + 3) + C*(2*a*c*(2*q + 1) - b^2*(p + 2*q + 2)))]*x^2, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2
- 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] && !IGtQ[q, 0]

```

### Rule 1066

```

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((A_) + (B_)*(x_) + (C_)*(x_
)^2)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[((B*c*f*(2*
p + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a +
b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x
^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) -
c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f
- e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)
*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e
*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e +
2*A*f)*(2*p + 2*q + 3)))]*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*
e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^
2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3)))]*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]

```

### Rule 1076

```

Int[((A_) + (B_)*(x_) + (C_)*(x_)^2)/(((a_) + (b_)*(x_) + (c_)*(x_)^2)
*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqr

```

t[d + e\*x + f\*x^2], x], x] + Dist[1/c, Int[(A\*c - a\*C + (B\*c - b\*C)\*x)/((a + b\*x + c\*x^2)\*Sqrt[d + e\*x + f\*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^3} dx &= \frac{(3+10x)(3-x+2x^2)^{5/2}}{62(2+3x+5x^2)^2} - \frac{1}{62} \int \frac{(3-x+2x^2)^{3/2} \left(-\frac{195}{2} + 35x + 40x^2\right)}{(2+3x+5x^2)^2} dx \\
 &= \frac{(3+10x)(3-x+2x^2)^{5/2}}{62(2+3x+5x^2)^2} + \frac{(769+2336x)(3-x+2x^2)^{3/2}}{3844(2+3x+5x^2)} + \frac{\int \frac{\left(\frac{66735}{4} - 7375x - 25840x^2\right) \sqrt{3-x+2x^2}}{2+3x+5x^2} dx}{9610} \\
 &= \frac{(11359-12920x)\sqrt{3-x+2x^2}}{48050} + \frac{(3+10x)(3-x+2x^2)^{5/2}}{62(2+3x+5x^2)^2} + \frac{(769+2336x)(3-x+2x^2)^{3/2}}{3844(2+3x+5x^2)} \\
 &= \frac{(11359-12920x)\sqrt{3-x+2x^2}}{48050} + \frac{(3+10x)(3-x+2x^2)^{5/2}}{62(2+3x+5x^2)^2} + \frac{(769+2336x)(3-x+2x^2)^{3/2}}{3844(2+3x+5x^2)} \\
 &= \frac{(11359-12920x)\sqrt{3-x+2x^2}}{48050} + \frac{(3+10x)(3-x+2x^2)^{5/2}}{62(2+3x+5x^2)^2} + \frac{(769+2336x)(3-x+2x^2)^{3/2}}{3844(2+3x+5x^2)} \\
 &= \frac{(11359-12920x)\sqrt{3-x+2x^2}}{48050} + \frac{(3+10x)(3-x+2x^2)^{5/2}}{62(2+3x+5x^2)^2} + \frac{(769+2336x)(3-x+2x^2)^{3/2}}{3844(2+3x+5x^2)} \\
 &= \frac{(11359-12920x)\sqrt{3-x+2x^2}}{48050} + \frac{(3+10x)(3-x+2x^2)^{5/2}}{62(2+3x+5x^2)^2} + \frac{(769+2336x)(3-x+2x^2)^{3/2}}{3844(2+3x+5x^2)}
 \end{aligned}$$

**Mathematica [C]** time = 2.00, size = 1009, normalized size = 3.59

$$12599950\sqrt{286-22i\sqrt{31}} \tanh^{-1}\left(\frac{-4i\sqrt{31}x+22x+i\sqrt{31}-63}{2\sqrt{286-22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)x^4 - 12290525i\sqrt{682(13-i\sqrt{31})} \tanh^{-1}\left(\frac{-4i\sqrt{31}x+22x+i\sqrt{31}-63}{2\sqrt{286-22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (153804640*Sqrt[3 - x + 2*x^2] + 474815220*x*Sqrt[3 - x + 2*x^2] + 64020704
0*x^2*Sqrt[3 - x + 2*x^2] + 662597100*x^3*Sqrt[3 - x + 2*x^2] + 1906624*Sqr
t[2]*(2 + 3*x + 5*x^2)^2*ArcSinh[(-1 + 4*x)/Sqrt[23]] - I*Sqrt[286 + (22*I)
*Sqrt[31]]*(-503998*I + 491621*Sqrt[31])*(2 + 3*x + 5*x^2)^2*ArcTanh[(63 +
I*Sqrt[31] - 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3
- x + 2*x^2])) - (1966484*I)*Sqrt[682*(13 - I*Sqrt[31])]*ArcTanh[(-63 + I*
Sqrt[31] + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 -
x + 2*x^2))] + 2015992*Sqrt[286 - (22*I)*Sqrt[31]]*ArcTanh[(-63 + I*Sqrt[3
1] + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2
*x^2))] - (5899452*I)*Sqrt[682*(13 - I*Sqrt[31])]*x*ArcTanh[(-63 + I*Sqrt[3
1] + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2
*x^2))] + 6047976*Sqrt[286 - (22*I)*Sqrt[31]]*x*ArcTanh[(-63 + I*Sqrt[31] +
22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2
])] - (14257009*I)*Sqrt[682*(13 - I*Sqrt[31])]*x^2*ArcTanh[(-63 + I*Sqrt[31
] + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*
x^2))] + 14615942*Sqrt[286 - (22*I)*Sqrt[31]]*x^2*ArcTanh[(-63 + I*Sqrt[31]
+ 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x
^2))] - (14748630*I)*Sqrt[682*(13 - I*Sqrt[31])]*x^3*ArcTanh[(-63 + I*Sqrt[
31] + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x +
2*x^2))] + 15119940*Sqrt[286 - (22*I)*Sqrt[31]]*x^3*ArcTanh[(-63 + I*Sqrt[3
1] + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2
*x^2))] - (12290525*I)*Sqrt[682*(13 - I*Sqrt[31])]*x^4*ArcTanh[(-63 + I*Sqr
t[31] + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x
+ 2*x^2))] + 12599950*Sqrt[286 - (22*I)*Sqrt[31]]*x^4*ArcTanh[(-63 + I*Sqrt
[31] + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x +
2*x^2)))]/(59582000*(2 + 3*x + 5*x^2)^2)
```

**fricas [B]** time = 2.98, size = 2240, normalized size = 7.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/758714159921174808909075728000*(3184949732636*3868444992270541948232^(1/4)
)*sqrt(1999081657)*sqrt(62)*sqrt(2)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*s
qrt(3531015707557*sqrt(2) + 4997704142500)*arctan(1/45354848806294031039917
89624695893204150231*(2850690442882*sqrt(1999081657))*(2*3868444992270541948
232^(3/4)*sqrt(62)*(2627559914*x^7 - 10187615527*x^6 + 21362956024*x^5 - 34
451465819*x^4 + 17321103240*x^3 - 8320757400*x^2 - sqrt(2)*(1893366636*x^7
- 7237484076*x^6 + 15226003533*x^5 - 24262105817*x^4 + 12127036096*x^3 - 56
```

$$\begin{aligned}
& 64787848*x^2 - 13367586816*x + 9338025600) - 18676051200*x + 13367586816) + \\
& 61971531367*3868444992270541948232^{(1/4)}*\sqrt{62}*(400116332*x^7 - 6149336 \\
& 082*x^6 + 32552996440*x^5 - 74427496472*x^4 + 96235107840*x^3 - 61219656000 \\
& *x^2 - \sqrt{2}*(286685371*x^7 - 4395067059*x^6 + 23180544704*x^5 - 52748573 \\
& 780*x^4 + 68065744032*x^3 - 42544702944*x^2 - 48625837056*x + 34092306432) \\
& - 68184612864*x + 48625837056))*\sqrt{2*x^2 - x + 3}*\sqrt{3531015707557*\sqrt{ \\
& (2) + 4997704142500) + 12874924822431853972621854418491567805329688*\sqrt{31 \\
& )*\sqrt{2}*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 \\
& - 642048*x^3 - 98496*x^2 - \sqrt{2}*(8746*x^8 - 102335*x^7 + 396104*x^6 - 78 \\
& 3113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 115 \\
& 4304*x - 456192) - \sqrt{1999081657/828550919}*(\sqrt{1999081657}*(2*38684449 \\
& 92270541948232^{(3/4)}*\sqrt{62}*(9351066298*x^7 - 13433496653*x^6 + 433103458 \\
& 23*x^5 - 17374572240*x^4 + 20927636280*x^3 + 18483199488*x^2 - \sqrt{2}*(683 \\
& 9273266*x^7 - 9809465289*x^6 + 31524099699*x^5 - 12024617744*x^4 + 13914887 \\
& 256*x^3 + 14839341696*x^2 - 14839341696*x) - 18483199488*x) + 61971531367*3 \\
& 868444992270541948232^{(1/4)}*\sqrt{62}*(1427210918*x^7 - 18462714328*x^6 + 71 \\
& 210222920*x^5 - 92387041920*x^4 + 119489780160*x^3 + 68726817792*x^2 - \sqrt{ \\
& (2)*(1033310523*x^7 - 13365477772*x^6 + 51521534980*x^5 - 66583614528*x^4 + \\
& 85122955872*x^3 + 53108877312*x^2 - 53108877312*x) - 68726817792*x))*\sqrt{ \\
& 2*x^2 - x + 3}*\sqrt{3531015707557*\sqrt{2) + 4997704142500) + 45164233298567 \\
& 21284677540671884*\sqrt{31}*\sqrt{2}*(123408*x^8 - 914152*x^7 + 1578888*x^6 - \\
& 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - \sqrt{2}*(15550*x^8 - \\
& 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600 \\
& *x^2 - 1036800*x) + 3276288*x) + 205291969538941876576251848722*\sqrt{31}*(2 \\
& 54591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 742 \\
& 19328*x^3 - 168956928*x^2 - 15488*\sqrt{2}*(4*x^8 - 76*x^7 + 517*x^6 - 1536* \\
& x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*\sqrt{-(38684 \\
& 44992270541948232^{(1/4)}*\sqrt{1999081657}*\sqrt{62}*\sqrt{31}*\sqrt{2*x^2 - x + \\
& 3}*(\sqrt{2}*(2141441*x + 1076175) - 3217616*x - 1065266)*\sqrt{353101570755 \\
& 7*\sqrt{2) + 4997704142500) - 155990877430002205517374*x^2 - 140073440957553 \\
& 000872744*\sqrt{2}*(2*x^2 - x + 3) + 480706581467965980267826*x - 6366974588 \\
& 97968185785200)/x^2) + 146305963891271067870702891119222361424201*\sqrt{31}* \\
& (2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - \\
& 249300096*x^3 + 37981440*x^2 - 7744*\sqrt{2}*(1348*x^8 - 2692*x^7 + 9789*x^ \\
& 6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 22306406 \\
& 4*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 1 \\
& 3562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 31849 \\
& 49732636*3868444992270541948232^{(1/4)}*\sqrt{1999081657}*\sqrt{62}*\sqrt{2}*(25 \\
& *x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*\sqrt{3531015707557*\sqrt{2) + 49977041425 \\
& 00)*\arctan(1/4535484880629403103991789624695893204150231*(2850690442882*\sqrt{ \\
& (1999081657}*(2*3868444992270541948232^{(3/4)}*\sqrt{62}*(2627559914*x^7 - 10 \\
& 187615527*x^6 + 21362956024*x^5 - 34451465819*x^4 + 17321103240*x^3 - 83207 \\
& 57400*x^2 - \sqrt{2}*(1893366636*x^7 - 7237484076*x^6 + 15226003533*x^5 - 24 \\
& 262105817*x^4 + 12127036096*x^3 - 5664787848*x^2 - 13367586816*x + 93380256 \\
& 00) - 18676051200*x + 13367586816) + 61971531367*3868444992270541948232^{(1/
\end{aligned}$$

$$\begin{aligned}
& 4) * \sqrt{62} * (400116332 * x^7 - 6149336082 * x^6 + 32552996440 * x^5 - 74427496472 \\
& * x^4 + 96235107840 * x^3 - 61219656000 * x^2 - \sqrt{2} * (286685371 * x^7 - 4395067 \\
& 059 * x^6 + 23180544704 * x^5 - 52748573780 * x^4 + 68065744032 * x^3 - 42544702944 \\
& * x^2 - 48625837056 * x + 34092306432) - 68184612864 * x + 48625837056) * \sqrt{2} * \\
& x^2 - x + 3) * \sqrt{3531015707557 * \sqrt{2} + 4997704142500} - 1287492482243185 \\
& 3972621854418491567805329688 * \sqrt{31} * \sqrt{2} * (28180 * x^8 - 254666 * x^7 + 704 \\
& 270 * x^6 - 1385256 * x^5 + 1549144 * x^4 - 642048 * x^3 - 98496 * x^2 - \sqrt{2} * (874 \\
& 6 * x^8 - 102335 * x^7 + 396104 * x^6 - 783113 * x^5 + 1320710 * x^4 - 752088 * x^3 + 3 \\
& 96144 * x^2 + 546048 * x - 539136) + 1154304 * x - 456192) - \sqrt{1999081657/8285 \\
& 50919} * (\sqrt{1999081657} * (2 * 3868444992270541948232^{3/4} * \sqrt{62} * (93510662 \\
& 98 * x^7 - 13433496653 * x^6 + 43310345823 * x^5 - 17374572240 * x^4 + 20927636280 * \\
& x^3 + 18483199488 * x^2 - \sqrt{2} * (6839273266 * x^7 - 9809465289 * x^6 + 31524099 \\
& 699 * x^5 - 12024617744 * x^4 + 13914887256 * x^3 + 14839341696 * x^2 - 14839341696 \\
& * x) - 18483199488 * x) + 61971531367 * 3868444992270541948232^{1/4} * \sqrt{62} * (1 \\
& 427210918 * x^7 - 18462714328 * x^6 + 71210222920 * x^5 - 92387041920 * x^4 + 11948 \\
& 9780160 * x^3 + 68726817792 * x^2 - \sqrt{2} * (1033310523 * x^7 - 13365477772 * x^6 + \\
& 51521534980 * x^5 - 66583614528 * x^4 + 85122955872 * x^3 + 53108877312 * x^2 - 53 \\
& 108877312 * x) - 68726817792 * x) * \sqrt{2} * x^2 - x + 3) * \sqrt{3531015707557 * \sqrt{2} + \\
& 4997704142500} - 4516423329856721284677540671884 * \sqrt{31} * \sqrt{2} * (123 \\
& 408 * x^8 - 914152 * x^7 + 1578888 * x^6 - 3293072 * x^5 + 396480 * x^4 + 798336 * x^3 \\
& - 3822336 * x^2 - \sqrt{2} * (15550 * x^8 - 118051 * x^7 + 244047 * x^6 - 707374 * x^5 + \\
& 1053960 * x^4 - 1667952 * x^3 + 1209600 * x^2 - 1036800 * x) + 3276288 * x) - 205291 \\
& 969538941876576251848722 * \sqrt{31} * (254591 * x^8 - 4815126 * x^7 + 32303580 * x^6 \\
& - 90866808 * x^5 + 108781920 * x^4 - 74219328 * x^3 - 168956928 * x^2 - 15488 * \sqrt{2} * \\
& (4 * x^8 - 76 * x^7 + 517 * x^6 - 1536 * x^5 + 2385 * x^4 - 3618 * x^3 + 2268 * x^2 - \\
& 1944 * x) + 144820224 * x) * \sqrt{(3868444992270541948232^{1/4} * \sqrt{1999081657} \\
& * \sqrt{62} * \sqrt{31} * \sqrt{2} * x^2 - x + 3) * (\sqrt{2} * (2141441 * x + 1076175) - 321 \\
& 7616 * x - 1065266) * \sqrt{3531015707557 * \sqrt{2} + 4997704142500} + 15599087743 \\
& 0002205517374 * x^2 + 140073440957553000872744 * \sqrt{2} * (2 * x^2 - x + 3) - 4807 \\
& 06581467965980267826 * x + 636697458897968185785200) / x^2) - 14630596389127106 \\
& 7870702891119222361424201 * \sqrt{31} * (2828123 * x^8 - 9696916 * x^7 + 53385560 * x^ \\
& 6 - 142835344 * x^5 + 254146592 * x^4 - 249300096 * x^3 + 37981440 * x^2 - 7744 * \sqrt{2} * \\
& (1348 * x^8 - 2692 * x^7 + 9789 * x^6 - 10070 * x^5 + 15569 * x^4 - 5568 * x^3 + 1 \\
& 080 * x^2 + 4320 * x - 5184) + 223064064 * x - 94887936) / (2585191 * x^8 - 4661200 * \\
& x^7 + 14191920 * x^6 + 490880 * x^5 - 13562944 * x^4 + 44249088 * x^3 - 34615296 * x^ \\
& 2 - 24772608 * x + 18579456) + 3868444992270541948232^{1/4} * \sqrt{1999081657} \\
& * \sqrt{62} * \sqrt{31} * (124942603562500 * x^4 + 149931124275000 * x^3 + 14493342013 \\
& 2500 * x^2 - 3531015707557 * \sqrt{2} * (25 * x^4 + 30 * x^3 + 29 * x^2 + 12 * x + 4) + 59 \\
& 972449710000 * x + 19990816570000) * \sqrt{3531015707557 * \sqrt{2} + 4997704142500} \\
& ) * \log(3123565089062500/828550919 * (3868444992270541948232^{1/4} * \sqrt{1999081 \\
& 657} * \sqrt{62} * \sqrt{31} * \sqrt{2} * x^2 - x + 3) * (\sqrt{2} * (2141441 * x + 1076175) - \\
& 3217616 * x - 1065266) * \sqrt{3531015707557 * \sqrt{2} + 4997704142500} + 1559908 \\
& 77430002205517374 * x^2 + 140073440957553000872744 * \sqrt{2} * (2 * x^2 - x + 3) - \\
& 480706581467965980267826 * x + 636697458897968185785200) / x^2) - 3868444992270 \\
& 541948232^{1/4} * \sqrt{1999081657} * \sqrt{62} * \sqrt{31} * (124942603562500 * x^4 + 1
\end{aligned}$$

$49931124275000x^3 + 144933420132500x^2 - 3531015707557\sqrt{2}(25x^4 + 30x^3 + 29x^2 + 12x + 4) + 59972449710000x + 19990816570000)\sqrt{3531015707557\sqrt{2} + 4997704142500})\log(-3123565089062500/828550919*(3868444992270541948232^{(1/4)}\sqrt{1999081657})\sqrt{62})\sqrt{31})\sqrt{2x^2 - x + 3})$   
 $*(\sqrt{2}*(2141441x + 1076175) - 3217616x - 1065266)\sqrt{3531015707557\sqrt{2} + 4997704142500) - 155990877430002205517374x^2 - 140073440957553000872744\sqrt{2}*(2x^2 - x + 3) + 480706581467965980267826x - 636697458897968185785200)/x^2) + 12139426558738796942545211648\sqrt{2}*(25x^4 + 30x^3 + 29x^2 + 12x + 4)\log(-4\sqrt{2})\sqrt{2x^2 - x + 3}*(4x - 1) - 32x^2 + 16x - 25) + 86845533393682860541101280*(97155x^3 + 93872x^2 + 69621x + 22552)\sqrt{2x^2 - x + 3})/(25x^4 + 30x^3 + 29x^2 + 12x + 4)$

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(5/2)/(5\*x^2+3\*x+2)^3,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP  
 UT:sage2:=int(sage0,x):;OUTPUT:Francis algorithm failure for[-1.0,infinity,  
 infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,inf  
 inity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity  
 ]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai  
 lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity  
 ,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf  
 inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinit  
 y]Evaluation time: 59.56Done

**maple** [B] time = 0.38, size = 119458, normalized size = 425.12

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^(5/2)/(5\*x^2+3\*x+2)^3,x)

[Out] result too large to display

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - x + 3)^{\frac{5}{2}}}{(5x^2 + 3x + 2)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((2\*x^2-x+3)^(5/2)/(5\*x^2+3\*x+2)^3,x, algorithm="maxima")

[Out] integrate((2\*x^2 - x + 3)^(5/2)/(5\*x^2 + 3\*x + 2)^3, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(2x^2 - x + 3)^{5/2}}{(5x^2 + 3x + 2)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)^(5/2)/(3\*x + 5\*x^2 + 2)^3,x)

[Out] int((2\*x^2 - x + 3)^(5/2)/(3\*x + 5\*x^2 + 2)^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - x + 3)^{\frac{5}{2}}}{(5x^2 + 3x + 2)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2-x+3)\*\*(5/2)/(5\*x\*\*2+3\*x+2)\*\*3,x)

[Out] Integral((2\*x\*\*2 - x + 3)\*\*(5/2)/(5\*x\*\*2 + 3\*x + 2)\*\*3, x)

$$3.79 \quad \int \frac{(2+3x+5x^2)^4}{\sqrt{3-x+2x^2}} dx$$

**Optimal.** Leaf size=185

$$-\frac{15428243\sqrt{2x^2-x+3}x^2}{131072} + \frac{1572007407\sqrt{2x^2-x+3}x}{7340032} + \frac{16493087661\sqrt{2x^2-x+3}}{29360128} + \frac{625}{16}\sqrt{2x^2-x+3}x^7 + \frac{57}{4}$$

[Out] 2899366573/16777216\*arcsinh(1/23\*(1-4\*x)\*23^(1/2))\*2^(1/2)+16493087661/29360128\*(2\*x^2-x+3)^(1/2)+1572007407/7340032\*x\*(2\*x^2-x+3)^(1/2)-15428243/131072\*x^2\*(2\*x^2-x+3)^(1/2)-19750457/229376\*x^3\*(2\*x^2-x+3)^(1/2)+686531/6144\*x^4\*(2\*x^2-x+3)^(1/2)+2116475/10752\*x^5\*(2\*x^2-x+3)^(1/2)+57375/448\*x^6\*(2\*x^2-x+3)^(1/2)+625/16\*x^7\*(2\*x^2-x+3)^(1/2)

**Rubi [A]** time = 0.31, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$ , Rules used = {1661, 640, 619, 215}

$$\frac{625}{16}\sqrt{2x^2-x+3}x^7 + \frac{57375}{448}\sqrt{2x^2-x+3}x^6 + \frac{2116475\sqrt{2x^2-x+3}x^5}{10752} + \frac{686531\sqrt{2x^2-x+3}x^4}{6144} - \frac{19750457\sqrt{2x^2-x+3}x^3}{229376}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3\*x + 5\*x^2)^4/Sqrt[3 - x + 2\*x^2], x]

[Out] (16493087661\*Sqrt[3 - x + 2\*x^2])/29360128 + (1572007407\*x\*Sqrt[3 - x + 2\*x^2])/7340032 - (15428243\*x^2\*Sqrt[3 - x + 2\*x^2])/131072 - (19750457\*x^3\*Sqrt[3 - x + 2\*x^2])/229376 + (686531\*x^4\*Sqrt[3 - x + 2\*x^2])/6144 + (2116475\*x^5\*Sqrt[3 - x + 2\*x^2])/10752 + (57375\*x^6\*Sqrt[3 - x + 2\*x^2])/448 + (625\*x^7\*Sqrt[3 - x + 2\*x^2])/16 + (2899366573\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(8388608\*Sqrt[2])

**Rule 215**

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

**Rule 619**

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
  := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x]] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2 + 3x + 5x^2)^4}{\sqrt{3 - x + 2x^2}} dx &= \frac{625}{16} x^7 \sqrt{3 - x + 2x^2} + \frac{1}{16} \int \frac{256 + 1536x + 6016x^2 + 14976x^3 + 28176x^4 + 37440x^5 + 20480x^6}{\sqrt{3 - x + 2x^2}} dx \\
&= \frac{57375}{448} x^6 \sqrt{3 - x + 2x^2} + \frac{625}{16} x^7 \sqrt{3 - x + 2x^2} + \frac{1}{224} \int \frac{3584 + 21504x + 84224x^2 + 204800x^3 + 242816x^4 + 149760x^5 + 43008x^6 + 25800x^7}{\sqrt{3 - x + 2x^2}} dx \\
&= \frac{2116475x^5 \sqrt{3 - x + 2x^2}}{10752} + \frac{57375}{448} x^6 \sqrt{3 - x + 2x^2} + \frac{625}{16} x^7 \sqrt{3 - x + 2x^2} + \frac{1}{224} \int \frac{43008 + 258000x + 1044480x^2 + 2428160x^3 + 3584000x^4 + 3744000x^5 + 2428160x^6 + 1044480x^7 + 258000x^8}{\sqrt{3 - x + 2x^2}} dx \\
&= \frac{686531x^4 \sqrt{3 - x + 2x^2}}{6144} + \frac{2116475x^5 \sqrt{3 - x + 2x^2}}{10752} + \frac{57375}{448} x^6 \sqrt{3 - x + 2x^2} + \frac{625}{16} x^7 \sqrt{3 - x + 2x^2} + \frac{1}{224} \int \frac{43008 + 258000x + 1044480x^2 + 2428160x^3 + 3584000x^4 + 3744000x^5 + 2428160x^6 + 1044480x^7 + 258000x^8}{\sqrt{3 - x + 2x^2}} dx \\
&= -\frac{19750457x^3 \sqrt{3 - x + 2x^2}}{229376} + \frac{686531x^4 \sqrt{3 - x + 2x^2}}{6144} + \frac{2116475x^5 \sqrt{3 - x + 2x^2}}{10752} + \frac{57375}{448} x^6 \sqrt{3 - x + 2x^2} + \frac{625}{16} x^7 \sqrt{3 - x + 2x^2} + \frac{1}{224} \int \frac{43008 + 258000x + 1044480x^2 + 2428160x^3 + 3584000x^4 + 3744000x^5 + 2428160x^6 + 1044480x^7 + 258000x^8}{\sqrt{3 - x + 2x^2}} dx \\
&= -\frac{15428243x^2 \sqrt{3 - x + 2x^2}}{131072} - \frac{19750457x^3 \sqrt{3 - x + 2x^2}}{229376} + \frac{686531x^4 \sqrt{3 - x + 2x^2}}{6144} + \frac{2116475x^5 \sqrt{3 - x + 2x^2}}{10752} + \frac{57375}{448} x^6 \sqrt{3 - x + 2x^2} + \frac{625}{16} x^7 \sqrt{3 - x + 2x^2} + \frac{1}{224} \int \frac{43008 + 258000x + 1044480x^2 + 2428160x^3 + 3584000x^4 + 3744000x^5 + 2428160x^6 + 1044480x^7 + 258000x^8}{\sqrt{3 - x + 2x^2}} dx \\
&= \frac{1572007407x \sqrt{3 - x + 2x^2}}{7340032} - \frac{15428243x^2 \sqrt{3 - x + 2x^2}}{131072} - \frac{19750457x^3 \sqrt{3 - x + 2x^2}}{229376} + \frac{686531x^4 \sqrt{3 - x + 2x^2}}{6144} + \frac{2116475x^5 \sqrt{3 - x + 2x^2}}{10752} + \frac{57375}{448} x^6 \sqrt{3 - x + 2x^2} + \frac{625}{16} x^7 \sqrt{3 - x + 2x^2} + \frac{1}{224} \int \frac{43008 + 258000x + 1044480x^2 + 2428160x^3 + 3584000x^4 + 3744000x^5 + 2428160x^6 + 1044480x^7 + 258000x^8}{\sqrt{3 - x + 2x^2}} dx \\
&= \frac{16493087661 \sqrt{3 - x + 2x^2}}{29360128} + \frac{1572007407x \sqrt{3 - x + 2x^2}}{7340032} - \frac{15428243x^2 \sqrt{3 - x + 2x^2}}{131072} + \frac{686531x^4 \sqrt{3 - x + 2x^2}}{6144} + \frac{2116475x^5 \sqrt{3 - x + 2x^2}}{10752} + \frac{57375}{448} x^6 \sqrt{3 - x + 2x^2} + \frac{625}{16} x^7 \sqrt{3 - x + 2x^2} + \frac{1}{224} \int \frac{43008 + 258000x + 1044480x^2 + 2428160x^3 + 3584000x^4 + 3744000x^5 + 2428160x^6 + 1044480x^7 + 258000x^8}{\sqrt{3 - x + 2x^2}} dx \\
&= \frac{16493087661 \sqrt{3 - x + 2x^2}}{29360128} + \frac{1572007407x \sqrt{3 - x + 2x^2}}{7340032} - \frac{15428243x^2 \sqrt{3 - x + 2x^2}}{131072} + \frac{686531x^4 \sqrt{3 - x + 2x^2}}{6144} + \frac{2116475x^5 \sqrt{3 - x + 2x^2}}{10752} + \frac{57375}{448} x^6 \sqrt{3 - x + 2x^2} + \frac{625}{16} x^7 \sqrt{3 - x + 2x^2} + \frac{1}{224} \int \frac{43008 + 258000x + 1044480x^2 + 2428160x^3 + 3584000x^4 + 3744000x^5 + 2428160x^6 + 1044480x^7 + 258000x^8}{\sqrt{3 - x + 2x^2}} dx \\
&= \frac{16493087661 \sqrt{3 - x + 2x^2}}{29360128} + \frac{1572007407x \sqrt{3 - x + 2x^2}}{7340032} - \frac{15428243x^2 \sqrt{3 - x + 2x^2}}{131072} + \frac{686531x^4 \sqrt{3 - x + 2x^2}}{6144} + \frac{2116475x^5 \sqrt{3 - x + 2x^2}}{10752} + \frac{57375}{448} x^6 \sqrt{3 - x + 2x^2} + \frac{625}{16} x^7 \sqrt{3 - x + 2x^2} + \frac{1}{224} \int \frac{43008 + 258000x + 1044480x^2 + 2428160x^3 + 3584000x^4 + 3744000x^5 + 2428160x^6 + 1044480x^7 + 258000x^8}{\sqrt{3 - x + 2x^2}} dx
\end{aligned}$$

**Mathematica [A]** time = 0.24, size = 75, normalized size = 0.41

$$\frac{4\sqrt{2x^2 - x + 3} (3440640000x^7 + 11280384000x^6 + 17338163200x^5 + 9842108416x^4 - 7584175488x^3 - 1036774400x^2 + 1044480x - 25800)}{352321536}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x + 5\*x^2)^4/Sqrt[3 - x + 2\*x^2],x]

[Out] (4\*Sqrt[3 - x + 2\*x^2]\*(49479262983 + 18864088884\*x - 10367779296\*x^2 - 7584175488\*x^3 + 9842108416\*x^4 + 17338163200\*x^5 + 11280384000\*x^6 + 3440640000\*x^7) + 60886698033\*Sqrt[2]\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/352321536

**fricas** [A] time = 0.77, size = 88, normalized size = 0.48

$$\frac{1}{88080384} (3440640000 x^7 + 11280384000 x^6 + 17338163200 x^5 + 9842108416 x^4 - 7584175488 x^3 - 10367779296 x^2 + 18864088884 x - 10367779296)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^4/(2\*x^2-x+3)^(1/2),x, algorithm="fricas")

[Out] 1/88080384\*(3440640000\*x^7 + 11280384000\*x^6 + 17338163200\*x^5 + 9842108416\*x^4 - 7584175488\*x^3 - 10367779296\*x^2 + 18864088884\*x + 49479262983)\*sqrt(2\*x^2 - x + 3) + 2899366573/33554432\*sqrt(2)\*log(4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25)

**giac** [A] time = 0.50, size = 83, normalized size = 0.45

$$\frac{1}{88080384} (4 (8 (4 (16 (100 (120 (140 x + 459) x + 84659) x + 4805717) x - 59251371) x - 323993103) x + 4716022221) x + 49479262983) \sqrt{2 x^2 - x + 3} + \frac{2899366573}{16777216} \sqrt{2} \log(-2 \sqrt{2} (\sqrt{2} x - \sqrt{2 x^2 - x + 3})) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^4/(2\*x^2-x+3)^(1/2),x, algorithm="giac")

[Out] 1/88080384\*(4\*(8\*(4\*(16\*(100\*(120\*(140\*x + 459)\*x + 84659)\*x + 4805717)\*x - 59251371)\*x - 323993103)\*x + 4716022221)\*x + 49479262983)\*sqrt(2\*x^2 - x + 3) + 2899366573/16777216\*sqrt(2)\*log(-2\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))) + 1)

**maple** [A] time = 0.02, size = 147, normalized size = 0.79

$$\frac{625\sqrt{2x^2-x+3}x^7}{16} + \frac{57375\sqrt{2x^2-x+3}x^6}{448} + \frac{2116475\sqrt{2x^2-x+3}x^5}{10752} + \frac{686531\sqrt{2x^2-x+3}x^4}{6144} - \frac{19750457\sqrt{2x^2-x+3}x^3}{229376} + \frac{19750457\sqrt{2x^2-x+3}x^2}{229376} - \frac{19750457\sqrt{2x^2-x+3}x}{229376} + \frac{19750457\sqrt{2x^2-x+3}}{229376}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+3\*x+2)^4/(2\*x^2-x+3)^(1/2),x)

[Out] 16493087661/29360128\*(2\*x^2-x+3)^(1/2)-2899366573/16777216\*2^(1/2)\*arcsinh(4/23\*23^(1/2)\*(x-1/4))+625/16\*x^7\*(2\*x^2-x+3)^(1/2)+57375/448\*x^6\*(2\*x^2-x+3)^(1/2)+2116475/10752\*x^5\*(2\*x^2-x+3)^(1/2)+686531/6144\*x^4\*(2\*x^2-x+3)^(1/2)

/2)-19750457/229376\*x^3\*(2\*x^2-x+3)^(1/2)-15428243/131072\*x^2\*(2\*x^2-x+3)^(1/2)+1572007407/7340032\*x\*(2\*x^2-x+3)^(1/2)

**maxima** [A] time = 1.00, size = 148, normalized size = 0.80

$$\frac{625}{16} \sqrt{2x^2 - x + 3} x^7 + \frac{57375}{448} \sqrt{2x^2 - x + 3} x^6 + \frac{2116475}{10752} \sqrt{2x^2 - x + 3} x^5 + \frac{686531}{6144} \sqrt{2x^2 - x + 3} x^4 - \frac{19750457}{229376} \sqrt{2x^2 - x + 3} x^3 - \frac{15428243}{131072} \sqrt{2x^2 - x + 3} x^2 + \frac{1572007407}{7340032} \sqrt{2x^2 - x + 3} x - \frac{2899366573}{16777216} \sqrt{2x^2 - x + 3} - \frac{16493087661}{29360128} \sqrt{2x^2 - x + 3} \operatorname{arcsinh}\left(\frac{1}{23} \sqrt{23} (4x - 1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^4/(2\*x^2-x+3)^(1/2),x, algorithm="maxima")

[Out] 625/16\*sqrt(2\*x^2 - x + 3)\*x^7 + 57375/448\*sqrt(2\*x^2 - x + 3)\*x^6 + 2116475/10752\*sqrt(2\*x^2 - x + 3)\*x^5 + 686531/6144\*sqrt(2\*x^2 - x + 3)\*x^4 - 19750457/229376\*sqrt(2\*x^2 - x + 3)\*x^3 - 15428243/131072\*sqrt(2\*x^2 - x + 3)\*x^2 + 1572007407/7340032\*sqrt(2\*x^2 - x + 3)\*x - 2899366573/16777216\*sqrt(2\*x^2 - x + 3) - 16493087661/29360128\*sqrt(2\*x^2 - x + 3)\*arcsinh(1/23\*sqrt(23)\*(4\*x - 1))

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(5x^2 + 3x + 2)^4}{\sqrt{2x^2 - x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x + 5\*x^2 + 2)^4/(2\*x^2 - x + 3)^(1/2), x)

[Out] int((3\*x + 5\*x^2 + 2)^4/(2\*x^2 - x + 3)^(1/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x^2 + 3x + 2)^4}{\sqrt{2x^2 - x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*2+3\*x+2)\*\*4/(2\*x\*\*2-x+3)\*\*(1/2), x)

[Out] Integral((5\*x\*\*2 + 3\*x + 2)\*\*4/sqrt(2\*x\*\*2 - x + 3), x)

$$3.80 \quad \int \frac{(2+3x+5x^2)^3}{\sqrt{3-x+2x^2}} dx$$

**Optimal.** Leaf size=143

$$\frac{3387\sqrt{2x^2-x+3}x^2}{1024} - \frac{372783\sqrt{2x^2-x+3}x}{8192} - \frac{203373\sqrt{2x^2-x+3}}{32768} + \frac{125}{12}\sqrt{2x^2-x+3}x^5 + \frac{1355}{48}\sqrt{2x^2-x+3}$$

[Out]  $-9267707/131072*\operatorname{arcsinh}(1/23*(1-4*x)*23^{(1/2)})*2^{(1/2)}-203373/32768*(2*x^2-x+3)^{(1/2)}-372783/8192*x*(2*x^2-x+3)^{(1/2)}-3387/1024*x^2*(2*x^2-x+3)^{(1/2)}+8185/256*x^3*(2*x^2-x+3)^{(1/2)}+1355/48*x^4*(2*x^2-x+3)^{(1/2)}+125/12*x^5*(2*x^2-x+3)^{(1/2)}$

**Rubi [A]** time = 0.17, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$ , Rules used = {1661, 640, 619, 215}

$$\frac{125}{12}\sqrt{2x^2-x+3}x^5 + \frac{1355}{48}\sqrt{2x^2-x+3}x^4 + \frac{8185}{256}\sqrt{2x^2-x+3}x^3 - \frac{3387\sqrt{2x^2-x+3}x^2}{1024} - \frac{372783\sqrt{2x^2-x+3}x}{8192}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3\*x + 5\*x^2)^3/Sqrt[3 - x + 2\*x^2], x]

[Out]  $(-203373*\operatorname{Sqrt}[3-x+2*x^2])/32768 - (372783*x*\operatorname{Sqrt}[3-x+2*x^2])/8192 - (3387*x^2*\operatorname{Sqrt}[3-x+2*x^2])/1024 + (8185*x^3*\operatorname{Sqrt}[3-x+2*x^2])/256 + (1355*x^4*\operatorname{Sqrt}[3-x+2*x^2])/48 + (125*x^5*\operatorname{Sqrt}[3-x+2*x^2])/12 - (9267707*\operatorname{ArcSinh}[(1-4*x)/\operatorname{Sqrt}[23]])/(65536*\operatorname{Sqrt}[2])$

### Rule 215

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

### Rule 619

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^(p)), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

### Rule 640

Int[((d\_) + (e\_)\*(x\_))\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b

$*e)/(2*c), \text{Int}[(a + b*x + c*x^2)^p, x], x] /; \text{FreeQ}[\{a, b, c, d, e, p\}, x]$   
 $\&\& \text{NeQ}[2*c*d - b*e, 0] \&\& \text{NeQ}[p, -1]$

### Rule 1661

$\text{Int}[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x\_Symbol] := \text{With}[\{q =$   
 $\text{Expon}[Pq, x], e = \text{Coeff}[Pq, x, \text{Expon}[Pq, x]]\}, \text{Simp}[(e*x^(q - 1)*(a + b*x +$   
 $c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + \text{Dist}[1/(c*(q + 2*p + 1)), \text{Int}[(a +$   
 $b*x + c*x^2)^p*\text{ExpandToSum}[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*$   
 $e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x]] /; \text{FreeQ}[\{a, b, c,$   
 $p\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& !\text{LeQ}[p, -1]$

### Rubi steps

$$\begin{aligned} \int \frac{(2 + 3x + 5x^2)^3}{\sqrt{3 - x + 2x^2}} dx &= \frac{125}{12} x^5 \sqrt{3 - x + 2x^2} + \frac{1}{12} \int \frac{96 + 432x + 1368x^2 + 2484x^3 + 1545x^4 + \frac{6775x^5}{2}}{\sqrt{3 - x + 2x^2}} dx \\ &= \frac{1355}{48} x^4 \sqrt{3 - x + 2x^2} + \frac{125}{12} x^5 \sqrt{3 - x + 2x^2} + \frac{1}{120} \int \frac{960 + 4320x + 13680x^2 - 15810x^3}{\sqrt{3 - x + 2x^2}} dx \\ &= \frac{8185}{256} x^3 \sqrt{3 - x + 2x^2} + \frac{1355}{48} x^4 \sqrt{3 - x + 2x^2} + \frac{125}{12} x^5 \sqrt{3 - x + 2x^2} + \frac{1}{960} \int \frac{7680 + 372783x}{\sqrt{3 - x + 2x^2}} dx \\ &= -\frac{3387x^2 \sqrt{3 - x + 2x^2}}{1024} + \frac{8185}{256} x^3 \sqrt{3 - x + 2x^2} + \frac{1355}{48} x^4 \sqrt{3 - x + 2x^2} + \frac{125}{12} x^5 \sqrt{3 - x + 2x^2} \\ &= -\frac{372783x \sqrt{3 - x + 2x^2}}{8192} - \frac{3387x^2 \sqrt{3 - x + 2x^2}}{1024} + \frac{8185}{256} x^3 \sqrt{3 - x + 2x^2} + \frac{1355}{48} x^4 \sqrt{3 - x + 2x^2} \\ &= -\frac{203373 \sqrt{3 - x + 2x^2}}{32768} - \frac{372783x \sqrt{3 - x + 2x^2}}{8192} - \frac{3387x^2 \sqrt{3 - x + 2x^2}}{1024} + \frac{8185}{256} x^3 \sqrt{3 - x + 2x^2} \\ &= -\frac{203373 \sqrt{3 - x + 2x^2}}{32768} - \frac{372783x \sqrt{3 - x + 2x^2}}{8192} - \frac{3387x^2 \sqrt{3 - x + 2x^2}}{1024} + \frac{8185}{256} x^3 \sqrt{3 - x + 2x^2} \\ &= -\frac{203373 \sqrt{3 - x + 2x^2}}{32768} - \frac{372783x \sqrt{3 - x + 2x^2}}{8192} - \frac{3387x^2 \sqrt{3 - x + 2x^2}}{1024} + \frac{8185}{256} x^3 \sqrt{3 - x + 2x^2} \end{aligned}$$



**Mathematica [A]** time = 0.13, size = 65, normalized size = 0.45

$$\frac{4\sqrt{2x^2 - x + 3} (1024000x^5 + 2775040x^4 + 3143040x^3 - 325152x^2 - 4473396x - 610119) - 27803121\sqrt{2} \sinh}{393216}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x + 5\*x^2)^3/Sqrt[3 - x + 2\*x^2], x]

[Out] (4\*Sqrt[3 - x + 2\*x^2]\*(-610119 - 4473396\*x - 325152\*x^2 + 3143040\*x^3 + 2775040\*x^4 + 1024000\*x^5) - 27803121\*Sqrt[2]\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/393216

**fricas [A]** time = 0.68, size = 78, normalized size = 0.55

$$\frac{1}{98304} (1024000x^5 + 2775040x^4 + 3143040x^3 - 325152x^2 - 4473396x - 610119)\sqrt{2x^2 - x + 3} + \frac{9267707}{262144}\sqrt{2} \log$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^3/(2\*x^2-x+3)^(1/2), x, algorithm="fricas")

[Out] 1/98304\*(1024000\*x^5 + 2775040\*x^4 + 3143040\*x^3 - 325152\*x^2 - 4473396\*x - 610119)\*sqrt(2\*x^2 - x + 3) + 9267707/262144\*sqrt(2)\*log(-4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25)

**giac [A]** time = 0.52, size = 73, normalized size = 0.51

$$\frac{1}{98304} (4(8(20(16(100x + 271)x + 4911)x - 10161)x - 1118349)x - 610119)\sqrt{2x^2 - x + 3} - \frac{9267707}{131072}\sqrt{2} \log$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^3/(2\*x^2-x+3)^(1/2), x, algorithm="giac")

[Out] 1/98304\*(4\*(8\*(20\*(16\*(100\*x + 271)\*x + 4911)\*x - 10161)\*x - 1118349)\*x - 610119)\*sqrt(2\*x^2 - x + 3) - 9267707/131072\*sqrt(2)\*log(-2\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 1)

**maple [A]** time = 0.01, size = 113, normalized size = 0.79

$$\frac{125\sqrt{2x^2 - x + 3} x^5}{12} + \frac{1355\sqrt{2x^2 - x + 3} x^4}{48} + \frac{8185\sqrt{2x^2 - x + 3} x^3}{256} - \frac{3387\sqrt{2x^2 - x + 3} x^2}{1024} - \frac{372783\sqrt{2x^2 - x + 3}}{8192}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x)`

[Out]  $-203373/32768*(2*x^2-x+3)^{(1/2)}+9267707/131072*2^{(1/2)}*\operatorname{arcsinh}(4/23*23^{(1/2)}*(x-1/4))+125/12*(2*x^2-x+3)^{(1/2)}*x^5+1355/48*(2*x^2-x+3)^{(1/2)}*x^4+8185/256*(2*x^2-x+3)^{(1/2)}*x^3-3387/1024*(2*x^2-x+3)^{(1/2)}*x^2-372783/8192*(2*x^2-x+3)^{(1/2)}*x$

**maxima** [A] time = 0.98, size = 114, normalized size = 0.80

$$\frac{125}{12} \sqrt{2x^2 - x + 3} x^5 + \frac{1355}{48} \sqrt{2x^2 - x + 3} x^4 + \frac{8185}{256} \sqrt{2x^2 - x + 3} x^3 - \frac{3387}{1024} \sqrt{2x^2 - x + 3} x^2 - \frac{372783}{8192} \sqrt{2x^2 - x + 3} x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x, algorithm="maxima")`

[Out]  $125/12*\operatorname{sqrt}(2*x^2 - x + 3)*x^5 + 1355/48*\operatorname{sqrt}(2*x^2 - x + 3)*x^4 + 8185/256*\operatorname{sqrt}(2*x^2 - x + 3)*x^3 - 3387/1024*\operatorname{sqrt}(2*x^2 - x + 3)*x^2 - 372783/8192*\operatorname{sqrt}(2*x^2 - x + 3)*x + 9267707/131072*\operatorname{sqrt}(2)*\operatorname{arcsinh}(1/23*\operatorname{sqrt}(23)*(4*x - 1)) - 203373/32768*\operatorname{sqrt}(2*x^2 - x + 3)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(5x^2 + 3x + 2)^3}{\sqrt{2x^2 - x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^(1/2),x)`

[Out] `int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^(1/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x^2 + 3x + 2)^3}{\sqrt{2x^2 - x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**2+3*x+2)**3/(2*x**2-x+3)**(1/2),x)`

[Out] `Integral((5*x**2 + 3*x + 2)**3/sqrt(2*x**2 - x + 3), x)`

$$3.81 \quad \int \frac{(2+3x+5x^2)^2}{\sqrt{3-x+2x^2}} dx$$

**Optimal.** Leaf size=101

$$\frac{655}{96} \sqrt{2x^2 - x + 3} x^2 + \frac{3443}{768} \sqrt{2x^2 - x + 3} x - \frac{11373 \sqrt{2x^2 - x + 3}}{1024} + \frac{25}{8} \sqrt{2x^2 - x + 3} x^3 + \frac{30725 \sinh^{-1} \left( \frac{1-4x}{\sqrt{23}} \right)}{2048 \sqrt{2}}$$

[Out] 30725/4096\*arcsinh(1/23\*(1-4\*x)\*23^(1/2))\*2^(1/2)-11373/1024\*(2\*x^2-x+3)^(1/2)+3443/768\*x\*(2\*x^2-x+3)^(1/2)+655/96\*x^2\*(2\*x^2-x+3)^(1/2)+25/8\*x^3\*(2\*x^2-x+3)^(1/2)

**Rubi [A]** time = 0.09, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$ , Rules used = {1661, 640, 619, 215}

$$\frac{25}{8} \sqrt{2x^2 - x + 3} x^3 + \frac{655}{96} \sqrt{2x^2 - x + 3} x^2 + \frac{3443}{768} \sqrt{2x^2 - x + 3} x - \frac{11373 \sqrt{2x^2 - x + 3}}{1024} + \frac{30725 \sinh^{-1} \left( \frac{1-4x}{\sqrt{23}} \right)}{2048 \sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3\*x + 5\*x^2)^2/Sqrt[3 - x + 2\*x^2], x]

[Out] (-11373\*Sqrt[3 - x + 2\*x^2])/1024 + (3443\*x\*Sqrt[3 - x + 2\*x^2])/768 + (655\*x^2\*Sqrt[3 - x + 2\*x^2])/96 + (25\*x^3\*Sqrt[3 - x + 2\*x^2])/8 + (30725\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(2048\*Sqrt[2])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] :> Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 619

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] :> Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^(p)), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 640

Int[((d\_) + (e\_)\*(x\_))\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]

&& NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

### Rule 1661

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(q + 2\*p + 1)), x] + Dist[1/(c\*(q + 2\*p + 1)), Int[(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*(q + 2\*p + 1)\*Pq - a\*e\*(q - 1)\*x^(q - 2) - b\*e\*(q + p)\*x^(q - 1) - c\*e\*(q + 2\*p + 1)\*x^q, x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

### Rubi steps

$$\begin{aligned}
 \int \frac{(2 + 3x + 5x^2)^2}{\sqrt{3 - x + 2x^2}} dx &= \frac{25}{8} x^3 \sqrt{3 - x + 2x^2} + \frac{1}{8} \int \frac{32 + 96x + 7x^2 + \frac{655x^3}{2}}{\sqrt{3 - x + 2x^2}} dx \\
 &= \frac{655}{96} x^2 \sqrt{3 - x + 2x^2} + \frac{25}{8} x^3 \sqrt{3 - x + 2x^2} + \frac{1}{48} \int \frac{192 - 1389x + \frac{3443x^2}{4}}{\sqrt{3 - x + 2x^2}} dx \\
 &= \frac{3443}{768} x \sqrt{3 - x + 2x^2} + \frac{655}{96} x^2 \sqrt{3 - x + 2x^2} + \frac{25}{8} x^3 \sqrt{3 - x + 2x^2} + \frac{1}{192} \int \frac{-\frac{7257}{4} - \frac{3411}{8}}{\sqrt{3 - x + 2x^2}} dx \\
 &= -\frac{11373 \sqrt{3 - x + 2x^2}}{1024} + \frac{3443}{768} x \sqrt{3 - x + 2x^2} + \frac{655}{96} x^2 \sqrt{3 - x + 2x^2} + \frac{25}{8} x^3 \sqrt{3 - x + 2x^2} \\
 &= -\frac{11373 \sqrt{3 - x + 2x^2}}{1024} + \frac{3443}{768} x \sqrt{3 - x + 2x^2} + \frac{655}{96} x^2 \sqrt{3 - x + 2x^2} + \frac{25}{8} x^3 \sqrt{3 - x + 2x^2} \\
 &= -\frac{11373 \sqrt{3 - x + 2x^2}}{1024} + \frac{3443}{768} x \sqrt{3 - x + 2x^2} + \frac{655}{96} x^2 \sqrt{3 - x + 2x^2} + \frac{25}{8} x^3 \sqrt{3 - x + 2x^2}
 \end{aligned}$$

**Mathematica** [A] time = 0.08, size = 55, normalized size = 0.54

$$\frac{4\sqrt{2x^2 - x + 3} (9600x^3 + 20960x^2 + 13772x - 34119) + 92175\sqrt{2} \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{12288}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x + 5\*x^2)^2/Sqrt[3 - x + 2\*x^2], x]

[Out]  $(4\sqrt{3-x+2x^2})(-34119+13772x+20960x^2+9600x^3)+92175\sqrt{2}\operatorname{ArcSinh}[(1-4x)/\sqrt{23}]/12288$

**fricas** [A] time = 0.73, size = 68, normalized size = 0.67

$$\frac{1}{3072}(9600x^3+20960x^2+13772x-34119)\sqrt{2x^2-x+3}+\frac{30725}{8192}\sqrt{2}\log\left(4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x, algorithm="fricas")`

[Out]  $1/3072*(9600*x^3+20960*x^2+13772*x-34119)*\sqrt{2*x^2-x+3}+30725/8192*\sqrt{2}*\log(4*\sqrt{2}*\sqrt{2*x^2-x+3}*(4*x-1)-32*x^2+16*x-25)$

**giac** [A] time = 0.49, size = 63, normalized size = 0.62

$$\frac{1}{3072}(4(40(60x+131)x+3443)x-34119)\sqrt{2x^2-x+3}+\frac{30725}{4096}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x, algorithm="giac")`

[Out]  $1/3072*(4*(40*(60*x+131)*x+3443)*x-34119)*\sqrt{2*x^2-x+3}+30725/4096*\sqrt{2}*\log(-2*\sqrt{2}*(\sqrt{2}*x-\sqrt{2*x^2-x+3}))+1)$

**maple** [A] time = 0.01, size = 79, normalized size = 0.78

$$\frac{25\sqrt{2x^2-x+3}x^3}{8}+\frac{655\sqrt{2x^2-x+3}x^2}{96}+\frac{3443\sqrt{2x^2-x+3}x}{768}-\frac{30725\sqrt{2}\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{4096}-\frac{11373\sqrt{2x^2-x+3}}{1024}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x)`

[Out]  $25/8*(2*x^2-x+3)^(1/2)*x^3+655/96*(2*x^2-x+3)^(1/2)*x^2+3443/768*(2*x^2-x+3)^(1/2)*x-11373/1024*(2*x^2-x+3)^(1/2)-30725/4096*2^(1/2)*\operatorname{arcsinh}(4/23*23^(1/2)*(x-1/4))$

**maxima** [A] time = 0.97, size = 80, normalized size = 0.79

$$\frac{25}{8}\sqrt{2x^2-x+3}x^3+\frac{655}{96}\sqrt{2x^2-x+3}x^2+\frac{3443}{768}\sqrt{2x^2-x+3}x-\frac{30725}{4096}\sqrt{2}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right)-\frac{11373}{1024}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^2/(2\*x^2-x+3)^(1/2),x, algorithm="maxima")

[Out] 25/8\*sqrt(2\*x^2 - x + 3)\*x^3 + 655/96\*sqrt(2\*x^2 - x + 3)\*x^2 + 3443/768\*sqrt(2\*x^2 - x + 3)\*x - 30725/4096\*sqrt(2)\*arcsinh(1/23\*sqrt(23)\*(4\*x - 1)) - 11373/1024\*sqrt(2\*x^2 - x + 3)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(5x^2 + 3x + 2)^2}{\sqrt{2x^2 - x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x + 5\*x^2 + 2)^2/(2\*x^2 - x + 3)^(1/2),x)

[Out] int((3\*x + 5\*x^2 + 2)^2/(2\*x^2 - x + 3)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x^2 + 3x + 2)^2}{\sqrt{2x^2 - x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*2+3\*x+2)\*\*2/(2\*x\*\*2-x+3)\*\*(1/2),x)

[Out] Integral((5\*x\*\*2 + 3\*x + 2)\*\*2/sqrt(2\*x\*\*2 - x + 3), x)

$$3.82 \quad \int \frac{2+3x+5x^2}{\sqrt{3-x+2x^2}} dx$$

Optimal. Leaf size=59

$$\frac{5}{4}\sqrt{2x^2-x+3}x + \frac{39}{16}\sqrt{2x^2-x+3} + \frac{17 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}$$

[Out] 17/64\*arcsinh(1/23\*(1-4\*x)\*23^(1/2))\*2^(1/2)+39/16\*(2\*x^2-x+3)^(1/2)+5/4\*x\*(2\*x^2-x+3)^(1/2)

**Rubi** [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$ , Rules used = {1661, 640, 619, 215}

$$\frac{5}{4}\sqrt{2x^2-x+3}x + \frac{39}{16}\sqrt{2x^2-x+3} + \frac{17 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3\*x + 5\*x^2)/Sqrt[3 - x + 2\*x^2], x]

[Out] (39\*Sqrt[3 - x + 2\*x^2])/16 + (5\*x\*Sqrt[3 - x + 2\*x^2])/4 + (17\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(32\*Sqrt[2])

Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] :> Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^(p)), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x]] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

### Rubi steps

$$\begin{aligned} \int \frac{2 + 3x + 5x^2}{\sqrt{3 - x + 2x^2}} dx &= \frac{5}{4}x\sqrt{3 - x + 2x^2} + \frac{1}{4} \int \frac{-7 + \frac{39x}{2}}{\sqrt{3 - x + 2x^2}} dx \\ &= \frac{39}{16}\sqrt{3 - x + 2x^2} + \frac{5}{4}x\sqrt{3 - x + 2x^2} - \frac{17}{32} \int \frac{1}{\sqrt{3 - x + 2x^2}} dx \\ &= \frac{39}{16}\sqrt{3 - x + 2x^2} + \frac{5}{4}x\sqrt{3 - x + 2x^2} - \frac{17 \operatorname{Subst} \left( \int \frac{1}{\sqrt{1 + \frac{x^2}{23}}} dx, x, -1 + 4x \right)}{32\sqrt{46}} \\ &= \frac{39}{16}\sqrt{3 - x + 2x^2} + \frac{5}{4}x\sqrt{3 - x + 2x^2} + \frac{17 \sinh^{-1} \left( \frac{1 - 4x}{\sqrt{23}} \right)}{32\sqrt{2}} \end{aligned}$$

**Mathematica** [A] time = 0.04, size = 45, normalized size = 0.76

$$\frac{1}{64} \left( 4\sqrt{2x^2 - x + 3} (20x + 39) + 17\sqrt{2} \sinh^{-1} \left( \frac{1 - 4x}{\sqrt{23}} \right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)/Sqrt[3 - x + 2*x^2], x]
```

```
[Out] (4*(39 + 20*x)*Sqrt[3 - x + 2*x^2] + 17*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]]
)/64
```

**fricas** [A] time = 0.81, size = 58, normalized size = 0.98

$$\frac{1}{16} \sqrt{2x^2 - x + 3} (20x + 39) + \frac{17}{128} \sqrt{2} \log \left( 4\sqrt{2} \sqrt{2x^2 - x + 3} (4x - 1) - 32x^2 + 16x - 25 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(1/2), x, algorithm="fricas")
```



[Out]  $\frac{1}{16}\sqrt{2x^2 - x + 3}(20x + 39) + \frac{17}{128}\sqrt{2}\log(4\sqrt{2})\sqrt{2x^2 - x + 3}(4x - 1) - 32x^2 + 16x - 25)$

**giac** [A] time = 0.53, size = 53, normalized size = 0.90

$$\frac{1}{16}\sqrt{2x^2 - x + 3}(20x + 39) + \frac{17}{64}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x - \sqrt{2x^2 - x + 3}\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x, algorithm="giac")`

[Out]  $\frac{1}{16}\sqrt{2x^2 - x + 3}(20x + 39) + \frac{17}{64}\sqrt{2}\log(-2\sqrt{2})(\sqrt{2}x - \sqrt{2x^2 - x + 3}) + 1)$

**maple** [A] time = 0.01, size = 45, normalized size = 0.76

$$\frac{5\sqrt{2x^2 - x + 3}x}{4} - \frac{17\sqrt{2}\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{64} + \frac{39\sqrt{2x^2 - x + 3}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x)`

[Out]  $\frac{5}{4}(2x^2-x+3)^{(1/2)}x + \frac{39}{16}(2x^2-x+3)^{(1/2)} - \frac{17}{64}2^{(1/2)}\operatorname{arcsinh}\left(\frac{4}{23}23^{(1/2)}(x-1/4)\right)$

**maxima** [A] time = 0.96, size = 46, normalized size = 0.78

$$\frac{5}{4}\sqrt{2x^2 - x + 3}x - \frac{17}{64}\sqrt{2}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x - 1)\right) + \frac{39}{16}\sqrt{2x^2 - x + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x, algorithm="maxima")`

[Out]  $\frac{5}{4}\sqrt{2x^2 - x + 3}x - \frac{17}{64}\sqrt{2}\operatorname{arcsinh}\left(\frac{1}{23}\sqrt{23}\right)(4x - 1) + \frac{39}{16}\sqrt{2x^2 - x + 3}$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{5x^2 + 3x + 2}{\sqrt{2x^2 - x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)/(2*x^2 - x + 3)^(1/2), x)
```

```
[Out] int((3*x + 5*x^2 + 2)/(2*x^2 - x + 3)^(1/2), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^2 + 3x + 2}{\sqrt{2x^2 - x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)/(2*x**2-x+3)**(1/2), x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)/sqrt(2*x**2 - x + 3), x)
```

$$3.83 \quad \int \frac{1}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx$$

**Optimal.** Leaf size=148

$$\sqrt{\frac{1}{682}(13+10\sqrt{2})} \tan^{-1} \left( \frac{\sqrt{\frac{11}{31(13+10\sqrt{2})}}((13+10\sqrt{2})x+3\sqrt{2}+7)}{\sqrt{2x^2-x+3}} \right) - \sqrt{\frac{1}{682}(10\sqrt{2}-13)} \tanh^{-1} \left( \frac{\sqrt{\frac{11}{31(13+10\sqrt{2})}}((13+10\sqrt{2})x+3\sqrt{2}+7)}{\sqrt{2x^2-x+3}} \right)$$

[Out]  $-1/682*\operatorname{arctanh}(1/31*(7+x*(13-10*2^{(1/2)})-3*2^{(1/2)})*341^{(1/2)/(-13+10*2^{(1/2)})^{(1/2)/(2*x^2-x+3)^{(1/2)}}*(-8866+6820*2^{(1/2)})^{(1/2)}+1/682*\operatorname{arctan}(1/31*(7+3*2^{(1/2)}+x*(13+10*2^{(1/2)}))*341^{(1/2)/(13+10*2^{(1/2)})^{(1/2)/(2*x^2-x+3)^{(1/2)}}*(8866+6820*2^{(1/2)})^{(1/2)})$

**Rubi [A]** time = 0.31, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$ , Rules used = {986, 1029, 204, 206}

$$\sqrt{\frac{1}{682}(13+10\sqrt{2})} \tan^{-1} \left( \frac{\sqrt{\frac{11}{31(13+10\sqrt{2})}}((13+10\sqrt{2})x+3\sqrt{2}+7)}{\sqrt{2x^2-x+3}} \right) - \sqrt{\frac{1}{682}(10\sqrt{2}-13)} \tanh^{-1} \left( \frac{\sqrt{\frac{11}{31(13+10\sqrt{2})}}((13+10\sqrt{2})x+3\sqrt{2}+7)}{\sqrt{2x^2-x+3}} \right)$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[3 - x + 2\*x^2]\*(2 + 3\*x + 5\*x^2)), x]

[Out]  $\operatorname{Sqrt}[(13+10*\operatorname{Sqrt}[2])/682]*\operatorname{ArcTan}[(\operatorname{Sqrt}[11/(31*(13+10*\operatorname{Sqrt}[2]))])*(7+3*\operatorname{Sqrt}[2]+(13+10*\operatorname{Sqrt}[2])*x)]/\operatorname{Sqrt}[3-x+2*x^2] - \operatorname{Sqrt}[(-13+10*\operatorname{Sqrt}[2])/682]*\operatorname{ArcTanh}[(\operatorname{Sqrt}[11/(31*(-13+10*\operatorname{Sqrt}[2]))])*(7-3*\operatorname{Sqrt}[2]+(13-10*\operatorname{Sqrt}[2])*x)]/\operatorname{Sqrt}[3-x+2*x^2]$

**Rule 204**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 206**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]]/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 986

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e - b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e - b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*g*(g*b - 2*a*h), Subst[Int[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f), 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{3-x+2x^2} (2+3x+5x^2)} dx &= -\frac{\int \frac{11-11\sqrt{2}-11x}{\sqrt{3-x+2x^2} (2+3x+5x^2)} dx}{22\sqrt{2}} + \frac{\int \frac{11+11\sqrt{2}-11x}{\sqrt{3-x+2x^2} (2+3x+5x^2)} dx}{22\sqrt{2}} \\ &= -\left(\frac{1}{2} \left(11(20-13\sqrt{2})\right)\right) \text{Subst}\left(\int \frac{1}{-3751(13-10\sqrt{2})-11x^2} dx, x, \frac{11(7-}{\sqrt{\frac{1}{682} (13+10\sqrt{2})}} \tan^{-1}\left(\frac{\sqrt{\frac{11}{31(13+10\sqrt{2})}} (7+3\sqrt{2}+(13+10\sqrt{2})x)}{\sqrt{3-x+2x^2}}\right)\right) \end{aligned}$$

Mathematica [C] time = 0.30, size = 176, normalized size = 1.19

$$\frac{\sqrt{13+i\sqrt{31}} (\sqrt{31}+13i) \tanh^{-1}\left(\frac{(-22-4i\sqrt{31})x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right) + \sqrt{13-i\sqrt{31}} (\sqrt{31}-13i) \tanh^{-1}\left(\frac{(-22+4i\sqrt{31})x-i\sqrt{31}}{2\sqrt{286-22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)}{20\sqrt{682}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[3 - x + 2\*x^2]\*(2 + 3\*x + 5\*x^2)),x]

[Out] 
$$-1/20*(\text{Sqrt}[13 + I*\text{Sqrt}[31]]*(13*I + \text{Sqrt}[31])*\text{ArcTanh}[(63 + I*\text{Sqrt}[31] + (-22 - (4*I)*\text{Sqrt}[31])*x)/(2*\text{Sqrt}[286 + (22*I)*\text{Sqrt}[31]]*\text{Sqrt}[3 - x + 2*x^2])]) + \text{Sqrt}[13 - I*\text{Sqrt}[31]]*(-13*I + \text{Sqrt}[31])*\text{ArcTanh}[(63 - I*\text{Sqrt}[31] + (-22 + (4*I)*\text{Sqrt}[31])*x)/(2*\text{Sqrt}[286 - (22*I)*\text{Sqrt}[31]]*\text{Sqrt}[3 - x + 2*x^2])])/\text{Sqrt}[682]$$

**fricas** [B] time = 2.25, size = 2002, normalized size = 13.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5\*x^2+3\*x+2)/(2\*x^2-x+3)^(1/2),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/845680*\text{sqrt}(341)*200^{(1/4)}*\text{sqrt}(31)*\text{sqrt}(5)*\text{sqrt}(13*\text{sqrt}(2) + 20)*(13*\text{sqrt}(2) - 20)*\log(1240*(\text{sqrt}(341)*200^{(1/4)}*\text{sqrt}(31)*\text{sqrt}(5)*\text{sqrt}(2*x^2 - x + 3)*(\text{sqrt}(2)*(4*x - 1) - 3*x - 5)*\text{sqrt}(13*\text{sqrt}(2) + 20) + 7595*x^2 + 6820*\text{sqrt}(2)*(2*x^2 - x + 3) - 23405*x + 31000)/x^2) + 1/845680*\text{sqrt}(341)*200^{(1/4)}*\text{sqrt}(31)*\text{sqrt}(5)*\text{sqrt}(13*\text{sqrt}(2) + 20)*(13*\text{sqrt}(2) - 20)*\log(-1240*(\text{sqrt}(341)*200^{(1/4)}*\text{sqrt}(31)*\text{sqrt}(5)*\text{sqrt}(2*x^2 - x + 3)*(\text{sqrt}(2)*(4*x - 1) - 3*x - 5)*\text{sqrt}(13*\text{sqrt}(2) + 20) - 7595*x^2 - 6820*\text{sqrt}(2)*(2*x^2 - x + 3) + 23405*x - 31000)/x^2) - 1/6820*\text{sqrt}(341)*200^{(1/4)}*\text{sqrt}(5)*\text{sqrt}(2)*\text{sqrt}(13*\text{sqrt}(2) + 20)*\arctan(1/2762875*(14260*\text{sqrt}(341)*\text{sqrt}(5)*\text{sqrt}(2*x^2 - x + 3)*(11*200^{(3/4)}*(8056*x^7 - 28976*x^6 + 61838*x^5 - 93342*x^4 + 45376*x^3 - 18288*x^2 - \text{sqrt}(2)*(4702*x^7 - 19541*x^6 + 40352*x^5 - 68777*x^4 + 35480*x^3 - 19080*x^2 - 34560*x + 27648) - 55296*x + 34560) + 5*200^{(1/4)}*(18463*x^7 - 280047*x^6 + 1453472*x^5 - 3238500*x^4 + 4140576*x^3 - 2378592*x^2 - \text{sqrt}(2)*(11418*x^7 - 177633*x^6 + 957180*x^5 - 2237548*x^4 + 2920320*x^3 - 2005920*x^2 - 1990656*x + 1534464) - 3068928*x + 1990656))*\text{sqrt}(13*\text{sqrt}(2) + 20) + 7843000*\text{sqrt}(31)*\text{sqrt}(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - \text{sqrt}(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*\text{sqrt}(310)*(\text{sqrt}(341)*\text{sqrt}(5)*\text{sqrt}(2*x^2 - x + 3)*(11*200^{(3/4)}*(30876*x^7 - 44014*x^6 + 139674*x^5 - 42464*x^4 + 38736*x^3 + 89856*x^2 - \text{sqrt}(2)*(15454*x^7 - 22399*x^6 + 73509*x^5 - 37360*x^4 + 52200*x^3 + 13824*x^2 - 13824*x) - 89856*x) + 5*200^{(1/4)}*(69479*x^7 - 898236*x^6 + 3454740*x^5 - 4394304*x^4 + 5347296*x^3 + 4478976*x^2 - \text{sqrt}(2)*(38627*x^7 - 500012*x^6 + 1934180*x^5 - 2560320*x^4 + 3506400*x^3 + 1202688*x^2 - 1202688*x) - 4478976*x))*\text{sqrt}(13*\text{sqrt}(2) + 20) + 550*\text{sqrt}(31)*\text{sqrt}(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - \text{sqrt}(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) + 25*\text{sqrt}(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*\text{sqrt}(2)*(4*x^8 - 7 \end{aligned}$$

```

6*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144
820224*x))*sqrt(-(sqrt(341)*200^(1/4)*sqrt(31)*sqrt(5)*sqrt(2*x^2 - x + 3)*
(sqrt(2)*(4*x - 1) - 3*x - 5)*sqrt(13*sqrt(2) + 20) - 7595*x^2 - 6820*sqrt(
2)*(2*x^2 - x + 3) + 23405*x - 31000)/x^2) + 89125*sqrt(31)*(2828123*x^8 -
9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3
+ 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 +
15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936)
)/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 4
4249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) - 1/6820*sqrt(341)*200
^(1/4)*sqrt(5)*sqrt(2)*sqrt(13*sqrt(2) + 20)*arctan(1/2762875*(14260*sqrt(3
41)*sqrt(5)*sqrt(2*x^2 - x + 3)*(11*200^(3/4)*(8056*x^7 - 28976*x^6 + 61838
*x^5 - 93342*x^4 + 45376*x^3 - 18288*x^2 - sqrt(2)*(4702*x^7 - 19541*x^6 +
40352*x^5 - 68777*x^4 + 35480*x^3 - 19080*x^2 - 34560*x + 27648) - 55296*x
+ 34560) + 5*200^(1/4)*(18463*x^7 - 280047*x^6 + 1453472*x^5 - 3238500*x^4
+ 4140576*x^3 - 2378592*x^2 - sqrt(2)*(11418*x^7 - 177633*x^6 + 957180*x^5
- 2237548*x^4 + 2920320*x^3 - 2005920*x^2 - 1990656*x + 1534464) - 3068928*
x + 1990656))*sqrt(13*sqrt(2) + 20) - 7843000*sqrt(31)*sqrt(2)*(28180*x^8 -
254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x
^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4
- 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*sq
qrt(310)*(sqrt(341)*sqrt(5)*sqrt(2*x^2 - x + 3)*(11*200^(3/4)*(30876*x^7 -
44014*x^6 + 139674*x^5 - 42464*x^4 + 38736*x^3 + 89856*x^2 - sqrt(2)*(15454
*x^7 - 22399*x^6 + 73509*x^5 - 37360*x^4 + 52200*x^3 + 13824*x^2 - 13824*x)
- 89856*x) + 5*200^(1/4)*(69479*x^7 - 898236*x^6 + 3454740*x^5 - 4394304*x
^4 + 5347296*x^3 + 4478976*x^2 - sqrt(2)*(38627*x^7 - 500012*x^6 + 1934180*
x^5 - 2560320*x^4 + 3506400*x^3 + 1202688*x^2 - 1202688*x) - 4478976*x))*sq
rt(13*sqrt(2) + 20) - 550*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 15788
88*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(155
50*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 +
1209600*x^2 - 1036800*x) + 3276288*x) - 25*sqrt(31)*(254591*x^8 - 4815126*
x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 16895692
8*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 361
8*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt((sqrt(341)*200^(1/4)*sqrt(3
1)*sqrt(5)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(4*x - 1) - 3*x - 5)*sqrt(13*sqrt(2
) + 20) + 7595*x^2 + 6820*sqrt(2)*(2*x^2 - x + 3) - 23405*x + 31000)/x^2) -
89125*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 +
254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 26
92*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x -
5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6
+ 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18
579456))

```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5\*x^2+3\*x+2)/(2\*x^2-x+3)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP  
 UT:sage2:=int(sage0,x)::OUTPUT:Francis algorithm failure for[-1.0,infinity,  
 infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infini  
 ty]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity  
 ]proot error [1.0,infinity,infinity,infinity,infinity]Evaluation time: 7.57  
 Done

maple [B] time = 0.00, size = 684, normalized size = 4.62

$$\sqrt{\frac{8(x+\sqrt{2}-1)^2}{(-x+\sqrt{2}+1)^2} + \frac{3\sqrt{2}(x+\sqrt{2}-1)^2}{(-x+\sqrt{2}+1)^2} + 8 - 3\sqrt{2}} \sqrt{2} \left( 465124\sqrt{2} \operatorname{arctanh} \left( \frac{31 \sqrt{\frac{8(x+\sqrt{2}-1)^2}{(-x+\sqrt{2}+1)^2} + \frac{3\sqrt{2}(x+\sqrt{2}-1)^2}{(-x+\sqrt{2}+1)^2} + 8 - 3\sqrt{2}}}{2\sqrt{-8866+6820\sqrt{2}}} \right) - 8668 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(5\*x^2+3\*x+2)/(2\*x^2-x+3)^(1/2),x)

[Out] 1/21142\*(8\*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3\*2^(1/2)\*(x+2^(1/2)-1)^2/(-x+2  
 ^2+8-3\*2^(1/2))^2)^(1/2)\*2^(1/2)\*(369\*2^(1/2)\*(-8866+6820\*2^(1/2))^2  
 ^2)\*(-775687+549362\*2^(1/2))^2)\*arctan(1/11692487\*(-775687+549362\*2^(1/2)  
 ))^(1/2)\*(-23\*(8+3\*2^(1/2))\*(-23\*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24\*2^(1/2  
 )-41))^2\*(6485\*2^(1/2)\*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+10368\*(x+2^(1/2  
 )-1)^2/(-x+2^(1/2)+1)^2+22379\*2^(1/2)+32016)/(23\*(x+2^(1/2)-1)^4/(-x+2^(1/2  
 )+1)^4+82\*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23)\*(x+2^(1/2)-1)/(-x+2^(1/2)+1  
 \*(8+3\*2^(1/2)))+520\*(-8866+6820\*2^(1/2))^2)\*(-775687+549362\*2^(1/2))^2  
 ^2)\*arctan(1/11692487\*(-775687+549362\*2^(1/2))^2)\*(-23\*(8+3\*2^(1/2))\*(-23  
 \*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24\*2^(1/2)-41))^2\*(6485\*2^(1/2)\*(x+2^(  
 1/2)-1)^2/(-x+2^(1/2)+1)^2+10368\*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+22379\*2^(  
 1/2)+32016)/(23\*(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)^4+82\*(x+2^(1/2)-1)^2/(-x+2^(  
 1/2)+1)^2+23)\*(x+2^(1/2)-1)/(-x+2^(1/2)+1)\*(8+3\*2^(1/2)))+465124\*arctanh(3  
 1/2\*(8\*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3\*2^(1/2)\*(x+2^(1/2)-1)^2/(-x+2^(1/  
 2)+1)^2+8-3\*2^(1/2))^2/(-8866+6820\*2^(1/2))^2)\*2^(1/2)-866822\*arctan  
 h(31/2\*(8\*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3\*2^(1/2)\*(x+2^(1/2)-1)^2/(-x+2  
 ^2+8-3\*2^(1/2))^2/(-8866+6820\*2^(1/2))^2)/((8\*(x+2^(1/2)-  
 1)^2/(-x+2^(1/2)+1)^2+3\*2^(1/2)\*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3\*2^(1/2

))/((1+(x+2^(1/2)-1)/(-x+2^(1/2)+1))^2)^(1/2)/((1+(x+2^(1/2)-1)/(-x+2^(1/2)+1))/((8+3\*2^(1/2)))/(-8866+6820\*2^(1/2)))^(1/2)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(5x^2 + 3x + 2)\sqrt{2x^2 - x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5\*x^2+3\*x+2)/(2\*x^2-x+3)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((5\*x^2 + 3\*x + 2)\*sqrt(2\*x^2 - x + 3)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2x^2 - x + 3} (5x^2 + 3x + 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((2\*x^2 - x + 3)^(1/2)\*(3\*x + 5\*x^2 + 2)),x)

[Out] int(1/((2\*x^2 - x + 3)^(1/2)\*(3\*x + 5\*x^2 + 2)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^2 - x + 3} (5x^2 + 3x + 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5\*x\*\*2+3\*x+2)/(2\*x\*\*2-x+3)\*\*(1/2),x)

[Out] Integral(1/(sqrt(2\*x\*\*2 - x + 3)\*(5\*x\*\*2 + 3\*x + 2)), x)



$$3.84 \quad \int \frac{1}{\sqrt{3-x+2x^2} (2+3x+5x^2)^2} dx$$

**Optimal.** Leaf size=188

$$\frac{\sqrt{2x^2 - x + 3} (65x + 4)}{682 (5x^2 + 3x + 2)} + \frac{\sqrt{\frac{1}{682} (2343727 + 1678700\sqrt{2})} \tan^{-1} \left( \frac{\sqrt{\frac{11}{31(2343727+1678700\sqrt{2})}} ((5751+3935\sqrt{2})x+1816\sqrt{2}+2119)}{\sqrt{2x^2-x+3}} \right)}{1364}$$

[Out] 1/682\*(4+65\*x)\*(2\*x^2-x+3)^(1/2)/(5\*x^2+3\*x+2)-1/930248\*arctanh(1/31\*(2119+x\*(5751-3935\*2^(1/2))-1816\*2^(1/2))\*341^(1/2)/(-2343727+1678700\*2^(1/2))^(1/2)/(2\*x^2-x+3)^(1/2))\*(-1598421814+1144873400\*2^(1/2))^(1/2)+1/930248\*arctan(1/31\*(2119+1816\*2^(1/2)+x\*(5751+3935\*2^(1/2)))\*341^(1/2)/(2343727+1678700\*2^(1/2))^(1/2)/(2\*x^2-x+3)^(1/2))\*(1598421814+1144873400\*2^(1/2))^(1/2)

**Rubi [A]** time = 0.43, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {974, 1035, 1029, 206, 204}

$$\frac{\sqrt{2x^2 - x + 3} (65x + 4)}{682 (5x^2 + 3x + 2)} + \frac{\sqrt{\frac{1}{682} (2343727 + 1678700\sqrt{2})} \tan^{-1} \left( \frac{\sqrt{\frac{11}{31(2343727+1678700\sqrt{2})}} ((5751+3935\sqrt{2})x+1816\sqrt{2}+2119)}{\sqrt{2x^2-x+3}} \right)}{1364}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[3 - x + 2\*x^2]\*(2 + 3\*x + 5\*x^2)^2), x]

[Out] ((4 + 65\*x)\*Sqrt[3 - x + 2\*x^2])/(682\*(2 + 3\*x + 5\*x^2)) + (Sqrt[(2343727 + 1678700\*Sqrt[2])/682]\*ArcTan[(Sqrt[11/(31\*(2343727 + 1678700\*Sqrt[2]))])\*(2119 + 1816\*Sqrt[2] + (5751 + 3935\*Sqrt[2])\*x)]/Sqrt[3 - x + 2\*x^2]])/1364 - (Sqrt[(-2343727 + 1678700\*Sqrt[2])/682]\*ArcTanh[(Sqrt[11/(31\*(-2343727 + 1678700\*Sqrt[2]))])\*(2119 - 1816\*Sqrt[2] + (5751 - 3935\*Sqrt[2])\*x)]/Sqrt[3 - x + 2\*x^2]])/1364

**Rule 204**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :- Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 206**

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rule 974

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((d_) + (e_)*(x_) + (f_)*(x
_)^2)^(q_), x_Symbol] := Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q,
0]
```

### Rule 1029

```
Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) +
(e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := Dist[-2*g*(g*b - 2*a*h), Subst[Int
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g*
b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

### Rule 1035

```
Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) +
(e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d
- a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{3-x+2x^2} (2+3x+5x^2)^2} dx &= \frac{(4+65x)\sqrt{3-x+2x^2}}{682(2+3x+5x^2)} - \frac{\int \frac{-1826+\frac{2255x}{2}}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{7502} \\
&= \frac{(4+65x)\sqrt{3-x+2x^2}}{682(2+3x+5x^2)} - \frac{\int \frac{\frac{121}{2}(537-332\sqrt{2})-\frac{121}{2}(127-205\sqrt{2})x}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{165044\sqrt{2}} + \frac{\int \frac{121}{2}(537-332\sqrt{2})}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{136} \\
&= \frac{(4+65x)\sqrt{3-x+2x^2}}{682(2+3x+5x^2)} - \frac{1}{496} \left( 11 \left( 3357400 - 2343727\sqrt{2} \right) \right) \text{Subst} \left( \int \frac{121}{2}(537-332\sqrt{2})}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx \right) \\
&= \frac{(4+65x)\sqrt{3-x+2x^2}}{682(2+3x+5x^2)} + \frac{\sqrt{\frac{1}{682} (2343727 + 1678700\sqrt{2})} \tan^{-1} \left( \frac{\sqrt{31(2343727 + 1678700\sqrt{2})}}{\sqrt{31-x+2x^2}} \right)}{136}
\end{aligned}$$

**Mathematica [C]** time = 1.00, size = 287, normalized size = 1.53

$$25 \left[ \frac{i\sqrt{286+22i\sqrt{31}} (224\sqrt{31}+1023i) \tanh^{-1} \left( \frac{(-22-4i\sqrt{31})x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}} \sqrt{2x^2-x+3}} \right)}{(\sqrt{31}-13i)^2} + \frac{10i \left( 1364(\sqrt{31}+13i)(65x+4)\sqrt{2x^2-x+3} - 5\sqrt{286-22i\sqrt{31}}(787\sqrt{31}-10ix+\sqrt{31}+3i)(5\sqrt{31-x+2x^2}) \right)}{(\sqrt{31}+13i)^2(10ix+\sqrt{31}+3i)(5\sqrt{31-x+2x^2})} \right]$$

116281

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[3 - x + 2\*x^2]\*(2 + 3\*x + 5\*x^2)^2), x]

[Out] (25\*((I\*Sqrt[286 + (22\*I)\*Sqrt[31]]\*(1023\*I + 224\*Sqrt[31])\*ArcTanh[(63 + I\*Sqrt[31] + (-22 - (4\*I)\*Sqrt[31])\*x]/(2\*Sqrt[286 + (22\*I)\*Sqrt[31]]\*Sqrt[3 - x + 2\*x^2])))/(-13\*I + Sqrt[31])^2 + ((10\*I)\*(1364\*(13\*I + Sqrt[31])\*(4 + 65\*x)\*Sqrt[3 - x + 2\*x^2] - 5\*Sqrt[286 - (22\*I)\*Sqrt[31]]\*(-1271\*I + 787\*Sqrt[31] + (-22 + (4\*I)\*Sqrt[31])\*x)/(2\*Sqrt[286 - (22\*I)\*Sqrt[31]]\*Sqrt[3 - x + 2\*x^2])))/((13\*I + Sqrt[31])^2\*(3\*I + Sqrt[31] + (10\*I)\*x)\*(-4\*I + 8\*Sqrt[31] + 5\*(-13\*I + Sqrt[31])\*x)))/116281

**fricas [B]** time = 2.60, size = 2102, normalized size = 11.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5\*x^2+3\*x+2)^2/(2\*x^2-x+3)^(1/2),x, algorithm="fricas")

[Out] 1/263043507934399808\*(8422204\*563606738^(1/4)\*sqrt(33574)\*sqrt(341)\*sqrt(2)  
\*(5\*x^2 + 3\*x + 2)\*sqrt(2343727\*sqrt(2) + 3357400)\*arctan(1/710190022151725  
4683789\*(47876524\*sqrt(33574)\*(22\*563606738^(3/4)\*sqrt(341)\*(2950932\*x^7 -  
11691762\*x^6 + 24397746\*x^5 - 40053004\*x^4 + 20309552\*x^3 - 10145376\*x^2 -  
sqrt(2)\*(2248634\*x^7 - 8421787\*x^6 + 17801494\*x^5 - 27869789\*x^4 + 13808040  
\*x^3 - 6172200\*x^2 - 15724800\*x + 10596096) - 21192192\*x + 15724800) + 5203  
97\*563606738^(1/4)\*sqrt(341)\*(226651\*x^7 - 3496629\*x^6 + 18614024\*x^5 - 428  
60780\*x^4 + 55586592\*x^3 - 36274464\*x^2 - sqrt(2)\*(168871\*x^7 - 2579646\*x^6  
+ 13533020\*x^5 - 30582616\*x^4 + 39345120\*x^3 - 23947200\*x^2 - 28449792\*x +  
19450368) - 38900736\*x + 28449792))\*sqrt(2\*x^2 - x + 3)\*sqrt(2343727\*sqrt(  
2) + 3357400) + 20160232886887690715272\*sqrt(31)\*sqrt(2)\*(28180\*x^8 - 25466  
6\*x^7 + 704270\*x^6 - 1385256\*x^5 + 1549144\*x^4 - 642048\*x^3 - 98496\*x^2 - s  
qrt(2)\*(8746\*x^8 - 102335\*x^7 + 396104\*x^6 - 783113\*x^5 + 1320710\*x^4 - 752  
088\*x^3 + 396144\*x^2 + 546048\*x - 539136) + 1154304\*x - 456192) - 2\*sqrt(33  
574/2191)\*(sqrt(33574)\*(22\*563606738^(3/4)\*sqrt(341)\*(10257392\*x^7 - 147733  
68\*x^6 + 47877288\*x^5 - 20710528\*x^4 + 26321472\*x^3 + 17079552\*x^2 - sqrt(2  
)\*(8292238\*x^7 - 11867543\*x^6 + 37968813\*x^5 - 13449840\*x^4 + 14570280\*x^3  
+ 20176128\*x^2 - 20176128\*x) - 17079552\*x) + 520397\*563606738^(1/4)\*sqrt(34  
1)\*(795513\*x^7 - 10292932\*x^6 + 39734380\*x^5 - 51864768\*x^4 + 68281632\*x^3  
+ 34255872\*x^2 - 8\*sqrt(2)\*(77213\*x^7 - 998548\*x^6 + 3846220\*x^5 - 4943520\*x  
^4 + 6215760\*x^3 + 4318272\*x^2 - 4318272\*x) - 34255872\*x))\*sqrt(2\*x^2 - x  
+ 3)\*sqrt(2343727\*sqrt(2) + 3357400) + 421088065768678\*sqrt(31)\*sqrt(2)\*(12  
3408\*x^8 - 914152\*x^7 + 1578888\*x^6 - 3293072\*x^5 + 396480\*x^4 + 798336\*x^3  
- 3822336\*x^2 - sqrt(2)\*(15550\*x^8 - 118051\*x^7 + 244047\*x^6 - 707374\*x^5  
+ 1053960\*x^4 - 1667952\*x^3 + 1209600\*x^2 - 1036800\*x) + 3276288\*x) + 19140  
366625849\*sqrt(31)\*(254591\*x^8 - 4815126\*x^7 + 32303580\*x^6 - 90866808\*x^5  
+ 108781920\*x^4 - 74219328\*x^3 - 168956928\*x^2 - 15488\*sqrt(2)\*(4\*x^8 - 76\*x  
^7 + 517\*x^6 - 1536\*x^5 + 2385\*x^4 - 3618\*x^3 + 2268\*x^2 - 1944\*x) + 14482  
0224\*x))\*sqrt(-(563606738^(1/4)\*sqrt(33574)\*sqrt(341)\*sqrt(31)\*sqrt(2\*x^2 -  
x + 3)\*(sqrt(2)\*(1123\*x + 898) - 2021\*x - 225)\*sqrt(2343727\*sqrt(2) + 3357  
400) - 1731948347213\*x^2 - 1555218924028\*sqrt(2)\*(2\*x^2 - x + 3) + 53372285  
80187\*x - 7069176927400)/x^2) + 229093555532814667219\*sqrt(31)\*(2828123\*x^8  
- 9696916\*x^7 + 53385560\*x^6 - 142835344\*x^5 + 254146592\*x^4 - 249300096\*x  
^3 + 37981440\*x^2 - 7744\*sqrt(2)\*(1348\*x^8 - 2692\*x^7 + 9789\*x^6 - 10070\*x^5  
+ 15569\*x^4 - 5568\*x^3 + 1080\*x^2 + 4320\*x - 5184) + 223064064\*x - 948879  
36))/(2585191\*x^8 - 4661200\*x^7 + 14191920\*x^6 + 490880\*x^5 - 13562944\*x^4  
+ 44249088\*x^3 - 34615296\*x^2 - 24772608\*x + 18579456) + 8422204\*563606738  
^(1/4)\*sqrt(33574)\*sqrt(341)\*sqrt(2)\*(5\*x^2 + 3\*x + 2)\*sqrt(2343727\*sqrt(2)  
+ 3357400)\*arctan(1/7101900221517254683789\*(47876524\*sqrt(33574)\*(22\*56360  
6738^(3/4)\*sqrt(341)\*(2950932\*x^7 - 11691762\*x^6 + 24397746\*x^5 - 40053004\*x  
^4 + 20309552\*x^3 - 10145376\*x^2 - sqrt(2)\*(2248634\*x^7 - 8421787\*x^6 + 17

$$\begin{aligned}
& 801494x^5 - 27869789x^4 + 13808040x^3 - 6172200x^2 - 15724800x + 10596 \\
& 096) - 21192192x + 15724800) + 520397*563606738^{(1/4)}*\sqrt{341}*(226651x^7 \\
& - 3496629x^6 + 18614024x^5 - 42860780x^4 + 55586592x^3 - 36274464x^2 \\
& - \sqrt{2}*(168871x^7 - 2579646x^6 + 13533020x^5 - 30582616x^4 + 393451 \\
& 20x^3 - 23947200x^2 - 28449792x + 19450368) - 38900736x + 28449792))*\sqrt{2} \\
& \sqrt{2x^2 - x + 3}*\sqrt{2343727*\sqrt{2} + 3357400} - 20160232886887690715272 \\
& *\sqrt{31}*\sqrt{2}*(28180x^8 - 254666x^7 + 704270x^6 - 1385256x^5 + 1549 \\
& 144x^4 - 642048x^3 - 98496x^2 - \sqrt{2}*(8746x^8 - 102335x^7 + 396104x^6 \\
& - 783113x^5 + 1320710x^4 - 752088x^3 + 396144x^2 + 546048x - 53913 \\
& 6) + 1154304x - 456192) - 2*\sqrt{33574/2191}*(\sqrt{33574}*(22*563606738^{(3 \\
& /4)}*\sqrt{341}*(10257392x^7 - 14773368x^6 + 47877288x^5 - 20710528x^4 + \\
& 26321472x^3 + 17079552x^2 - \sqrt{2}*(8292238x^7 - 11867543x^6 + 3796881 \\
& 3x^5 - 13449840x^4 + 14570280x^3 + 20176128x^2 - 20176128x) - 17079552 \\
& *x) + 520397*563606738^{(1/4)}*\sqrt{341}*(795513x^7 - 10292932x^6 + 3973438 \\
& 0x^5 - 51864768x^4 + 68281632x^3 + 34255872x^2 - 8*\sqrt{2}*(77213x^7 - \\
& 998548x^6 + 3846220x^5 - 4943520x^4 + 6215760x^3 + 4318272x^2 - 43182 \\
& 72x) - 34255872x))*\sqrt{2}*\sqrt{2x^2 - x + 3}*\sqrt{2343727*\sqrt{2} + 3357400} - \\
& 421088065768678*\sqrt{31}*\sqrt{2}*(123408x^8 - 914152x^7 + 1578888x^6 - 3 \\
& 293072x^5 + 396480x^4 + 798336x^3 - 3822336x^2 - \sqrt{2}*(15550x^8 - 1 \\
& 18051x^7 + 244047x^6 - 707374x^5 + 1053960x^4 - 1667952x^3 + 1209600x^2 \\
& - 1036800x) + 3276288x) - 19140366625849*\sqrt{31}*(254591x^8 - 481512 \\
& 6x^7 + 32303580x^6 - 90866808x^5 + 108781920x^4 - 74219328x^3 - 168956 \\
& 928x^2 - 15488*\sqrt{2}*(4x^8 - 76x^7 + 517x^6 - 1536x^5 + 2385x^4 - 3 \\
& 618x^3 + 2268x^2 - 1944x) + 144820224x))*\sqrt{(563606738^{(1/4)}*\sqrt{335 \\
& 74}*\sqrt{341}*\sqrt{31}*\sqrt{2x^2 - x + 3}*(\sqrt{2}*(1123x + 898) - 2021x \\
& - 225)*\sqrt{2343727*\sqrt{2} + 3357400} + 1731948347213x^2 + 1555218924028 \\
& *\sqrt{2}*(2x^2 - x + 3) - 5337228580187x + 7069176927400)/x^2) - 22909355 \\
& 5532814667219*\sqrt{31}*(2828123x^8 - 9696916x^7 + 53385560x^6 - 14283534 \\
& 4x^5 + 254146592x^4 - 249300096x^3 + 37981440x^2 - 7744*\sqrt{2}*(1348x^8 \\
& - 2692x^7 + 9789x^6 - 10070x^5 + 15569x^4 - 5568x^3 + 1080x^2 + 43 \\
& 20x - 5184) + 223064064x - 94887936))/(2585191x^8 - 4661200x^7 + 141919 \\
& 20x^6 + 490880x^5 - 13562944x^4 + 44249088x^3 - 34615296x^2 - 24772608 \\
& *x + 18579456) + 563606738^{(1/4)}*\sqrt{33574}*\sqrt{341}*\sqrt{31}*(16787000x^2 \\
& - 2343727*\sqrt{2}*(5x^2 + 3x + 2) + 10072200x + 6714800)*\sqrt{234372 \\
& 7*\sqrt{2} + 3357400}*\log(335740000/2191*(563606738^{(1/4)}*\sqrt{33574}*\sqrt{3 \\
& 41}*\sqrt{31}*\sqrt{2x^2 - x + 3}*(\sqrt{2}*(1123x + 898) - 2021x - 225)*\sqrt{2} \\
& \sqrt{2343727*\sqrt{2} + 3357400} + 1731948347213x^2 + 1555218924028*\sqrt{2}*( \\
& 2x^2 - x + 3) - 5337228580187x + 7069176927400)/x^2) - 563606738^{(1/4)}*\sqrt{33574} \\
& *\sqrt{341}*\sqrt{31}*(16787000x^2 - 2343727*\sqrt{2}*(5x^2 + 3x + \\
& 2) + 10072200x + 6714800)*\sqrt{2343727*\sqrt{2} + 3357400}*\log(-335740000/ \\
& 2191*(563606738^{(1/4)}*\sqrt{33574}*\sqrt{341}*\sqrt{31}*\sqrt{2x^2 - x + 3}*(\sqrt{2} \\
& *(1123x + 898) - 2021x - 225)*\sqrt{2343727*\sqrt{2} + 3357400} - 173 \\
& 1948347213x^2 - 1555218924028*\sqrt{2}*(2x^2 - x + 3) + 5337228580187x - \\
& 7069176927400)/x^2) + 385694293158944*\sqrt{2x^2 - x + 3}*(65x + 4))/(5x^2 \\
& + 3x + 2)
\end{aligned}$$

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5\*x^2+3\*x+2)^2/(2\*x^2-x+3)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP  
 UT:sage2:=int(sage0,x)::OUTPUT:Francis algorithm failure for[-1.0,infinity,  
 infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,inf  
 inity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity  
 ]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai  
 lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity  
 ,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf  
 inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinity  
 ]Evaluation time: 27.71Done

**maple** [B] time = 0.01, size = 5225, normalized size = 27.79

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(5\*x^2+3\*x+2)^2/(2\*x^2-x+3)^(1/2),x)

[Out] result too large to display

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(5x^2 + 3x + 2)^2 \sqrt{2x^2 - x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5\*x^2+3\*x+2)^2/(2\*x^2-x+3)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((5\*x^2 + 3\*x + 2)^2\*sqrt(2\*x^2 - x + 3)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2x^2 - x + 3} (5x^2 + 3x + 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((2\*x^2 - x + 3)^(1/2)\*(3\*x + 5\*x^2 + 2)^2),x)

[Out] `int(1/((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^2 - x + 3} (5x^2 + 3x + 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(5*x**2+3*x+2)**2/(2*x**2-x+3)**(1/2), x)`

[Out] `Integral(1/(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2)**2), x)`

$$3.85 \quad \int \frac{1}{\sqrt{3-x+2x^2} (2+3x+5x^2)^3} dx$$

Optimal. Leaf size=223

$$\frac{\sqrt{2x^2 - x + 3} (65x + 4)}{1364 (5x^2 + 3x + 2)^2} + \frac{(86265x + 26794)\sqrt{2x^2 - x + 3}}{1860496 (5x^2 + 3x + 2)} + \frac{25\sqrt{\frac{1}{682} (6414867847 + 4536374600\sqrt{2})} \tan^{-1} \left( \frac{\sqrt{\frac{1}{31}}}{372} \right)}{372}$$

[Out] 1/1364\*(4+65\*x)\*(2\*x^2-x+3)^(1/2)/(5\*x^2+3\*x+2)^2+1/1860496\*(26794+86265\*x)\*(2\*x^2-x+3)^(1/2)/(5\*x^2+3\*x+2)-25/2537716544\*arctanh(1/31\*(123161+x\*(294669-208915\*2^(1/2))-85754\*2^(1/2))\*341^(1/2)/(-6414867847+4536374600\*2^(1/2)))^(1/2)/(2\*x^2-x+3)^(1/2))\*(-4374939871654+3093807477200\*2^(1/2))^(1/2)+25/2537716544\*arctan(1/31\*(123161+85754\*2^(1/2)+x\*(294669+208915\*2^(1/2)))\*341^(1/2)/(6414867847+4536374600\*2^(1/2)))^(1/2)/(2\*x^2-x+3)^(1/2))\*(4374939871654+3093807477200\*2^(1/2))^(1/2)

**Rubi [A]** time = 0.47, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {974, 1060, 1035, 1029, 206, 204}

$$\frac{\sqrt{2x^2 - x + 3} (65x + 4)}{1364 (5x^2 + 3x + 2)^2} + \frac{(86265x + 26794)\sqrt{2x^2 - x + 3}}{1860496 (5x^2 + 3x + 2)} + \frac{25\sqrt{\frac{1}{682} (6414867847 + 4536374600\sqrt{2})} \tan^{-1} \left( \frac{\sqrt{\frac{1}{31}}}{372} \right)}{372}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[3 - x + 2\*x^2]\*(2 + 3\*x + 5\*x^2)^3), x]

[Out] ((4 + 65\*x)\*Sqrt[3 - x + 2\*x^2])/(1364\*(2 + 3\*x + 5\*x^2)^2) + ((26794 + 86265\*x)\*Sqrt[3 - x + 2\*x^2])/(1860496\*(2 + 3\*x + 5\*x^2)) + (25\*Sqrt[(6414867847 + 4536374600\*Sqrt[2])/682]\*ArcTan[(Sqrt[11/(31\*(6414867847 + 4536374600\*Sqrt[2]))])\*(123161 + 85754\*Sqrt[2] + (294669 + 208915\*Sqrt[2])\*x)]/Sqrt[3 - x + 2\*x^2]])/3720992 - (25\*Sqrt[(-6414867847 + 4536374600\*Sqrt[2])/682]\*ArcTanH[(Sqrt[11/(31\*(-6414867847 + 4536374600\*Sqrt[2]))])\*(123161 - 85754\*Sqrt[2] + (294669 - 208915\*Sqrt[2])\*x)]/Sqrt[3 - x + 2\*x^2]])/3720992

Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :-> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[



a, 0] || LtQ[b, 0])

### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 974

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_)\*((d\_) + (e\_)\*(x\_) + (f\_)\*(x\_)^2)^(q\_), x\_Symbol] := Simp[((2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f) + c\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*x)\*(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^(q + 1))/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1)), x] - Dist[1/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1)), Int[(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^q\*Simp[2\*c\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1) - (2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(a\*f\*(p + 1) - c\*d\*(p + 2)) - e\*(b^2\*c\*e - 2\*a\*c^2\*e - b^3\*f - b\*c\*(c\*d - 3\*a\*f))\*(p + q + 2) + (2\*f\*(2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f))\*(p + q + 2) - (2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(b\*f\*(p + 1) - c\*e\*(2\*p + q + 4)))\*x + c\*f\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(2\*p + 2\*q + 5)\*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && LtQ[p, -1] && NeQ[(c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f), 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]

### Rule 1029

Int[((g\_) + (h\_)\*(x\_))/(((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)\*Sqrt[(d\_) + (e\_)\*(x\_) + (f\_)\*(x\_)^2]), x\_Symbol] := Dist[-2\*g\*(g\*b - 2\*a\*h), Subst[Int[1/Simp[g\*(g\*b - 2\*a\*h)\*(b^2 - 4\*a\*c) - (b\*d - a\*e)\*x^2, x], x], x, Simp[g\*b - 2\*a\*h - (b\*h - 2\*g\*c)\*x, x]/Sqrt[d + e\*x + f\*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && NeQ[b\*d - a\*e, 0] && EqQ[h^2\*(b\*d - a\*e) - 2\*g\*h\*(c\*d - a\*f) + g^2\*(c\*e - b\*f), 0]

### Rule 1035

Int[((g\_) + (h\_)\*(x\_))/(((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)\*Sqrt[(d\_) + (e\_)\*(x\_) + (f\_)\*(x\_)^2]), x\_Symbol] := With[{q = Rt[(c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f), 2]}, Dist[1/(2\*q), Int[Simp[h\*(b\*d - a\*e) - g\*(c\*d - a\*f - q) - (g\*(c\*e - b\*f) - h\*(c\*d - a\*f + q))\*x, x]/((a + b\*x + c\*x^2)\*Sqrt[d + e\*x + f\*x^2]), x], x] - Dist[1/(2\*q), Int[Simp[h\*(b\*d - a\*e) - g\*(c\*d - a\*f + q) - (g\*(c\*e - b\*f) - h\*(c\*d - a\*f - q))\*x, x]/((a + b\*x + c\*x^2)\*Sqrt[d + e\*x + f\*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ

$[b^2 - 4ac, 0] \&\& \text{NeQ}[e^2 - 4df, 0] \&\& \text{NeQ}[bd - ae, 0] \&\& \text{NegQ}[b^2 - 4ac]$

### Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{3-x+2x^2} (2+3x+5x^2)^3} dx &= \frac{(4+65x)\sqrt{3-x+2x^2}}{1364(2+3x+5x^2)^2} - \frac{\int \frac{-5775+\frac{6479x}{2}-2860x^2}{\sqrt{3-x+2x^2}(2+3x+5x^2)^2} dx}{15004} \\
&= \frac{(4+65x)\sqrt{3-x+2x^2}}{1364(2+3x+5x^2)^2} + \frac{(26794+86265x)\sqrt{3-x+2x^2}}{1860496(2+3x+5x^2)} - \frac{\int \frac{\frac{28220225}{2}}{\sqrt{3-x+2x^2}}}{1125} \\
&= \frac{(4+65x)\sqrt{3-x+2x^2}}{1364(2+3x+5x^2)^2} + \frac{(26794+86265x)\sqrt{3-x+2x^2}}{1860496(2+3x+5x^2)} - \frac{\int \frac{\frac{33275}{4}(261)}{\sqrt{3-x+2x^2}}}{(6875)(90)} \\
&= \frac{(4+65x)\sqrt{3-x+2x^2}}{1364(2+3x+5x^2)^2} + \frac{(26794+86265x)\sqrt{3-x+2x^2}}{1860496(2+3x+5x^2)} - \frac{(6875)(90)}{25\sqrt{\frac{1}{682}}(6)} \\
&= \frac{(4+65x)\sqrt{3-x+2x^2}}{1364(2+3x+5x^2)^2} + \frac{(26794+86265x)\sqrt{3-x+2x^2}}{1860496(2+3x+5x^2)} + \frac{25\sqrt{\frac{1}{682}}(6)}{1125}
\end{aligned}$$

**Mathematica [C]** time = 6.23, size = 1277, normalized size = 5.73

result too large to display

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[3 - x + 2\*x^2]\*(2 + 3\*x + 5\*x^2)^3), x]

[Out] (2500\*Sqrt[3 - x + 2\*x^2])/(341\*Sqrt[31]\*(13\*I + Sqrt[31])\*(3 - I\*Sqrt[31] + 10\*x)^2) + (7500\*Sqrt[3 - x + 2\*x^2])/(10571\*(13 - I\*Sqrt[31])\*(3 - I\*Sqrt[31] + 10\*x)) - (2500\*Sqrt[3 - x + 2\*x^2])/(341\*Sqrt[31]\*(13\*I - Sqrt[31])\*(3 + I\*Sqrt[31] + 10\*x)^2) + (7500\*Sqrt[3 - x + 2\*x^2])/(10571\*(13 + I\*Sqrt[31])\*(3 + I\*Sqrt[31] + 10\*x)) - (375\*Sqrt[(2\*(13 - I\*Sqrt[31]))/11]\*(11 - (2\*I)\*Sqrt[31])\*ArcTanh[(63 - I\*Sqrt[31] - 2\*(11 - (2\*I)\*Sqrt[31])\*x]/(2\*Sqrt[22\*(13 - I\*Sqrt[31]))\*Sqrt[3 - x + 2\*x^2]])/(10571\*(13\*I + Sqrt[31])^2) - (750\*Sqrt[(2\*(13 - I\*Sqrt[31]))/341])\*ArcTanh[(63 - I\*Sqrt[31] - 2\*(11 - (2\*I)\*Sqrt[31])\*x]/(2\*Sqrt[22\*(13 - I\*Sqrt[31]))\*Sqrt[3 - x + 2\*x^2]])/(961\*(13\*I + Sqrt[31])) + (750\*Sqrt[(2\*(13 + I\*Sqrt[31]))/341])\*ArcTanh[(63 + I\*Sqrt[31] - 2\*(11 + (2\*I)\*Sqrt[31])\*x]/(2\*Sqrt[22\*(13 + I\*Sqrt[31]))\*Sqrt[3 - x + 2\*x^2]])/(961\*(13 - I\*Sqrt[31]))

$$\begin{aligned} & (3 - x + 2x^2)]]) / (961 * (13 * I - \text{Sqrt}[31])) - (375 * \text{Sqrt}[(2 * (13 + I * \text{Sqrt}[31])) \\ & / 11] * (11 + (2 * I) * \text{Sqrt}[31]) * \text{ArcTanh}[(63 + I * \text{Sqrt}[31] - 2 * (11 + (2 * I) * \text{Sqrt}[31] \\ & ) * x) / (2 * \text{Sqrt}[22 * (13 + I * \text{Sqrt}[31])] * \text{Sqrt}[3 - x + 2 * x^2])]) / (10571 * (13 * I - \text{S} \\ & \text{qrt}[31])^2) + (((500 * I) / 31) * (((-20 * (-3 + I * \text{Sqrt}[31]) + 10 * (27 - (4 * I) * \text{Sqrt}[ \\ & 31])) * \text{Sqrt}[3 - x + 2 * x^2]) / ((300 - 10 * (-3 + I * \text{Sqrt}[31]) + 2 * (-3 + I * \text{Sqrt}[31] \\ & )^2) * (-3 + I * \text{Sqrt}[31] - 10 * x)) + (2 * \text{Sqrt}[22 * (13 - I * \text{Sqrt}[31])] * (20 * (-3 + I \\ & * \text{Sqrt}[31]) + 10 * (27 - (4 * I) * \text{Sqrt}[31]) - 2 * (600 + 2 * (-3 + I * \text{Sqrt}[31]) * (27 - \\ & (4 * I) * \text{Sqrt}[31]))) * \text{ArcTanh}[(-63 + I * \text{Sqrt}[31] - (-10 + 4 * (-3 + I * \text{Sqrt}[31])) * x \\ & ) / (2 * \text{Sqrt}[22 * (13 - I * \text{Sqrt}[31])] * \text{Sqrt}[3 - x + 2 * x^2])]) / ((300 - 10 * (-3 + I * \text{S} \\ & \text{qrt}[31]) + 2 * (-3 + I * \text{Sqrt}[31])^2) * (1200 - 40 * (-3 + I * \text{Sqrt}[31]) + 8 * (-3 + I * \\ & \text{Sqrt}[31])^2)))) / (\text{Sqrt}[31] * (300 - 10 * (-3 + I * \text{Sqrt}[31]) + 2 * (-3 + I * \text{Sqrt}[31]) \\ & ^2)) + (((500 * I) / 31) * (((20 * (3 + I * \text{Sqrt}[31]) - 10 * (-27 - (4 * I) * \text{Sqrt}[31])) * \text{S} \\ & \text{qrt}[3 - x + 2 * x^2]) / ((300 + 10 * (3 + I * \text{Sqrt}[31]) + 2 * (3 + I * \text{Sqrt}[31])^2) * (3 + \\ & I * \text{Sqrt}[31] + 10 * x)) + (2 * \text{Sqrt}[22 * (13 + I * \text{Sqrt}[31])] * (-20 * (3 + I * \text{Sqrt}[31]) \\ & - 10 * (-27 - (4 * I) * \text{Sqrt}[31]) - 2 * (600 + 2 * (3 + I * \text{Sqrt}[31]) * (-27 - (4 * I) * \text{Sqrt}[ \\ & 31]))) * \text{ArcTanh}[(63 + I * \text{Sqrt}[31] - (10 + 4 * (3 + I * \text{Sqrt}[31])) * x) / (2 * \text{Sqrt}[22 * \\ & (13 + I * \text{Sqrt}[31])] * \text{Sqrt}[3 - x + 2 * x^2])]) / ((300 + 10 * (3 + I * \text{Sqrt}[31]) + 2 * ( \\ & 3 + I * \text{Sqrt}[31])^2) * (1200 + 40 * (3 + I * \text{Sqrt}[31]) + 8 * (3 + I * \text{Sqrt}[31])^2)))) / ( \\ & \text{Sqrt}[31] * (300 + 10 * (3 + I * \text{Sqrt}[31]) + 2 * (3 + I * \text{Sqrt}[31])^2)) \end{aligned}$$

**fricas [B]** time = 2.37, size = 2183, normalized size = 9.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5\*x^2+3\*x+2)^3/(2\*x^2-x+3)^(1/2),x, algorithm="fricas")

[Out]  $-1/212344000027477426346822144 * (46113488900 * 4115738902305032^{(1/4)} * \text{sqrt}(22681873) * \text{sqrt}(341) * \text{sqrt}(2) * (25 * x^4 + 30 * x^3 + 29 * x^2 + 12 * x + 4) * \text{sqrt}(6414867847 * \text{sqrt}(2) + 9072749200) * \text{arctan}(1/3836668309294009530058322373948769 * (64688701796 * \text{sqrt}(22681873) * (11 * 4115738902305032^{(3/4)} * \text{sqrt}(341) * (160344708 * x^7 - 615873378 * x^6 + 1294230774 * x^5 - 2070733376 * x^4 + 1037098288 * x^3 - 489164544 * x^2 - \text{sqrt}(2) * (112700446 * x^7 - 434839553 * x^6 + 912850886 * x^5 - 1466127691 * x^4 + 735661560 * x^3 - 350098200 * x^2 - 799200000 * x + 567316224) - 1134632448 * x + 799200000) + 703138063 * 4115738902305032^{(1/4)} * \text{sqrt}(341) * (12162569 * x^7 - 186616851 * x^6 + 985490056 * x^5 - 2246141620 * x^4 + 2900382048 * x^3 - 1823848416 * x^2 - \text{sqrt}(2) * (8564099 * x^7 - 131508024 * x^6 + 695288980 * x^5 - 1587105104 * x^4 + 2050714080 * x^3 - 1296806400 * x^2 - 1457077248 * x + 1033108992) - 2066217984 * x + 1457077248)) * \text{sqrt}(2 * x^2 - x + 3) * \text{sqrt}(6414867847 * \text{sqrt}(2) + 9072749200) + 10891187458641059311133302222822312 * \text{sqrt}(31) * \text{sqrt}(2) * (28180 * x^8 - 254666 * x^7 + 704270 * x^6 - 1385256 * x^5 + 1549144 * x^4 - 642048 * x^3 - 98496 * x^2 - \text{sqrt}(2) * (8746 * x^8 - 102335 * x^7 + 396104 * x^6 - 783113 * x^5 + 1320710 * x^4 - 752088 * x^3 + 396144 * x^2 + 546048 * x - 539136) + 1154304 * x - 456192) - 2 * \text{sqrt}(45363746/479849) * (\text{sqrt}(22681873) * (11 * 4115738902305032^{(3/4)} * \text{sqrt}(341) * (576322648 * x^7 - 827050092 * x^6 + 2660713572 * x^5 - 1032439232 * x^4 + 12116047$

$$\begin{aligned}
& 68x^3 + 1213394688x^2 - \sqrt{2}*(403157522x^7 - 578844217x^6 + 18641293 \\
& 47x^5 - 735062160x^4 + 873708120x^3 + 823986432x^2 - 823986432x) - 121 \\
& 3394688x) + 703138063*4115738902305032^{(1/4)}*\sqrt{341}*(43684647x^7 - 565 \\
& 067708x^6 + 2178643220x^5 - 2819241792x^4 + 3618371808x^3 + 2197767168x \\
& x^2 - 2*\sqrt{2}*(15328963x^7 - 198290348x^6 + 764653220x^5 - 990717120x \\
& ^4 + 1276256160x^3 + 755350272x^2 - 755350272x) - 2197767168x))*\sqrt{2} \\
& x^2 - x + 3)*\sqrt{6414867847*\sqrt{2} + 9072749200} + 1683630550043672623393 \\
& 22*\sqrt{31}*\sqrt{2}*(123408x^8 - 914152x^7 + 1578888x^6 - 3293072x^5 + \\
& 396480x^4 + 798336x^3 - 3822336x^2 - \sqrt{2}*(15550x^8 - 118051x^7 + 2 \\
& 44047x^6 - 707374x^5 + 1053960x^4 - 1667952x^3 + 1209600x^2 - 1036800x \\
& x) + 3276288x) + 7652866136562148288151*\sqrt{31}*(254591x^8 - 4815126x^7 \\
& + 32303580x^6 - 90866808x^5 + 108781920x^4 - 74219328x^3 - 168956928x \\
& ^2 - 15488*\sqrt{2}*(4x^8 - 76x^7 + 517x^6 - 1536x^5 + 2385x^4 - 3618x \\
& ^3 + 2268x^2 - 1944x) + 144820224x))*\sqrt{-(4115738902305032^{(1/4)}*\sqrt{2} \\
& 22681873)*\sqrt{341}*\sqrt{31}*\sqrt{2}*(2x^2 - x + 3)*(sqrt{2}*(67187x + 26012) \\
& - 93199x - 41175)*\sqrt{6414867847*\sqrt{2} + 9072749200} - 512510746420187 \\
& 753x^2 - 460213731479352268*\sqrt{2}*(2x^2 - x + 3) + 1579369851213231647x \\
& x - 2091880597633419400)/x^2) + 123763493848193855808332979804799*\sqrt{31} \\
& (2828123x^8 - 9696916x^7 + 53385560x^6 - 142835344x^5 + 254146592x^4 - \\
& 249300096x^3 + 37981440x^2 - 7744*\sqrt{2}*(1348x^8 - 2692x^7 + 9789x^6 \\
& - 10070x^5 + 15569x^4 - 5568x^3 + 1080x^2 + 4320x - 5184) + 22306406 \\
& 4x - 94887936))/(2585191x^8 - 4661200x^7 + 14191920x^6 + 490880x^5 - 1 \\
& 3562944x^4 + 44249088x^3 - 34615296x^2 - 24772608x + 18579456)) + 46113 \\
& 488900*4115738902305032^{(1/4)}*\sqrt{22681873}*\sqrt{341}*\sqrt{2}*(25x^4 + 30 \\
& *x^3 + 29x^2 + 12x + 4)*\sqrt{6414867847*\sqrt{2} + 9072749200}*\arctan(1/38 \\
& 36668309294009530058322373948769*(64688701796*\sqrt{22681873}*(11*4115738902 \\
& 305032^{(3/4)}*\sqrt{341}*(160344708x^7 - 615873378x^6 + 1294230774x^5 - 20 \\
& 70733376x^4 + 1037098288x^3 - 489164544x^2 - \sqrt{2}*(112700446x^7 - 43 \\
& 4839553x^6 + 912850886x^5 - 1466127691x^4 + 735661560x^3 - 350098200x^2 \\
& - 799200000x + 567316224) - 1134632448x + 799200000) + 703138063*411573 \\
& 8902305032^{(1/4)}*\sqrt{341}*(12162569x^7 - 186616851x^6 + 985490056x^5 - \\
& 2246141620x^4 + 2900382048x^3 - 1823848416x^2 - \sqrt{2}*(8564099x^7 - 1 \\
& 31508024x^6 + 695288980x^5 - 1587105104x^4 + 2050714080x^3 - 1296806400 \\
& *x^2 - 1457077248x + 1033108992) - 2066217984x + 1457077248))*\sqrt{2} \\
& x^2 - x + 3)*\sqrt{6414867847*\sqrt{2} + 9072749200} - 10891187458641059311133302 \\
& 222822312*\sqrt{31}*\sqrt{2}*(28180x^8 - 254666x^7 + 704270x^6 - 1385256x \\
& ^5 + 1549144x^4 - 642048x^3 - 98496x^2 - \sqrt{2}*(8746x^8 - 102335x^7 \\
& + 396104x^6 - 783113x^5 + 1320710x^4 - 752088x^3 + 396144x^2 + 546048x \\
& x - 539136) + 1154304x - 456192) - 2*\sqrt{45363746/479849}*(sqrt{22681873} \\
& *(11*4115738902305032^{(3/4)}*\sqrt{341}*(576322648x^7 - 827050092x^6 + 2660 \\
& 713572x^5 - 1032439232x^4 + 1211604768x^3 + 1213394688x^2 - \sqrt{2}*(40 \\
& 3157522x^7 - 578844217x^6 + 1864129347x^5 - 735062160x^4 + 873708120x^3 \\
& + 823986432x^2 - 823986432x) - 1213394688x) + 703138063*41157389023050 \\
& 32^{(1/4)}*\sqrt{341}*(43684647x^7 - 565067708x^6 + 2178643220x^5 - 2819241 \\
& 792x^4 + 3618371808x^3 + 2197767168x^2 - 2*\sqrt{2}*(15328963x^7 - 19829
\end{aligned}$$

```

0348*x^6 + 764653220*x^5 - 990717120*x^4 + 1276256160*x^3 + 755350272*x^2 -
755350272*x) - 2197767168*x)*sqrt(2*x^2 - x + 3)*sqrt(6414867847*sqrt(2)
+ 9072749200) - 168363055004367262339322*sqrt(31)*sqrt(2)*(123408*x^8 - 914
152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2
- sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4
- 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) - 7652866136562148288
151*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 1087
81920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 +
517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x
))*sqrt((4115738902305032^(1/4)*sqrt(22681873)*sqrt(341)*sqrt(31)*sqrt(2*x^
2 - x + 3)*(sqrt(2)*(67187*x + 26012) - 93199*x - 41175)*sqrt(6414867847*sq
rt(2) + 9072749200) + 512510746420187753*x^2 + 460213731479352268*sqrt(2)*(
2*x^2 - x + 3) - 1579369851213231647*x + 2091880597633419400)/x^2) - 123763
493848193855808332979804799*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*
x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sq
rt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 +
1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 466120
0*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*
x^2 - 24772608*x + 18579456) - 25*4115738902305032^(1/4)*sqrt(22681873)*sq
rt(341)*sqrt(31)*(226818730000*x^4 + 272182476000*x^3 + 263109726800*x^2 -
6414867847*sqrt(2)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4) + 108872990400*x +
36290996800)*sqrt(6414867847*sqrt(2) + 9072749200)*log(1134093650000000/47
9849*(4115738902305032^(1/4)*sqrt(22681873)*sqrt(341)*sqrt(31)*sqrt(2*x^2 -
x + 3)*(sqrt(2)*(67187*x + 26012) - 93199*x - 41175)*sqrt(6414867847*sqrt(
2) + 9072749200) + 512510746420187753*x^2 + 460213731479352268*sqrt(2)*(2*x
^2 - x + 3) - 1579369851213231647*x + 2091880597633419400)/x^2) + 25*411573
8902305032^(1/4)*sqrt(22681873)*sqrt(341)*sqrt(31)*(226818730000*x^4 + 2721
82476000*x^3 + 263109726800*x^2 - 6414867847*sqrt(2)*(25*x^4 + 30*x^3 + 29*
x^2 + 12*x + 4) + 108872990400*x + 36290996800)*sqrt(6414867847*sqrt(2) + 9
072749200)*log(-1134093650000000/479849*(4115738902305032^(1/4)*sqrt(226818
73)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(67187*x + 26012) - 931
99*x - 41175)*sqrt(6414867847*sqrt(2) + 9072749200) - 512510746420187753*x^
2 - 460213731479352268*sqrt(2)*(2*x^2 - x + 3) + 1579369851213231647*x - 20
91880597633419400)/x^2) - 114133005406879362464*(431325*x^3 + 392765*x^2 +
341572*x + 59044)*sqrt(2*x^2 - x + 3))/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4
)

```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5\*x^2+3\*x+2)^3/(2\*x^2-x+3)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP

```

UT:sage2:=int(sage0,x);OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,inf
inity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity
]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai
lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity
,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf
inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinit
y]Evaluation time: 42.09Done

```

**maple** [B] time = 0.01, size = 13040, normalized size = 58.48

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x)
```

```
[Out] result too large to display
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(5x^2 + 3x + 2)^3 \sqrt{2x^2 - x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)^3*sqrt(2*x^2 - x + 3)), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{2x^2 - x + 3} (5x^2 + 3x + 2)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^3),x)
```

```
[Out] int(1/((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^3), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^2 - x + 3} (5x^2 + 3x + 2)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+3*x+2)**3/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2)**3), x)
```



$$3.86 \quad \int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{3/2}} dx$$

Optimal. Leaf size=166

$$\frac{111315\sqrt{2x^2-x+3}x^2}{2048} - \frac{8992487\sqrt{2x^2-x+3}x}{16384} - \frac{31009685\sqrt{2x^2-x+3}}{65536} - \frac{14641(79x+101)}{1472\sqrt{2x^2-x+3}} + \frac{625}{24}\sqrt{2x^2-x+3}$$

[Out] -310445587/262144\*arcsinh(1/23\*(1-4\*x)\*23^(1/2))\*2^(1/2)-14641/1472\*(101+79\*x)/(2\*x^2-x+3)^(1/2)-31009685/65536\*(2\*x^2-x+3)^(1/2)-8992487/16384\*x\*(2\*x^2-x+3)^(1/2)-111315/2048\*x^2\*(2\*x^2-x+3)^(1/2)+79425/512\*x^3\*(2\*x^2-x+3)^(1/2)+10075/96\*x^4\*(2\*x^2-x+3)^(1/2)+625/24\*x^5\*(2\*x^2-x+3)^(1/2)

Rubi [A] time = 0.20, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {1660, 1661, 640, 619, 215}

$$\frac{625}{24}\sqrt{2x^2-x+3}x^5 + \frac{10075}{96}\sqrt{2x^2-x+3}x^4 + \frac{79425}{512}\sqrt{2x^2-x+3}x^3 - \frac{111315\sqrt{2x^2-x+3}x^2}{2048} - \frac{8992487\sqrt{2x^2-x+3}x}{16384}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3\*x + 5\*x^2)^4/(3 - x + 2\*x^2)^(3/2), x]

[Out] (-14641\*(101 + 79\*x))/(1472\*Sqrt[3 - x + 2\*x^2]) - (31009685\*Sqrt[3 - x + 2\*x^2])/65536 - (8992487\*x\*Sqrt[3 - x + 2\*x^2])/16384 - (111315\*x^2\*Sqrt[3 - x + 2\*x^2])/2048 + (79425\*x^3\*Sqrt[3 - x + 2\*x^2])/512 + (10075\*x^4\*Sqrt[3 - x + 2\*x^2])/96 + (625\*x^5\*Sqrt[3 - x + 2\*x^2])/24 - (310445587\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(131072\*Sqrt[2])

### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] :> Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

### Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

### Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]
```

### Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(2 + 3x + 5x^2)^4}{(3 - x + 2x^2)^{3/2}} dx &= -\frac{14641(101 + 79x)}{1472\sqrt{3 - x + 2x^2}} + \frac{2}{23} \int \frac{\frac{2821893}{256} - \frac{661181x}{128} - \frac{488267x^2}{64} + \frac{143635x^3}{32} + \frac{213325x^4}{16} + \frac{83375x^5}{8}}{\sqrt{3 - x + 2x^2}} \\
&= -\frac{14641(101 + 79x)}{1472\sqrt{3 - x + 2x^2}} + \frac{625}{24}x^5\sqrt{3 - x + 2x^2} + \frac{1}{138} \int \frac{\frac{8465679}{64} - \frac{1983543x}{32} - \frac{1464801x^2}{16} + \frac{4}{16}}{\sqrt{3 - x + 2x^2}} \\
&= -\frac{14641(101 + 79x)}{1472\sqrt{3 - x + 2x^2}} + \frac{10075}{96}x^4\sqrt{3 - x + 2x^2} + \frac{625}{24}x^5\sqrt{3 - x + 2x^2} + \int \frac{\frac{42328395}{32} - \frac{99177}{16}}{\sqrt{3 - x + 2x^2}} \\
&= -\frac{14641(101 + 79x)}{1472\sqrt{3 - x + 2x^2}} + \frac{79425}{512}x^3\sqrt{3 - x + 2x^2} + \frac{10075}{96}x^4\sqrt{3 - x + 2x^2} + \frac{625}{24}x^5\sqrt{3 - x + 2x^2} \\
&= -\frac{14641(101 + 79x)}{1472\sqrt{3 - x + 2x^2}} - \frac{111315x^2\sqrt{3 - x + 2x^2}}{2048} + \frac{79425}{512}x^3\sqrt{3 - x + 2x^2} + \frac{10075}{96}x^4\sqrt{3 - x + 2x^2} \\
&= -\frac{14641(101 + 79x)}{1472\sqrt{3 - x + 2x^2}} - \frac{8992487x\sqrt{3 - x + 2x^2}}{16384} - \frac{111315x^2\sqrt{3 - x + 2x^2}}{2048} + \frac{79425}{512}x^3\sqrt{3 - x + 2x^2} \\
&= -\frac{14641(101 + 79x)}{1472\sqrt{3 - x + 2x^2}} - \frac{31009685\sqrt{3 - x + 2x^2}}{65536} - \frac{8992487x\sqrt{3 - x + 2x^2}}{16384} - \frac{111315x^2\sqrt{3 - x + 2x^2}}{2048} \\
&= -\frac{14641(101 + 79x)}{1472\sqrt{3 - x + 2x^2}} - \frac{31009685\sqrt{3 - x + 2x^2}}{65536} - \frac{8992487x\sqrt{3 - x + 2x^2}}{16384} - \frac{111315x^2\sqrt{3 - x + 2x^2}}{2048} \\
&= -\frac{14641(101 + 79x)}{1472\sqrt{3 - x + 2x^2}} - \frac{31009685\sqrt{3 - x + 2x^2}}{65536} - \frac{8992487x\sqrt{3 - x + 2x^2}}{16384} - \frac{111315x^2\sqrt{3 - x + 2x^2}}{2048}
\end{aligned}$$

**Mathematica [A]** time = 0.37, size = 95, normalized size = 0.57

$$\sqrt{2x^2 - x + 3} \left( \frac{625x^5}{24} + \frac{10075x^4}{96} + \frac{79425x^3}{512} - \frac{111315x^2}{2048} - \frac{14641(79x + 101)}{1472(2x^2 - x + 3)} - \frac{8992487x}{16384} - \frac{31009685}{65536} \right) + \dots$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x + 5\*x^2)^4/(3 - x + 2\*x^2)^(3/2), x]

[Out]  $\text{Sqrt}[3 - x + 2*x^2]*(-31009685/65536 - (8992487*x)/16384 - (111315*x^2)/2048 + (79425*x^3)/512 + (10075*x^4)/96 + (625*x^5)/24 - (14641*(101 + 79*x))/(1472*(3 - x + 2*x^2)) + (310445587*\text{ArcSinh}[(-1 + 4*x)/\text{Sqrt}[23]])/(131072*\text{Sqrt}[2])$

**fricas** [A] time = 0.91, size = 112, normalized size = 0.67

$$\frac{21420745503 \sqrt{2} (2x^2 - x + 3) \log(-4 \sqrt{2} \sqrt{2x^2 - x + 3} (4x - 1) - 32x^2 + 16x - 25) + 8 (235520000x^7 + 831385600x^6 + 1281670400x^5 + 230669760x^4 - 2613624504x^3 - 2534760678x^2 - 8859305979x - 10961697147) \sqrt{2x^2 - x + 3}}{(2x^2 - x + 3)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(3/2),x, algorithm="fricas")`

[Out]  $\frac{1}{36175872} * (21420745503 * \text{sqrt}(2) * (2*x^2 - x + 3) * \log(-4 * \text{sqrt}(2) * \text{sqrt}(2*x^2 - x + 3) * (4*x - 1) - 32*x^2 + 16*x - 25) + 8 * (235520000*x^7 + 831385600*x^6 + 1281670400*x^5 + 230669760*x^4 - 2613624504*x^3 - 2534760678*x^2 - 8859305979*x - 10961697147) * \text{sqrt}(2*x^2 - x + 3)) / (2*x^2 - x + 3)$

**giac** [A] time = 0.25, size = 82, normalized size = 0.49

$$-\frac{310445587}{262144} \sqrt{2} \log\left(-2 \sqrt{2} \left(\sqrt{2} x - \sqrt{2x^2 - x + 3}\right) + 1\right) + \frac{(46(4(40(20(16(100x + 353)x + 8707)x + 31341)x - 14204481)x - 55103493)x - 8859305979)x - 10961697147)}{452198 \sqrt{2x^2 - x + 3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(3/2),x, algorithm="giac")`

[Out]  $-310445587/262144 * \text{sqrt}(2) * \log(-2 * \text{sqrt}(2) * (\text{sqrt}(2) * x - \text{sqrt}(2*x^2 - x + 3)) + 1) + 1/4521984 * ((46 * (4 * (40 * (20 * (16 * (100*x + 353) * x + 8707) * x + 31341) * x - 14204481) * x - 55103493) * x - 8859305979) * x - 10961697147) / \text{sqrt}(2*x^2 - x + 3))$

**maple** [A] time = 0.03, size = 166, normalized size = 1.00

$$\frac{625x^7}{12\sqrt{2x^2 - x + 3}} + \frac{8825x^6}{48\sqrt{2x^2 - x + 3}} + \frac{217675x^5}{768\sqrt{2x^2 - x + 3}} + \frac{52235x^4}{1024\sqrt{2x^2 - x + 3}} - \frac{4734827x^3}{8192\sqrt{2x^2 - x + 3}} - \frac{18367831x^2}{32768\sqrt{2x^2 - x + 3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^2+3*x+2)^4/(2*x^2-x+3)^(3/2),x)`

[Out]  $-1217267299/524288 / (2*x^2 - x + 3)^{1/2} + 1234044515/12058624 * (4*x - 1) / (2*x^2 - x + 3)^{1/2} + 310445587/262144 * 2^{1/2} * \text{arcsinh}(4/23 * 23^{1/2} * (x - 1/4)) + 217675/768 * \sqrt{2x^2 - x + 3}$

$$x^5/(2x^2-x+3)^{(1/2)}+52235/1024*x^4/(2x^2-x+3)^{(1/2)}-4734827/8192*x^3/(2x^2-x+3)^{(1/2)}+625/12*x^7/(2x^2-x+3)^{(1/2)}+8825/48*x^6/(2x^2-x+3)^{(1/2)}-18367831/32768*x^2/(2x^2-x+3)^{(1/2)}-310445587/131072*x/(2x^2-x+3)^{(1/2)}$$

**maxima** [A] time = 0.98, size = 148, normalized size = 0.89

$$\frac{625x^7}{12\sqrt{2x^2-x+3}} + \frac{8825x^6}{48\sqrt{2x^2-x+3}} + \frac{217675x^5}{768\sqrt{2x^2-x+3}} + \frac{52235x^4}{1024\sqrt{2x^2-x+3}} - \frac{4734827x^3}{8192\sqrt{2x^2-x+3}} - \frac{18367831x^2}{32768\sqrt{2x^2-x+3}} - \frac{310445587x}{131072\sqrt{2x^2-x+3}} - \frac{310445587}{131072\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^4/(2\*x^2-x+3)^(3/2),x, algorithm="maxima")

[Out] 625/12\*x^7/sqrt(2\*x^2 - x + 3) + 8825/48\*x^6/sqrt(2\*x^2 - x + 3) + 217675/768\*x^5/sqrt(2\*x^2 - x + 3) + 52235/1024\*x^4/sqrt(2\*x^2 - x + 3) - 4734827/8192\*x^3/sqrt(2\*x^2 - x + 3) - 18367831/32768\*x^2/sqrt(2\*x^2 - x + 3) + 310445587/262144\*sqrt(2)\*arcsinh(1/23\*sqrt(23)\*(4\*x - 1)) - 2953101993/1507328\*x/sqrt(2\*x^2 - x + 3) - 3653899049/1507328/sqrt(2\*x^2 - x + 3)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(5x^2 + 3x + 2)^4}{(2x^2 - x + 3)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x + 5\*x^2 + 2)^4/(2\*x^2 - x + 3)^(3/2),x)

[Out] int((3\*x + 5\*x^2 + 2)^4/(2\*x^2 - x + 3)^(3/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x^2 + 3x + 2)^4}{(2x^2 - x + 3)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*2+3\*x+2)\*\*4/(2\*x\*\*2-x+3)\*\*(3/2),x)

[Out] Integral((5\*x\*\*2 + 3\*x + 2)\*\*4/(2\*x\*\*2 - x + 3)\*\*(3/2), x)

$$3.87 \quad \int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{3/2}} dx$$

**Optimal.** Leaf size=124

$$\frac{1825}{64} \sqrt{2x^2 - x + 3} x^2 + \frac{15565}{512} \sqrt{2x^2 - x + 3} x - \frac{181561 \sqrt{2x^2 - x + 3}}{2048} - \frac{1331(17 - 45x)}{368 \sqrt{2x^2 - x + 3}} + \frac{125}{16} \sqrt{2x^2 - x + 3} x^3 + \dots$$

[Out] 1168881/8192\*arcsinh(1/23\*(1-4\*x)\*23^(1/2))\*2^(1/2)-1331/368\*(17-45\*x)/(2\*x^2-x+3)^(1/2)-181561/2048\*(2\*x^2-x+3)^(1/2)+15565/512\*x\*(2\*x^2-x+3)^(1/2)+1825/64\*x^2\*(2\*x^2-x+3)^(1/2)+125/16\*x^3\*(2\*x^2-x+3)^(1/2)

**Rubi [A]** time = 0.13, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {1660, 1661, 640, 619, 215}

$$\frac{125}{16} \sqrt{2x^2 - x + 3} x^3 + \frac{1825}{64} \sqrt{2x^2 - x + 3} x^2 + \frac{15565}{512} \sqrt{2x^2 - x + 3} x - \frac{181561 \sqrt{2x^2 - x + 3}}{2048} - \frac{1331(17 - 45x)}{368 \sqrt{2x^2 - x + 3}} + \dots$$

Antiderivative was successfully verified.

[In] Int[(2 + 3\*x + 5\*x^2)^3/(3 - x + 2\*x^2)^(3/2), x]

[Out] (-1331\*(17 - 45\*x))/(368\*sqrt[3 - x + 2\*x^2]) - (181561\*sqrt[3 - x + 2\*x^2])/2048 + (15565\*x\*sqrt[3 - x + 2\*x^2])/512 + (1825\*x^2\*sqrt[3 - x + 2\*x^2])/64 + (125\*x^3\*sqrt[3 - x + 2\*x^2])/16 + (1168881\*ArcSinh[(1 - 4\*x)/sqrt[23]])/(4096\*sqrt[2])

### Rule 215

Int[1/sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

### Rule 619

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

### Rule 640

Int[((d\_) + (e\_)\*(x\_))\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b

$\ast e)/(2\ast c)$ , Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

### Rule 1660

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 1]}, Simp[((b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] + Dist[1/((p + 1)\*(b^2 - 4\*a\*c)), Int[(a + b\*x + c\*x^2)^(p + 1)\*ExpandToSum[(p + 1)\*(b^2 - 4\*a\*c)\*Q - (2\*p + 3)\*(2\*c\*f - b\*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && LtQ[p, -1]

### Rule 1661

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(q + 2\*p + 1)), x] + Dist[1/(c\*(q + 2\*p + 1)), Int[(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*(q + 2\*p + 1)\*Pq - a\*e\*(q - 1)\*x^(q - 2) - b\*e\*(q + p)\*x^(q - 1) - c\*e\*(q + 2\*p + 1)\*x^q, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

### Rubi steps

$$\begin{aligned}
\int \frac{(2 + 3x + 5x^2)^3}{(3 - x + 2x^2)^{3/2}} dx &= -\frac{1331(17 - 45x)}{368\sqrt{3 - x + 2x^2}} + \frac{2}{23} \int \frac{-\frac{110285}{64} - \frac{19067x}{32} + \frac{22195x^2}{16} + \frac{13225x^3}{8} + \frac{2875x^4}{4}}{\sqrt{3 - x + 2x^2}} dx \\
&= -\frac{1331(17 - 45x)}{368\sqrt{3 - x + 2x^2}} + \frac{125}{16}x^3\sqrt{3 - x + 2x^2} + \frac{1}{92} \int \frac{-\frac{110285}{8} - \frac{19067x}{4} + \frac{18515x^2}{4} + \frac{125925x^3}{8}}{\sqrt{3 - x + 2x^2}} \\
&= -\frac{1331(17 - 45x)}{368\sqrt{3 - x + 2x^2}} + \frac{1825}{64}x^2\sqrt{3 - x + 2x^2} + \frac{125}{16}x^3\sqrt{3 - x + 2x^2} + \frac{1}{552} \int \frac{-\frac{330855}{4} - \frac{110285x}{4} + \frac{18515x^2}{4} + \frac{125925x^3}{8}}{\sqrt{3 - x + 2x^2}} \\
&= -\frac{1331(17 - 45x)}{368\sqrt{3 - x + 2x^2}} + \frac{15565}{512}x\sqrt{3 - x + 2x^2} + \frac{1825}{64}x^2\sqrt{3 - x + 2x^2} + \frac{125}{16}x^3\sqrt{3 - x + 2x^2} \\
&= -\frac{1331(17 - 45x)}{368\sqrt{3 - x + 2x^2}} - \frac{181561\sqrt{3 - x + 2x^2}}{2048} + \frac{15565}{512}x\sqrt{3 - x + 2x^2} + \frac{1825}{64}x^2\sqrt{3 - x + 2x^2} \\
&= -\frac{1331(17 - 45x)}{368\sqrt{3 - x + 2x^2}} - \frac{181561\sqrt{3 - x + 2x^2}}{2048} + \frac{15565}{512}x\sqrt{3 - x + 2x^2} + \frac{1825}{64}x^2\sqrt{3 - x + 2x^2} \\
&= -\frac{1331(17 - 45x)}{368\sqrt{3 - x + 2x^2}} - \frac{181561\sqrt{3 - x + 2x^2}}{2048} + \frac{15565}{512}x\sqrt{3 - x + 2x^2} + \frac{1825}{64}x^2\sqrt{3 - x + 2x^2}
\end{aligned}$$

**Mathematica [A]** time = 0.23, size = 65, normalized size = 0.52

$$\frac{4(736000x^5 + 2318400x^4 + 2624760x^3 - 5754186x^2 + 16138403x - 15423965)}{\sqrt{2x^2 - x + 3}} - 26884263\sqrt{2} \sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)$$

188416

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x + 5\*x^2)^3/(3 - x + 2\*x^2)^(3/2), x]

[Out] ((4\*(-15423965 + 16138403\*x - 5754186\*x^2 + 2624760\*x^3 + 2318400\*x^4 + 736000\*x^5))/Sqrt[3 - x + 2\*x^2] - 26884263\*Sqrt[2]\*ArcSinh[(-1 + 4\*x)/Sqrt[23]])/188416

**fricas [A]** time = 0.94, size = 102, normalized size = 0.82

$$\frac{26884263\sqrt{2}(2x^2 - x + 3)\log\left(4\sqrt{2}\sqrt{2x^2 - x + 3}(4x - 1) - 32x^2 + 16x - 25\right) + 8(736000x^5 + 2318400x^4 + 2624760x^3 - 5754186x^2 + 16138403x - 15423965)}{376832(2x^2 - x + 3)}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^3/(2\*x^2-x+3)^(3/2),x, algorithm="fricas")

[Out]  $\frac{1}{376832} \cdot (26884263 \cdot \sqrt{2}) \cdot (2x^2 - x + 3) \cdot \log(4 \cdot \sqrt{2} \cdot \sqrt{2x^2 - x + 3}) \cdot (4x - 1) - 32x^2 + 16x - 25) + 8 \cdot (736000x^5 + 2318400x^4 + 2624760x^3 - 5754186x^2 + 16138403x - 15423965) \cdot \sqrt{2x^2 - x + 3}) / (2x^2 - x + 3)$

**giac** [A] time = 0.28, size = 72, normalized size = 0.58

$$\frac{1168881}{8192} \sqrt{2} \log\left(-2 \sqrt{2} \left(\sqrt{2}x - \sqrt{2x^2 - x + 3}\right) + 1\right) + \frac{(46(20(40(20x + 63)x + 2853)x - 125091)x + 16138403)x - 15423965}{47104 \sqrt{2x^2 - x + 3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^3/(2\*x^2-x+3)^(3/2),x, algorithm="giac")

[Out]  $\frac{1168881}{8192} \cdot \sqrt{2} \cdot \log(-2 \cdot \sqrt{2} \cdot (\sqrt{2}x - \sqrt{2x^2 - x + 3}) + 1) + \frac{1}{47104} \cdot ((46 \cdot (20 \cdot (40 \cdot (20x + 63) \cdot x + 2853) \cdot x - 125091) \cdot x + 16138403) \cdot x - 15423965) / \sqrt{2x^2 - x + 3}$

**maple** [A] time = 0.01, size = 132, normalized size = 1.06

$$\frac{125x^5}{8\sqrt{2x^2 - x + 3}} + \frac{1575x^4}{32\sqrt{2x^2 - x + 3}} + \frac{14265x^3}{256\sqrt{2x^2 - x + 3}} - \frac{125091x^2}{1024\sqrt{2x^2 - x + 3}} + \frac{1168881x}{4096\sqrt{2x^2 - x + 3}} - \frac{1168881\sqrt{2}}{8192} \operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x - 1) - \sqrt{2x^2 - x + 3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+3\*x+2)^3/(2\*x^2-x+3)^(3/2),x)

[Out]  $-5130399/16384 \cdot (2x^2 - x + 3)^{(1/2)} + 5392543/376832 \cdot (4x - 1) \cdot (2x^2 - x + 3)^{(1/2)} - 168881/8192 \cdot 2^{(1/2)} \cdot \operatorname{arsinh}(4/23 \cdot 23^{(1/2)} \cdot (x - 1/4)) + 125/8 \cdot (2x^2 - x + 3)^{(1/2)} \cdot x^5 + 1575/32 \cdot (2x^2 - x + 3)^{(1/2)} \cdot x^4 + 14265/256 \cdot (2x^2 - x + 3)^{(1/2)} \cdot x^3 - 125091/1024 \cdot (2x^2 - x + 3)^{(1/2)} \cdot x^2 + 1168881/4096 \cdot (2x^2 - x + 3)^{(1/2)} \cdot x$

**maxima** [A] time = 0.97, size = 114, normalized size = 0.92

$$\frac{125x^5}{8\sqrt{2x^2 - x + 3}} + \frac{1575x^4}{32\sqrt{2x^2 - x + 3}} + \frac{14265x^3}{256\sqrt{2x^2 - x + 3}} - \frac{125091x^2}{1024\sqrt{2x^2 - x + 3}} - \frac{1168881}{8192} \sqrt{2} \operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x - 1) - \sqrt{2x^2 - x + 3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^3/(2\*x^2-x+3)^(3/2),x, algorithm="maxima")

[Out]  $125/8*x^5/\sqrt{2*x^2 - x + 3} + 1575/32*x^4/\sqrt{2*x^2 - x + 3} + 14265/256*x^3/\sqrt{2*x^2 - x + 3} - 125091/1024*x^2/\sqrt{2*x^2 - x + 3} - 1168881/8192*\sqrt{2}*\operatorname{arcsinh}(1/23*\sqrt{23}*(4*x - 1)) + 16138403/47104*x/\sqrt{2*x^2 - x + 3} - 15423965/47104/\sqrt{2*x^2 - x + 3}$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(5x^2 + 3x + 2)^3}{(2x^2 - x + 3)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^(3/2), x)`

[Out] `int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^(3/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x^2 + 3x + 2)^3}{(2x^2 - x + 3)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**2+3*x+2)**3/(2*x**2-x+3)**(3/2), x)`

[Out] `Integral((5*x**2 + 3*x + 2)**3/(2*x**2 - x + 3)**(3/2), x)`

$$3.88 \quad \int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{3/2}} dx$$

Optimal. Leaf size=82

$$\frac{121(19-7x)}{92\sqrt{2x^2-x+3}} + \frac{25}{8}x\sqrt{2x^2-x+3} + \frac{415}{32}\sqrt{2x^2-x+3} - \frac{223 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}}$$

[Out] -223/128\*arcsinh(1/23\*(1-4\*x)\*23^(1/2))\*2^(1/2)+121/92\*(19-7\*x)/(2\*x^2-x+3)^(1/2)+415/32\*(2\*x^2-x+3)^(1/2)+25/8\*x\*(2\*x^2-x+3)^(1/2)

**Rubi [A]** time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {1660, 1661, 640, 619, 215}

$$\frac{121(19-7x)}{92\sqrt{2x^2-x+3}} + \frac{25}{8}x\sqrt{2x^2-x+3} + \frac{415}{32}\sqrt{2x^2-x+3} - \frac{223 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3\*x + 5\*x^2)^2/(3 - x + 2\*x^2)^(3/2), x]

[Out] (121\*(19 - 7\*x))/(92\*sqrt[3 - x + 2\*x^2]) + (415\*sqrt[3 - x + 2\*x^2])/32 + (25\*x\*sqrt[3 - x + 2\*x^2])/8 - (223\*ArcSinh[(1 - 4\*x)/sqrt[23]])/(64\*sqrt[2])

Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] :> Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]

&& NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

### Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]
```

### Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(2 + 3x + 5x^2)^2}{(3 - x + 2x^2)^{3/2}} dx &= \frac{121(19 - 7x)}{92\sqrt{3 - x + 2x^2}} + \frac{2}{23} \int \frac{\frac{1173}{16} + \frac{1955x}{8} + \frac{575x^2}{4}}{\sqrt{3 - x + 2x^2}} dx \\
&= \frac{121(19 - 7x)}{92\sqrt{3 - x + 2x^2}} + \frac{25}{8}x\sqrt{3 - x + 2x^2} + \frac{1}{46} \int \frac{-138 + \frac{9545x}{8}}{\sqrt{3 - x + 2x^2}} dx \\
&= \frac{121(19 - 7x)}{92\sqrt{3 - x + 2x^2}} + \frac{415}{32}\sqrt{3 - x + 2x^2} + \frac{25}{8}x\sqrt{3 - x + 2x^2} + \frac{223}{64} \int \frac{1}{\sqrt{3 - x + 2x^2}} dx \\
&= \frac{121(19 - 7x)}{92\sqrt{3 - x + 2x^2}} + \frac{415}{32}\sqrt{3 - x + 2x^2} + \frac{25}{8}x\sqrt{3 - x + 2x^2} + \frac{223 \operatorname{Subst}\left(\int \frac{1}{\sqrt{1 + \frac{x^2}{23}}} dx, x, \right)}{64\sqrt{46}} \\
&= \frac{121(19 - 7x)}{92\sqrt{3 - x + 2x^2}} + \frac{415}{32}\sqrt{3 - x + 2x^2} + \frac{25}{8}x\sqrt{3 - x + 2x^2} - \frac{223 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}}
\end{aligned}$$

**Mathematica [A]** time = 0.14, size = 55, normalized size = 0.67

$$\frac{4600x^3 + 16790x^2 - 9421x + 47027}{736\sqrt{2x^2 - x + 3}} + \frac{223 \sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x + 5\*x^2)^2/(3 - x + 2\*x^2)^(3/2), x]

[Out] (47027 - 9421\*x + 16790\*x^2 + 4600\*x^3)/(736\*Sqrt[3 - x + 2\*x^2]) + (223\*ArcSinh[(-1 + 4\*x)/Sqrt[23]])/(64\*Sqrt[2])

**fricas [A]** time = 0.88, size = 92, normalized size = 1.12

$$\frac{5129\sqrt{2}(2x^2 - x + 3)\log\left(-4\sqrt{2}\sqrt{2x^2 - x + 3}(4x - 1) - 32x^2 + 16x - 25\right) + 8(4600x^3 + 16790x^2 - 9421x + 47027)\sqrt{2x^2 - x + 3}}{5888(2x^2 - x + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^2/(2\*x^2-x+3)^(3/2), x, algorithm="fricas")

[Out] 1/5888\*(5129\*sqrt(2)\*(2\*x^2 - x + 3)\*log(-4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25) + 8\*(4600\*x^3 + 16790\*x^2 - 9421\*x + 47027)\*sqrt(2\*x^2 - x + 3))/(2\*x^2 - x + 3)

**giac [A]** time = 0.24, size = 62, normalized size = 0.76

$$-\frac{223}{128}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x - \sqrt{2x^2 - x + 3}\right) + 1\right) + \frac{(230(20x + 73)x - 9421)x + 47027}{736\sqrt{2x^2 - x + 3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^2/(2\*x^2-x+3)^(3/2), x, algorithm="giac")

[Out] -223/128\*sqrt(2)\*log(-2\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 1) + 1/736\*((230\*(20\*x + 73)\*x - 9421)\*x + 47027)/sqrt(2\*x^2 - x + 3)

**maple [A]** time = 0.01, size = 98, normalized size = 1.20

$$\frac{25x^3}{4\sqrt{2x^2 - x + 3}} + \frac{365x^2}{16\sqrt{2x^2 - x + 3}} - \frac{223x}{64\sqrt{2x^2 - x + 3}} + \frac{223\sqrt{2} \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x - \frac{1}{4}\right)}{23}\right)}{128} + \frac{15761}{256\sqrt{2x^2 - x + 3}} - \frac{1371}{5888\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^2+3*x+2)^2/(2*x^2-x+3)^(3/2),x)`

[Out]  $25/4/(2*x^2-x+3)^{(1/2)}*x^3+365/16/(2*x^2-x+3)^{(1/2)}*x^2-223/64/(2*x^2-x+3)^{(1/2)}*x+15761/256/(2*x^2-x+3)^{(1/2)}-13713/5888*(4*x-1)/(2*x^2-x+3)^{(1/2)}+223/128*2^{(1/2)}*\operatorname{arcsinh}(4/23*23^{(1/2)}*(x-1/4))$

**maxima** [A] time = 0.96, size = 80, normalized size = 0.98

$$\frac{25x^3}{4\sqrt{2x^2-x+3}} + \frac{365x^2}{16\sqrt{2x^2-x+3}} + \frac{223}{128}\sqrt{2}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{9421x}{736\sqrt{2x^2-x+3}} + \frac{47027}{736\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(3/2),x, algorithm="maxima")`

[Out]  $25/4*x^3/\operatorname{sqrt}(2*x^2-x+3) + 365/16*x^2/\operatorname{sqrt}(2*x^2-x+3) + 223/128*\operatorname{sqrt}(2)*\operatorname{arcsinh}(1/23*\operatorname{sqrt}(23)*(4*x-1)) - 9421/736*x/\operatorname{sqrt}(2*x^2-x+3) + 47027/736/\operatorname{sqrt}(2*x^2-x+3)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(5x^2 + 3x + 2)^2}{(2x^2 - x + 3)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3)^(3/2),x)`

[Out] `int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3)^(3/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x^2 + 3x + 2)^2}{(2x^2 - x + 3)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**2+3*x+2)**2/(2*x**2-x+3)**(3/2),x)`

[Out] `Integral((5*x**2 + 3*x + 2)**2/(2*x**2 - x + 3)**(3/2), x)`

$$3.89 \quad \int \frac{2+3x+5x^2}{(3-x+2x^2)^{3/2}} dx$$

Optimal. Leaf size=45

$$-\frac{11(3x+5)}{23\sqrt{2x^2-x+3}} - \frac{5 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2\sqrt{2}}$$

[Out]  $-5/4*\operatorname{arcsinh}(1/23*(1-4*x)*23^{(1/2)})*2^{(1/2)}-11/23*(5+3*x)/(2*x^2-x+3)^{(1/2)}$

Rubi [A] time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$ , Rules used = {1660, 12, 619, 215}

$$-\frac{11(3x+5)}{23\sqrt{2x^2-x+3}} - \frac{5 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^{(3/2)}, x]$

[Out]  $(-11*(5 + 3*x))/(23*\operatorname{Sqrt}[3 - x + 2*x^2]) - (5*\operatorname{ArcSinh}[(1 - 4*x)/\operatorname{Sqrt}[23]])/(2*\operatorname{Sqrt}[2])$

### Rule 12

$\operatorname{Int}[(a_*)(u_), x\_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \ \&\& \ !\operatorname{Match} Q[u, (b_*)(v_)] /; \operatorname{FreeQ}[b, x]$

### Rule 215

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_*) + (b_*)(x_)^2], x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{ArcSinh}[(\operatorname{Rt}[b, 2]*x)/\operatorname{Sqrt}[a]]/\operatorname{Rt}[b, 2], x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \ \operatorname{GtQ}[a, 0] \ \&\& \ \operatorname{PosQ}[b]$

### Rule 619

$\operatorname{Int}[((a_*) + (b_*)(x_) + (c_*)(x_)^2)^{(p_)}, x\_Symbol] \rightarrow \operatorname{Dist}[1/(2*c*((-4*c)/(b^2 - 4*a*c))^{(p_)}, \operatorname{Subst}[\operatorname{Int}[\operatorname{Simp}[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; \operatorname{FreeQ}\{a, b, c, p\}, x \ \&\& \ \operatorname{GtQ}[4*a - b^2/c, 0]$

### Rule 1660

$\operatorname{Int}[(Pq_*)((a_*) + (b_*)(x_) + (c_*)(x_)^2)^{(p_)}, x\_Symbol] \rightarrow \operatorname{With}\{Q = \operatorname{PolynomialQuotient}[Pq, a + b*x + c*x^2, x], f = \operatorname{Coeff}[\operatorname{PolynomialRemainder}[P$

```

q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]], Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]

```

### Rubi steps

$$\begin{aligned}
\int \frac{2 + 3x + 5x^2}{(3 - x + 2x^2)^{3/2}} dx &= -\frac{11(5 + 3x)}{23\sqrt{3 - x + 2x^2}} + \frac{2}{23} \int \frac{115}{4\sqrt{3 - x + 2x^2}} dx \\
&= -\frac{11(5 + 3x)}{23\sqrt{3 - x + 2x^2}} + \frac{5}{2} \int \frac{1}{\sqrt{3 - x + 2x^2}} dx \\
&= -\frac{11(5 + 3x)}{23\sqrt{3 - x + 2x^2}} + \frac{5 \operatorname{Subst}\left(\int \frac{1}{\sqrt{1 + \frac{x^2}{23}}} dx, x, -1 + 4x\right)}{2\sqrt{46}} \\
&= -\frac{11(5 + 3x)}{23\sqrt{3 - x + 2x^2}} - \frac{5 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2\sqrt{2}}
\end{aligned}$$

**Mathematica** [A] time = 0.08, size = 45, normalized size = 1.00

$$\frac{5 \sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{2\sqrt{2}} - \frac{11(3x+5)}{23\sqrt{2x^2-x+3}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x + 5\*x^2)/(3 - x + 2\*x^2)^(3/2), x]

[Out] (-11\*(5 + 3\*x))/(23\*Sqrt[3 - x + 2\*x^2]) + (5\*ArcSinh[(-1 + 4\*x)/Sqrt[23]])/(2\*Sqrt[2])

**fricas** [B] time = 0.80, size = 82, normalized size = 1.82

$$\frac{115\sqrt{2}(2x^2 - x + 3) \log\left(-4\sqrt{2}\sqrt{2x^2 - x + 3}(4x - 1) - 32x^2 + 16x - 25\right) - 88\sqrt{2x^2 - x + 3}(3x + 5)}{184(2x^2 - x + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((5\*x^2+3\*x+2)/(2\*x^2-x+3)^(3/2),x, algorithm="fricas")

[Out] 1/184\*(115\*sqrt(2)\*(2\*x^2 - x + 3)\*log(-4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25) - 88\*sqrt(2\*x^2 - x + 3)\*(3\*x + 5))/(2\*x^2 - x + 3)

**giac** [A] time = 0.23, size = 53, normalized size = 1.18

$$-\frac{5}{4}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x - \sqrt{2x^2 - x + 3}\right) + 1\right) - \frac{11(3x + 5)}{23\sqrt{2x^2 - x + 3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)/(2\*x^2-x+3)^(3/2),x, algorithm="giac")

[Out] -5/4\*sqrt(2)\*log(-2\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 1) - 11/23\*(3\*x + 5)/sqrt(2\*x^2 - x + 3)

**maple** [A] time = 0.01, size = 64, normalized size = 1.42

$$-\frac{5x}{2\sqrt{2x^2 - x + 3}} + \frac{5\sqrt{2}\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x - \frac{1}{4}\right)}{23}\right)}{4} - \frac{17}{8\sqrt{2x^2 - x + 3}} + \frac{\frac{49x}{46} - \frac{49}{184}}{\sqrt{2x^2 - x + 3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+3\*x+2)/(2\*x^2-x+3)^(3/2),x)

[Out] -5/2/(2\*x^2-x+3)^(1/2)\*x-17/8/(2\*x^2-x+3)^(1/2)+49/184\*(4\*x-1)/(2\*x^2-x+3)^(1/2)+5/4\*2^(1/2)\*arcsinh(4/23\*23^(1/2)\*(x-1/4))

**maxima** [A] time = 0.95, size = 46, normalized size = 1.02

$$\frac{5}{4}\sqrt{2}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x - 1)\right) - \frac{33x}{23\sqrt{2x^2 - x + 3}} - \frac{55}{23\sqrt{2x^2 - x + 3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)/(2\*x^2-x+3)^(3/2),x, algorithm="maxima")

[Out] 5/4\*sqrt(2)\*arcsinh(1/23\*sqrt(23)\*(4\*x - 1)) - 33/23\*x/sqrt(2\*x^2 - x + 3) - 55/23/sqrt(2\*x^2 - x + 3)

**mupad** [B] time = 0.23, size = 87, normalized size = 1.93

$$\frac{5\sqrt{2}\ln\left(\sqrt{2x^2 - x + 3} + \frac{\sqrt{2}\left(2x - \frac{1}{2}\right)}{2}\right)}{4} + \frac{3(2x - 12)}{23\sqrt{2x^2 - x + 3}} - \frac{10\left(\frac{11x}{2} + \frac{3}{2}\right)}{23\sqrt{2x^2 - x + 3}} + \frac{16x - 4}{23\sqrt{2x^2 - x + 3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x + 5*x^2 + 2)/(2*x^2 - x + 3)^(3/2), x)`

[Out]  $(5 \cdot 2^{1/2} \cdot \log((2x^2 - x + 3)^{1/2} + (2^{1/2} \cdot (2x - 1/2))/2))/4 + (3 \cdot (2x - 12))/(23 \cdot (2x^2 - x + 3)^{1/2}) - (10 \cdot ((11x)/2 + 3/2))/(23 \cdot (2x^2 - x + 3)^{1/2}) + (16x - 4)/(23 \cdot (2x^2 - x + 3)^{1/2})$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^2 + 3x + 2}{(2x^2 - x + 3)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**2+3*x+2)/(2*x**2-x+3)**(3/2), x)`

[Out] `Integral((5*x**2 + 3*x + 2)/(2*x**2 - x + 3)**(3/2), x)`

$$3.90 \quad \int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)} dx$$

Optimal. Leaf size=176

$$\frac{13-6x}{253\sqrt{2x^2-x+3}} + \frac{1}{22}\sqrt{\frac{1}{682}(247+500\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{\frac{11}{31(247+500\sqrt{2})}}((69+65\sqrt{2})x+4\sqrt{2}+61)}{\sqrt{2x^2-x+3}}\right) - \frac{1}{22}\sqrt{\frac{1}{682}}$$

[Out] 1/253\*(13-6\*x)/(2\*x^2-x+3)^(1/2)-1/15004\*arctanh(1/31\*(61+x\*(69-65\*2^(1/2))-4\*2^(1/2))\*341^(1/2)/(-247+500\*2^(1/2))^(1/2)/(2\*x^2-x+3)^(1/2))\*(-168454+341000\*2^(1/2))^(1/2)+1/15004\*arctan(1/31\*(61+4\*2^(1/2)+x\*(69+65\*2^(1/2)))\*341^(1/2)/(247+500\*2^(1/2))^(1/2)/(2\*x^2-x+3)^(1/2))\*(168454+341000\*2^(1/2))^(1/2)

Rubi [A] time = 0.41, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {974, 1035, 1029, 206, 204}

$$\frac{13-6x}{253\sqrt{2x^2-x+3}} + \frac{1}{22}\sqrt{\frac{1}{682}(247+500\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{\frac{11}{31(247+500\sqrt{2})}}((69+65\sqrt{2})x+4\sqrt{2}+61)}{\sqrt{2x^2-x+3}}\right) - \frac{1}{22}\sqrt{\frac{1}{682}}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - x + 2\*x^2)^(3/2)\*(2 + 3\*x + 5\*x^2)),x]

[Out] (13 - 6\*x)/(253\*Sqrt[3 - x + 2\*x^2]) + (Sqrt[(247 + 500\*Sqrt[2])/682]\*ArcTan[(Sqrt[11/(31\*(247 + 500\*Sqrt[2]))])\*(61 + 4\*Sqrt[2] + (69 + 65\*Sqrt[2])\*x)]/Sqrt[3 - x + 2\*x^2])/22 - (Sqrt[(-247 + 500\*Sqrt[2])/682]\*ArcTanh[(Sqrt[11/(31\*(-247 + 500\*Sqrt[2]))])\*(61 - 4\*Sqrt[2] + (69 - 65\*Sqrt[2])\*x)]/Sqrt[3 - x + 2\*x^2])/22

Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

Q[a, 0] || LtQ[b, 0])

### Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]
```

### Rule 1029

```
Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]], x_Symbol] := Dist[-2*g*(g*b - 2*a*h), Subst[Int[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f), 0]
```

### Rule 1035

```
Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]], x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 - 4*a*c]
```

### Rubi steps

$$\begin{aligned}
\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)} dx &= \frac{13-6x}{253\sqrt{3-x+2x^2}} - \frac{\int \frac{-1012-\frac{1265x}{2}}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{2783} \\
&= \frac{13-6x}{253\sqrt{3-x+2x^2}} + \frac{\int \frac{\frac{2783}{2}(3+8\sqrt{2})-\frac{2783}{2}(13-5\sqrt{2})x}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{61226\sqrt{2}} - \frac{\int \frac{\frac{2783}{2}(3-8\sqrt{2})-\frac{2783}{2}}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{61226\sqrt{2}} \\
&= \frac{13-6x}{253\sqrt{3-x+2x^2}} - \frac{1}{8} \left( 253(1000-247\sqrt{2}) \right) \text{Subst} \left( \int \frac{-\frac{240097759}{4}(247+500\sqrt{2})}{\sqrt{31(247+500\sqrt{2})}} dx \right) \\
&= \frac{13-6x}{253\sqrt{3-x+2x^2}} + \frac{1}{22} \sqrt{\frac{1}{682}(247+500\sqrt{2})} \tan^{-1} \left( \sqrt{\frac{11}{31(247+500\sqrt{2})}} \right)
\end{aligned}$$

**Mathematica [C]** time = 1.29, size = 202, normalized size = 1.15

$$\frac{-\frac{27280(6x-13)}{\sqrt{2x^2-x+3}} - 23\sqrt{682(13+i\sqrt{31})}(13\sqrt{31}+69i) \tanh^{-1}\left(\frac{(-22-4i\sqrt{31})x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right) - 23\sqrt{682(13-i\sqrt{31})}(13\sqrt{31}-69i) \tanh^{-1}\left(\frac{(-22+4i\sqrt{31})x-i\sqrt{31}+63}{2\sqrt{286-22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)}{6901840}$$

Antiderivative was successfully verified.

[In] Integrate[1/((3-x+2\*x^2)^(3/2)\*(2+3\*x+5\*x^2)),x]

[Out] ((-27280\*(-13+6\*x))/Sqrt[3-x+2\*x^2]-23\*Sqrt[682\*(13+I\*Sqrt[31])]\*(69\*I+13\*Sqrt[31])\*ArcTanh[(63+I\*Sqrt[31]+(-22-(4\*I)\*Sqrt[31])\*x)/(2\*Sqrt[286+(22\*I)\*Sqrt[31]]\*Sqrt[3-x+2\*x^2])]-23\*Sqrt[682\*(13-I\*Sqrt[31])]\*(-69\*I+13\*Sqrt[31])\*ArcTanh[(63-I\*Sqrt[31]+(-22+(4\*I)\*Sqrt[31])\*x)/(2\*Sqrt[286-(22\*I)\*Sqrt[31]]\*Sqrt[3-x+2\*x^2])])/6901840

**fricas [B]** time = 2.34, size = 2083, normalized size = 11.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^(3/2)/(5\*x^2+3\*x+2),x, algorithm="fricas")

[Out] -1/50921775520\*(339388\*sqrt(341)\*50^(1/4)\*sqrt(10)\*sqrt(2)\*(2\*x^2-x+3)\*sqrt(247\*sqrt(2)+1000)\*arctan(1/328782125\*(14260\*sqrt(341)\*sqrt(10)\*sqrt(2)

$$\begin{aligned}
& 2*x^2 - x + 3)*(22*50^{(3/4)}*(57708*x^7 - 181278*x^6 + 400374*x^5 - 525676*x^4 \\
& + 235088*x^3 - 46944*x^2 - \sqrt{2}*(20846*x^7 - 109153*x^6 + 215386*x^5 - 427391*x^4 \\
& + 234360*x^3 - 156600*x^2 - 172800*x + 186624) - 373248*x + 172800) + 5*50^{(1/4)}*(125839*x^7 - 1864281*x^6 + 9323336*x^5 - 19725020*x^4 + \\
& 24624288*x^3 - 10862496*x^2 - \sqrt{2}*(56119*x^7 - 908994*x^6 + 5175980*x^5 - 12895624*x^4 \\
& + 17261280*x^3 - 14184000*x^2 - 10533888*x + 9994752) - 19989504*x + 10533888)) * \sqrt{247*\sqrt{2} + 1000} + 933317000*\sqrt{31}*\sqrt{2} \\
& *(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - \sqrt{2}*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 \\
& + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*\sqrt{310/119}*(\sqrt{341}*\sqrt{10}*\sqrt{2*x^2 - x + 3}*(22*50^{(3/4)}*(246848*x^7 - 348192*x^6 + 1080672*x^5 - 178432*x^4 - 18432*x^3 + 10298 \\
& 88*x^2 - \sqrt{2}*(46522*x^7 - 71117*x^6 + 257247*x^5 - 273360*x^4 + 484920*x^3 - 269568*x^2 + 269568*x) - 1029888*x) + 5*50^{(1/4)}*(516957*x^7 - 667694 \\
& 8*x^6 + 25569820*x^5 - 31522752*x^4 + 34450848*x^3 + 46199808*x^2 - 4*\sqrt{2}*(38689*x^7 - 502244*x^6 + 1967660*x^5 - 2828160*x^4 + 4711680*x^3 - 1689 \\
& 984*x^2 + 1689984*x) - 46199808*x)) * \sqrt{247*\sqrt{2} + 1000} + 65450*\sqrt{31}*\sqrt{2}*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 \\
& + 798336*x^3 - 3822336*x^2 - \sqrt{2}*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276 \\
& 288*x) + 2975*\sqrt{31}*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*\sqrt{2}*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 1 \\
& 44820224*x)) * \sqrt{-(\sqrt{341}*50^{(1/4)}*\sqrt{31}*\sqrt{10}*\sqrt{2*x^2 - x + 3})*(\sqrt{2}*(37*x - 38) + x - 75)*\sqrt{247*\sqrt{2} + 1000} - 903805*x^2 - 81 \\
& 1580*\sqrt{2}*(2*x^2 - x + 3) + 2785195*x - 3689000)/x^2} + 10605875*\sqrt{31}*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*\sqrt{2}*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064 \\
& 064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 339 \\
& 388*\sqrt{341}*50^{(1/4)}*\sqrt{10}*\sqrt{2}*(2*x^2 - x + 3)*\sqrt{247*\sqrt{2} + 1000}*\arctan(1/328782125*(14260*\sqrt{341}*\sqrt{10}*\sqrt{2*x^2 - x + 3}*(22* \\
& 50^{(3/4)}*(57708*x^7 - 181278*x^6 + 400374*x^5 - 525676*x^4 + 235088*x^3 - 46944*x^2 - \sqrt{2}*(20846*x^7 - 109153*x^6 + 215386*x^5 - 427391*x^4 + 2343 \\
& 60*x^3 - 156600*x^2 - 172800*x + 186624) - 373248*x + 172800) + 5*50^{(1/4)}*(125839*x^7 - 1864281*x^6 + 9323336*x^5 - 19725020*x^4 + 24624288*x^3 - 108 \\
& 62496*x^2 - \sqrt{2}*(56119*x^7 - 908994*x^6 + 5175980*x^5 - 12895624*x^4 + 17261280*x^3 - 14184000*x^2 - 10533888*x + 9994752) - 19989504*x + 10533888 \\
& )) * \sqrt{247*\sqrt{2} + 1000} - 933317000*\sqrt{31}*\sqrt{2}*(28180*x^8 - 25466 \\
& 6*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - \sqrt{2}*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752 \\
& 088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*\sqrt{310/119}*(\sqrt{341}*\sqrt{10}*\sqrt{2*x^2 - x + 3}*(22*50^{(3/4)}*(246848*x^7 - 3 \\
& 48192*x^6 + 1080672*x^5 - 178432*x^4 - 18432*x^3 + 1029888*x^2 - \sqrt{2}*(4
\end{aligned}$$

$$\begin{aligned}
& 6522x^7 - 71117x^6 + 257247x^5 - 273360x^4 + 484920x^3 - 269568x^2 + \\
& 269568x - 1029888x + 5 \cdot 50^{1/4} (516957x^7 - 6676948x^6 + 25569820x^5 - \\
& 31522752x^4 + 34450848x^3 + 46199808x^2 - 4\sqrt{2}(38689x^7 - 502 \\
& 244x^6 + 1967660x^5 - 2828160x^4 + 4711680x^3 - 1689984x^2 + 1689984x \\
& ) - 46199808x) \sqrt{247\sqrt{2} + 1000} - 65450\sqrt{31}\sqrt{2}(123408x^8 - \\
& 914152x^7 + 1578888x^6 - 3293072x^5 + 396480x^4 + 798336x^3 - 38 \\
& 22336x^2 - \sqrt{2}(15550x^8 - 118051x^7 + 244047x^6 - 707374x^5 + 105 \\
& 3960x^4 - 1667952x^3 + 1209600x^2 - 1036800x) + 3276288x) - 2975\sqrt{31} \\
& (254591x^8 - 4815126x^7 + 32303580x^6 - 90866808x^5 + 108781920x^4 - \\
& 74219328x^3 - 168956928x^2 - 15488\sqrt{2}(4x^8 - 76x^7 + 517x^6 - \\
& 1536x^5 + 2385x^4 - 3618x^3 + 2268x^2 - 1944x) + 144820224x) \sqrt{(\sqrt{341} \\
& \cdot 50^{1/4} \sqrt{31} \sqrt{10} \sqrt{2x^2 - x + 3} (\sqrt{2}(37x - 38) + x - 75) \sqrt{247\sqrt{2} + 1000} \\
& + 903805x^2 + 811580\sqrt{2}(2x^2 - x + 3) - 2785195x + 3689000)/x^2} - 10605875\sqrt{31} \\
& (2828123x^8 - 9696916x^7 + 53385560x^6 - 142835344x^5 + 254146592x^4 - 249300096x^3 + 3 \\
& 7981440x^2 - 7744\sqrt{2}(1348x^8 - 2692x^7 + 9789x^6 - 10070x^5 + 15 \\
& 569x^4 - 5568x^3 + 1080x^2 + 4320x - 5184) + 223064064x - 94887936) / ( \\
& 2585191x^8 - 4661200x^7 + 14191920x^6 + 490880x^5 - 13562944x^4 + 4424 \\
& 9088x^3 - 34615296x^2 - 24772608x + 18579456) - 23\sqrt{341} \cdot 50^{1/4} \sqrt{31} \\
& \sqrt{10} (2000x^2 - 247\sqrt{2}(2x^2 - x + 3) - 1000x + 3000) \sqrt{247\sqrt{2} + 1000} \\
& \log(3100000/119(\sqrt{341} \cdot 50^{1/4} \sqrt{31} \sqrt{10} \sqrt{2x^2 - x + 3} (\sqrt{2}(37x - 38) + x - 75) \\
& \sqrt{247\sqrt{2} + 1000} + 903805x^2 + 811580\sqrt{2}(2x^2 - x + 3) - 2785195x + 3689000)/x^2) \\
& + 23\sqrt{341} \cdot 50^{1/4} \sqrt{31} \sqrt{10} (2000x^2 - 247\sqrt{2}(2x^2 - x + 3) - 1000x + 3000) \\
& \sqrt{247\sqrt{2} + 1000} \log(-3100000/119(\sqrt{341} \cdot 50^{1/4} \sqrt{31} \sqrt{10} \sqrt{2x^2 - x + 3} \\
& (\sqrt{2}(37x - 38) + x - 75) \sqrt{247\sqrt{2} + 1000} - 903805x^2 - 811580\sqrt{2}(2x^2 - x + 3) \\
& + 2785195x - 3689000)/x^2) + 201271840\sqrt{2x^2 - x + 3} (6x - 13) / (2x^2 - x + 3)
\end{aligned}$$

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^(3/2)/(5\*x^2+3\*x+2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command: INP UT:sage2:=int(sage0,x)::OUTPUT:Francis algorithm failure for[-1.0,infinity, infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf

inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinity]Evaluation time: 9.49Done

**maple [B]** time = 0.03, size = 718, normalized size = 4.08

$$\sqrt{\frac{8(x+\sqrt{2}-1)^2}{(-x+\sqrt{2}+1)^2} + \frac{3\sqrt{2}(x+\sqrt{2}-1)^2}{(-x+\sqrt{2}+1)^2} + 8 - 3\sqrt{2}} \sqrt{2} \left( 1712502\sqrt{2} \operatorname{arctanh} \left( \frac{31 \sqrt{\frac{8(x+\sqrt{2}-1)^2}{(-x+\sqrt{2}+1)^2} + \frac{3\sqrt{2}(x+\sqrt{2}-1)^2}{(-x+\sqrt{2}+1)^2} + 8 - 3\sqrt{2}}}{2\sqrt{-8866+6820\sqrt{2}}} \right) - 6617446 \operatorname{arctanh} \left( \frac{31}{2\sqrt{-8866+6820\sqrt{2}}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x)`

[Out]  $\frac{1}{465124} \left( 8(x+2^{1/2}-1)^2/(-x+2^{1/2}+1)^2 + 3 \cdot 2^{1/2} \cdot (x+2^{1/2}-1)^2/(-x+2^{1/2}+1)^2 + 8 - 3 \cdot 2^{1/2} \right)^{1/2} \cdot 2^{1/2} \cdot (2197 \cdot 2^{1/2} \cdot (-8866+6820 \cdot 2^{1/2}))^{1/2} \cdot (-775687+549362 \cdot 2^{1/2})^{1/2} \cdot \arctan\left(\frac{1}{11692487} \cdot (-775687+549362 \cdot 2^{1/2})^{1/2}\right)^{1/2} \cdot (-23 \cdot (8+3 \cdot 2^{1/2})) \cdot (-23 \cdot (x+2^{1/2}-1)^2/(-x+2^{1/2}+1)^2 + 24 \cdot 2^{1/2} \cdot (x+2^{1/2}-1)^2/(-x+2^{1/2}+1)^2 + 22379 \cdot 2^{1/2} + 32016) / (23 \cdot (x+2^{1/2}-1)^4/(-x+2^{1/2}+1)^4 + 82 \cdot (x+2^{1/2}-1)^2/(-x+2^{1/2}+1)^2 + 23) \cdot (x+2^{1/2}-1) / (-x+2^{1/2}+1) \cdot (8+3 \cdot 2^{1/2}) + 3070 \cdot (-8866+6820 \cdot 2^{1/2})^{1/2} \cdot (-775687+549362 \cdot 2^{1/2})^{1/2} \cdot \arctan\left(\frac{1}{11692487} \cdot (-775687+549362 \cdot 2^{1/2})^{1/2}\right)^{1/2} \cdot (-23 \cdot (8+3 \cdot 2^{1/2})) \cdot (-23 \cdot (x+2^{1/2}-1)^2/(-x+2^{1/2}+1)^2 + 24 \cdot 2^{1/2} \cdot (x+2^{1/2}-1)^2/(-x+2^{1/2}+1)^2 + 22379 \cdot 2^{1/2} + 32016) / (23 \cdot (x+2^{1/2}-1)^4/(-x+2^{1/2}+1)^4 + 82 \cdot (x+2^{1/2}-1)^2/(-x+2^{1/2}+1)^2 + 23) \cdot (x+2^{1/2}-1) / (-x+2^{1/2}+1) \cdot (8+3 \cdot 2^{1/2}) + 1712502 \cdot \operatorname{arctanh}\left(\frac{31 \cdot 2^{1/2} \cdot (8 \cdot (x+2^{1/2}-1)^2/(-x+2^{1/2}+1)^2 + 3 \cdot 2^{1/2} \cdot (x+2^{1/2}-1)^2/(-x+2^{1/2}+1)^2 + 8 - 3 \cdot 2^{1/2})^{1/2}}{(-8866+6820 \cdot 2^{1/2})^{1/2}}\right)^{1/2} \cdot 2^{1/2} - 6617446 \cdot \operatorname{arctanh}\left(\frac{31 \cdot 2^{1/2} \cdot (8 \cdot (x+2^{1/2}-1)^2/(-x+2^{1/2}+1)^2 + 3 \cdot 2^{1/2} \cdot (x+2^{1/2}-1)^2/(-x+2^{1/2}+1)^2 + 8 - 3 \cdot 2^{1/2})^{1/2}}{(-8866+6820 \cdot 2^{1/2})^{1/2}}\right)^{1/2} / ((8 \cdot (x+2^{1/2}-1)^2/(-x+2^{1/2}+1)^2 + 3 \cdot 2^{1/2} \cdot (x+2^{1/2}-1)^2/(-x+2^{1/2}+1)^2 + 8 - 3 \cdot 2^{1/2})^{1/2}) / (1 + (x+2^{1/2}-1)/(-x+2^{1/2}+1))^{1/2} / (1 + (x+2^{1/2}-1)/(-x+2^{1/2}+1)) / (8+3 \cdot 2^{1/2}) / (-8866+6820 \cdot 2^{1/2})^{1/2} + 1/22 / (2 \cdot x^2 - x + 3)^{1/2} - 3/506 \cdot (4 \cdot x - 1) / (2 \cdot x^2 - x + 3)^{1/2}$

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(5x^2 + 3x + 2)(2x^2 - x + 3)^{\frac{3}{2}}} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^(3/2)/(5\*x^2+3\*x+2),x, algorithm="maxima")

[Out] integrate(1/((5\*x^2 + 3\*x + 2)\*(2\*x^2 - x + 3)^(3/2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(2x^2 - x + 3)^{3/2} (5x^2 + 3x + 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((2\*x^2 - x + 3)^(3/2)\*(3\*x + 5\*x^2 + 2)),x)

[Out] int(1/((2\*x^2 - x + 3)^(3/2)\*(3\*x + 5\*x^2 + 2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(2x^2 - x + 3)^{\frac{3}{2}} (5x^2 + 3x + 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x\*\*2-x+3)\*\*(3/2)/(5\*x\*\*2+3\*x+2),x)

[Out] Integral(1/((2\*x\*\*2 - x + 3)\*\*(3/2)\*(5\*x\*\*2 + 3\*x + 2)), x)

$$3.91 \quad \int \frac{1}{(3-x+2x^2)^{3/2} (2+3x+5x^2)^2} dx$$

**Optimal.** Leaf size=211

$$\frac{6315 - 2306x}{345092\sqrt{2x^2 - x + 3}} + \frac{65x + 4}{682\sqrt{2x^2 - x + 3} (5x^2 + 3x + 2)} + \frac{\sqrt{\frac{1}{682} (129694447 + 103775000\sqrt{2})} \tan^{-1} \left( \frac{\sqrt{31(129694447 + 103775000\sqrt{2})}}{31(129694447 + 103775000\sqrt{2})} \right)}{30008}$$

[Out] 1/345092\*(-6315+2306\*x)/(2\*x^2-x+3)^(1/2)+1/682\*(4+65\*x)/(5\*x^2+3\*x+2)/(2\*x^2-x+3)^(1/2)-1/20465456\*arctanh(1/31\*(12611+x\*(45519-29065\*2^(1/2))-16454\*2^(1/2))\*341^(1/2)/(-129694447+103775000\*2^(1/2))^(1/2)/(2\*x^2-x+3)^(1/2))\*(-88451612854+70774550000\*2^(1/2))^(1/2)+1/20465456\*arctan(1/31\*(12611+16454\*2^(1/2)+x\*(45519+29065\*2^(1/2)))\*341^(1/2)/(129694447+103775000\*2^(1/2))^(1/2)/(2\*x^2-x+3)^(1/2))\*(88451612854+70774550000\*2^(1/2))^(1/2)

**Rubi [A]** time = 0.47, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {974, 1060, 1035, 1029, 206, 204}

$$\frac{6315 - 2306x}{345092\sqrt{2x^2 - x + 3}} + \frac{65x + 4}{682\sqrt{2x^2 - x + 3} (5x^2 + 3x + 2)} + \frac{\sqrt{\frac{1}{682} (129694447 + 103775000\sqrt{2})} \tan^{-1} \left( \frac{\sqrt{31(129694447 + 103775000\sqrt{2})}}{31(129694447 + 103775000\sqrt{2})} \right)}{30008}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - x + 2\*x^2)^(3/2)\*(2 + 3\*x + 5\*x^2)^2), x]

[Out] -(6315 - 2306\*x)/(345092\*Sqrt[3 - x + 2\*x^2]) + (4 + 65\*x)/(682\*Sqrt[3 - x + 2\*x^2]\*(2 + 3\*x + 5\*x^2)) + (Sqrt[(129694447 + 103775000\*Sqrt[2])/682]\*ArcTan[(Sqrt[11/(31\*(129694447 + 103775000\*Sqrt[2]))]\*(12611 + 16454\*Sqrt[2] + (45519 + 29065\*Sqrt[2])\*x))/Sqrt[3 - x + 2\*x^2]])/30008 - (Sqrt[(-12969447 + 103775000\*Sqrt[2])/682]\*ArcTanh[(Sqrt[11/(31\*(-129694447 + 103775000\*Sqrt[2]))]\*(12611 - 16454\*Sqrt[2] + (45519 - 29065\*Sqrt[2])\*x))/Sqrt[3 - x + 2\*x^2]])/30008

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 974

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_)\*((d\_) + (e\_)\*(x\_) + (f\_)\*(x\_)^2)^(q\_), x\_Symbol] := Simp[((2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f) + c\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*x)\*(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^(q + 1))/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1)), x] - Dist[1/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1)), Int[(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^q\*Simp[2\*c\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1) - (2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(a\*f\*(p + 1) - c\*d\*(p + 2)) - e\*(b^2\*c\*e - 2\*a\*c^2\*e - b^3\*f - b\*c\*(c\*d - 3\*a\*f))\*(p + q + 2) + (2\*f\*(2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f))\*(p + q + 2) - (2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(b\*f\*(p + 1) - c\*e\*(2\*p + q + 4)))\*x + c\*f\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(2\*p + 2\*q + 5)\*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && LtQ[p, -1] && NeQ[(c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f), 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]

Rule 1029

Int[((g\_) + (h\_)\*(x\_))/(((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)\*Sqrt[(d\_) + (e\_)\*(x\_) + (f\_)\*(x\_)^2]), x\_Symbol] := Dist[-2\*g\*(g\*b - 2\*a\*h), Subst[Int[1/Simp[g\*(g\*b - 2\*a\*h)\*(b^2 - 4\*a\*c) - (b\*d - a\*e)\*x^2, x], x], x, Simp[g\*b - 2\*a\*h - (b\*h - 2\*g\*c)\*x, x]/Sqrt[d + e\*x + f\*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && NeQ[b\*d - a\*e, 0] && EqQ[h^2\*(b\*d - a\*e) - 2\*g\*h\*(c\*d - a\*f) + g^2\*(c\*e - b\*f), 0]

Rule 1035

Int[((g\_) + (h\_)\*(x\_))/(((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)\*Sqrt[(d\_) + (e\_)\*(x\_) + (f\_)\*(x\_)^2]), x\_Symbol] := With[{q = Rt[(c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f), 2]}, Dist[1/(2\*q), Int[Simp[h\*(b\*d - a\*e) - g\*(c\*d - a\*f - q) - (g\*(c\*e - b\*f) - h\*(c\*d - a\*f + q))\*x, x]/((a + b\*x + c\*x^2)\*Sqrt[d + e\*x + f\*x^2]), x], x] - Dist[1/(2\*q), Int[Simp[h\*(b\*d - a\*e) - g\*(c\*d - a\*f + q) - (g\*(c\*e - b\*f) - h\*(c\*d - a\*f - q))\*x, x]/((a + b\*x + c\*x^2)\*Sqrt[d + e\*x + f\*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && NeQ[b\*d - a\*e, 0] && NegQ[b^2 - 4\*a\*c]

Rule 1060

```

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)^2} dx &= \frac{4+65x}{682\sqrt{3-x+2x^2}(2+3x+5x^2)} - \int \frac{-1782+\frac{3333x}{2}-2860x^2}{(3-x+2x^2)^{3/2}(2+3x+5x^2)} dx \\
&= -\frac{6315-2306x}{345092\sqrt{3-x+2x^2}} + \frac{4+65x}{682\sqrt{3-x+2x^2}(2+3x+5x^2)} - \int \frac{-\frac{300}{\sqrt{3-x+2x^2}}}{2} \\
&= -\frac{6315-2306x}{345092\sqrt{3-x+2x^2}} + \frac{4+65x}{682\sqrt{3-x+2x^2}(2+3x+5x^2)} + \int \frac{\frac{30613}{4}}{4} \\
&= -\frac{6315-2306x}{345092\sqrt{3-x+2x^2}} + \frac{4+65x}{682\sqrt{3-x+2x^2}(2+3x+5x^2)} - \frac{(253(2))}{\sqrt{\frac{1}{682}}(1)} \\
&= -\frac{6315-2306x}{345092\sqrt{3-x+2x^2}} + \frac{4+65x}{682\sqrt{3-x+2x^2}(2+3x+5x^2)} + \frac{\sqrt{\frac{1}{682}}(1)}{1}
\end{aligned}$$

**Mathematica [C]** time = 1.53, size = 740, normalized size = 3.51

$$100 \left( \frac{682((22-4i\sqrt{31})x+i\sqrt{31}+52)}{(\sqrt{31}+13i)(10ix+\sqrt{31}+3i)\sqrt{2x^2-x+3}} + \frac{682((22+4i\sqrt{31})x-i\sqrt{31}+52)}{(\sqrt{31}-13i)(-10ix+\sqrt{31}-3i)\sqrt{2x^2-x+3}} + \frac{22(2(11\sqrt{31}-62i)x+52\sqrt{31}+31i)}{(\sqrt{31}+13i)\sqrt{2x^2-x+3}} + \frac{22(2(11\sqrt{31}+62i)x+52\sqrt{31}-31i)}{(\sqrt{31}-13i)\sqrt{2x^2-x+3}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/((3 - x + 2\*x^2)^(3/2)\*(2 + 3\*x + 5\*x^2)^2), x]

[Out] (100\*((682\*(52 + I\*Sqrt[31] + (22 - (4\*I)\*Sqrt[31])\*x))/((13\*I + Sqrt[31])\* (3\*I + Sqrt[31] + (10\*I)\*x)\*Sqrt[3 - x + 2\*x^2]) + (682\*(52 - I\*Sqrt[31] + (22 + (4\*I)\*Sqrt[31])\*x))/((-13\*I + Sqrt[31])\*(-3\*I + Sqrt[31] - (10\*I)\*x)\* Sqrt[3 - x + 2\*x^2]) + (22\*(31\*I + 52\*Sqrt[31] + 2\*(-62\*I + 11\*Sqrt[31])\*x) )/((13\*I + Sqrt[31])\*Sqrt[3 - x + 2\*x^2]) + (22\*(-31\*I + 52\*Sqrt[31] + 2\*(6 2\*I + 11\*Sqrt[31])\*x))/((-13\*I + Sqrt[31])\*Sqrt[3 - x + 2\*x^2]) + ((575\*I)\* Sqrt[682\*(13 + I\*Sqrt[31])]\*ArcTanh[(63 + I\*Sqrt[31] + (-22 - (4\*I)\*Sqrt[31 ])\*x]/(2\*Sqrt[286 + (22\*I)\*Sqrt[31]]\*Sqrt[3 - x + 2\*x^2])))/(-13\*I + Sqrt[3

$$\begin{aligned} & 1])^2 + (155*(44*(16353 + (581*I)*\text{Sqrt}[31]))*\text{Sqrt}[3 - x + 2*x^2] + 345*\text{Sqrt}[ \\ & 286 + (22*I)*\text{Sqrt}[31]]*(-29 + (17*I)*\text{Sqrt}[31] + 10*(11 + (2*I)*\text{Sqrt}[31])*x) \\ & *\text{ArcTanh}[(63 + I*\text{Sqrt}[31] + (-22 - (4*I)*\text{Sqrt}[31])*x)/(2*\text{Sqrt}[286 + (22*I)* \\ & \text{Sqrt}[31]]*\text{Sqrt}[3 - x + 2*x^2])))/(22*(-13*I + \text{Sqrt}[31])^3*(-3*I + \text{Sqrt}[31] \\ & - (10*I)*x)) + (155*(44*(16353 - (581*I)*\text{Sqrt}[31]))*\text{Sqrt}[3 - x + 2*x^2] + 3 \\ & 45*\text{Sqrt}[286 - (22*I)*\text{Sqrt}[31]]*(29 + (17*I)*\text{Sqrt}[31] + (-110 + (20*I)*\text{Sqrt}[ \\ & 31])*x)*\text{ArcTanh}[(-63 + I*\text{Sqrt}[31] + (22 - (4*I)*\text{Sqrt}[31])*x)/(2*\text{Sqrt}[286 - \\ & (22*I)*\text{Sqrt}[31]]*\text{Sqrt}[3 - x + 2*x^2])))/(22*(13*I + \text{Sqrt}[31])^3*(3*I + \text{Sqr} \\ & t[31] + (10*I)*x)) - ((575*I)*\text{Sqrt}[682*(13 - I*\text{Sqrt}[31])]*\text{ArcTanh}[(63 - I*S \\ & \text{qrt}[31] + (-22 + (4*I)*\text{Sqrt}[31])*x)/(2*\text{Sqrt}[286 - (22*I)*\text{Sqrt}[31]]*\text{Sqrt}[3 - \\ & x + 2*x^2)]))/(13*I + \text{Sqrt}[31])^2)/2674463 \end{aligned}$$

**fricas [B]** time = 2.53, size = 2173, normalized size = 10.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^(3/2)/(5\*x^2+3\*x+2)^2,x, algorithm="fricas")

[Out] 1/29889247038841109870720\*(35183643812\*3446160200^(1/4)\*sqrt(20755)\*sqrt(341)\*sqrt(2)\*(10\*x^4 + x^3 + 16\*x^2 + 7\*x + 6)\*sqrt(129694447\*sqrt(2) + 207550000)\*arctan(1/2437871055247532640924125\*(59193260\*sqrt(20755)\*(11\*3446160200^(3/4)\*sqrt(341)\*(20748108\*x^7 - 87744678\*x^6 + 180517074\*x^5 - 311740976\*x^4 + 161753488\*x^3 - 89046144\*x^2 - sqrt(2)\*(18515146\*x^7 - 65709803\*x^6 + 140687186\*x^5 - 209710441\*x^4 + 101256360\*x^3 - 39198600\*x^2 - 126316800\*x + 76909824) - 153819648\*x + 126316800) + 643405\*3446160200^(1/4)\*sqrt(341)\*(1637219\*x^7 - 25548801\*x^6 + 138274456\*x^5 - 324967420\*x^4 + 425065248\*x^3 - 297030816\*x^2 - sqrt(2)\*(1361849\*x^7 - 20608224\*x^6 + 106575580\*x^5 - 236322704\*x^4 + 301502880\*x^3 - 169632000\*x^2 - 225358848\*x + 143534592) - 287069184\*x + 225358848))\*sqrt(2\*x^2 - x + 3)\*sqrt(129694447\*sqrt(2) + 207550000) + 6920408156831705561333000\*sqrt(31)\*sqrt(2)\*(28180\*x^8 - 254666\*x^7 + 704270\*x^6 - 1385256\*x^5 + 1549144\*x^4 - 642048\*x^3 - 98496\*x^2 - sqrt(2)\*(8746\*x^8 - 102335\*x^7 + 396104\*x^6 - 783113\*x^5 + 1320710\*x^4 - 752088\*x^3 + 396144\*x^2 + 546048\*x - 539136) + 1154304\*x - 456192) - 2\*sqrt(41510/397951)\*(sqrt(20755)\*(11\*3446160200^(3/4)\*sqrt(341)\*(66710248\*x^7 - 96938292\*x^6 + 319739772\*x^5 - 172116032\*x^4 + 247423968\*x^3 + 38700288\*x^2 - sqrt(2)\*(71827622\*x^7 - 102266467\*x^6 + 323714097\*x^5 - 93357360\*x^4 + 79054920\*x^3 + 219532032\*x^2 - 219532032\*x) - 38700288\*x) + 643405\*3446160200^(1/4)\*sqrt(341)\*(5462397\*x^7 - 70721108\*x^6 + 273784220\*x^5 - 364358592\*x^4 + 506287008\*x^3 + 144903168\*x^2 - 2\*sqrt(2)\*(2586013\*x^7 - 33428948\*x^6 + 128512220\*x^5 - 162918720\*x^4 + 196126560\*x^3 + 173705472\*x^2 - 173705472\*x) - 144903168\*x))\*sqrt(2\*x^2 - x + 3)\*sqrt(129694447\*sqrt(2) + 207550000) + 116912097033204550\*sqrt(31)\*sqrt(2)\*(123408\*x^8 - 914152\*x^7 + 1578888\*x^6 - 3293072\*x^5 + 396480\*x^4 + 798336\*x^3 - 3822336\*x^2 - sqrt(2)\*(15550\*x^8 - 118051\*x^7 + 244047\*x^6 - 707374\*x^5 + 1053960\*x^4 - 1667952\*x^3 + 1209600\*x^2

$$\begin{aligned}
& - 1036800*x) + 3276288*x) + 5314186228782025*\sqrt{31}*(254591*x^8 - 481512 \\
& 6*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956 \\
& 928*x^2 - 15488*\sqrt{2}*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3 \\
& 618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*\sqrt{-(3446160200^{(1/4)}*\sqrt{2} \\
& 0755)*\sqrt{341}*\sqrt{31}*\sqrt{2*x^2 - x + 3}*(\sqrt{2}*(6137*x + 12812) - 18 \\
& 949*x + 6675)*\sqrt{129694447*\sqrt{2} + 207550000) - 388930324332445*x^2 - 3 \\
& 49243556543420*\sqrt{2}*(2*x^2 - x + 3) + 1198540387228555*x - 1587470711561 \\
& 000)/x^2) + 78641001782178472287875*\sqrt{31}*(2828123*x^8 - 9696916*x^7 + 5 \\
& 3385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 \\
& - 7744*\sqrt{2}*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 55 \\
& 68*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 \\
& - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 3 \\
& 4615296*x^2 - 24772608*x + 18579456) + 35183643812*3446160200^{(1/4)}*\sqrt{2} \\
& 0755)*\sqrt{341}*\sqrt{2}*(10*x^4 + x^3 + 16*x^2 + 7*x + 6)*\sqrt{129694447*\sqrt{2} \\
& + 207550000)*\arctan(1/2437871055247532640924125*(59193260*\sqrt{20755} \\
& *(11*3446160200^{(3/4)}*\sqrt{341}*(20748108*x^7 - 87744678*x^6 + 180517074*x^ \\
& 5 - 311740976*x^4 + 161753488*x^3 - 89046144*x^2 - \sqrt{2}*(18515146*x^7 - \\
& 65709803*x^6 + 140687186*x^5 - 209710441*x^4 + 101256360*x^3 - 39198600*x^2 \\
& - 126316800*x + 76909824) - 153819648*x + 126316800) + 643405*3446160200^{( \\
& 1/4)}*\sqrt{341}*(1637219*x^7 - 25548801*x^6 + 138274456*x^5 - 324967420*x^4 \\
& + 425065248*x^3 - 297030816*x^2 - \sqrt{2}*(1361849*x^7 - 20608224*x^6 + 106 \\
& 575580*x^5 - 236322704*x^4 + 301502880*x^3 - 169632000*x^2 - 225358848*x + \\
& 143534592) - 287069184*x + 225358848))*\sqrt{2*x^2 - x + 3}*\sqrt{129694447*\sqrt{2} \\
& + 207550000) - 6920408156831705561333000*\sqrt{31}*\sqrt{2}*(28180*x^8 \\
& - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496 \\
& *x^2 - \sqrt{2}*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x \\
& ^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2 \\
& *\sqrt{41510/397951}*(\sqrt{20755}*(11*3446160200^{(3/4)}*\sqrt{341}*(66710248*x \\
& ^7 - 96938292*x^6 + 319739772*x^5 - 172116032*x^4 + 247423968*x^3 + 3870028 \\
& 8*x^2 - \sqrt{2}*(71827622*x^7 - 102266467*x^6 + 323714097*x^5 - 93357360*x^ \\
& 4 + 79054920*x^3 + 219532032*x^2 - 219532032*x) - 38700288*x) + 643405*3446 \\
& 160200^{(1/4)}*\sqrt{341}*(5462397*x^7 - 70721108*x^6 + 273784220*x^5 - 364358 \\
& 592*x^4 + 506287008*x^3 + 144903168*x^2 - 2*\sqrt{2}*(2586013*x^7 - 33428948 \\
& *x^6 + 128512220*x^5 - 162918720*x^4 + 196126560*x^3 + 173705472*x^2 - 1737 \\
& 05472*x) - 144903168*x))*\sqrt{2*x^2 - x + 3}*\sqrt{129694447*\sqrt{2} + 20755 \\
& 0000) - 116912097033204550*\sqrt{31}*\sqrt{2}*(123408*x^8 - 914152*x^7 + 1578 \\
& 888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - \sqrt{2}*(15 \\
& 550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 \\
& + 1209600*x^2 - 1036800*x) + 3276288*x) - 5314186228782025*\sqrt{31}*(254591 \\
& *x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328 \\
& *x^3 - 168956928*x^2 - 15488*\sqrt{2}*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + \\
& 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*\sqrt{(3446160200^{ \\
& (1/4)}*\sqrt{20755}*\sqrt{341}*\sqrt{31}*\sqrt{2*x^2 - x + 3}*(\sqrt{2}*(6137*x + \\
& 12812) - 18949*x + 6675)*\sqrt{129694447*\sqrt{2} + 207550000) + 38893032433 \\
& 2445*x^2 + 349243556543420*\sqrt{2}*(2*x^2 - x + 3) - 1198540387228555*x + 1
\end{aligned}$$

```

587470711561000)/x^2) - 78641001782178472287875*sqrt(31)*(2828123*x^8 - 969
6916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 3
7981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15
569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/
(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 4424
9088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 23*3446160200^(1/4)*sqr
t(20755)*sqrt(341)*sqrt(31)*(2075500000*x^4 + 207550000*x^3 + 3320800000*x^
2 - 129694447*sqrt(2)*(10*x^4 + x^3 + 16*x^2 + 7*x + 6) + 1452850000*x + 12
45300000)*sqrt(129694447*sqrt(2) + 207550000)*log(1037750000000/397951*(344
6160200^(1/4)*sqrt(20755)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(
6137*x + 12812) - 18949*x + 6675)*sqrt(129694447*sqrt(2) + 207550000) + 388
930324332445*x^2 + 349243556543420*sqrt(2)*(2*x^2 - x + 3) - 11985403872285
55*x + 1587470711561000)/x^2) - 23*3446160200^(1/4)*sqrt(20755)*sqrt(341)*s
qrt(31)*(2075500000*x^4 + 207550000*x^3 + 3320800000*x^2 - 129694447*sqrt(2
)*(10*x^4 + x^3 + 16*x^2 + 7*x + 6) + 1452850000*x + 1245300000)*sqrt(12969
4447*sqrt(2) + 207550000)*log(-1037750000000/397951*(3446160200^(1/4)*sqrt(
20755)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(6137*x + 12812) - 1
8949*x + 6675)*sqrt(129694447*sqrt(2) + 207550000) - 388930324332445*x^2 -
349243556543420*sqrt(2)*(2*x^2 - x + 3) + 1198540387228555*x - 158747071156
1000)/x^2) + 86612402022768160*(11530*x^3 - 24657*x^2 + 18557*x - 10606)*sq
rt(2*x^2 - x + 3))/(10*x^4 + x^3 + 16*x^2 + 7*x + 6)

```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]root error [1.0,infinity,infinity,infinity,infi
nity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity
]root error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai
lure for[-1.0,infinity,infinity,infinity,infinity]root error [1.0,infinity
,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf
inity,infinity,infinity]root error [1.0,infinity,infinity,infinity,infinity]
Evaluation time: 36.15Done
```

**maple** [B] time = 0.10, size = 5942, normalized size = 28.16

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x)
```



[Out] result too large to display

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(5x^2 + 3x + 2)^2 (2x^2 - x + 3)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^(3/2)/(5\*x^2+3\*x+2)^2,x, algorithm="maxima")

[Out] integrate(1/((5\*x^2 + 3\*x + 2)^2\*(2\*x^2 - x + 3)^(3/2)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(2x^2 - x + 3)^{3/2} (5x^2 + 3x + 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((2\*x^2 - x + 3)^(3/2)\*(3\*x + 5\*x^2 + 2)^2),x)

[Out] int(1/((2\*x^2 - x + 3)^(3/2)\*(3\*x + 5\*x^2 + 2)^2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(2x^2 - x + 3)^{\frac{3}{2}} (5x^2 + 3x + 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x\*\*2-x+3)\*\*(3/2)/(5\*x\*\*2+3\*x+2)\*\*2,x)

[Out] Integral(1/((2\*x\*\*2 - x + 3)\*\*(3/2)\*(5\*x\*\*2 + 3\*x + 2)\*\*2), x)

$$3.92 \quad \int \frac{1}{(3-x+2x^2)^{3/2} (2+3x+5x^2)^3} dx$$

**Optimal.** Leaf size=246

$$-\frac{4353943 - 6508666x}{941410976\sqrt{2x^2 - x + 3}} + \frac{5(17315x + 7318)}{1860496\sqrt{2x^2 - x + 3} (5x^2 + 3x + 2)} + \frac{65x + 4}{1364\sqrt{2x^2 - x + 3} (5x^2 + 3x + 2)^2} + \frac{3\sqrt{\frac{1}{682}}}{1}$$

[Out] 1/941410976\*(-4353943+6508666\*x)/(2\*x^2-x+3)^(1/2)+1/1364\*(4+65\*x)/(5\*x^2+3\*x+2)^2/(2\*x^2-x+3)^(1/2)+5/1860496\*(7318+17315\*x)/(5\*x^2+3\*x+2)/(2\*x^2-x+3)^(1/2)-3/55829763968\*arctanh(1/31\*(5538393+x\*(13785797-9662095\*2^(1/2)))-4123702\*2^(1/2))\*341^(1/2)/(-13874275807943+9819738650000\*2^(1/2))^(1/2)/(2\*x^2-x+3)^(1/2))\*(-9462256101017126+6697061759300000\*2^(1/2))^(1/2)+3/55829763968\*arctan(1/31\*(5538393+4123702\*2^(1/2)+x\*(13785797+9662095\*2^(1/2)))\*341^(1/2)/(13874275807943+9819738650000\*2^(1/2))^(1/2)/(2\*x^2-x+3)^(1/2))\*(9462256101017126+6697061759300000\*2^(1/2))^(1/2)

**Rubi [A]** time = 0.53, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {974, 1060, 1035, 1029, 206, 204}

$$-\frac{4353943 - 6508666x}{941410976\sqrt{2x^2 - x + 3}} + \frac{5(17315x + 7318)}{1860496\sqrt{2x^2 - x + 3} (5x^2 + 3x + 2)} + \frac{65x + 4}{1364\sqrt{2x^2 - x + 3} (5x^2 + 3x + 2)^2} + \frac{3\sqrt{\frac{1}{682}}}{1}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - x + 2\*x^2)^(3/2)\*(2 + 3\*x + 5\*x^2)^3), x]

[Out] -(4353943 - 6508666\*x)/(941410976\*Sqrt[3 - x + 2\*x^2]) + (4 + 65\*x)/(1364\*Sqrt[3 - x + 2\*x^2]\*(2 + 3\*x + 5\*x^2)^2) + (5\*(7318 + 17315\*x))/(1860496\*Sqrt[3 - x + 2\*x^2]\*(2 + 3\*x + 5\*x^2)) + (3\*Sqrt[(13874275807943 + 9819738650000\*Sqrt[2])/682]\*ArcTan[(Sqrt[11/(31\*(13874275807943 + 9819738650000\*Sqrt[2])])]\*(5538393 + 4123702\*Sqrt[2] + (13785797 + 9662095\*Sqrt[2])\*x))/Sqrt[3 - x + 2\*x^2]])/81861824 - (3\*Sqrt[(-13874275807943 + 9819738650000\*Sqrt[2])/682]\*ArcTanh[(Sqrt[11/(31\*(-13874275807943 + 9819738650000\*Sqrt[2])])]\*(5538393 - 4123702\*Sqrt[2] + (13785797 - 9662095\*Sqrt[2])\*x))/Sqrt[3 - x + 2\*x^2]])/81861824

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

### Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

### Rule 974

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]
```

### Rule 1029

```
Int[((g_) + (h_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := Dist[-2*g*(g*b - 2*a*h), Subst[Int[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f), 0]
```

### Rule 1035

```
Int[((g_) + (h_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
```

```
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

### Rule 1060

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((A_) + (B_)*(x_) + (C_)*(x_)
^2)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x +
c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
+ q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !
IGtQ[q, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{1}{(3-x+2x^2)^{3/2} (2+3x+5x^2)^3} dx &= \frac{4+65x}{1364\sqrt{3-x+2x^2} (2+3x+5x^2)^2} - \frac{\int \frac{-5731+\frac{7557x}{2}-5720x^2}{(3-x+2x^2)^{3/2} (2+3x+5x^2)^2} dx}{15004} \\
&= \frac{4+65x}{1364\sqrt{3-x+2x^2} (2+3x+5x^2)^2} + \frac{5(7318+17315x)}{1860496\sqrt{3-x+2x^2} (2+3x+5x^2)^2} \\
&= -\frac{4353943-6508666x}{941410976\sqrt{3-x+2x^2}} + \frac{4+65x}{1364\sqrt{3-x+2x^2} (2+3x+5x^2)^2} + \frac{5(7318+17315x)}{1860496\sqrt{3-x+2x^2} (2+3x+5x^2)^2} \\
&= -\frac{4353943-6508666x}{941410976\sqrt{3-x+2x^2}} + \frac{4+65x}{1364\sqrt{3-x+2x^2} (2+3x+5x^2)^2} + \frac{5(7318+17315x)}{1860496\sqrt{3-x+2x^2} (2+3x+5x^2)^2} \\
&= -\frac{4353943-6508666x}{941410976\sqrt{3-x+2x^2}} + \frac{4+65x}{1364\sqrt{3-x+2x^2} (2+3x+5x^2)^2} + \frac{5(7318+17315x)}{1860496\sqrt{3-x+2x^2} (2+3x+5x^2)^2} \\
&= -\frac{4353943-6508666x}{941410976\sqrt{3-x+2x^2}} + \frac{4+65x}{1364\sqrt{3-x+2x^2} (2+3x+5x^2)^2} + \frac{5(7318+17315x)}{1860496\sqrt{3-x+2x^2} (2+3x+5x^2)^2}
\end{aligned}$$

**Mathematica [C]** time = 2.21, size = 231, normalized size = 0.94

$$69\sqrt{286-22i\sqrt{31}} (13785797\sqrt{31} + 14026539i) \tan^{-1} \left( \frac{-2(2\sqrt{31}+11i)x+\sqrt{31}+63i}{2\sqrt{286-22i\sqrt{31}} \sqrt{2x^2-x+3}} \right) - 69i\sqrt{286+22i\sqrt{31}} (13785797\sqrt{31} + 14026539i)$$

25

Antiderivative was successfully verified.

[In] Integrate[1/((3-x+2\*x^2)^(3/2)\*(2+3\*x+5\*x^2)^3),x]

[Out] ((27280\*(22374044+161806828\*x+175833195\*x^2+277167774\*x^3+86411405\*x^4+162716650\*x^5))/(Sqrt[3-x+2\*x^2]\*(2+3\*x+5\*x^2)^2)+69\*Sqrt[286-(22\*I)\*Sqrt[31]]\*(14026539\*I+13785797\*Sqrt[31])\*ArcTan[(63\*I+Sqrt[31]-2\*(11\*I+2\*Sqrt[31])\*x)/(2\*Sqrt[286-(22\*I)\*Sqrt[31]]\*Sqrt[3-x+2\*x^2])])

$$2*x^2)) - (69*I)*\text{Sqrt}[286 + (22*I)*\text{Sqrt}[31]]*(-14026539*I + 13785797*\text{Sqrt}[31])*\text{ArcTanh}[(63 + I*\text{Sqrt}[31] + (-22 - (4*I)*\text{Sqrt}[31])*x)/(2*\text{Sqrt}[286 + (22*I)*\text{Sqrt}[31]]*\text{Sqrt}[3 - x + 2*x^2)])]/25681691425280$$

**fricas [B]** time = 3.04, size = 2263, normalized size = 9.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^(3/2)/(5\*x^2+3\*x+2)^3,x, algorithm="fricas")

[Out] 1/33652296632397026886019646994897920\*(920746859815884\*1928545343086076450^(1/4)\*sqrt(1963947730)\*sqrt(341)\*sqrt(2)\*(50\*x^6 + 35\*x^5 + 103\*x^4 + 85\*x^3 + 83\*x^2 + 32\*x + 12)\*sqrt(13874275807943\*sqrt(2) + 19639477300000)\*arctan(1/2252270155289097943751876925347228391692375\*(2800589462980\*sqrt(1963947730)\*(22\*1928545343086076450^(3/4)\*sqrt(341)\*(7361410004\*x^7 - 28555361914\*x^6 + 59872788262\*x^5 - 96593638888\*x^4 + 48573560944\*x^3 - 23355012672\*x^2 - sqrt(2)\*(5311119598\*x^7 - 20292577289\*x^6 + 42695479118\*x^5 - 68006818683\*x^4 + 33985514680\*x^3 - 15860251800\*x^2 - 37489478400\*x + 26167456512) - 52334913024\*x + 37489478400) + 30441189815\*1928545343086076450^(1/4)\*sqrt(341)\*(560592897\*x^7 - 8616399363\*x^6 + 45618625128\*x^5 - 104316505460\*x^4 + 134890825824\*x^3 - 85859939808\*x^2 - sqrt(2)\*(402019087\*x^7 - 6162703212\*x^6 + 32499503540\*x^5 - 73942829952\*x^4 + 95407993440\*x^3 - 59600016000\*x^2 - 68177562624\*x + 47773380096) - 95546760192\*x + 68177562624))\*sqrt(2\*x^2 - x + 3)\*sqrt(13874275807943\*sqrt(2) + 19639477300000) + 6393541085981955453231134497759874144159000\*sqrt(31)\*sqrt(2)\*(28180\*x^8 - 254666\*x^7 + 704270\*x^6 - 1385256\*x^5 + 1549144\*x^4 - 642048\*x^3 - 98496\*x^2 - sqrt(2)\*(8746\*x^8 - 102335\*x^7 + 396104\*x^6 - 783113\*x^5 + 1320710\*x^4 - 752088\*x^3 + 396144\*x^2 + 546048\*x - 539136) + 1154304\*x - 456192) - 2\*sqrt(1963947730/3471424919)\*(sqrt(1963947730)\*(22\*1928545343086076450^(3/4)\*sqrt(341)\*(26184810824\*x^7 - 37618468196\*x^6 + 121297463436\*x^5 - 48741866816\*x^4 + 58784153184\*x^3 + 51583129344\*x^2 - sqrt(2)\*(19194187986\*x^7 - 27528525721\*x^6 + 88457613411\*x^5 - 33685377680\*x^4 + 38926767960\*x^3 + 41764674816\*x^2 - 41764674816\*x) - 51583129344\*x) + 30441189815\*1928545343086076450^(1/4)\*sqrt(341)\*(1998926311\*x^7 - 25858659004\*x^6 + 99738083860\*x^5 - 129415692096\*x^4 + 167446420704\*x^3 + 96037622784\*x^2 - 22\*sqrt(2)\*(65886479\*x^7 - 852213084\*x^6 + 3285070260\*x^5 - 4244909760\*x^4 + 5424792480\*x^3 + 3393259776\*x^2 - 3393259776\*x) - 96037622784\*x))\*sqrt(2\*x^2 - x + 3)\*sqrt(13874275807943\*sqrt(2) + 19639477300000) + 2282926923240949861309948624550\*sqrt(31)\*sqrt(2)\*(123408\*x^8 - 914152\*x^7 + 1578888\*x^6 - 3293072\*x^5 + 396480\*x^4 + 798336\*x^3 - 3822336\*x^2 - sqrt(2)\*(15550\*x^8 - 118051\*x^7 + 244047\*x^6 - 707374\*x^5 + 1053960\*x^4 - 1667952\*x^3 + 1209600\*x^2 - 1036800\*x) + 3276288\*x) + 103769405601861357332270392025\*sqrt(31)\*(254591\*x^8 - 4815126\*x^7 + 32303580\*x^6 - 90866808\*x^5 + 108781920\*x^4 - 74219328\*x^3 - 168956928\*x^2 - 15488\*sqrt(2)\*(4\*x^8 - 76\*x^7 + 517\*x^6 - 1536\*x^5 + 2385\*x^4 - 3618\*x^3 + 2268\*x^2 - 1944

$$\begin{aligned}
& *x) + 144820224*x)) * \sqrt{-(1928545343086076450^{1/4} * \sqrt{1963947730} * \sqrt{341} * \sqrt{31} * \sqrt{2*x^2 - x + 3}) * (\sqrt{2} * (2995431*x + 1523456) - 4518887*x - 1471975) * \sqrt{(13874275807943 * \sqrt{2}) + 19639477300000)} - 16051926912456} \\
& 8199977215*x^2 - 144139751866959199979540 * \sqrt{2} * (2*x^2 - x + 3) + 4946614 \\
& 21179791799929785*x - 655180690304359999907000) / x^2) + 72653875977067675604 \\
& 899255656362206183625 * \sqrt{31} * (2828123*x^8 - 9696916*x^7 + 53385560*x^6 - \\
& 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744 * \sqrt{2}) \\
& * (1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080* \\
& x^2 + 4320*x - 5184) + 223064064*x - 94887936) / (2585191*x^8 - 4661200*x^7 \\
& + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - \\
& 24772608*x + 18579456) + 920746859815884 * 1928545343086076450^{1/4} * \sqrt{19} \\
& 63947730 * \sqrt{341} * \sqrt{2} * (50*x^6 + 35*x^5 + 103*x^4 + 85*x^3 + 83*x^2 + \\
& 32*x + 12) * \sqrt{(13874275807943 * \sqrt{2}) + 19639477300000)} * \arctan(1/225227015 \\
& 5289097943751876925347228391692375 * (2800589462980 * \sqrt{1963947730}) * (22 * 1928 \\
& 545343086076450^{3/4} * \sqrt{341} * (7361410004*x^7 - 28555361914*x^6 + 5987278 \\
& 8262*x^5 - 96593638888*x^4 + 48573560944*x^3 - 23355012672*x^2 - \sqrt{2} * (5 \\
& 311119598*x^7 - 20292577289*x^6 + 42695479118*x^5 - 68006818683*x^4 + 33985 \\
& 514680*x^3 - 15860251800*x^2 - 37489478400*x + 26167456512) - 52334913024*x \\
& + 37489478400) + 30441189815 * 1928545343086076450^{1/4} * \sqrt{341} * (56059289 \\
& 7*x^7 - 8616399363*x^6 + 45618625128*x^5 - 104316505460*x^4 + 134890825824* \\
& x^3 - 85859939808*x^2 - \sqrt{2} * (402019087*x^7 - 6162703212*x^6 + 324995035 \\
& 40*x^5 - 73942829952*x^4 + 95407993440*x^3 - 59600016000*x^2 - 68177562624* \\
& x + 47773380096) - 95546760192*x + 68177562624) * \sqrt{2*x^2 - x + 3} * \sqrt{(1 \\
& 3874275807943 * \sqrt{2}) + 19639477300000)} - 639354108598195545323113449775987 \\
& 4144159000 * \sqrt{31} * \sqrt{2} * (28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256* \\
& x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - \sqrt{2} * (8746*x^8 - 102335*x^7 \\
& + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048 \\
& *x - 539136) + 1154304*x - 456192) - 2 * \sqrt{1963947730/3471424919} * (\sqrt{19} \\
& 63947730) * (22 * 1928545343086076450^{3/4} * \sqrt{341} * (26184810824*x^7 - 376184 \\
& 68196*x^6 + 121297463436*x^5 - 48741866816*x^4 + 58784153184*x^3 + 51583129 \\
& 344*x^2 - \sqrt{2} * (19194187986*x^7 - 27528525721*x^6 + 88457613411*x^5 - 33 \\
& 685377680*x^4 + 38926767960*x^3 + 41764674816*x^2 - 41764674816*x) - 515831 \\
& 29344*x) + 30441189815 * 1928545343086076450^{1/4} * \sqrt{341} * (1998926311*x^7 \\
& - 25858659004*x^6 + 99738083860*x^5 - 129415692096*x^4 + 167446420704*x^3 + \\
& 96037622784*x^2 - 22 * \sqrt{2} * (65886479*x^7 - 852213084*x^6 + 3285070260*x^ \\
& 5 - 4244909760*x^4 + 5424792480*x^3 + 3393259776*x^2 - 3393259776*x) - 9603 \\
& 7622784*x) * \sqrt{2*x^2 - x + 3} * \sqrt{(13874275807943 * \sqrt{2}) + 1963947730000} \\
& 0) - 2282926923240949861309948624550 * \sqrt{31} * \sqrt{2} * (123408*x^8 - 914152* \\
& x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - \sqrt{2} * (15550*x^8 \\
& - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 16 \\
& 67952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) - 10376940560186135733227 \\
& 0392025 * \sqrt{31} * (254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + \\
& 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488 * \sqrt{2} * (4*x^8 - 76*x^ \\
& 7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 1448202 \\
& 24*x) * \sqrt{((1928545343086076450^{1/4} * \sqrt{1963947730} * \sqrt{341} * \sqrt{31}) *
\end{aligned}$$

```

sqrt(2*x^2 - x + 3)*(sqrt(2)*(2995431*x + 1523456) - 4518887*x - 1471975)*s
qrt(13874275807943*sqrt(2) + 19639477300000) + 160519269124568199977215*x^2
+ 144139751866959199979540*sqrt(2)*(2*x^2 - x + 3) - 494661421179791799929
785*x + 655180690304359999907000)/x^2) - 7265387597706767560489925565636220
6183625*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5
+ 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2
692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x -
5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6
+ 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 1
8579456)) + 69*1928545343086076450^(1/4)*sqrt(1963947730)*sqrt(341)*sqrt(31
)*(981973865000000*x^6 + 687381705500000*x^5 + 2022866161900000*x^4 + 16693
55570500000*x^3 + 1630076615900000*x^2 - 13874275807943*sqrt(2)*(50*x^6 + 3
5*x^5 + 103*x^4 + 85*x^3 + 83*x^2 + 32*x + 12) + 628463273600000*x + 235673
727600000)*sqrt(13874275807943*sqrt(2) + 19639477300000)*log(17675529570000
00000/3471424919*(1928545343086076450^(1/4)*sqrt(1963947730)*sqrt(341)*sqrt
(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(2995431*x + 1523456) - 4518887*x - 14719
75)*sqrt(13874275807943*sqrt(2) + 19639477300000) + 16051926912456819997721
5*x^2 + 144139751866959199979540*sqrt(2)*(2*x^2 - x + 3) - 4946614211797917
99929785*x + 655180690304359999907000)/x^2) - 69*1928545343086076450^(1/4)*
sqrt(1963947730)*sqrt(341)*sqrt(31)*(981973865000000*x^6 + 687381705500000*
x^5 + 2022866161900000*x^4 + 1669355570500000*x^3 + 1630076615900000*x^2 -
13874275807943*sqrt(2)*(50*x^6 + 35*x^5 + 103*x^4 + 85*x^3 + 83*x^2 + 32*x
+ 12) + 628463273600000*x + 235673727600000)*sqrt(13874275807943*sqrt(2) +
19639477300000)*log(-1767552957000000000/3471424919*(1928545343086076450^(1
/4)*sqrt(1963947730)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(29954
31*x + 1523456) - 4518887*x - 1471975)*sqrt(13874275807943*sqrt(2) + 196394
77300000) - 160519269124568199977215*x^2 - 144139751866959199979540*sqrt(2)
*(2*x^2 - x + 3) + 494661421179791799929785*x - 655180690304359999907000)/x
^2) + 35746658463005881594925920*(162716650*x^5 + 86411405*x^4 + 277167774*
x^3 + 175833195*x^2 + 161806828*x + 22374044)*sqrt(2*x^2 - x + 3))/(50*x^6
+ 35*x^5 + 103*x^4 + 85*x^3 + 83*x^2 + 32*x + 12)

```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infini
ty]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity]
]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai
lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity
```



,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinity]Evaluation time: 69.9Done

**maple** [B] time = 0.21, size = 18981, normalized size = 77.16

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2\*x^2-x+3)^(3/2)/(5\*x^2+3\*x+2)^3,x)

[Out] result too large to display

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(5x^2 + 3x + 2)^3 (2x^2 - x + 3)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^(3/2)/(5\*x^2+3\*x+2)^3,x, algorithm="maxima")

[Out] integrate(1/((5\*x^2 + 3\*x + 2)^3\*(2\*x^2 - x + 3)^(3/2)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(2x^2 - x + 3)^{\frac{3}{2}} (5x^2 + 3x + 2)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((2\*x^2 - x + 3)^(3/2)\*(3\*x + 5\*x^2 + 2)^3),x)

[Out] int(1/((2\*x^2 - x + 3)^(3/2)\*(3\*x + 5\*x^2 + 2)^3), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(2x^2 - x + 3)^{\frac{3}{2}} (5x^2 + 3x + 2)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x\*\*2-x+3)\*\*(3/2)/(5\*x\*\*2+3\*x+2)\*\*3,x)

[Out] Integral(1/((2\*x\*\*2 - x + 3)\*\*(3/2)\*(5\*x\*\*2 + 3\*x + 2)\*\*3), x)

$$3.93 \quad \int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{5/2}} dx$$

**Optimal.** Leaf size=147

$$\frac{38375}{384} \sqrt{2x^2 - x + 3} x^2 + \frac{526075 \sqrt{2x^2 - x + 3} x}{3072} - \frac{1308645 \sqrt{2x^2 - x + 3}}{4096} + \frac{1331(116368x + 7409)}{101568 \sqrt{2x^2 - x + 3}} - \frac{14641(79x + 101)}{4416 (2x^2 - x + 3)^{3/2}}$$

[Out] -14641/4416\*(101+79\*x)/(2\*x^2-x+3)^(3/2)+16955197/16384\*arcsinh(1/23\*(1-4\*x)\*23^(1/2))\*2^(1/2)+1331/101568\*(7409+116368\*x)/(2\*x^2-x+3)^(1/2)-1308645/4096\*(2\*x^2-x+3)^(1/2)+526075/3072\*x\*(2\*x^2-x+3)^(1/2)+38375/384\*x^2\*(2\*x^2-x+3)^(1/2)+625/32\*x^3\*(2\*x^2-x+3)^(1/2)

**Rubi [A]** time = 0.17, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {1660, 1661, 640, 619, 215}

$$\frac{625}{32} \sqrt{2x^2 - x + 3} x^3 + \frac{38375}{384} \sqrt{2x^2 - x + 3} x^2 + \frac{526075 \sqrt{2x^2 - x + 3} x}{3072} - \frac{1308645 \sqrt{2x^2 - x + 3}}{4096} + \frac{1331(116368x + 7409)}{101568 \sqrt{2x^2 - x + 3}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3\*x + 5\*x^2)^4/(3 - x + 2\*x^2)^(5/2), x]

[Out] (-14641\*(101 + 79\*x))/(4416\*(3 - x + 2\*x^2)^(3/2)) + (1331\*(7409 + 116368\*x))/(101568\*Sqrt[3 - x + 2\*x^2]) - (1308645\*Sqrt[3 - x + 2\*x^2])/4096 + (526075\*x\*Sqrt[3 - x + 2\*x^2])/3072 + (38375\*x^2\*Sqrt[3 - x + 2\*x^2])/384 + (625\*x^3\*Sqrt[3 - x + 2\*x^2])/32 + (16955197\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(8192\*Sqrt[2])

### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

### Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

### Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]
```

### Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(2 + 3x + 5x^2)^4}{(3 - x + 2x^2)^{5/2}} dx &= -\frac{14641(101 + 79x)}{4416(3 - x + 2x^2)^{3/2}} + \frac{2}{69} \int \frac{\frac{3839123}{256} - \frac{1983543x}{128} - \frac{1464801x^2}{64} + \frac{430905x^3}{32} + \frac{639975x^4}{16} + \frac{250}{16}}{(3 - x + 2x^2)^{3/2}} dx \\
&= -\frac{14641(101 + 79x)}{4416(3 - x + 2x^2)^{3/2}} + \frac{1331(7409 + 116368x)}{101568\sqrt{3 - x + 2x^2}} + \frac{4}{1587} \int \frac{-\frac{141812733}{256} - \frac{1880595x}{16} + \frac{15512925x^2}{64} + \frac{3372375x^3}{16}}{\sqrt{3 - x + 2x^2}} dx \\
&= -\frac{14641(101 + 79x)}{4416(3 - x + 2x^2)^{3/2}} + \frac{1331(7409 + 116368x)}{101568\sqrt{3 - x + 2x^2}} + \frac{625}{32} x^3 \sqrt{3 - x + 2x^2} + \frac{\int \frac{-\frac{141812733}{32} - \frac{1880595x}{16} + \frac{15512925x^2}{64} + \frac{3372375x^3}{16}}{\sqrt{3 - x + 2x^2}} dx}{1587} \\
&= -\frac{14641(101 + 79x)}{4416(3 - x + 2x^2)^{3/2}} + \frac{1331(7409 + 116368x)}{101568\sqrt{3 - x + 2x^2}} + \frac{38375}{384} x^2 \sqrt{3 - x + 2x^2} + \frac{625}{32} x^3 \sqrt{3 - x + 2x^2} \\
&= -\frac{14641(101 + 79x)}{4416(3 - x + 2x^2)^{3/2}} + \frac{1331(7409 + 116368x)}{101568\sqrt{3 - x + 2x^2}} + \frac{526075x\sqrt{3 - x + 2x^2}}{3072} + \frac{38375}{384} x^2 \sqrt{3 - x + 2x^2} \\
&= -\frac{14641(101 + 79x)}{4416(3 - x + 2x^2)^{3/2}} + \frac{1331(7409 + 116368x)}{101568\sqrt{3 - x + 2x^2}} - \frac{1308645\sqrt{3 - x + 2x^2}}{4096} + \frac{526075x\sqrt{3 - x + 2x^2}}{3072} \\
&= -\frac{14641(101 + 79x)}{4416(3 - x + 2x^2)^{3/2}} + \frac{1331(7409 + 116368x)}{101568\sqrt{3 - x + 2x^2}} - \frac{1308645\sqrt{3 - x + 2x^2}}{4096} + \frac{526075x\sqrt{3 - x + 2x^2}}{3072} \\
&= -\frac{14641(101 + 79x)}{4416(3 - x + 2x^2)^{3/2}} + \frac{1331(7409 + 116368x)}{101568\sqrt{3 - x + 2x^2}} - \frac{1308645\sqrt{3 - x + 2x^2}}{4096} + \frac{526075x\sqrt{3 - x + 2x^2}}{3072}
\end{aligned}$$

**Mathematica [A]** time = 0.51, size = 75, normalized size = 0.51

$$\frac{507840000x^7 + 2090608000x^6 + 3504730800x^5 - 5076781260x^4 + 39848900984x^3 - 36481630395x^2 + 4988380000x - 500000}{6500352(2x^2 - x + 3)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x + 5\*x^2)^4/(3 - x + 2\*x^2)^(5/2), x]

[Out]  $(-18974698519 + 49883864262*x - 36481630395*x^2 + 39848900984*x^3 - 5076781260*x^4 + 3504730800*x^5 + 2090608000*x^6 + 507840000*x^7)/(6500352*(3 - x + 2*x^2)^{(3/2)}) - (16955197*\text{ArcSinh}[(-1 + 4*x)/\text{Sqrt}[23]])/(8192*\text{Sqrt}[2])$

**fricas** [A] time = 0.65, size = 132, normalized size = 0.90

$$\frac{26907897639 \sqrt{2} (4x^4 - 4x^3 + 13x^2 - 6x + 9) \log(4\sqrt{2}\sqrt{2x^2 - x + 3}(4x - 1) - 32x^2 + 16x - 25) + 8(507840000x^7 + 2090608000x^6 + 3504730800x^5 - 5076781260x^4 + 39848900984x^3 - 36481630395x^2 + 49883864262x - 18974698519)\sqrt{2x^2 - x + 3}}{(4x^4 - 4x^3 + 13x^2 - 6x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(5/2),x, algorithm="fricas")`

[Out]  $\frac{1}{52002816} * (26907897639 * \text{sqrt}(2) * (4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9) * \log(4 * \text{sqrt}(2) * \text{sqrt}(2*x^2 - x + 3) * (4*x - 1) - 32*x^2 + 16*x - 25) + 8 * (507840000*x^7 + 2090608000*x^6 + 3504730800*x^5 - 5076781260*x^4 + 39848900984*x^3 - 36481630395*x^2 + 49883864262*x - 18974698519) * \text{sqrt}(2*x^2 - x + 3)) / (4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)$

**giac** [A] time = 0.25, size = 81, normalized size = 0.55

$$\frac{16955197}{16384} \sqrt{2} \log\left(-2\sqrt{2}\left(\sqrt{2}x - \sqrt{2x^2 - x + 3}\right) + 1\right) + \frac{((4(2645(20(40(60x + 247)x + 16563)x - 479847)x - 62225246)x - 36481630395)x + 49883864262)x - 18974698519)}{6500352(2x^2 - x + 3)^{(3/2)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(5/2),x, algorithm="giac")`

[Out]  $\frac{16955197}{16384} * \text{sqrt}(2) * \log(-2 * \text{sqrt}(2) * (\text{sqrt}(2) * x - \text{sqrt}(2*x^2 - x + 3))) + 1) + \frac{1}{6500352} * (((4 * (2645 * (20 * (40 * (60*x + 247) * x + 16563) * x - 479847) * x + 9962225246) * x - 36481630395) * x + 49883864262) * x - 18974698519) / (2*x^2 - x + 3)^{(3/2)})$

**maple** [A] time = 0.03, size = 214, normalized size = 1.46

$$\frac{625x^7}{8(2x^2 - x + 3)^{\frac{3}{2}}} + \frac{30875x^6}{96(2x^2 - x + 3)^{\frac{3}{2}}} + \frac{138025x^5}{256(2x^2 - x + 3)^{\frac{3}{2}}} - \frac{799745x^4}{1024(2x^2 - x + 3)^{\frac{3}{2}}} + \frac{16955197x^3}{12288(2x^2 - x + 3)^{\frac{3}{2}}} - \frac{62225246x^2}{16384(2x^2 - x + 3)^{\frac{3}{2}}} + \frac{49883864262x}{16384(2x^2 - x + 3)^{\frac{3}{2}}} - \frac{18974698519}{16384(2x^2 - x + 3)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^2+3*x+2)^4/(2*x^2-x+3)^(5/2),x)`

[Out]  $\frac{16955197}{32768} * (2*x^2 - x + 3)^{(1/2)} - \frac{2149616639}{524288} * (2*x^2 - x + 3)^{(3/2)} + \frac{992926033}{13000704} * (4*x - 1) / (2*x^2 - x + 3)^{(1/2)} - \frac{16955197}{16384} * 2^{(1/2)} * \text{arcsinh}(4/23 * \text{sqrt}(2x^2 - x + 3))$

$$23^{(1/2)}*(x-1/4))+5141612725/36175872*(4*x-1)/(2*x^2-x+3)^{(3/2)}+625/8*x^7/(2*x^2-x+3)^{(3/2)}+30875/96*x^6/(2*x^2-x+3)^{(3/2)}+138025/256*x^5/(2*x^2-x+3)^{(3/2)}-799745/1024*x^4/(2*x^2-x+3)^{(3/2)}+16955197/12288*x^3/(2*x^2-x+3)^{(3/2)}-67488035/16384*x^2/(2*x^2-x+3)^{(3/2)}+55167267/131072*x/(2*x^2-x+3)^{(3/2)}+16955197/8192/(2*x^2-x+3)^{(1/2)}*x$$

**maxima** [B] time = 1.00, size = 253, normalized size = 1.72

$$\frac{625x^7}{8(2x^2-x+3)^{\frac{3}{2}}} + \frac{30875x^6}{96(2x^2-x+3)^{\frac{3}{2}}} + \frac{138025x^5}{256(2x^2-x+3)^{\frac{3}{2}}} - \frac{799745x^4}{1024(2x^2-x+3)^{\frac{3}{2}}} - \frac{16955197}{13000704}x \left( \frac{284x}{\sqrt{2x^2-x+3}} - \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^4/(2\*x^2-x+3)^(5/2),x, algorithm="maxima")

[Out] 625/8\*x^7/(2\*x^2 - x + 3)^(3/2) + 30875/96\*x^6/(2\*x^2 - x + 3)^(3/2) + 138025/256\*x^5/(2\*x^2 - x + 3)^(3/2) - 799745/1024\*x^4/(2\*x^2 - x + 3)^(3/2) - 16955197/13000704\*x\*(284\*x/sqrt(2\*x^2 - x + 3) - 3174\*x^2/(2\*x^2 - x + 3)^(3/2) - 71/sqrt(2\*x^2 - x + 3) + 805\*x/(2\*x^2 - x + 3)^(3/2) - 3243/(2\*x^2 - x + 3)^(3/2)) - 16955197/16384\*sqrt(2)\*arcsinh(1/23\*sqrt(23)\*(4\*x - 1)) + 1203818987/6500352\*sqrt(2\*x^2 - x + 3) + 3536205583/3250176\*x/sqrt(2\*x^2 - x + 3) - 2638851/512\*x^2/(2\*x^2 - x + 3)^(3/2) + 257773037/1083392/sqrt(2\*x^2 - x + 3) + 29484067/23552\*x/(2\*x^2 - x + 3)^(3/2) - 374445479/70656/(2\*x^2 - x + 3)^(3/2)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(5x^2 + 3x + 2)^4}{(2x^2 - x + 3)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x + 5\*x^2 + 2)^4/(2\*x^2 - x + 3)^(5/2),x)

[Out] int((3\*x + 5\*x^2 + 2)^4/(2\*x^2 - x + 3)^(5/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x^2 + 3x + 2)^4}{(2x^2 - x + 3)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**4/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)**4/(2*x**2 - x + 3)**(5/2), x)
```

$$3.94 \quad \int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{5/2}} dx$$

**Optimal.** Leaf size=105

$$\frac{121(10679 - 6744x)}{8464\sqrt{2x^2 - x + 3}} + \frac{125}{16}x\sqrt{2x^2 - x + 3} + \frac{3175}{64}\sqrt{2x^2 - x + 3} - \frac{1331(17 - 45x)}{1104(2x^2 - x + 3)^{3/2}} - \frac{7495 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{128\sqrt{2}}$$

[Out] -1331/1104\*(17-45\*x)/(2\*x^2-x+3)^(3/2)-7495/256\*arcsinh(1/23\*(1-4\*x)\*23^(1/2))\*2^(1/2)+121/8464\*(10679-6744\*x)/(2\*x^2-x+3)^(1/2)+3175/64\*(2\*x^2-x+3)^(1/2)+125/16\*x\*(2\*x^2-x+3)^(1/2)

**Rubi [A]** time = 0.11, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {1660, 1661, 640, 619, 215}

$$\frac{121(10679 - 6744x)}{8464\sqrt{2x^2 - x + 3}} + \frac{125}{16}x\sqrt{2x^2 - x + 3} + \frac{3175}{64}\sqrt{2x^2 - x + 3} - \frac{1331(17 - 45x)}{1104(2x^2 - x + 3)^{3/2}} - \frac{7495 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{128\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3\*x + 5\*x^2)^3/(3 - x + 2\*x^2)^(5/2), x]

[Out] (-1331\*(17 - 45\*x))/(1104\*(3 - x + 2\*x^2)^(3/2)) + (121\*(10679 - 6744\*x))/(8464\*Sqrt[3 - x + 2\*x^2]) + (3175\*Sqrt[3 - x + 2\*x^2])/64 + (125\*x\*Sqrt[3 - x + 2\*x^2])/16 - (7495\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(128\*Sqrt[2])

### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^(p)), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

### Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b



```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

### Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]
```

### Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(2 + 3x + 5x^2)^3}{(3 - x + 2x^2)^{5/2}} dx &= -\frac{1331(17 - 45x)}{1104(3 - x + 2x^2)^{3/2}} + \frac{2}{69} \int \frac{-\frac{91275}{64} - \frac{57201x}{32} + \frac{66585x^2}{16} + \frac{39675x^3}{8} + \frac{8625x^4}{4}}{(3 - x + 2x^2)^{3/2}} dx \\
&= -\frac{1331(17 - 45x)}{1104(3 - x + 2x^2)^{3/2}} + \frac{121(10679 - 6744x)}{8464\sqrt{3 - x + 2x^2}} + \frac{4 \int \frac{\frac{1452105}{64} + \frac{277725x}{8} + \frac{198375x^2}{16}}{\sqrt{3-x+2x^2}} dx}{1587} \\
&= -\frac{1331(17 - 45x)}{1104(3 - x + 2x^2)^{3/2}} + \frac{121(10679 - 6744x)}{8464\sqrt{3 - x + 2x^2}} + \frac{125}{16} x \sqrt{3 - x + 2x^2} + \frac{\int \frac{\frac{214245}{4} + \frac{5038725x}{32}}{\sqrt{3-x+2x^2}} dx}{1587} \\
&= -\frac{1331(17 - 45x)}{1104(3 - x + 2x^2)^{3/2}} + \frac{121(10679 - 6744x)}{8464\sqrt{3 - x + 2x^2}} + \frac{3175}{64} \sqrt{3 - x + 2x^2} + \frac{125}{16} x \sqrt{3 - x + 2x^2} \\
&= -\frac{1331(17 - 45x)}{1104(3 - x + 2x^2)^{3/2}} + \frac{121(10679 - 6744x)}{8464\sqrt{3 - x + 2x^2}} + \frac{3175}{64} \sqrt{3 - x + 2x^2} + \frac{125}{16} x \sqrt{3 - x + 2x^2}
\end{aligned}$$

**Mathematica [A]** time = 0.34, size = 65, normalized size = 0.62

$$\frac{3174000x^5 + 16980900x^4 - 29423976x^3 + 101546529x^2 - 62463282x + 89784565}{101568(2x^2 - x + 3)^{3/2}} + \frac{7495 \sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{128\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x + 5\*x^2)^3/(3 - x + 2\*x^2)^(5/2), x]

[Out] (89784565 - 62463282\*x + 101546529\*x^2 - 29423976\*x^3 + 16980900\*x^4 + 3174000\*x^5)/(101568\*(3 - x + 2\*x^2)^(3/2)) + (7495\*ArcSinh[(-1 + 4\*x)/Sqrt[23]])/(128\*Sqrt[2])

**fricas [A]** time = 0.60, size = 122, normalized size = 1.16

$$\frac{11894565 \sqrt{2} (4x^4 - 4x^3 + 13x^2 - 6x + 9) \log(-4\sqrt{2}\sqrt{2x^2 - x + 3}(4x - 1) - 32x^2 + 16x - 25) + 8(3174000x^5 + 16980900x^4 - 29423976x^3 + 101546529x^2 - 62463282x + 89784565)}{812544(4x^4 - 4x^3 + 13x^2 - 6x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^3/(2\*x^2-x+3)^(5/2),x, algorithm="fricas")

[Out] 1/812544\*(11894565\*sqrt(2)\*(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)\*log(-4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25) + 8\*(3174000\*x^5 + 16980900\*x^4 - 29423976\*x^3 + 101546529\*x^2 - 62463282\*x + 89784565)\*sqrt(2\*x^2 - x + 3))/(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)

**giac** [A] time = 0.27, size = 72, normalized size = 0.69

$$-\frac{7495}{256} \sqrt{2} \log\left(-2 \sqrt{2} \left(\sqrt{2} x - \sqrt{2x^2 - x + 3}\right) + 1\right) + \frac{3((4(13225(20x + 107)x - 2451998)x + 33848843)x - 20821094)x + 89784565)}{101568(2x^2 - x + 3)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^3/(2\*x^2-x+3)^(5/2),x, algorithm="giac")

[Out] -7495/256\*sqrt(2)\*log(-2\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 1) + 1/101568\*(3\*((4\*(13225\*(20\*x + 107)\*x - 2451998)\*x + 33848843)\*x - 20821094)\*x + 89784565)/(2\*x^2 - x + 3)^(3/2)

**maple** [B] time = 0.01, size = 180, normalized size = 1.71

$$\frac{125x^5}{4(2x^2 - x + 3)^{\frac{3}{2}}} + \frac{2675x^4}{16(2x^2 - x + 3)^{\frac{3}{2}}} - \frac{7495x^3}{192(2x^2 - x + 3)^{\frac{3}{2}}} + \frac{222809x^2}{256(2x^2 - x + 3)^{\frac{3}{2}}} - \frac{281177x}{2048(2x^2 - x + 3)^{\frac{3}{2}}} - \frac{7495}{128\sqrt{2x^2 - x + 3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+3\*x+2)^3/(2\*x^2-x+3)^(5/2),x)

[Out] -7495/512/(2\*x^2-x+3)^(1/2)+20961031/24576/(2\*x^2-x+3)^(3/2)-3391139/203136\*(4\*x-1)/(2\*x^2-x+3)^(1/2)+7495/256\*2^(1/2)\*arcsinh(4/23\*23^(1/2)\*(x-1/4))-14081711/565248\*(4\*x-1)/(2\*x^2-x+3)^(3/2)+125/4/(2\*x^2-x+3)^(3/2)\*x^5+2675/16/(2\*x^2-x+3)^(3/2)\*x^4-7495/192/(2\*x^2-x+3)^(3/2)\*x^3+222809/256/(2\*x^2-x+3)^(3/2)\*x^2-281177/2048/(2\*x^2-x+3)^(3/2)\*x-7495/128/(2\*x^2-x+3)^(1/2)\*x

**maxima** [B] time = 1.02, size = 219, normalized size = 2.09

$$\frac{125x^5}{4(2x^2 - x + 3)^{\frac{3}{2}}} + \frac{2675x^4}{16(2x^2 - x + 3)^{\frac{3}{2}}} + \frac{7495}{203136} x \left( \frac{284x}{\sqrt{2x^2 - x + 3}} - \frac{3174x^2}{(2x^2 - x + 3)^{\frac{3}{2}}} - \frac{71}{\sqrt{2x^2 - x + 3}} + \frac{805}{(2x^2 - x + 3)^{\frac{3}{2}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^3/(2\*x^2-x+3)^(5/2),x, algorithm="maxima")

[Out] 125/4\*x^5/(2\*x^2 - x + 3)^(3/2) + 2675/16\*x^4/(2\*x^2 - x + 3)^(3/2) + 7495/203136\*x\*(284\*x/sqrt(2\*x^2 - x + 3) - 3174\*x^2/(2\*x^2 - x + 3)^(3/2) - 71/sqrt(2\*x^2 - x + 3) + 805\*x/(2\*x^2 - x + 3)^(3/2) - 3243/(2\*x^2 - x + 3)^(3/2)) + 7495/256\*sqrt(2)\*arcsinh(1/23\*sqrt(23)\*(4\*x - 1)) - 532145/101568\*sqrt(2\*x^2 - x + 3) - 4515389/50784\*x/sqrt(2\*x^2 - x + 3) + 7197/8\*x^2/(2\*x^2 - x + 3)^(3/2) + 396211/50784/sqrt(2\*x^2 - x + 3) - 269783/1104\*x/(2\*x^2 - x + 3)^(3/2) + 1002137/1104/(2\*x^2 - x + 3)^(3/2)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(5x^2 + 3x + 2)^3}{(2x^2 - x + 3)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x + 5\*x^2 + 2)^3/(2\*x^2 - x + 3)^(5/2),x)

[Out] int((3\*x + 5\*x^2 + 2)^3/(2\*x^2 - x + 3)^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x^2 + 3x + 2)^3}{(2x^2 - x + 3)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*2+3\*x+2)\*\*3/(2\*x\*\*2-x+3)\*\*(5/2),x)

[Out] Integral((5\*x\*\*2 + 3\*x + 2)\*\*3/(2\*x\*\*2 - x + 3)\*\*(5/2), x)

$$3.95 \quad \int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{5/2}} dx$$

Optimal. Leaf size=68

$$\frac{121(19-7x)}{276(2x^2-x+3)^{3/2}} - \frac{11(2336x+7351)}{6348\sqrt{2x^2-x+3}} - \frac{25 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{2}}$$

[Out] 121/276\*(19-7\*x)/(2\*x^2-x+3)^(3/2)-25/8\*arcsinh(1/23\*(1-4\*x)\*23^(1/2))\*2^(1/2)-11/6348\*(7351+2336\*x)/(2\*x^2-x+3)^(1/2)

**Rubi [A]** time = 0.06, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$ , Rules used = {1660, 12, 619, 215}

$$\frac{121(19-7x)}{276(2x^2-x+3)^{3/2}} - \frac{11(2336x+7351)}{6348\sqrt{2x^2-x+3}} - \frac{25 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3\*x + 5\*x^2)^2/(3 - x + 2\*x^2)^(5/2), x]

[Out] (121\*(19 - 7\*x))/(276\*(3 - x + 2\*x^2)^(3/2)) - (11\*(7351 + 2336\*x))/(6348\*sqrt[3 - x + 2\*x^2]) - (25\*ArcSinh[(1 - 4\*x)/sqrt[23]])/(4\*sqrt[2])

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] :> Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(2 + 3x + 5x^2)^2}{(3 - x + 2x^2)^{5/2}} dx &= \frac{121(19 - 7x)}{276(3 - x + 2x^2)^{3/2}} + \frac{2}{69} \int \frac{\frac{131}{16} + \frac{5865x}{8} + \frac{1725x^2}{4}}{(3 - x + 2x^2)^{3/2}} dx \\
&= \frac{121(19 - 7x)}{276(3 - x + 2x^2)^{3/2}} - \frac{11(7351 + 2336x)}{6348\sqrt{3 - x + 2x^2}} + \frac{4 \int \frac{39675}{16\sqrt{3 - x + 2x^2}} dx}{1587} \\
&= \frac{121(19 - 7x)}{276(3 - x + 2x^2)^{3/2}} - \frac{11(7351 + 2336x)}{6348\sqrt{3 - x + 2x^2}} + \frac{25}{4} \int \frac{1}{\sqrt{3 - x + 2x^2}} dx \\
&= \frac{121(19 - 7x)}{276(3 - x + 2x^2)^{3/2}} - \frac{11(7351 + 2336x)}{6348\sqrt{3 - x + 2x^2}} + \frac{25 \operatorname{Subst}\left(\int \frac{1}{\sqrt{1 + \frac{x^2}{23}}} dx, x, -1 + 4x\right)}{4\sqrt{46}} \\
&= \frac{121(19 - 7x)}{276(3 - x + 2x^2)^{3/2}} - \frac{11(7351 + 2336x)}{6348\sqrt{3 - x + 2x^2}} - \frac{25 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{2}}
\end{aligned}$$

**Mathematica [A]** time = 0.24, size = 55, normalized size = 0.81

$$\frac{25 \sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{4\sqrt{2}} - \frac{11(2336x^3 + 6183x^2 + 714x + 8623)}{3174(2x^2 - x + 3)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x + 5\*x^2)^2/(3 - x + 2\*x^2)^(5/2), x]

[Out] (-11\*(8623 + 714\*x + 6183\*x^2 + 2336\*x^3))/(3174\*(3 - x + 2\*x^2)^(3/2)) + (25\*ArcSinh[(-1 + 4\*x)/Sqrt[23]])/(4\*Sqrt[2])

**fricas** [B] time = 0.68, size = 112, normalized size = 1.65

$$\frac{39675 \sqrt{2} (4x^4 - 4x^3 + 13x^2 - 6x + 9) \log(-4\sqrt{2}\sqrt{2x^2 - x + 3}(4x - 1) - 32x^2 + 16x - 25) - 88(2336x^3 + 6183x^2 + 714x + 8623)\sqrt{2x^2 - x + 3}}{25392(4x^4 - 4x^3 + 13x^2 - 6x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^2/(2\*x^2-x+3)^(5/2),x, algorithm="fricas")

[Out] 1/25392\*(39675\*sqrt(2)\*(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)\*log(-4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25) - 88\*(2336\*x^3 + 6183\*x^2 + 714\*x + 8623)\*sqrt(2\*x^2 - x + 3))/(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)

**giac** [A] time = 0.27, size = 61, normalized size = 0.90

$$-\frac{25}{8} \sqrt{2} \log\left(-2\sqrt{2}\left(\sqrt{2}x - \sqrt{2x^2 - x + 3}\right) + 1\right) - \frac{11\left(\left(\left(2336x + 6183\right)x + 714\right)x + 8623\right)}{3174\left(2x^2 - x + 3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^2/(2\*x^2-x+3)^(5/2),x, algorithm="giac")

[Out] -25/8\*sqrt(2)\*log(-2\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 1) - 11/3174\*(((2336\*x + 6183)\*x + 714)\*x + 8623)/(2\*x^2 - x + 3)^(3/2)

**maple** [B] time = 0.01, size = 146, normalized size = 2.15

$$\frac{25x^3}{6(2x^2 - x + 3)^{\frac{3}{2}}} - \frac{145x^2}{8(2x^2 - x + 3)^{\frac{3}{2}}} - \frac{319x}{64(2x^2 - x + 3)^{\frac{3}{2}}} - \frac{25x}{4\sqrt{2x^2 - x + 3}} + \frac{25\sqrt{2} \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x - \frac{1}{4}\right)}{23}\right)}{8} - \frac{1}{768(2x^2 - x + 3)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+3\*x+2)^2/(2\*x^2-x+3)^(5/2),x)

[Out] -25/6/(2\*x^2-x+3)^(3/2)\*x^3-145/8/(2\*x^2-x+3)^(3/2)\*x^2-319/64/(2\*x^2-x+3)^(3/2)\*x-15775/768/(2\*x^2-x+3)^(3/2)+8493/5888\*(4\*x-1)/(2\*x^2-x+3)^(3/2)+2267/2116\*(4\*x-1)/(2\*x^2-x+3)^(1/2)-25/4/(2\*x^2-x+3)^(1/2)\*x-25/16/(2\*x^2-x+3)^(1/2)+25/8\*2^(1/2)\*arcsinh(4/23\*23^(1/2)\*(x-1/4))

**maxima** [B] time = 0.98, size = 185, normalized size = 2.72

$$\frac{25}{6348} x \left( \frac{284x}{\sqrt{2x^2 - x + 3}} - \frac{3174x^2}{(2x^2 - x + 3)^{\frac{3}{2}}} - \frac{71}{\sqrt{2x^2 - x + 3}} + \frac{805x}{(2x^2 - x + 3)^{\frac{3}{2}}} - \frac{3243}{(2x^2 - x + 3)^{\frac{3}{2}}} \right) + \frac{25}{8} \sqrt{2} \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x - \frac{1}{4}\right)}{23}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^2/(2\*x^2-x+3)^(5/2),x, algorithm="maxima")

[Out] 25/6348\*x\*(284\*x/sqrt(2\*x^2 - x + 3) - 3174\*x^2/(2\*x^2 - x + 3)^(3/2) - 71/sqrt(2\*x^2 - x + 3) + 805\*x/(2\*x^2 - x + 3)^(3/2) - 3243/(2\*x^2 - x + 3)^(3/2)) + 25/8\*sqrt(2)\*arcsinh(1/23\*sqrt(23)\*(4\*x - 1)) - 1775/3174\*sqrt(2\*x^2 - x + 3) + 1017/529\*x/sqrt(2\*x^2 - x + 3) - 15\*x^2/(2\*x^2 - x + 3)^(3/2) - 6413/3174/sqrt(2\*x^2 - x + 3) - 1/138\*x/(2\*x^2 - x + 3)^(3/2) - 2593/138/(2\*x^2 - x + 3)^(3/2)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(5x^2 + 3x + 2)^2}{(2x^2 - x + 3)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x + 5\*x^2 + 2)^2/(2\*x^2 - x + 3)^(5/2),x)

[Out] int((3\*x + 5\*x^2 + 2)^2/(2\*x^2 - x + 3)^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x^2 + 3x + 2)^2}{(2x^2 - x + 3)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*2+3\*x+2)\*\*2/(2\*x\*\*2-x+3)\*\*(5/2),x)

[Out] Integral((5\*x\*\*2 + 3\*x + 2)\*\*2/(2\*x\*\*2 - x + 3)\*\*(5/2), x)



$$3.96 \quad \int \frac{2+3x+5x^2}{(3-x+2x^2)^{5/2}} dx$$

Optimal. Leaf size=47

$$-\frac{71(1-4x)}{529\sqrt{2x^2-x+3}} - \frac{11(3x+5)}{69(2x^2-x+3)^{3/2}}$$

[Out]  $-11/69*(5+3*x)/(2*x^2-x+3)^{(3/2)}-71/529*(1-4*x)/(2*x^2-x+3)^{(1/2)}$

**Rubi [A]** time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$ , Rules used = {1660, 12, 613}

$$-\frac{71(1-4x)}{529\sqrt{2x^2-x+3}} - \frac{11(3x+5)}{69(2x^2-x+3)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3\*x + 5\*x^2)/(3 - x + 2\*x^2)^(5/2), x]

[Out]  $(-11*(5 + 3*x))/(69*(3 - x + 2*x^2)^{(3/2)}) - (71*(1 - 4*x))/(529*\text{Sqrt}[3 - x + 2*x^2])$

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 613

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-3/2), x\_Symbol] := Simp[(-2\*(b + 2\*c\*x))/((b^2 - 4\*a\*c)\*Sqrt[a + b\*x + c\*x^2]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 1660

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 1]}, Simp[((b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] + Dist[1/((p + 1)\*(b^2 - 4\*a\*c)), Int[(a + b\*x + c\*x^2)^(p + 1)\*ExpandToSum[(p + 1)\*(b^2 - 4\*a\*c)\*Q - (2\*p + 3)\*(2\*c\*f - b\*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2

- 4\*a\*c, 0] && LtQ[p, -1]

### Rubi steps

$$\begin{aligned} \int \frac{2+3x+5x^2}{(3-x+2x^2)^{5/2}} dx &= -\frac{11(5+3x)}{69(3-x+2x^2)^{3/2}} + \frac{2}{69} \int \frac{213}{4(3-x+2x^2)^{3/2}} dx \\ &= -\frac{11(5+3x)}{69(3-x+2x^2)^{3/2}} + \frac{71}{46} \int \frac{1}{(3-x+2x^2)^{3/2}} dx \\ &= -\frac{11(5+3x)}{69(3-x+2x^2)^{3/2}} - \frac{71(1-4x)}{529\sqrt{3-x+2x^2}} \end{aligned}$$

**Mathematica [A]** time = 0.10, size = 33, normalized size = 0.70

$$\frac{2(852x^3 - 639x^2 + 1005x - 952)}{1587(2x^2 - x + 3)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x + 5\*x^2)/(3 - x + 2\*x^2)^(5/2), x]

[Out] (2\*(-952 + 1005\*x - 639\*x^2 + 852\*x^3))/(1587\*(3 - x + 2\*x^2)^(3/2))

**fricas [A]** time = 0.57, size = 51, normalized size = 1.09

$$\frac{2(852x^3 - 639x^2 + 1005x - 952)\sqrt{2x^2 - x + 3}}{1587(4x^4 - 4x^3 + 13x^2 - 6x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)/(2\*x^2-x+3)^(5/2), x, algorithm="fricas")

[Out] 2/1587\*(852\*x^3 - 639\*x^2 + 1005\*x - 952)\*sqrt(2\*x^2 - x + 3)/(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)

**giac [A]** time = 0.23, size = 29, normalized size = 0.62

$$\frac{2(3(71(4x-3)x+335)x-952)}{1587(2x^2-x+3)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)/(2\*x^2-x+3)^(5/2),x, algorithm="giac")

[Out] 2/1587\*(3\*(71\*(4\*x - 3)\*x + 335)\*x - 952)/(2\*x^2 - x + 3)^(3/2)

**maple** [A] time = 0.00, size = 30, normalized size = 0.64

$$\frac{\frac{568}{529}x^3 - \frac{426}{529}x^2 + \frac{670}{529}x - \frac{1904}{1587}}{(2x^2 - x + 3)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+3\*x+2)/(2\*x^2-x+3)^(5/2),x)

[Out] 2/1587/(2\*x^2-x+3)^(3/2)\*(852\*x^3-639\*x^2+1005\*x-952)

**maxima** [A] time = 0.43, size = 59, normalized size = 1.26

$$\frac{284x}{529\sqrt{2x^2-x+3}} - \frac{71}{529\sqrt{2x^2-x+3}} - \frac{11x}{23(2x^2-x+3)^{\frac{3}{2}}} - \frac{55}{69(2x^2-x+3)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)/(2\*x^2-x+3)^(5/2),x, algorithm="maxima")

[Out] 284/529\*x/sqrt(2\*x^2 - x + 3) - 71/529/sqrt(2\*x^2 - x + 3) - 11/23\*x/(2\*x^2 - x + 3)^(3/2) - 55/69/(2\*x^2 - x + 3)^(3/2)

**mupad** [B] time = 0.09, size = 29, normalized size = 0.62

$$\frac{2(852x^3 - 639x^2 + 1005x - 952)}{1587(2x^2 - x + 3)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x + 5\*x^2 + 2)/(2\*x^2 - x + 3)^(5/2),x)

[Out] (2\*(1005\*x - 639\*x^2 + 852\*x^3 - 952))/(1587\*(2\*x^2 - x + 3)^(3/2))

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^2 + 3x + 2}{(2x^2 - x + 3)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)/(2*x**2 - x + 3)**(5/2), x)
```

$$3.97 \quad \int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)} dx$$

**Optimal.** Leaf size=199

$$\frac{3603 - 658x}{128018\sqrt{2x^2 - x + 3}} + \frac{13 - 6x}{759(2x^2 - x + 3)^{3/2}} + \frac{1}{484} \sqrt{\frac{1}{682} (25000\sqrt{2} - 15457)} \tan^{-1} \left( \frac{\sqrt{\frac{11}{31(25000\sqrt{2} - 15457)}} \left( (247 + 345\sqrt{2x^2 - x + 3}) \sqrt{2x^2 - x + 3} - (15457 + 25000\sqrt{2x^2 - x + 3}) \right)}{\sqrt{2x^2 - x + 3}} \right)$$

[Out] 1/759\*(13-6\*x)/(2\*x^2-x+3)^(3/2)+1/128018\*(3603-658\*x)/(2\*x^2-x+3)^(1/2)+1/330088\*arctan(1/31\*(443-98\*2^(1/2)+x\*(247+345\*2^(1/2)))\*341^(1/2)/(-15457+25000\*2^(1/2))^(1/2)/(2\*x^2-x+3)^(1/2))\*(-10541674+17050000\*2^(1/2))^(1/2)-1/330088\*arctanh(1/31\*(443+x\*(247-345\*2^(1/2))+98\*2^(1/2))\*341^(1/2)/(15457+25000\*2^(1/2))^(1/2)/(2\*x^2-x+3)^(1/2))\*(10541674+17050000\*2^(1/2))^(1/2)

**Rubi [A]** time = 0.46, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {974, 1060, 1035, 1029, 206, 204}

$$\frac{3603 - 658x}{128018\sqrt{2x^2 - x + 3}} + \frac{13 - 6x}{759(2x^2 - x + 3)^{3/2}} + \frac{1}{484} \sqrt{\frac{1}{682} (25000\sqrt{2} - 15457)} \tan^{-1} \left( \frac{\sqrt{\frac{11}{31(25000\sqrt{2} - 15457)}} \left( (247 + 345\sqrt{2x^2 - x + 3}) \sqrt{2x^2 - x + 3} - (15457 + 25000\sqrt{2x^2 - x + 3}) \right)}{\sqrt{2x^2 - x + 3}} \right)$$

Antiderivative was successfully verified.

[In] Int[1/((3 - x + 2\*x^2)^(5/2)\*(2 + 3\*x + 5\*x^2)),x]

[Out] (13 - 6\*x)/(759\*(3 - x + 2\*x^2)^(3/2)) + (3603 - 658\*x)/(128018\*Sqrt[3 - x + 2\*x^2]) + (Sqrt[(-15457 + 25000\*Sqrt[2])/682]\*ArcTan[(Sqrt[11/(31\*(-15457 + 25000\*Sqrt[2]))])\*(443 - 98\*Sqrt[2] + (247 + 345\*Sqrt[2])\*x)]/Sqrt[3 - x + 2\*x^2])/484 - (Sqrt[(15457 + 25000\*Sqrt[2])/682]\*ArcTanh[(Sqrt[11/(31\*(15457 + 25000\*Sqrt[2]))])\*(443 + 98\*Sqrt[2] + (247 - 345\*Sqrt[2])\*x)]/Sqrt[3 - x + 2\*x^2])/484

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

Q[a, 0] || LtQ[b, 0])

### Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]
```

### Rule 1029

```
Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]], x_Symbol] := Dist[-2*g*(g*b - 2*a*h), Subst[Int[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f), 0]
```

### Rule 1035

```
Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]], x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 - 4*a*c]
```

### Rule 1060

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
```

```

)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x +
  c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
  a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
  ^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
  2*a*(c*d - a*f)))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
  f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
  - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
  *B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
  2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
  (c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
  *c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
  + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
  2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
  A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
  (b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
  c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
  p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
  NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
  )^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && !LtQ[q, -1]) && !
  IGtQ[q, 0]

```

### Rubi steps

$$\begin{aligned}
\int \frac{1}{(3-x+2x^2)^{5/2} (2+3x+5x^2)} dx &= \frac{13-6x}{759(3-x+2x^2)^{3/2}} - \frac{\int \frac{-2772-\frac{3003x}{2}+660x^2}{(3-x+2x^2)^{3/2}(2+3x+5x^2)} dx}{8349} \\
&= \frac{13-6x}{759(3-x+2x^2)^{3/2}} + \frac{3603-658x}{128018\sqrt{3-x+2x^2}} - \frac{\int \frac{\frac{5184729}{2}-\frac{12481755x}{4}}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{23235267} \\
&= \frac{13-6x}{759(3-x+2x^2)^{3/2}} + \frac{3603-658x}{128018\sqrt{3-x+2x^2}} + \frac{\int \frac{-\frac{2112297}{4}(11-54\sqrt{2})-\frac{2112297}{4}}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{511175874\sqrt{2}} \\
&= \frac{13-6x}{759(3-x+2x^2)^{3/2}} + \frac{3603-658x}{128018\sqrt{3-x+2x^2}} - \frac{1}{32} \left( 17457 \left( 50000 - 15457\sqrt{2} \right) \right) \\
&= \frac{13-6x}{759(3-x+2x^2)^{3/2}} + \frac{3603-658x}{128018\sqrt{3-x+2x^2}} + \frac{1}{484} \sqrt{\frac{1}{682}} \left( -15457 + 25117\sqrt{2} \right)
\end{aligned}$$

**Mathematica [C]** time = 0.87, size = 218, normalized size = 1.10

$$\frac{\sqrt{\frac{1}{682}} (13 + i\sqrt{31}) (119\sqrt{31} + 247i) \tanh^{-1} \left( \frac{(-22-4i\sqrt{31})x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}} \sqrt{2x^2-x+3}} \right)}{9680} + \frac{\sqrt{\frac{1}{682}} (13 - i\sqrt{31}) (119\sqrt{31} - 247i) \tanh^{-1} \left( \frac{(-22+4i\sqrt{31})x+i\sqrt{31}+63}{2\sqrt{286-22i\sqrt{31}} \sqrt{2x^2-x+3}} \right)}{9680}$$

Antiderivative was successfully verified.

[In] Integrate[1/((3 - x + 2\*x^2)^(5/2)\*(2 + 3\*x + 5\*x^2)), x]

[Out] (39005 - 19767\*x + 23592\*x^2 - 3948\*x^3)/(384054\*(3 - x + 2\*x^2)^(3/2)) - (Sqrt[(13 + I\*Sqrt[31])/682]\*(247\*I + 119\*Sqrt[31])\*ArcTanh[(63 + I\*Sqrt[31] + (-22 - (4\*I)\*Sqrt[31])\*x)/(2\*Sqrt[286 + (22\*I)\*Sqrt[31]]\*Sqrt[3 - x + 2\*x^2])])/9680 + (Sqrt[(13 - I\*Sqrt[31])/682]\*(-247\*I + 119\*Sqrt[31])\*ArcTanh[(-63 + I\*Sqrt[31] + (22 - (4\*I)\*Sqrt[31])\*x)/(2\*Sqrt[286 - (22\*I)\*Sqrt[31]]\*Sqrt[3 - x + 2\*x^2])])/9680

**fricas [B]** time = 2.04, size = 2133, normalized size = 10.72

result too large to display



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^(5/2)/(5\*x^2+3\*x+2),x, algorithm="fricas")

[Out] 1/370971467791584000\*(1123856268\*sqrt(341)\*200^(1/4)\*sqrt(2)\*(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)\*sqrt(-772850000\*sqrt(2) + 2500000000)\*arctan(-1/7889389562500\*(71300\*sqrt(341)\*sqrt(2\*x^2 - x + 3)\*(11\*200^(3/4)\*(347404\*x^7 - 907814\*x^6 + 2112962\*x^5 - 2166688\*x^4 + 787344\*x^3 + 304128\*x^2 - sqrt(2)\*(35898\*x^7 - 441939\*x^6 + 782418\*x^5 - 2117233\*x^4 + 1272680\*x^3 - 1081800\*x^2 - 518400\*x + 1043712) - 2087424\*x + 518400) + 5\*200^(1/4)\*(712757\*x^7 - 10233303\*x^6 + 48529768\*x^5 - 94500260\*x^4 + 113086944\*x^3 - 22282848\*x^2 - sqrt(2)\*(158647\*x^7 - 2935272\*x^6 + 19428740\*x^5 - 55765712\*x^4 + 78380640\*x^3 - 84096000\*x^2 - 37407744\*x + 53208576) - 106417152\*x + 37407744))\*sqrt(-772850000\*sqrt(2) + 2500000000) + 22395686500000\*sqrt(31)\*sqrt(2)\*(28180\*x^8 - 254666\*x^7 + 704270\*x^6 - 1385256\*x^5 + 1549144\*x^4 - 642048\*x^3 - 98496\*x^2 - sqrt(2)\*(8746\*x^8 - 102335\*x^7 + 396104\*x^6 - 783113\*x^5 + 1320710\*x^4 - 752088\*x^3 + 396144\*x^2 + 546048\*x - 539136) + 1154304\*x - 456192) - sqrt(310/5711)\*(sqrt(341)\*sqrt(2\*x^2 - x + 3)\*(11\*200^(3/4)\*(1665224\*x^7 - 2325796\*x^6 + 7065036\*x^5 - 196416\*x^4 - 2176416\*x^3 + 8895744\*x^2 + sqrt(2)\*(167914\*x^7 - 195429\*x^6 + 331239\*x^5 + 1685680\*x^4 - 3693960\*x^3 + 4195584\*x^2 - 4195584\*x) - 8895744\*x) + 5\*200^(1/4)\*(3246491\*x^7 - 41888524\*x^6 + 159670660\*x^5 - 190080576\*x^4 + 180496224\*x^3 + 376648704\*x^2 - 2\*sqrt(2)\*(40239\*x^7 - 558044\*x^6 + 2804660\*x^5 - 9524160\*x^4 + 34843680\*x^3 - 74006784\*x^2 + 74006784\*x) - 376648704\*x))\*sqrt(-772850000\*sqrt(2) + 2500000000) + 314105000\*sqrt(31)\*sqrt(2)\*(123408\*x^8 - 914152\*x^7 + 1578888\*x^6 - 3293072\*x^5 + 396480\*x^4 + 798336\*x^3 - 3822336\*x^2 - sqrt(2)\*(15550\*x^8 - 118051\*x^7 + 244047\*x^6 - 707374\*x^5 + 1053960\*x^4 - 1667952\*x^3 + 1209600\*x^2 - 1036800\*x) + 3276288\*x) + 14277500\*sqrt(31)\*(254591\*x^8 - 4815126\*x^7 + 32303580\*x^6 - 90866808\*x^5 + 108781920\*x^4 - 74219328\*x^3 - 168956928\*x^2 - 15488\*sqrt(2)\*(4\*x^8 - 76\*x^7 + 517\*x^6 - 1536\*x^5 + 2385\*x^4 - 3618\*x^3 + 2268\*x^2 - 1944\*x) + 144820224\*x))\*sqrt(-(sqrt(341)\*200^(1/4)\*sqrt(31)\*sqrt(2\*x^2 - x + 3)\*(sqrt(2)\*(281\*x - 444) + 163\*x - 725)\*sqrt(-772850000\*sqrt(2) + 2500000000) - 4337504500\*x^2 - 3894902000\*sqrt(2)\*(2\*x^2 - x + 3) + 13366595500\*x - 17704100000)/x^2) + 254496437500\*sqrt(31)\*(2828123\*x^8 - 9696916\*x^7 + 53385560\*x^6 - 142835344\*x^5 + 254146592\*x^4 - 249300096\*x^3 + 37981440\*x^2 - 7744\*sqrt(2)\*(1348\*x^8 - 2692\*x^7 + 9789\*x^6 - 10070\*x^5 + 15569\*x^4 - 5568\*x^3 + 1080\*x^2 + 4320\*x - 5184) + 223064064\*x - 94887936)) / (2585191\*x^8 - 4661200\*x^7 + 14191920\*x^6 + 490880\*x^5 - 13562944\*x^4 + 44249088\*x^3 - 34615296\*x^2 - 24772608\*x + 18579456) + 1123856268\*sqrt(341)\*200^(1/4)\*sqrt(2)\*(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)\*sqrt(-772850000\*sqrt(2) + 2500000000)\*arctan(-1/7889389562500\*(71300\*sqrt(341)\*sqrt(2\*x^2 - x + 3)\*(11\*200^(3/4)\*(347404\*x^7 - 907814\*x^6 + 2112962\*x^5 - 2166688\*x^4 + 787344\*x^3 + 304128\*x^2 - sqrt(2)\*(35898\*x^7 - 441939\*x^6 + 782418\*x^5 - 2117233\*x^4 + 1272680\*x^3 - 1081800\*x^2 - 518400\*x + 1043712) - 2087424\*x + 518400) + 5\*200^(1/4)\*(712757\*x^7 - 10233303\*x^6 + 48529768\*x^5 - 94500260\*x^4 +

$$\begin{aligned}
& 113086944*x^3 - 22282848*x^2 - \sqrt{2}*(158647*x^7 - 2935272*x^6 + 1942874 \\
& 0*x^5 - 55765712*x^4 + 78380640*x^3 - 84096000*x^2 - 37407744*x + 53208576) \\
& - 106417152*x + 37407744))*\sqrt{-772850000*\sqrt{2} + 2500000000} - 2239568 \\
& 6500000*\sqrt{31}*\sqrt{2}*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 \\
& + 1549144*x^4 - 642048*x^3 - 98496*x^2 - \sqrt{2}*(8746*x^8 - 102335*x^7 + \\
& 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x \\
& - 539136) + 1154304*x - 456192) - \sqrt{310/5711}*(\sqrt{341}*\sqrt{2*x^2 - x \\
& + 3}*(11*200^{(3/4)}*(1665224*x^7 - 2325796*x^6 + 7065036*x^5 - 196416*x^4 - \\
& 2176416*x^3 + 8895744*x^2 + \sqrt{2}*(167914*x^7 - 195429*x^6 + 331239*x^5 + \\
& 1685680*x^4 - 3693960*x^3 + 4195584*x^2 - 4195584*x) - 8895744*x) + 5*200^ \\
& (1/4)*(3246491*x^7 - 41888524*x^6 + 159670660*x^5 - 190080576*x^4 + 1804962 \\
& 24*x^3 + 376648704*x^2 - 2*\sqrt{2}*(40239*x^7 - 558044*x^6 + 2804660*x^5 - \\
& 9524160*x^4 + 34843680*x^3 - 74006784*x^2 + 74006784*x) - 376648704*x))*\sqrt{ \\
& (-772850000*\sqrt{2} + 2500000000) - 314105000*\sqrt{31}*\sqrt{2}*(123408*x^8 \\
& - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 38223 \\
& 36*x^2 - \sqrt{2}*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 105396 \\
& 0*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) - 14277500*\sqrt{ \\
& (31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^ \\
& 4 - 74219328*x^3 - 168956928*x^2 - 15488*\sqrt{2}*(4*x^8 - 76*x^7 + 517*x^6 \\
& - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*\sqrt{ \\
& (\sqrt{341}*200^{(1/4)}*\sqrt{31}*\sqrt{2*x^2 - x + 3}*(\sqrt{2}*(281*x - 444) + \\
& 163*x - 725))*\sqrt{-772850000*\sqrt{2} + 2500000000} + 4337504500*x^2 + 38949 \\
& 02000*\sqrt{2}*(2*x^2 - x + 3) - 13366595500*x + 17704100000)/x^2) - 2544964 \\
& 37500*\sqrt{31}*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + \\
& 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*\sqrt{2}*(1348*x^8 - 269 \\
& 2*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5 \\
& 184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + \\
& 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 185 \\
& 79456)) + 1587*\sqrt{341}*200^{(1/4)}*\sqrt{31}*(200000*x^4 - 200000*x^3 + 6500 \\
& 00*x^2 + 15457*\sqrt{2}*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9) - 300000*x + 4500 \\
& 00)*\sqrt{-772850000*\sqrt{2} + 2500000000}*\log(77500000/5711*(\sqrt{341}*200^ \\
& (1/4)*\sqrt{31}*\sqrt{2*x^2 - x + 3}*(\sqrt{2}*(281*x - 444) + 163*x - 725))*\sqrt{ \\
& (-772850000*\sqrt{2} + 2500000000) + 4337504500*x^2 + 3894902000*\sqrt{2}*( \\
& 2*x^2 - x + 3) - 13366595500*x + 17704100000)/x^2) - 1587*\sqrt{341}*200^{(1/ \\
& 4)}*\sqrt{31}*(200000*x^4 - 200000*x^3 + 650000*x^2 + 15457*\sqrt{2}*(4*x^4 - \\
& 4*x^3 + 13*x^2 - 6*x + 9) - 300000*x + 450000)*\sqrt{-772850000*\sqrt{2} + 25 \\
& 00000000)*\log(-77500000/5711*(\sqrt{341}*200^{(1/4)}*\sqrt{31}*\sqrt{2*x^2 - x + \\
& 3}*(\sqrt{2}*(281*x - 444) + 163*x - 725))*\sqrt{-772850000*\sqrt{2} + 2500000 \\
& 000) - 4337504500*x^2 - 3894902000*\sqrt{2}*(2*x^2 - x + 3) + 13366595500*x \\
& - 17704100000)/x^2) - 965935696000*(3948*x^3 - 23592*x^2 + 19767*x - 39005) \\
& *\sqrt{2*x^2 - x + 3))/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
\end{aligned}$$

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^(5/2)/(5\*x^2+3\*x+2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP  
 UT:sage2:=int(sage0,x)::OUTPUT:Francis algorithm failure for[-1.0,infinity,  
 infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,inf  
 inity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity  
 ]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai  
 lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity  
 ,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf  
 inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinit  
 y]Evaluation time: 16.02Done

**maple [B]** time = 0.04, size = 751, normalized size = 3.77

$$\sqrt{\frac{8(x+\sqrt{2}-1)^2}{(-x+\sqrt{2}+1)^2} + \frac{3\sqrt{2}(x+\sqrt{2}-1)^2}{(-x+\sqrt{2}+1)^2} + 8 - 3\sqrt{2}} \sqrt{2} \left( -993674\sqrt{2} \operatorname{arctanh} \left( \frac{31 \sqrt{\frac{8(x+\sqrt{2}-1)^2}{(-x+\sqrt{2}+1)^2} + \frac{3\sqrt{2}(x+\sqrt{2}-1)^2}{(-x+\sqrt{2}+1)^2} + 8 - 3\sqrt{2}}}{2\sqrt{-8866+6820\sqrt{2}}} \right) - 426 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2\*x^2-x+3)^(5/2)/(5\*x^2+3\*x+2),x)

[Out] 1/10232728\*(8\*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3\*2^(1/2)\*(x+2^(1/2)-1)^2/(-  
 x+2^(1/2)+1)^2+8-3\*2^(1/2))^(1/2)\*2^(1/2)\*(10111\*2^(1/2)\*(-8866+6820\*2^(1/2  
 ))^(1/2)\*(-775687+549362\*2^(1/2))^(1/2)\*arctan(1/11692487\*(-775687+549362\*2  
 ^2^(1/2))^(1/2)\*(-23\*(8+3\*2^(1/2)))\*(-23\*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24\*2  
 ^2^(1/2)-41))^(1/2)\*(6485\*2^(1/2)\*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+10368\*(x+2  
 ^2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+22379\*2^(1/2)+32016)/(23\*(x+2^(1/2)-1)^4/(-x+2  
 ^2^(1/2)+1)^4+82\*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23)\*(x+2^(1/2)-1)/(-x+2^(1/  
 2)+1)\*(8+3\*2^(1/2)))+13910\*(-8866+6820\*2^(1/2))^(1/2)\*(-775687+549362\*2^(1/  
 2))^(1/2)\*arctan(1/11692487\*(-775687+549362\*2^(1/2))^(1/2)\*(-23\*(8+3\*2^(1/2  
 )))\*(-23\*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24\*2^(1/2)-41))^(1/2)\*(6485\*2^(1/2  
 ))\*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+10368\*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+2  
 2379\*2^(1/2)+32016)/(23\*(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)^4+82\*(x+2^(1/2)-1)^2  
 /(-x+2^(1/2)+1)^2+23)\*(x+2^(1/2)-1)/(-x+2^(1/2)+1)\*(8+3\*2^(1/2)))-993674\*ar  
 ctanh(31/2\*(8\*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3\*2^(1/2)\*(x+2^(1/2)-1)^2/(-  
 x+2^(1/2)+1)^2+8-3\*2^(1/2))^(1/2)/(-8866+6820\*2^(1/2))^(1/2)\*2^(1/2)-42685  
 698\*arctanh(31/2\*(8\*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3\*2^(1/2)\*(x+2^(1/2)-1

)<sup>2</sup>/(-x+2<sup>(1/2)</sup>+1)<sup>2</sup>+8-3\*2<sup>(1/2)</sup>)<sup>(1/2)</sup>/(-8866+6820\*2<sup>(1/2)</sup>)<sup>(1/2)</sup>)/((8\*(x+2<sup>(1/2)</sup>-1)<sup>2</sup>/(-x+2<sup>(1/2)</sup>+1)<sup>2</sup>+3\*2<sup>(1/2)</sup>\*(x+2<sup>(1/2)</sup>-1)<sup>2</sup>/(-x+2<sup>(1/2)</sup>+1)<sup>2</sup>+8-3\*2<sup>(1/2)</sup>)/(1+(x+2<sup>(1/2)</sup>-1)/(-x+2<sup>(1/2)</sup>+1))<sup>(1/2)</sup>/(1+(x+2<sup>(1/2)</sup>-1)/(-x+2<sup>(1/2)</sup>+1)))/(8+3\*2<sup>(1/2)</sup>)/(-8866+6820\*2<sup>(1/2)</sup>)<sup>(1/2)</sup>+13/484/(2\*x<sup>2</sup>-x+3)<sup>(1/2)</sup>-329/256036\*(4\*x-1)/(2\*x<sup>2</sup>-x+3)<sup>(1/2)</sup>+1/66/(2\*x<sup>2</sup>-x+3)<sup>(3/2)</sup>-1/506\*(4\*x-1)/(2\*x<sup>2</sup>-x+3)<sup>(3/2)</sup>)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(5x^2 + 3x + 2)(2x^2 - x + 3)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x<sup>2</sup>-x+3)<sup>(5/2)</sup>/(5\*x<sup>2</sup>+3\*x+2), x, algorithm="maxima")

[Out] integrate(1/((5\*x<sup>2</sup> + 3\*x + 2)\*(2\*x<sup>2</sup> - x + 3)<sup>(5/2)</sup>), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(2x^2 - x + 3)^{\frac{5}{2}} (5x^2 + 3x + 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((2\*x<sup>2</sup> - x + 3)<sup>(5/2)</sup>\*(3\*x + 5\*x<sup>2</sup> + 2)), x)

[Out] int(1/((2\*x<sup>2</sup> - x + 3)<sup>(5/2)</sup>\*(3\*x + 5\*x<sup>2</sup> + 2)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(2x^2 - x + 3)^{\frac{5}{2}} (5x^2 + 3x + 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x\*\*2-x+3)\*\*(5/2)/(5\*x\*\*2+3\*x+2), x)

[Out] Integral(1/((2\*x\*\*2 - x + 3)\*\*(5/2)\*(5\*x\*\*2 + 3\*x + 2)), x)

$$3.98 \quad \int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)^2} dx$$

Optimal. Leaf size=234

$$\frac{15101 - 8654x}{1035276(2x^2 - x + 3)^{3/2}} - \frac{1352542x + 3133427}{523849656\sqrt{2x^2 - x + 3}} + \frac{65x + 4}{682(2x^2 - x + 3)^{3/2}(5x^2 + 3x + 2)} + \frac{625\sqrt{\frac{1}{682}(30463 + 23600\sqrt{2})}}{682(2x^2 - x + 3)^{3/2}(5x^2 + 3x + 2)}$$

[Out] 1/1035276\*(-15101+8654\*x)/(2\*x^2-x+3)^(3/2)+1/682\*(4+65\*x)/(2\*x^2-x+3)^(3/2)/(5\*x^2+3\*x+2)+1/523849656\*(-3133427-1352542\*x)/(2\*x^2-x+3)^(1/2)-625/450240032\*arctanh(1/31\*(203+x\*(687-445\*2^(1/2))-242\*2^(1/2))\*341^(1/2)/(-30463+23600\*2^(1/2)))^(1/2)/(2\*x^2-x+3)^(1/2))\*(-20775766+16095200\*2^(1/2))^(1/2)+625/450240032\*arctan(1/31\*(203+242\*2^(1/2)+x\*(687+445\*2^(1/2))))\*341^(1/2)/(30463+23600\*2^(1/2))^(1/2)/(2\*x^2-x+3)^(1/2))\*(20775766+16095200\*2^(1/2))^(1/2)

**Rubi [A]** time = 0.54, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {974, 1060, 1035, 1029, 206, 204}

$$\frac{15101 - 8654x}{1035276(2x^2 - x + 3)^{3/2}} - \frac{1352542x + 3133427}{523849656\sqrt{2x^2 - x + 3}} + \frac{65x + 4}{682(2x^2 - x + 3)^{3/2}(5x^2 + 3x + 2)} + \frac{625\sqrt{\frac{1}{682}(30463 + 23600\sqrt{2})}}{682(2x^2 - x + 3)^{3/2}(5x^2 + 3x + 2)}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - x + 2\*x^2)^(5/2)\*(2 + 3\*x + 5\*x^2)^2), x]

[Out] -(15101 - 8654\*x)/(1035276\*(3 - x + 2\*x^2)^(3/2)) - (3133427 + 1352542\*x)/(523849656\*Sqrt[3 - x + 2\*x^2]) + (4 + 65\*x)/(682\*(3 - x + 2\*x^2)^(3/2)\*(2 + 3\*x + 5\*x^2)) + (625\*Sqrt[(30463 + 23600\*Sqrt[2])/682]\*ArcTan[(Sqrt[11/(31\*(30463 + 23600\*Sqrt[2]))])\*(203 + 242\*Sqrt[2] + (687 + 445\*Sqrt[2])\*x)]/Sqrt[3 - x + 2\*x^2])/660176 - (625\*Sqrt[(-30463 + 23600\*Sqrt[2])/682]\*ArcTanh[(Sqrt[11/(31\*(-30463 + 23600\*Sqrt[2]))])\*(203 - 242\*Sqrt[2] + (687 - 445\*Sqrt[2])\*x)]/Sqrt[3 - x + 2\*x^2])/660176

Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[

a, 0] || LtQ[b, 0])

### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[Rt[-b, 2]\*x]/Rt[a, 2])]/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 974

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_)\*((d\_) + (e\_)\*(x\_) + (f\_)\*(x\_)^2)^(q\_), x\_Symbol] := Simp[((2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f) + c\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*x)\*(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^(q + 1))/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1)), x] - Dist[1/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1)), Int[(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^q\*Simp[2\*c\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1) - (2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(a\*f\*(p + 1) - c\*d\*(p + 2)) - e\*(b^2\*c\*e - 2\*a\*c^2\*e - b^3\*f - b\*c\*(c\*d - 3\*a\*f))\*(p + q + 2) + (2\*f\*(2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f))\*(p + q + 2) - (2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(b\*f\*(p + 1) - c\*e\*(2\*p + q + 4)))\*x + c\*f\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(2\*p + 2\*q + 5)\*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && LtQ[p, -1] && NeQ[(c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f), 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]

### Rule 1029

Int[((g\_) + (h\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)\*Sqrt[(d\_) + (e\_)\*(x\_) + (f\_)\*(x\_)^2]], x\_Symbol] := Dist[-2\*g\*(g\*b - 2\*a\*h), Subst[Int[1/Simp[g\*(g\*b - 2\*a\*h)\*(b^2 - 4\*a\*c) - (b\*d - a\*e)\*x^2, x], x], x, Simp[g\*b - 2\*a\*h - (b\*h - 2\*g\*c)\*x, x]/Sqrt[d + e\*x + f\*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && NeQ[b\*d - a\*e, 0] && EqQ[h^2\*(b\*d - a\*e) - 2\*g\*h\*(c\*d - a\*f) + g^2\*(c\*e - b\*f), 0]

### Rule 1035

Int[((g\_) + (h\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)\*Sqrt[(d\_) + (e\_)\*(x\_) + (f\_)\*(x\_)^2]], x\_Symbol] := With[{q = Rt[(c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f), 2]}, Dist[1/(2\*q), Int[Simp[h\*(b\*d - a\*e) - g\*(c\*d - a\*f - q) - (g\*(c\*e - b\*f) - h\*(c\*d - a\*f + q))\*x, x]/((a + b\*x + c\*x^2)\*Sqrt[d + e\*x + f\*x^2]), x], x] - Dist[1/(2\*q), Int[Simp[h\*(b\*d - a\*e) - g\*(c\*d - a\*f + q) - (g\*(c\*e - b\*f) - h\*(c\*d - a\*f - q))\*x, x]/((a + b\*x + c\*x^2)\*Sqrt[d + e\*x + f\*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ

$[b^2 - 4ac, 0] \ \&\& \ \text{NeQ}[e^2 - 4df, 0] \ \&\& \ \text{NeQ}[bd - ae, 0] \ \&\& \ \text{NegQ}[b^2 - 4ac]$

### Rule 1060

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((A_) + (B_)*(x_) + (C_)*(x_)^2)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{1}{(3-x+2x^2)^{5/2} (2+3x+5x^2)^2} dx &= \frac{4+65x}{682(3-x+2x^2)^{3/2} (2+3x+5x^2)} - \frac{\int \frac{-1738 + \frac{4411x}{2} - 5720x^2}{(3-x+2x^2)^{5/2} (2+3x+5x^2)} dx}{7502} \\
&= -\frac{15101-8654x}{1035276(3-x+2x^2)^{3/2}} + \frac{4+65x}{682(3-x+2x^2)^{3/2} (2+3x+5x^2)} - \int \dots \\
&= -\frac{15101-8654x}{1035276(3-x+2x^2)^{3/2}} - \frac{3133427+1352542x}{523849656\sqrt{3-x+2x^2}} + \frac{\dots}{682(3-x+2x^2)^{3/2}} \\
&= -\frac{15101-8654x}{1035276(3-x+2x^2)^{3/2}} - \frac{3133427+1352542x}{523849656\sqrt{3-x+2x^2}} + \frac{\dots}{682(3-x+2x^2)^{3/2}} \\
&= -\frac{15101-8654x}{1035276(3-x+2x^2)^{3/2}} - \frac{3133427+1352542x}{523849656\sqrt{3-x+2x^2}} + \frac{\dots}{682(3-x+2x^2)^{3/2}} \\
&= -\frac{15101-8654x}{1035276(3-x+2x^2)^{3/2}} - \frac{3133427+1352542x}{523849656\sqrt{3-x+2x^2}} + \frac{\dots}{682(3-x+2x^2)^{3/2}}
\end{aligned}$$

**Mathematica** [C] time = 1.15, size = 296, normalized size = 1.26

$$198375i\sqrt{286+22i\sqrt{31}} (687\sqrt{31}+31i)\sqrt{2x^2-x+3} (10x^4+x^3+16x^2+7x+6) \tanh^{-1}\left(\frac{(-22-4i\sqrt{31})x+i\sqrt{31}}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/((3-x+2\*x^2)^(5/2)\*(2+3\*x+5\*x^2)^2),x]

[Out] -1/2858123723136\*(5456\*(31010342-5712309\*x+84671384\*x^2+2879479\*x^3+32686812\*x^4+13525420\*x^5)+(198375\*I)\*Sqrt[286+(22\*I)\*Sqrt[31]]\*(31\*I+687\*Sqrt[31])\*Sqrt[3-x+2\*x^2]\*(6+7\*x+16\*x^2+x^3+10\*x^4)\*ArcTanh[(63+I\*Sqrt[31]+(-22-(4\*I)\*Sqrt[31])\*x)/(2\*Sqrt[286+(22\*I)\*Sqrt[31]]\*Sqrt[3-x+2\*x^2])]



```
[31]]*Sqrt[3 - x + 2*x^2]]) + 198375*Sqrt[286 - (22*I)*Sqrt[31]]*(31 + (687
*I)*Sqrt[31])*Sqrt[3 - x + 2*x^2]*(6 + 7*x + 16*x^2 + x^3 + 10*x^4)*ArcTanh
[(-63 + I*Sqrt[31] + (22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]
]*Sqrt[3 - x + 2*x^2]])]/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2))
```

**fricas [B]** time = 1.99, size = 2253, normalized size = 9.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/25604335602537914112*(301208632500*6962^(1/4)*sqrt(341)*sqrt(118)*sqrt(2)
*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18)*sqrt(30463*sqrt(2) +
47200)*arctan(1/11117215998613*(168268*sqrt(118)*(22*6962^(3/4)*sqrt(341)*(
321084*x^7 - 1338894*x^6 + 2762802*x^5 - 4721048*x^4 + 2438224*x^3 - 131731
2*x^2 - sqrt(2)*(277258*x^7 - 994619*x^6 + 2123978*x^5 - 3198193*x^4 + 1552
680*x^3 - 621000*x^2 - 1900800*x + 1181952) - 2363904*x + 1900800) + 1829*6
962^(1/4)*sqrt(341)*(25187*x^7 - 392073*x^6 + 2114488*x^5 - 4948060*x^4 + 6
460704*x^3 - 4452768*x^2 - sqrt(2)*(20477*x^7 - 310452*x^6 + 1610140*x^5 -
3584192*x^4 + 4580640*x^3 - 2620800*x^2 - 3400704*x + 2198016) - 4396032*x
+ 3400704))*sqrt(2*x^2 - x + 3)*sqrt(30463*sqrt(2) + 47200) + 3155854864122
4*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 154
9144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104
*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 5391
36) + 1154304*x - 456192) - 2*sqrt(118/79)*(sqrt(118)*(22*6962^(3/4)*sqrt(3
41)*(1050904*x^7 - 1523916*x^6 + 5005956*x^5 - 2572736*x^4 + 3615264*x^3 +
877824*x^2 - sqrt(2)*(1065206*x^7 - 1518091*x^6 + 4815081*x^5 - 1448880*x^4
+ 1303560*x^3 + 3131136*x^2 - 3131136*x) - 877824*x) + 1829*6962^(1/4)*sqr
t(341)*(84981*x^7 - 1100084*x^6 + 4256060*x^5 - 5639616*x^4 + 7745184*x^3 +
2571264*x^2 - 242*sqrt(2)*(319*x^7 - 4124*x^6 + 15860*x^5 - 20160*x^4 + 24
480*x^3 + 20736*x^2 - 20736*x) - 2571264*x))*sqrt(2*x^2 - x + 3)*sqrt(30463
*sqrt(2) + 47200) + 187549318*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1
578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*
(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x
^3 + 1209600*x^2 - 1036800*x) + 3276288*x) + 8524969*sqrt(31)*(254591*x^8 -
4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 -
168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*
x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(6962^(1/4)*sqrt(
341)*sqrt(118)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(101*x + 176) - 277*x
+ 75)*sqrt(30463*sqrt(2) + 47200) - 219481829*x^2 - 197085724*sqrt(2)*(2*x^
2 - x + 3) + 676362371*x - 895844200)/x^2) + 358619870923*sqrt(31)*(2828123
*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 2493000
96*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 1007
0*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94
```

$$\begin{aligned}
& 887936)) / (2585191x^8 - 4661200x^7 + 14191920x^6 + 490880x^5 - 13562944x^4 \\
& + 44249088x^3 - 34615296x^2 - 24772608x + 18579456)) + 301208632500 * \\
& 6962^{(1/4)} * \sqrt{341} * \sqrt{118} * \sqrt{2} * (20x^6 - 8x^5 + 61x^4 + x^3 + 53x^2 \\
& + 15x + 18) * \sqrt{30463\sqrt{2} + 47200} * \arctan(1/11117215998613 * (16826 \\
& 8 * \sqrt{118} * (22 * 6962^{(3/4)} * \sqrt{341} * (321084x^7 - 1338894x^6 + 2762802x^5 \\
& - 4721048x^4 + 2438224x^3 - 1317312x^2 - \sqrt{2} * (277258x^7 - 994619x^6 \\
& + 2123978x^5 - 3198193x^4 + 1552680x^3 - 621000x^2 - 1900800x + 1181952) \\
& - 2363904x + 1900800) + 1829 * 6962^{(1/4)} * \sqrt{341} * (25187x^7 - 392073x^6 \\
& + 2114488x^5 - 4948060x^4 + 6460704x^3 - 4452768x^2 - \sqrt{2} * (20477x^7 \\
& - 310452x^6 + 1610140x^5 - 3584192x^4 + 4580640x^3 - 2620800x^2 - 3400704x \\
& + 2198016) - 4396032x + 3400704)) * \sqrt{2x^2 - x + 3} * \sqrt{30463\sqrt{2} \\
& + 47200} - 31558548641224 * \sqrt{31} * \sqrt{2} * (28180x^8 - 254666x^7 + 704270x^6 \\
& - 1385256x^5 + 1549144x^4 - 642048x^3 - 98496x^2 - \sqrt{2} * (8746x^8 - 102335x^7 \\
& + 396104x^6 - 783113x^5 + 1320710x^4 - 752088x^3 + 396144x^2 + 546048x - 539136) \\
& + 1154304x - 456192) - 2 * \sqrt{118/79} * (\sqrt{118} * (22 * 6962^{(3/4)} * \sqrt{341} * (1050904x^7 \\
& - 1523916x^6 + 5005956x^5 - 2572736x^4 + 3615264x^3 + 877824x^2 - \sqrt{2} * (1065206x^7 \\
& - 1518091x^6 + 4815081x^5 - 1448880x^4 + 1303560x^3 + 3131136x^2 - 3131136x) \\
& - 877824x) + 1829 * 6962^{(1/4)} * \sqrt{341} * (84981x^7 - 1100084x^6 + 4256060x^5 \\
& - 5639616x^4 + 7745184x^3 + 2571264x^2 - 242 * \sqrt{2} * (319x^7 - 4124x^6 \\
& + 15860x^5 - 20160x^4 + 24480x^3 + 20736x^2 - 20736x) - 2571264x)) * \sqrt{2x^2 - x + 3} \\
& * \sqrt{30463\sqrt{2} + 47200} - 187549318 * \sqrt{31} * \sqrt{2} * (123408x^8 - 914152x^7 \\
& + 1578888x^6 - 3293072x^5 + 396480x^4 + 798336x^3 - 3822336x^2 - \sqrt{2} * (15550x^8 \\
& - 118051x^7 + 244047x^6 - 707374x^5 + 1053960x^4 - 1667952x^3 + 1209600x^2 - 1036800x) \\
& + 3276288x) - 8524969 * \sqrt{31} * (254591x^8 - 4815126x^7 + 32303580x^6 - 9086680 \\
& 8x^5 + 108781920x^4 - 74219328x^3 - 168956928x^2 - 15488 * \sqrt{2} * (4x^8 - 76x^7 \\
& + 517x^6 - 1536x^5 + 2385x^4 - 3618x^3 + 2268x^2 - 1944x) + 144820224x) * \sqrt{(6962^{(1/4)} \\
& * \sqrt{341} * \sqrt{118} * \sqrt{31} * \sqrt{2x^2 - x + 3} * (\sqrt{2} * (101x + 176) - 277x + 75) \\
& * \sqrt{30463\sqrt{2} + 47200} + 219481829x^2 + 197085724 * \sqrt{2} * (2x^2 - x + 3) \\
& - 676362371x + 895844200) / x^2) - 358619870923 * \sqrt{31} * (2828123x^8 - 9696916x^7 \\
& + 53385560x^6 - 142835344x^5 + 254146592x^4 - 249300096x^3 + 37981440x^2 - 7744 * \sqrt{2} * (1348x^8 \\
& - 2692x^7 + 9789x^6 - 10070x^5 + 15569x^4 - 5568x^3 + 1080x^2 + 4320x - 5184) \\
& + 223064064x - 94887936)) / (2585191x^8 - 4661200x^7 + 14191920x^6 + 490880x^5 \\
& - 13562944x^4 + 44249088x^3 - 34615296x^2 - 24772608x + 18579456)) + 991875 * 6962^{(1/4)} \\
& * \sqrt{341} * \sqrt{118} * \sqrt{31} * (944000x^6 - 377600x^5 + 2879200x^4 + 47200x^3 + 2501600x^2 - 30463\sqrt{2} \\
& * (20x^6 - 8x^5 + 61x^4 + x^3 + 53x^2 + 15x + 18) + 708000x + 849600) * \sqrt{30463\sqrt{2} \\
& + 47200} * \log(7375000000000/79 * (6962^{(1/4)} * \sqrt{341} * \sqrt{118} * \sqrt{31} * \sqrt{2x^2 - x + 3} \\
& * (\sqrt{2} * (101x + 176) - 277x + 75) * \sqrt{30463\sqrt{2} + 47200} + 219481829x^2 + 197085724 * \sqrt{2} \\
& * (2x^2 - x + 3) - 676362371x + 895844200) / x^2) - 991875 * 6962^{(1/4)} * \sqrt{341} * \sqrt{118} * \sqrt{31} \\
& * (944000x^6 - 377600x^5 + 2879200x^4 + 47200x^3 + 2501600x^2 - 30463\sqrt{2} * (20x^6 - 8x^5 + 61x^4 \\
& + x^3 + 53x^2 + 15x + 18) + 70800
\end{aligned}$$

```
0*x + 849600)*sqrt(30463*sqrt(2) + 47200)*log(-7375000000000/79*(6962^(1/4)
*sqrt(341)*sqrt(118)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(101*x + 176) -
277*x + 75)*sqrt(30463*sqrt(2) + 47200) - 219481829*x^2 - 197085724*sqrt(2)
*(2*x^2 - x + 3) + 676362371*x - 895844200)/x^2) - 48877259552*(13525420*x^
5 + 32686812*x^4 + 2879479*x^3 + 84671384*x^2 - 5712309*x + 31010342)*sqrt(
2*x^2 - x + 3))/(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18)
```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,inf
inity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity
]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai
lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity
,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf
inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinit
y]Evaluation time: 18.73Done
```

**maple** [B] time = 0.10, size = 5975, normalized size = 25.53

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x)
```

```
[Out] result too large to display
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(5x^2 + 3x + 2)^2 (2x^2 - x + 3)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)^2*(2*x^2 - x + 3)^(5/2)), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(2x^2 - x + 3)^{5/2} (5x^2 + 3x + 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^2), x)`

[Out] `int(1/((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(2x^2 - x + 3)^{\frac{5}{2}} (5x^2 + 3x + 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2*x**2-x+3)**(5/2)/(5*x**2+3*x+2)**2, x)`

[Out] `Integral(1/((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2)**2), x)`

$$3.99 \quad \int \frac{1}{(3-x+2x^2)^{5/2} (2+3x+5x^2)^3} dx$$

**Optimal.** Leaf size=269

$$-\frac{1134826571 - 1504660754x}{476353953856\sqrt{2x^2 - x + 3}} + \frac{86885x + 46386}{1860496(2x^2 - x + 3)^{3/2}(5x^2 + 3x + 2)} - \frac{12280939 - 19536786x}{2824232928(2x^2 - x + 3)^{3/2}} + \frac{1}{1364}$$

[Out] 1/2824232928\*(-12280939+19536786\*x)/(2\*x^2-x+3)^(3/2)+1/1364\*(4+65\*x)/(2\*x^2-x+3)^(3/2)/(5\*x^2+3\*x+2)^2+1/1860496\*(46386+86885\*x)/(2\*x^2-x+3)^(3/2)/(5\*x^2+3\*x+2)+1/476353953856\*(-1134826571+1504660754\*x)/(2\*x^2-x+3)^(1/2)-35/1228254807296\*arctanh(1/31\*(1432939+x\*(6290431-3861685\*2^(1/2))-2428746\*2^(1/2))\*341^(1/2)/(-2243059557247+2011748500000\*2^(1/2))^(1/2)/(2\*x^2-x+3)^(1/2))\*(-1529766618042454+1372012477000000\*2^(1/2))^(1/2)+35/1228254807296\*arctan(1/31\*(1432939+2428746\*2^(1/2)+x\*(6290431+3861685\*2^(1/2)))\*341^(1/2)/(2243059557247+2011748500000\*2^(1/2))^(1/2)/(2\*x^2-x+3)^(1/2))\*(1529766618042454+1372012477000000\*2^(1/2))^(1/2)

**Rubi [A]** time = 0.59, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {974, 1060, 1035, 1029, 206, 204}

$$-\frac{1134826571 - 1504660754x}{476353953856\sqrt{2x^2 - x + 3}} + \frac{86885x + 46386}{1860496(2x^2 - x + 3)^{3/2}(5x^2 + 3x + 2)} - \frac{12280939 - 19536786x}{2824232928(2x^2 - x + 3)^{3/2}} + \frac{1}{1364}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - x + 2\*x^2)^(5/2)\*(2 + 3\*x + 5\*x^2)^3), x]

[Out] -(12280939 - 19536786\*x)/(2824232928\*(3 - x + 2\*x^2)^(3/2)) - (1134826571 - 1504660754\*x)/(476353953856\*Sqrt[3 - x + 2\*x^2]) + (4 + 65\*x)/(1364\*(3 - x + 2\*x^2)^(3/2)\*(2 + 3\*x + 5\*x^2)^2) + (46386 + 86885\*x)/(1860496\*(3 - x + 2\*x^2)^(3/2)\*(2 + 3\*x + 5\*x^2)) + (35\*Sqrt[(2243059557247 + 2011748500000\*Sqrt[2])/682]\*ArcTan[(Sqrt[11/(31\*(2243059557247 + 2011748500000\*Sqrt[2]))])\*(1432939 + 2428746\*Sqrt[2] + (6290431 + 3861685\*Sqrt[2])\*x)]/Sqrt[3 - x + 2\*x^2])/1800960128 - (35\*Sqrt[(-2243059557247 + 2011748500000\*Sqrt[2])/682]\*ArcTanh[(Sqrt[11/(31\*(-2243059557247 + 2011748500000\*Sqrt[2]))])\*(1432939 - 2428746\*Sqrt[2] + (6290431 - 3861685\*Sqrt[2])\*x)]/Sqrt[3 - x + 2\*x^2])/1800960128

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 974

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]
```

Rule 1029

```
Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := Dist[-2*g*(g*b - 2*a*h), Subst[Int[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f), 0]
```

Rule 1035

```
Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
```

```
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

### Rule 1060

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((A_) + (B_)*(x_) + (C_)*(x_
)^2)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
+ q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !
IGtQ[q, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{1}{(3-x+2x^2)^{5/2} (2+3x+5x^2)^3} dx &= \frac{4+65x}{1364 (3-x+2x^2)^{3/2} (2+3x+5x^2)^2} - \frac{\int \frac{-5687 + \frac{8635x}{2} - 8580x^2}{(3-x+2x^2)^{5/2} (2+3x+5x^2)^2} dx}{15004} \\
&= \frac{4+65x}{1364 (3-x+2x^2)^{3/2} (2+3x+5x^2)^2} + \frac{46386 + 86885x}{1860496 (3-x+2x^2)^{3/2} (2+3x+5x^2)} \\
&= -\frac{12280939 - 19536786x}{2824232928 (3-x+2x^2)^{3/2}} + \frac{4+65x}{1364 (3-x+2x^2)^{3/2} (2+3x+5x^2)^2} \\
&= -\frac{12280939 - 19536786x}{2824232928 (3-x+2x^2)^{3/2}} - \frac{1134826571 - 1504660754x}{476353953856 \sqrt{3-x+2x^2}} + \frac{1}{1364} \\
&= -\frac{12280939 - 19536786x}{2824232928 (3-x+2x^2)^{3/2}} - \frac{1134826571 - 1504660754x}{476353953856 \sqrt{3-x+2x^2}} + \frac{1}{1364} \\
&= -\frac{12280939 - 19536786x}{2824232928 (3-x+2x^2)^{3/2}} - \frac{1134826571 - 1504660754x}{476353953856 \sqrt{3-x+2x^2}} + \frac{1}{1364} \\
&= -\frac{12280939 - 19536786x}{2824232928 (3-x+2x^2)^{3/2}} - \frac{1134826571 - 1504660754x}{476353953856 \sqrt{3-x+2x^2}} + \frac{1}{1364}
\end{aligned}$$

**Mathematica [C]** time = 1.96, size = 242, normalized size = 0.90

$$11109\sqrt{286 + 22i\sqrt{31}} (4541903 - 6290431i\sqrt{31}) \tanh^{-1} \left( \frac{(-22-4i\sqrt{31})x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}} \sqrt{2x^2-x+3}} \right) - 11109i\sqrt{286 - 22i\sqrt{31}} (6290431 + 4541903i\sqrt{31})$$

Antiderivative was successfully verified.

[In] Integrate[1/((3 - x + 2\*x^2)^(5/2)\*(2 + 3\*x + 5\*x^2)^3), x]



```
[Out] ((5456*(9739335532 + 218659985088*x + 178650961091*x^2 + 519223213785*x^3 +
174241614961*x^4 + 592923725931*x^5 - 12234606480*x^6 + 225699113100*x^7))
/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^2) + 11109*Sqrt[286 + (22*I)*Sqrt
[31]]*(4541903 - (6290431*I)*Sqrt[31])*ArcTanh[(63 + I*Sqrt[31] + (-22 - (4
*I)*Sqrt[31])*x]/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])) - (11
109*I)*Sqrt[286 - (22*I)*Sqrt[31]]*(-4541903*I + 6290431*Sqrt[31])*ArcTanh[
(-63 + I*Sqrt[31] + (22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]
*Sqrt[3 - x + 2*x^2]))]/7796961516715008
```

**fricas [B]** time = 1.99, size = 2343, normalized size = 8.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/611377875290135815296770157063555072*(2164988593398757980*129508224872072
^(1/4)*sqrt(4023497)*sqrt(341)*sqrt(2)*(100*x^8 + 20*x^7 + 321*x^6 + 172*x^
5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36)*sqrt(2243059557247*sqrt(2) + 4
023497000000)*arctan(1/452534011574628261925237033857859439*(11475013444*sq
rt(4023497)*(11*129508224872072^(3/4)*sqrt(341)*(2673027292*x^7 - 117686842
22*x^6 + 24008796626*x^5 - 42687622824*x^4 + 22428040912*x^3 - 12956821056*
x^2 - sqrt(2)*(2612082154*x^7 - 9010050347*x^6 + 19426337114*x^5 - 28170626
609*x^4 + 13394761640*x^3 - 4698131400*x^2 - 17594323200*x + 10110341376) -
20220682752*x + 17594323200) + 124728407*129508224872072^(1/4)*sqrt(341)*(
214583731*x^7 - 3372306249*x^6 + 18434388344*x^5 - 43845503580*x^4 + 576317
17152*x^3 - 41786349984*x^2 - sqrt(2)*(190078101*x^7 - 2862100476*x^6 + 146
88003420*x^5 - 32231022496*x^4 + 40927641120*x^3 - 21959568000*x^2 - 311565
03552*x + 19060075008) - 38120150016*x + 31156503552))*sqrt(2*x^2 - x + 3)*
sqrt(2243059557247*sqrt(2) + 4023497000000) + 12846126780182995822393825477
25536472*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^
5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 +
396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x
- 539136) + 1154304*x - 456192) - 2*sqrt(8046994/10139750351)*(sqrt(402349
7)*(11*129508224872072^(3/4)*sqrt(341)*(8140972152*x^7 - 11907581308*x^6 +
39777303828*x^5 - 24395365568*x^4 + 37103094432*x^3 - 1836165888*x^2 - sqrt
(2)*(10387383478*x^7 - 14753211883*x^6 + 46462095753*x^5 - 11926110640*x^4
+ 8224291080*x^3 + 34793549568*x^2 - 34793549568*x) + 1836165888*x) + 12472
8407*129508224872072^(1/4)*sqrt(341)*(692762453*x^7 - 8972954292*x^6 + 3480
3726780*x^5 - 46915651008*x^4 + 67421983392*x^3 + 10625375232*x^2 - 2*sqrt(
2)*(367903387*x^7 - 4754813452*x^6 + 18261523780*x^5 - 22991417280*x^4 + 27
054001440*x^3 + 26759248128*x^2 - 26759248128*x) - 10625375232*x))*sqrt(2*x
^2 - x + 3)*sqrt(2243059557247*sqrt(2) + 4023497000000) + 11194868609848920
9076292438*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 329307
2*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051
```

$$\begin{aligned}
& *x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - \\
& 1036800*x) + 3276288*x) + 5088576640840418594376929*\sqrt{31}*(254591*x^8 - \\
& 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - \\
& 168956928*x^2 - 15488*\sqrt{2}*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x \\
& ^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*\sqrt{-(129508224872072^( \\
& 1/4)*\sqrt{4023497}*\sqrt{341}*\sqrt{31}*\sqrt{2*x^2 - x + 3}*(\sqrt{2}*(643213* \\
& x + 2195288) - 2838501*x + 1552075)*\sqrt{2243059557247*\sqrt{2} + 4023497000 \\
& 000) - 1921101946251381781783*x^2 - 1725071135409404048948*\sqrt{2}*(2*x^2 - \\
& x + 3) + 5920130487427727531617*x - 7841232433679109313400)/x^2) + 1459787 \\
& 1341117040707265710769608369*\sqrt{31}*(2828123*x^8 - 9696916*x^7 + 53385560 \\
& *x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744* \\
& \sqrt{2}*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 \\
& + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 46612 \\
& 00*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296 \\
& *x^2 - 24772608*x + 18579456)) + 2164988593398757980*129508224872072^(1/4)* \\
& \sqrt{4023497}*\sqrt{341}*\sqrt{2}*(100*x^8 + 20*x^7 + 321*x^6 + 172*x^5 + 390 \\
& *x^4 + 236*x^3 + 241*x^2 + 84*x + 36)*\sqrt{2243059557247*\sqrt{2} + 40234970 \\
& 00000)*\arctan(1/452534011574628261925237033857859439*(11475013444*\sqrt{4023 \\
& 497}*(11*129508224872072^(3/4)*\sqrt{341}*(2673027292*x^7 - 11768684222*x^6 \\
& + 24008796626*x^5 - 42687622824*x^4 + 22428040912*x^3 - 12956821056*x^2 - s \\
& \sqrt{2}*(2612082154*x^7 - 9010050347*x^6 + 19426337114*x^5 - 28170626609*x^4 \\
& + 13394761640*x^3 - 4698131400*x^2 - 17594323200*x + 10110341376) - 202206 \\
& 82752*x + 17594323200) + 124728407*129508224872072^(1/4)*\sqrt{341}*(2145837 \\
& 31*x^7 - 3372306249*x^6 + 18434388344*x^5 - 43845503580*x^4 + 57631717152*x \\
& ^3 - 41786349984*x^2 - \sqrt{2}*(190078101*x^7 - 2862100476*x^6 + 1468800342 \\
& 0*x^5 - 32231022496*x^4 + 40927641120*x^3 - 21959568000*x^2 - 31156503552*x \\
& + 19060075008) - 38120150016*x + 31156503552))*\sqrt{2*x^2 - x + 3}*\sqrt{22 \\
& 43059557247*\sqrt{2} + 4023497000000) - 128461267801829958223938254772553647 \\
& 2*\sqrt{31}*\sqrt{2}*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 154 \\
& 9144*x^4 - 642048*x^3 - 98496*x^2 - \sqrt{2}*(8746*x^8 - 102335*x^7 + 396104 \\
& *x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 5391 \\
& 36) + 1154304*x - 456192) - 2*\sqrt{8046994/10139750351}*(\sqrt{4023497}*(11* \\
& 129508224872072^(3/4)*\sqrt{341}*(8140972152*x^7 - 11907581308*x^6 + 3977730 \\
& 3828*x^5 - 24395365568*x^4 + 37103094432*x^3 - 1836165888*x^2 - \sqrt{2}*(10 \\
& 387383478*x^7 - 14753211883*x^6 + 46462095753*x^5 - 11926110640*x^4 + 82242 \\
& 91080*x^3 + 34793549568*x^2 - 34793549568*x) + 1836165888*x) + 124728407*12 \\
& 9508224872072^(1/4)*\sqrt{341}*(692762453*x^7 - 8972954292*x^6 + 34803726780 \\
& *x^5 - 46915651008*x^4 + 67421983392*x^3 + 10625375232*x^2 - 2*\sqrt{2}*(367 \\
& 903387*x^7 - 4754813452*x^6 + 18261523780*x^5 - 22991417280*x^4 + 270540014 \\
& 40*x^3 + 26759248128*x^2 - 26759248128*x) - 10625375232*x))*\sqrt{2*x^2 - x \\
& + 3}*\sqrt{2243059557247*\sqrt{2} + 4023497000000) - 111948686098489209076292 \\
& 438*\sqrt{31}*\sqrt{2}*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + \\
& 396480*x^4 + 798336*x^3 - 3822336*x^2 - \sqrt{2}*(15550*x^8 - 118051*x^7 + \\
& 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800 \\
& *x) + 3276288*x) - 5088576640840418594376929*\sqrt{31}*(254591*x^8 - 4815126
\end{aligned}$$

```

*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 1689569
28*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 36
18*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt((129508224872072^(1/4)*sqr
t(4023497)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(643213*x + 2195
288) - 2838501*x + 1552075)*sqrt(2243059557247*sqrt(2) + 4023497000000) + 1
921101946251381781783*x^2 + 1725071135409404048948*sqrt(2)*(2*x^2 - x + 3)
- 5920130487427727531617*x + 7841232433679109313400)/x^2) - 145978713411170
40707265710769608369*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 1
42835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*
(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x
^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 +
14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 2
4772608*x + 18579456)) + 55545*129508224872072^(1/4)*sqrt(4023497)*sqrt(341
)*sqrt(31)*(4023497000000000*x^8 + 804699400000000*x^7 + 1291542537000000*x^6
+ 692041484000000*x^5 + 1569163830000000*x^4 + 949545292000000*x^3 + 96966
2777000000*x^2 - 2243059557247*sqrt(2)*(100*x^8 + 20*x^7 + 321*x^6 + 172*x^
5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36) + 337973748000000*x + 14484589
2000000)*sqrt(2243059557247*sqrt(2) + 4023497000000)*log(24643919125000000
0000/10139750351*(129508224872072^(1/4)*sqrt(4023497)*sqrt(341)*sqrt(31)*sq
rt(2*x^2 - x + 3)*(sqrt(2)*(643213*x + 2195288) - 2838501*x + 1552075)*sqrt
(2243059557247*sqrt(2) + 4023497000000) + 1921101946251381781783*x^2 + 1725
071135409404048948*sqrt(2)*(2*x^2 - x + 3) - 5920130487427727531617*x + 784
1232433679109313400)/x^2) - 55545*129508224872072^(1/4)*sqrt(4023497)*sqrt(
341)*sqrt(31)*(4023497000000000*x^8 + 804699400000000*x^7 + 1291542537000000*
x^6 + 692041484000000*x^5 + 1569163830000000*x^4 + 949545292000000*x^3 + 96
9662777000000*x^2 - 2243059557247*sqrt(2)*(100*x^8 + 20*x^7 + 321*x^6 + 172
*x^5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36) + 337973748000000*x + 14484
5892000000)*sqrt(2243059557247*sqrt(2) + 4023497000000)*log(-24643919125000
0000000/10139750351*(129508224872072^(1/4)*sqrt(4023497)*sqrt(341)*sqrt(31
)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(643213*x + 2195288) - 2838501*x + 1552075)*
sqrt(2243059557247*sqrt(2) + 4023497000000) - 1921101946251381781783*x^2 -
1725071135409404048948*sqrt(2)*(2*x^2 - x + 3) + 5920130487427727531617*x -
7841232433679109313400)/x^2) + 427817641581532204139104*(225699113100*x^7
- 12234606480*x^6 + 592923725931*x^5 + 174241614961*x^4 + 519223213785*x^3
+ 178650961091*x^2 + 218659985088*x + 9739335532)*sqrt(2*x^2 - x + 3))/(100
*x^8 + 20*x^7 + 321*x^6 + 172*x^5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36
)

```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^(5/2)/(5\*x^2+3\*x+2)^3,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP  
 UT:sage2:=int(sage0,x):;OUTPUT:Francis algorithm failure for[-1.0,infinity,  
 infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,inf  
 inity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity  
 ]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai  
 lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity  
 ,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf  
 inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinity  
 ]Evaluation time: 70.56Done

**maple** [B] time = 0.21, size = 19014, normalized size = 70.68

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2\*x^2-x+3)^(5/2)/(5\*x^2+3\*x+2)^3,x)

[Out] result too large to display

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(5x^2 + 3x + 2)^3 (2x^2 - x + 3)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^(5/2)/(5\*x^2+3\*x+2)^3,x, algorithm="maxima")

[Out] integrate(1/((5\*x^2 + 3\*x + 2)^3\*(2\*x^2 - x + 3)^(5/2)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(2x^2 - x + 3)^{\frac{5}{2}} (5x^2 + 3x + 2)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((2\*x^2 - x + 3)^(5/2)\*(3\*x + 5\*x^2 + 2)^3),x)

[Out] int(1/((2\*x^2 - x + 3)^(5/2)\*(3\*x + 5\*x^2 + 2)^3), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(2x^2 - x + 3)^{\frac{5}{2}} (5x^2 + 3x + 2)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**(5/2)/(5*x**2+3*x+2)**3,x)
```

```
[Out] Integral(1/((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2)**3), x)
```

### 3.100 $\int \sqrt{a + bx + cx^2} (d + ex + fx^2)^2 dx$

**Optimal.** Leaf size=436

$$\frac{(b^2 - 4ac) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) (8c^2(2a^2f^2 + 12abef + 5b^2(2df + e^2)) - 56b^2cf(af + be) - 32c^3(a(2df + e^2) + 4bde))}{1024c^{11/2}}$$

[Out] 1/960\*(640\*c^3\*d\*e-105\*b^3\*f^2+28\*b\*c\*f\*(7\*a\*f+10\*b\*e)-8\*c^2\*(32\*a\*e\*f+25\*b\*(2\*d\*f+e^2)))\*(c\*x^2+b\*x+a)^(3/2)/c^4+1/160\*(21\*b^2\*f^2-4\*c\*f\*(5\*a\*f+14\*b\*e)+40\*c^2\*(2\*d\*f+e^2))\*x\*(c\*x^2+b\*x+a)^(3/2)/c^3+1/20\*f\*(-3\*b\*f+8\*c\*e)\*x^2\*(c\*x^2+b\*x+a)^(3/2)/c^2+1/6\*f^2\*x^3\*(c\*x^2+b\*x+a)^(3/2)/c-1/1024\*(-4\*a\*c+b^2)\*(128\*c^4\*d^2+21\*b^4\*f^2-56\*b^2\*c\*f\*(a\*f+b\*e)-32\*c^3\*(4\*b\*d\*e+a\*(2\*d\*f+e^2))+8\*c^2\*(12\*a\*b\*e\*f+2\*a^2\*f^2+5\*b^2\*(2\*d\*f+e^2)))\*arctanh(1/2\*(2\*c\*x+b)/c^(1/2)/(c\*x^2+b\*x+a)^(1/2))/c^(11/2)+1/512\*(128\*c^4\*d^2+21\*b^4\*f^2-56\*b^2\*c\*f\*(a\*f+b\*e)-32\*c^3\*(4\*b\*d\*e+a\*(2\*d\*f+e^2))+8\*c^2\*(12\*a\*b\*e\*f+2\*a^2\*f^2+5\*b^2\*(2\*d\*f+e^2)))\*(2\*c\*x+b)\*(c\*x^2+b\*x+a)^(1/2)/c^5

**Rubi [A]** time = 0.79, antiderivative size = 436, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {1661, 640, 612, 621, 206}

$$\frac{(b + 2cx)\sqrt{a + bx + cx^2} (8c^2(2a^2f^2 + 12abef + 5b^2(2df + e^2)) - 56b^2cf(af + be) - 32c^3(a(2df + e^2) + 4bde))}{512c^5}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b\*x + c\*x^2]\*(d + e\*x + f\*x^2)^2,x]

[Out] ((128\*c^4\*d^2 + 21\*b^4\*f^2 - 56\*b^2\*c\*f\*(b\*e + a\*f) - 32\*c^3\*(4\*b\*d\*e + a\*(e^2 + 2\*d\*f)) + 8\*c^2\*(12\*a\*b\*e\*f + 2\*a^2\*f^2 + 5\*b^2\*(e^2 + 2\*d\*f)))\*(b + 2\*c\*x)\*Sqrt[a + b\*x + c\*x^2]/(512\*c^5) + ((640\*c^3\*d\*e - 105\*b^3\*f^2 + 28\*b\*c\*f\*(10\*b\*e + 7\*a\*f) - 8\*c^2\*(32\*a\*e\*f + 25\*b\*(e^2 + 2\*d\*f)))\*(a + b\*x + c\*x^2)^(3/2))/(960\*c^4) + ((21\*b^2\*f^2 - 4\*c\*f\*(14\*b\*e + 5\*a\*f) + 40\*c^2\*(e^2 + 2\*d\*f))\*x\*(a + b\*x + c\*x^2)^(3/2))/(160\*c^3) + (f\*(8\*c\*e - 3\*b\*f)\*x^2\*(a + b\*x + c\*x^2)^(3/2))/(20\*c^2) + (f^2\*x^3\*(a + b\*x + c\*x^2)^(3/2))/(6\*c) - ((b^2 - 4\*a\*c)\*(128\*c^4\*d^2 + 21\*b^4\*f^2 - 56\*b^2\*c\*f\*(b\*e + a\*f) - 32\*c^3\*(4\*b\*d\*e + a\*(e^2 + 2\*d\*f)) + 8\*c^2\*(12\*a\*b\*e\*f + 2\*a^2\*f^2 + 5\*b^2\*(e^2 + 2\*d\*f)))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])]/(1024\*c^(11/2))

**Rule 206**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && Gt

$Q[a, 0] \parallel LtQ[b, 0]$

### Rule 612

$\text{Int}[(a_.) + (b_.)(x_) + (c_.)(x_)^2]^{(p_.)}, x\_Symbol] \rightarrow \text{Simp}[(b + 2cx)(a + bx + cx^2)^p / (2c(2p + 1)), x] - \text{Dist}[(p(b^2 - 4ac)) / (2c(2p + 1)), \text{Int}[(a + bx + cx^2)^{p-1}, x], x] /;$  FreeQ[{a, b, c}, x] && NeQ[b^2 - 4ac, 0] && GtQ[p, 0] && IntegerQ[4p]

### Rule 621

$\text{Int}[1/\text{Sqrt}[(a_) + (b_.)(x_) + (c_.)(x_)^2], x\_Symbol] \rightarrow \text{Dist}[2, \text{Subst}[\text{Int}[1/(4c - x^2), x], x, (b + 2cx)/\text{Sqrt}[a + bx + cx^2]], x] /;$  FreeQ[{a, b, c}, x] && NeQ[b^2 - 4ac, 0]

### Rule 640

$\text{Int}[(d_.) + (e_.)(x_)]((a_.) + (b_.)(x_) + (c_.)(x_)^2)^{(p_.)}, x\_Symbol] \rightarrow \text{Simp}[(e(a + bx + cx^2)^{p+1}) / (2c(p + 1)), x] + \text{Dist}[(2cd - be) / (2c), \text{Int}[(a + bx + cx^2)^p, x], x] /;$  FreeQ[{a, b, c, d, e, p}, x] && NeQ[2cd - be, 0] && NeQ[p, -1]

### Rule 1661

$\text{Int}[(Pq_)((a_.) + (b_.)(x_) + (c_.)(x_)^2)^{(p_.)}, x\_Symbol] \rightarrow \text{With}[\{q = \text{Expon}[Pq, x], e = \text{Coeff}[Pq, x, \text{Expon}[Pq, x]]\}, \text{Simp}[(e x^{q-1})(a + bx + cx^2)^{p+1} / (c(q + 2p + 1)), x] + \text{Dist}[1 / (c(q + 2p + 1)), \text{Int}[(a + bx + cx^2)^p \text{ExpandToSum}[c(q + 2p + 1)Pq - a e^{q-1} x^{q-2} - b e^{q+p} x^{q-1} - c e^{q+2p+1} x^q, x], x], x] /;$  FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4ac, 0] && !LeQ[p, -1]

### Rubi steps

$$\begin{aligned}
\int \sqrt{a+bx+cx^2} (d+ex+fx^2)^2 dx &= \frac{f^2 x^3 (a+bx+cx^2)^{3/2}}{6c} + \frac{\int \sqrt{a+bx+cx^2} (6cd^2+12cdex-3(a^2-2cex^2-2cdx^2)) dx}{6c} \\
&= \frac{f(8ce-3bf)x^2 (a+bx+cx^2)^{3/2}}{20c^2} + \frac{f^2 x^3 (a+bx+cx^2)^{3/2}}{6c} + \frac{\int \sqrt{a+bx+cx^2} (6cd^2+12cdex-3(a^2-2cex^2-2cdx^2)) dx}{6c} \\
&= \frac{(21b^2 f^2 - 4cf(14be + 5af) + 40c^2 (e^2 + 2df)) x (a+bx+cx^2)^{3/2}}{160c^3} + \frac{f(8ce-3bf)x^2 (a+bx+cx^2)^{3/2}}{20c^2} \\
&= \frac{(640c^3 de - 105b^3 f^2 + 28bcf(10be + 7af) - 8c^2 (32aef + 25b(e^2 + 2df))) x (a+bx+cx^2)^{3/2}}{960c^4} + \frac{f(8ce-3bf)x^2 (a+bx+cx^2)^{3/2}}{20c^2} \\
&= \frac{(128c^4 d^2 + 21b^4 f^2 - 56b^2 cf(be + af) - 32c^3 (4bde + a(e^2 + 2df))) + 8c^2 (32aef + 25b(e^2 + 2df))}{512c^5} \\
&= \frac{(128c^4 d^2 + 21b^4 f^2 - 56b^2 cf(be + af) - 32c^3 (4bde + a(e^2 + 2df))) + 8c^2 (32aef + 25b(e^2 + 2df))}{512c^5} \\
&= \frac{(128c^4 d^2 + 21b^4 f^2 - 56b^2 cf(be + af) - 32c^3 (4bde + a(e^2 + 2df))) + 8c^2 (32aef + 25b(e^2 + 2df))}{512c^5}
\end{aligned}$$

**Mathematica [A]** time = 0.93, size = 657, normalized size = 1.51

$$\frac{-f^2 \left( -15(16a^2 c^2 - 56ab^2 c + 21b^4) \left( 2\sqrt{c} (b + 2cx) \sqrt{a + x(b + cx)} - (b^2 - 4ac) \tanh^{-1} \left( \frac{b+2cx}{2\sqrt{c} \sqrt{a+x(b+cx)}} \right) \right) + 16c^{3/2} \right)}{512c^5}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b\*x + c\*x^2]\*(d + e\*x + f\*x^2)^2,x]

[Out] (3840\*c^(9/2)\*d^2\*(b + 2\*c\*x)\*Sqrt[a + x\*(b + c\*x)] + 10240\*c^(9/2)\*d\*e\*(a + x\*(b + c\*x))^(3/2) + 3840\*c^(9/2)\*(e^2 + 2\*d\*f)\*x\*(a + x\*(b + c\*x))^(3/2) + 6144\*c^(9/2)\*e\*f\*x^2\*(a + x\*(b + c\*x))^(3/2) + 2560\*c^(9/2)\*f^2\*x^3\*(a + x\*(b + c\*x))^(3/2) - 1920\*c^4\*(b^2 - 4\*a\*c)\*d^2\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + x\*(b + c\*x)])] - 1920\*b\*c^3\*d\*e\*(2\*Sqrt[c]\*(b + 2\*c\*x)\*Sqrt[a + x\*(b + c\*x)] - (b^2 - 4\*a\*c)\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + x\*(b + c\*x)])]) + 8\*c\*e\*f\*(-16\*c^(3/2)\*(-35\*b^2 + 32\*a\*c + 42\*b\*c\*x)\*(a + x\*(b + c\*x))^(3/2) - 15\*b\*(7\*b^2 - 12\*a\*c)\*(2\*Sqrt[c]\*(b + 2\*c\*x)\*Sqrt[a + x\*(b + c\*x)] - (b^2 - 4\*a\*c)\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + x\*(b + c\*x)])])) - 40\*c^2\*(e^2 + 2\*d\*f)\*(80\*b\*c^(3/2)\*(a + x\*(b + c\*x))^(3/2) - 3\*(5\*b^2 - 4\*a\*c)\*(2\*Sqrt[c]\*(b + 2\*c\*x)\*Sqrt[a + x\*(b + c\*x)] - (b^2 - 4\*a\*c)\*A



$$\operatorname{rcTanh}\left[\frac{b + 2cx}{2\sqrt{c}\sqrt{a + x(b + cx)}}\right] - f^2 \frac{(2304b^7c^{7/2}x^2(a + x(b + cx))^{3/2} + 16c^{3/2}(105b^3 - 196ab^2c - 126b^2cx + 120a^2c^2x)(a + x(b + cx))^{3/2} - 15(21b^4 - 56ab^2c + 16a^2c^2)(2\sqrt{c}(b + 2cx)\sqrt{a + x(b + cx)} - (b^2 - 4ac)\operatorname{ArcTanh}\left[\frac{b + 2cx}{2\sqrt{c}\sqrt{a + x(b + cx)}}\right]))}{(15360c^{11/2})}$$

**fricas** [A] time = 1.32, size = 1269, normalized size = 2.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cx^2+bx+a)^(1/2)\*(f\*x^2+e\*x+d)^2,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & [-1/30720*(15*(128*(b^2c^4 - 4ac^5)d^2 - 128*(b^3c^3 - 4ab^2c^4)d^2e \\ & + 8*(5b^4c^2 - 24ab^2c^3 + 16a^2c^4)e^2 + (21b^6 - 140ab^4c + 240a^2b^2c^2 - 64a^3c^3)f^2 + 8*(2*(5b^4c^2 - 24ab^2c^3 + 16a^2c^4)d \\ & - (7b^5c - 40ab^3c^2 + 48a^2b^2c^3)e)f)\sqrt{c}\log(-8c^2x^2 - 8b^2cx - b^2 - 4\sqrt{c}(cx^2 + bx + a)(2cx + b)\sqrt{c} - 4ac) - \\ & 4*(1280c^6f^2x^5 + 1920b^5c^5d^2 + 128*(24c^6e^2f + b^5c^5f^2)x^4 + 16*(120c^6e^2 - (9b^2c^4 - 20ac^5)f^2 + 24*(10c^6d + b^5c^5e)f)x^3 \\ & - 640*(3b^2c^4 - 8ac^5)d^2e + 40*(15b^3c^3 - 52ab^2c^4)e^2 + (315b^5c - 1680ab^3c^2 + 1808a^2b^2c^3)f^2 + 8*(640c^6d^2e + 40b^5c^5e^2 \\ & + (21b^3c^3 - 68ab^2c^4)f^2 + 8*(10b^5c^5d - (7b^2c^4 - 16ac^5)e)f)x^2 + 8*(10*(15b^3c^3 - 52ab^2c^4)d - (105b^4c^2 - 460ab^2c^3 \\ & + 256a^2c^4)e)f + 2*(1920c^6d^2 + 640b^5c^5d^2e - 40*(5b^2c^4 - 12ac^5)e^2 - (105b^4c^2 - 448ab^2c^3 + 240a^2c^4)f^2 - 8*(10*(5 \\ & b^2c^4 - 12ac^5)d - (35b^3c^3 - 116ab^2c^4)e)f)x)\sqrt{cx^2 + bx + a})/c^6, 1/15360*(15*(128*(b^2c^4 - 4ac^5)d^2 - 128*(b^3c^3 - 4ab^2c^4)d^2e \\ & + 8*(5b^4c^2 - 24ab^2c^3 + 16a^2c^4)e^2 + (21b^6 - 140ab^4c + 240a^2b^2c^2 - 64a^3c^3)f^2 + 8*(2*(5b^4c^2 - 24ab^2c^3 + 16a^2c^4)d \\ & - (7b^5c - 40ab^3c^2 + 48a^2b^2c^3)e)f)\sqrt{-c})\operatorname{arctan}(1/2\sqrt{cx^2 + bx + a}(2cx + b)\sqrt{-c})/(c^2x^2 + b^2cx + ac) \\ & + 2*(1280c^6f^2x^5 + 1920b^5c^5d^2 + 128*(24c^6e^2f + b^5c^5f^2)x^4 + 16*(120c^6e^2 - (9b^2c^4 - 20ac^5)f^2 + 24*(10c^6d + b^5c^5e)f)x^3 \\ & - 640*(3b^2c^4 - 8ac^5)d^2e + 40*(15b^3c^3 - 52ab^2c^4)e^2 + (315b^5c - 1680ab^3c^2 + 1808a^2b^2c^3)f^2 + 8*(640c^6d^2e + 40b^5c^5e^2 \\ & + (21b^3c^3 - 68ab^2c^4)f^2 + 8*(10b^5c^5d - (7b^2c^4 - 16ac^5)e)f)x^2 + 8*(10*(15b^3c^3 - 52ab^2c^4)d - (105b^4c^2 - 460ab^2c^3 \\ & + 256a^2c^4)e)f + 2*(1920c^6d^2 + 640b^5c^5d^2e - 40*(5b^2c^4 - 12ac^5)e^2 - (105b^4c^2 - 448ab^2c^3 + 240a^2c^4)f^2 - 8*(10*(5 \\ & b^2c^4 - 12ac^5)d - (35b^3c^3 - 116ab^2c^4)e)f)x)\sqrt{cx^2 + bx + a})/c^6] \end{aligned}$$

**giac** [A] time = 0.36, size = 638, normalized size = 1.46

$$\frac{1}{7680} \sqrt{cx^2 + bx + a} \left( 2 \left( 4 \left( 2 \left( 8 \left( 10 f^2 x + \frac{bc^4 f^2 + 24 c^5 f e}{c^5} \right) x + \frac{240 c^5 d f - 9 b^2 c^3 f^2 + 20 a c^4 f^2 + 24 b c^4 f e + 120}{c^5} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(1/2)\*(f\*x^2+e\*x+d)^2,x, algorithm="giac")

[Out] 1/7680\*sqrt(c\*x^2 + b\*x + a)\*(2\*(4\*(2\*(8\*(10\*f^2\*x + (b\*c^4\*f^2 + 24\*c^5\*f\*e)/c^5)\*x + (240\*c^5\*d\*f - 9\*b^2\*c^3\*f^2 + 20\*a\*c^4\*f^2 + 24\*b\*c^4\*f\*e + 120\*c^5\*e^2)/c^5)\*x + (80\*b\*c^4\*d\*f + 21\*b^3\*c^2\*f^2 - 68\*a\*b\*c^3\*f^2 + 640\*c^5\*d\*e - 56\*b^2\*c^3\*f\*e + 128\*a\*c^4\*f\*e + 40\*b\*c^4\*e^2)/c^5)\*x + (1920\*c^5\*d^2 - 400\*b^2\*c^3\*d\*f + 960\*a\*c^4\*d\*f - 105\*b^4\*c\*f^2 + 448\*a\*b^2\*c^2\*f^2 - 240\*a^2\*c^3\*f^2 + 640\*b\*c^4\*d\*e + 280\*b^3\*c^2\*f\*e - 928\*a\*b\*c^3\*f\*e - 200\*b^2\*c^3\*e^2 + 480\*a\*c^4\*e^2)/c^5)\*x + (1920\*b\*c^4\*d^2 + 1200\*b^3\*c^2\*d\*f - 4160\*a\*b\*c^3\*d\*f + 315\*b^5\*f^2 - 1680\*a\*b^3\*c\*f^2 + 1808\*a^2\*b\*c^2\*f^2 - 1920\*b^2\*c^3\*d\*e + 5120\*a\*c^4\*d\*e - 840\*b^4\*c\*f\*e + 3680\*a\*b^2\*c^2\*f\*e - 2048\*a^2\*c^3\*f\*e + 600\*b^3\*c^2\*e^2 - 2080\*a\*b\*c^3\*e^2)/c^5) + 1/1024\*(128\*b^2\*c^4\*d^2 - 512\*a\*c^5\*d^2 + 80\*b^4\*c^2\*d\*f - 384\*a\*b^2\*c^3\*d\*f + 256\*a^2\*c^4\*d\*f + 21\*b^6\*f^2 - 140\*a\*b^4\*c\*f^2 + 240\*a^2\*b^2\*c^2\*f^2 - 64\*a^3\*c^3\*f^2 - 128\*b^3\*c^3\*d\*e + 512\*a\*b\*c^4\*d\*e - 56\*b^5\*c\*f\*e + 320\*a\*b^3\*c^2\*f\*e - 384\*a^2\*b\*c^3\*f\*e + 40\*b^4\*c^2\*e^2 - 192\*a\*b^2\*c^3\*e^2 + 128\*a^2\*c^4\*e^2)\*log(abs(-2\*(sqrt(c)\*x - sqrt(c\*x^2 + b\*x + a))\*sqrt(c) - b))/c^(11/2)

**maple** [B] time = 0.02, size = 1429, normalized size = 3.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+b\*x+a)^(1/2)\*(f\*x^2+e\*x+d)^2,x)

[Out] 1/6\*f^2\*x^3\*(c\*x^2+b\*x+a)^(3/2)/c-7/32\*f^2\*b^2/c^3\*a\*x\*(c\*x^2+b\*x+a)^(1/2)-7/20\*e\*f\*b/c^2\*x\*(c\*x^2+b\*x+a)^(3/2)-7/32\*e\*f\*b^3/c^3\*x\*(c\*x^2+b\*x+a)^(1/2)+2/3\*d\*e\*(c\*x^2+b\*x+a)^(3/2)/c+1/4\*d^2/c\*(c\*x^2+b\*x+a)^(1/2)\*b+1/2\*d^2/c^(1/2)\*ln((1/2\*b+c\*x)/c^(1/2)+(c\*x^2+b\*x+a)^(1/2))\*a-1/8\*d^2/c^(3/2)\*ln((1/2\*b+c\*x)/c^(1/2)+(c\*x^2+b\*x+a)^(1/2))\*b^2+1/4\*x\*(c\*x^2+b\*x+a)^(3/2)/c\*e^2-5/24\*b/c^2\*(c\*x^2+b\*x+a)^(3/2)\*e^2+5/64\*b^3/c^3\*(c\*x^2+b\*x+a)^(1/2)\*e^2-5/128\*b^4/c^(7/2)\*ln((1/2\*b+c\*x)/c^(1/2)+(c\*x^2+b\*x+a)^(1/2))\*e^2-1/8\*a^2/c^(3/2)\*ln((1/2\*b+c\*x)/c^(1/2)+(c\*x^2+b\*x+a)^(1/2))\*e^2+21/512\*f^2\*b^5/c^5\*(c\*x^2+b\*x+a)^(1/2)-21/1024\*f^2\*b^6/c^(11/2)\*ln((1/2\*b+c\*x)/c^(1/2)+(c\*x^2+b\*x+a)^(1/2))+1/16\*f^2\*a^3/c^(5/2)\*ln((1/2\*b+c\*x)/c^(1/2)+(c\*x^2+b\*x+a)^(1/2))-7/64\*f^2\*b^3/c^4\*(c\*x^2+b\*x+a)^(3/2)+3/8\*e\*f\*b/c^2\*a\*x\*(c\*x^2+b\*x+a)^(1/2)-5/16\*e\*f\*b^3/c^(7/2)\*ln((1/2\*b+c\*x)/c^(1/2)+(c\*x^2+b\*x+a)^(1/2))\*a+3/16\*e\*f\*b^2

$$\begin{aligned} & /c^3 a (c x^2 + b x + a)^{1/2} + 3/8 e f b /c^{5/2} a^2 \ln((1/2 b + c x)/c^{1/2} + (c x^2 + b x + a)^{1/2}) \\ & + 5/16 b^2 /c^2 x (c x^2 + b x + a)^{1/2} * d f + 3/8 b^2 /c^{5/2} \ln \\ & ((1/2 b + c x)/c^{1/2} + (c x^2 + b x + a)^{1/2}) * a d f - 1/4 a /c x (c x^2 + b x + a)^{1/2} \\ & * d f - 1/8 a /c^2 (c x^2 + b x + a)^{1/2} * b d f - 1/2 d e b /c x (c x^2 + b x + a)^{1/2} \\ & - 1/2 d e b /c^{3/2} \ln((1/2 b + c x)/c^{1/2} + (c x^2 + b x + a)^{1/2}) * a + 1/2 d^2 x \\ & * (c x^2 + b x + a)^{1/2} + 21/256 f^2 b^4 /c^4 x (c x^2 + b x + a)^{1/2} - 5/12 b /c^2 (c \\ & * x^2 + b x + a)^{3/2} * d f + 5/32 b^2 /c^2 x (c x^2 + b x + a)^{1/2} * e^2 + 5/32 b^3 /c^3 (c \\ & * x^2 + b x + a)^{1/2} * d f + 3/16 b^2 /c^{5/2} \ln((1/2 b + c x)/c^{1/2} + (c x^2 + b x + a)^{1/2}) \\ & * a * e^2 - 5/64 b^4 /c^{7/2} \ln((1/2 b + c x)/c^{1/2} + (c x^2 + b x + a)^{1/2}) \\ & * d f - 1/8 a /c x (c x^2 + b x + a)^{1/2} * e^2 - 1/16 a /c^2 (c x^2 + b x + a)^{1/2} * b * e^2 \\ & - 1/4 a^2 /c^{3/2} \ln((1/2 b + c x)/c^{1/2} + (c x^2 + b x + a)^{1/2}) * d f - 1/4 d e b^2 \\ & /c^2 (c x^2 + b x + a)^{1/2} + 35/256 f^2 b^4 /c^{9/2} \ln((1/2 b + c x)/c^{1/2} + (c x^2 + b x + a)^{1/2}) \\ & * a - 7/64 f^2 b^3 /c^4 a (c x^2 + b x + a)^{1/2} - 15/64 f^2 b^2 /c^{7/2} a^2 \ln((1/2 b + c x)/c^{1/2} + (c x^2 + b x + a)^{1/2}) \\ & + 49/240 f^2 b /c^3 a (c x^2 + b x + a)^{3/2} - 1/8 f^2 a /c^2 x (c x^2 + b x + a)^{3/2} + 1/16 f^2 a^2 /c^2 x (c \\ & * x^2 + b x + a)^{1/2} + 1/32 f^2 a^2 /c^3 (c x^2 + b x + a)^{1/2} * b + 2/5 e f x^2 (c x^2 \\ & + b x + a)^{3/2} /c + 7/24 e f b^2 /c^3 (c x^2 + b x + a)^{3/2} - 7/64 e f b^4 /c^4 (c x^2 \\ & + b x + a)^{1/2} + 7/128 e f b^5 /c^{9/2} \ln((1/2 b + c x)/c^{1/2} + (c x^2 + b x + a)^{1/2}) \\ & - 4/15 e f a /c^2 (c x^2 + b x + a)^{3/2} - 3/20 f^2 b /c^2 x^2 (c x^2 + b x + a)^{3/2} + 21/160 f^2 b^2 /c^3 x (c x^2 + b x + a)^{3/2} \\ & + 1/2 x (c x^2 + b x + a)^{3/2} /c d f + 1/8 d e b^3 /c^{5/2} \ln((1/2 b + c x)/c^{1/2} + (c x^2 + b x + a)^{1/2}) \end{aligned}$$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(1/2)\*(f\*x^2+e\*x+d)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details)Is 4\*a\*c-b^2 positive, negative or zero?

**mupad** [B] time = 5.31, size = 1299, normalized size = 2.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x + c\*x^2)^(1/2)\*(d + e\*x + f\*x^2)^2,x)

[Out]  $d^2(x/2 + b/(4c))(a + b x + c x^2)^{1/2} + (e^2 x (a + b x + c x^2)^{3/2})/(4c) + (a f^2 ((5 b ((\log((b + 2 c x)/c^{1/2} + 2(a + b x + c x^2)^{1/2})) * (b^3 - 4 a b c)) / (16 c^{5/2})) + ((8 c (a + c x^2) - 3 b^2 + 2 b c x) * (a + b x + c x^2)^{1/2}) / (24 c^2))) / (8 c) - (x (a + b x + c x^2)^{3/2}) / (4 c)$

```

+ (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) +
(a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(2*c) - (3*b
*f^2*((7*b*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^
3 - 4*a*b*c)))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x
+ c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*(
(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b
*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(10*c) - (2*a*((log
((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c)))/(16*c^(5
/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^
2)))/(5*c) + (x^2*(a + b*x + c*x^2)^(3/2))/(5*c)))/(4*c) + (f^2*x^3*(a + b*
x + c*x^2)^(3/2))/(6*c) - (a*e^2*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) +
(log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3
/2))))/(4*c) + (d^2*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c
- b^2/4))/(2*c^(3/2)) - (5*b*e^2*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x +
c*x^2)^(1/2))*(b^3 - 4*a*b*c)))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 +
2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (4*a*e*f*((log((b + 2*c
*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c)))/(16*c^(5/2)) + ((
8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(5*c
) - (5*b*d*f*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 -
4*a*b*c)))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*
x^2)^(1/2))/(24*c^2)))/(4*c) + (d*e*(8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a
+ b*x + c*x^2)^(1/2))/(12*c^2) + (d*f*x*(a + b*x + c*x^2)^(3/2))/(2*c) + (7
*b*e*f*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 -
4*a*b*c)))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*
x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*((x/2
+ b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x +
c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(5*c) + (2*e*f*x^2*(a +
b*x + c*x^2)^(3/2))/(5*c) - (a*d*f*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2)
) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c
^(3/2))))/(2*c) + (d*e*log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))
*(b^3 - 4*a*b*c))/(8*c^(5/2))

```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + bx + cx^2} (d + ex + fx^2)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+b\*x+a)\*\*(1/2)\*(f\*x\*\*2+e\*x+d)\*\*2,x)

[Out] Integral(sqrt(a + b\*x + c\*x\*\*2)\*(d + e\*x + f\*x\*\*2)\*\*2, x)

### 3.101 $\int \sqrt{a + bx + cx^2} (d + ex + fx^2) dx$

**Optimal.** Leaf size=175

$$\frac{(b^2 - 4ac) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) (-4c(af + 2be) + 5b^2f + 16c^2d)}{128c^{7/2}} + \frac{(b + 2cx)\sqrt{a + bx + cx^2} (-4acf + 5b^2f - 8bce + 16c^2d)}{64c^3}$$

[Out] 1/24\*(-5\*b\*f+8\*c\*e)\*(c\*x^2+b\*x+a)^(3/2)/c^2+1/4\*f\*x\*(c\*x^2+b\*x+a)^(3/2)/c-1/128\*(-4\*a\*c+b^2)\*(16\*c^2\*d+5\*b^2\*f-4\*c\*(a\*f+2\*b\*e))\*arctanh(1/2\*(2\*c\*x+b)/c^(1/2)/(c\*x^2+b\*x+a)^(1/2))/c^(7/2)+1/64\*(-4\*a\*c\*f+5\*b^2\*f-8\*b\*c\*e+16\*c^2\*d)\*(2\*c\*x+b)\*(c\*x^2+b\*x+a)^(1/2)/c^3

**Rubi [A]** time = 0.16, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {1661, 640, 612, 621, 206}

$$\frac{(b + 2cx)\sqrt{a + bx + cx^2} (-4acf + 5b^2f - 8bce + 16c^2d)}{64c^3} - \frac{(b^2 - 4ac) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) (-4c(af + 2be) + 5b^2f + 16c^2d)}{128c^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b\*x + c\*x^2]\*(d + e\*x + f\*x^2), x]

[Out] ((16\*c^2\*d - 8\*b\*c\*e + 5\*b^2\*f - 4\*a\*c\*f)\*(b + 2\*c\*x)\*Sqrt[a + b\*x + c\*x^2])/(64\*c^3) + ((8\*c\*e - 5\*b\*f)\*(a + b\*x + c\*x^2)^(3/2))/(24\*c^2) + (f\*x\*(a + b\*x + c\*x^2)^(3/2))/(4\*c) - ((b^2 - 4\*a\*c)\*(16\*c^2\*d + 5\*b^2\*f - 4\*c\*(2\*b\*e + a\*f))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(128\*c^(7/2))

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 612

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

#### Rule 621

```
Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 640

```
Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

### Rule 1661

```
Int[(Pq_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a + b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

### Rubi steps

$$\begin{aligned} \int \sqrt{a + bx + cx^2} (d + ex + fx^2) dx &= \frac{fx(a + bx + cx^2)^{3/2}}{4c} + \frac{\int \left(4cd - af + \frac{1}{2}(8ce - 5bf)x\right) \sqrt{a + bx + cx^2} dx}{4c} \\ &= \frac{(8ce - 5bf)(a + bx + cx^2)^{3/2}}{24c^2} + \frac{fx(a + bx + cx^2)^{3/2}}{4c} + \frac{(16c^2d - 8bce + 5bf^2)}{4c} \sqrt{a + bx + cx^2} \\ &= \frac{(16c^2d - 8bce + 5b^2f - 4acf)(b + 2cx)\sqrt{a + bx + cx^2}}{64c^3} + \frac{(8ce - 5bf)(a + bx + cx^2)^{3/2}}{24c^2} \\ &= \frac{(16c^2d - 8bce + 5b^2f - 4acf)(b + 2cx)\sqrt{a + bx + cx^2}}{64c^3} + \frac{(8ce - 5bf)(a + bx + cx^2)^{3/2}}{24c^2} \\ &= \frac{(16c^2d - 8bce + 5b^2f - 4acf)(b + 2cx)\sqrt{a + bx + cx^2}}{64c^3} + \frac{(8ce - 5bf)(a + bx + cx^2)^{3/2}}{24c^2} \end{aligned}$$

**Mathematica [A]** time = 0.27, size = 173, normalized size = 0.99

$$\frac{2\sqrt{c}\sqrt{a + x(b + cx)}(4bc(2c(6d + 2ex + fx^2) - 13af) + 8c^2(a(8e + 3fx) + 2cx(6d + 4ex + 3fx^2)) + 15b^3f - 2b^2c^2)}{384c^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b\*x + c\*x^2]\*(d + e\*x + f\*x^2),x]

[Out] (2\*Sqrt[c]\*Sqrt[a + x\*(b + c\*x)]\*(15\*b^3\*f - 2\*b^2\*c\*(12\*e + 5\*f\*x) + 4\*b\*c\*(-13\*a\*f + 2\*c\*(6\*d + 2\*e\*x + f\*x^2)) + 8\*c^2\*(a\*(8\*e + 3\*f\*x) + 2\*c\*x\*(6\*d + 4\*e\*x + 3\*f\*x^2))) - 3\*(b^2 - 4\*a\*c)\*(16\*c^2\*d + 5\*b^2\*f - 4\*c\*(2\*b\*e + a\*f))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + x\*(b + c\*x)])]/(384\*c^(7/2))

**fricas** [A] time = 0.77, size = 465, normalized size = 2.66

$$\frac{3 \left( 16 (b^2 c^2 - 4 a c^3) d - 8 (b^3 c - 4 a b c^2) e + (5 b^4 - 24 a b^2 c + 16 a^2 c^2) f \right) \sqrt{c} \log \left( -8 c^2 x^2 - 8 b c x - b^2 + 4 \sqrt{c x^2 + b x + a} \right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(1/2)\*(f\*x^2+e\*x+d),x, algorithm="fricas")

[Out] [1/768\*(3\*(16\*(b^2\*c^2 - 4\*a\*c^3)\*d - 8\*(b^3\*c - 4\*a\*b\*c^2)\*e + (5\*b^4 - 24\*a\*b^2\*c + 16\*a^2\*c^2)\*f)\*sqrt(c)\*log(-8\*c^2\*x^2 - 8\*b\*c\*x - b^2 + 4\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(c) - 4\*a\*c) + 4\*(48\*c^4\*f\*x^3 + 48\*b\*c^3\*d + 8\*(8\*c^4\*e + b\*c^3\*f)\*x^2 - 8\*(3\*b^2\*c^2 - 8\*a\*c^3)\*e + (15\*b^3\*c - 52\*a\*b\*c^2)\*f + 2\*(48\*c^4\*d + 8\*b\*c^3\*e - (5\*b^2\*c^2 - 12\*a\*c^3)\*f)\*x)\*sqrt(c\*x^2 + b\*x + a))/c^4, 1/384\*(3\*(16\*(b^2\*c^2 - 4\*a\*c^3)\*d - 8\*(b^3\*c - 4\*a\*b\*c^2)\*e + (5\*b^4 - 24\*a\*b^2\*c + 16\*a^2\*c^2)\*f)\*sqrt(-c)\*arctan(1/2\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(-c)/(c^2\*x^2 + b\*c\*x + a\*c)) + 2\*(48\*c^4\*f\*x^3 + 48\*b\*c^3\*d + 8\*(8\*c^4\*e + b\*c^3\*f)\*x^2 - 8\*(3\*b^2\*c^2 - 8\*a\*c^3)\*e + (15\*b^3\*c - 52\*a\*b\*c^2)\*f + 2\*(48\*c^4\*d + 8\*b\*c^3\*e - (5\*b^2\*c^2 - 12\*a\*c^3)\*f)\*x)\*sqrt(c\*x^2 + b\*x + a))/c^4]

**giac** [A] time = 0.30, size = 212, normalized size = 1.21

$$\frac{1}{192} \sqrt{c x^2 + b x + a} \left( 2 \left( 4 \left( 6 f x + \frac{b c^2 f + 8 c^3 e}{c^3} \right) x + \frac{48 c^3 d - 5 b^2 c f + 12 a c^2 f + 8 b c^2 e}{c^3} \right) x + \frac{48 b c^2 d + 15 b^3 f - 52 a b c^2 f - 24 b^2 c^2 e + 64 a^2 c^2 e}{c^3} \right) + \frac{1}{128} (16 b^2 c^2 d - 64 a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(1/2)\*(f\*x^2+e\*x+d),x, algorithm="giac")

[Out] 1/192\*sqrt(c\*x^2 + b\*x + a)\*(2\*(4\*(6\*f\*x + (b\*c^2\*f + 8\*c^3\*e)/c^3)\*x + (48\*c^3\*d - 5\*b^2\*c\*f + 12\*a\*c^2\*f + 8\*b\*c^2\*e)/c^3)\*x + (48\*b\*c^2\*d + 15\*b^3\*f - 52\*a\*b\*c\*f - 24\*b^2\*c\*e + 64\*a\*c^2\*e)/c^3) + 1/128\*(16\*b^2\*c^2\*d - 64\*a

$*c^3*d + 5*b^4*f - 24*a*b^2*c*f + 16*a^2*c^2*f - 8*b^3*c*e + 32*a*b*c^2*e)*$   
 $\log(\text{abs}(-2*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))*\text{sqrt}(c) - b))/c^{(7/2)}$

**maple [B]** time = 0.01, size = 453, normalized size = 2.59

$$-\frac{a^2 f \ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}} + \sqrt{cx^2 + bx + a}\right)}{8c^{\frac{3}{2}}} + \frac{3ab^2 f \ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}} + \sqrt{cx^2 + bx + a}\right)}{16c^{\frac{5}{2}}} - \frac{abe \ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}} + \sqrt{cx^2 + bx + a}\right)}{4c^{\frac{3}{2}}} + \frac{ad \ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}} + \sqrt{cx^2 + bx + a}\right)}{2c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x)`

[Out]  $1/4*f*x*(c*x^2+b*x+a)^{(3/2)}/c - 5/24*f*b/c^2*(c*x^2+b*x+a)^{(3/2)} + 5/32*f*b^2/c^2*(c*x^2+b*x+a)^{(1/2)}*x + 5/64*f*b^3/c^3*(c*x^2+b*x+a)^{(1/2)} + 3/16*f*b^2/c^{(5/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})$   
 $*a - 5/128*f*b^4/c^{(7/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)}) - 1/8*f*a/c*(c*x^2+b*x+a)^{(1/2)}*x - 1/16*f*a/c^2*(c*x^2+b*x+a)^{(1/2)}*b - 1/8*f*a^2/c^{(3/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})$   
 $+ 1/3*e*(c*x^2+b*x+a)^{(3/2)}/c - 1/4*e*b/c*(c*x^2+b*x+a)^{(1/2)}*x - 1/8*e*b^2/c^2*(c*x^2+b*x+a)^{(1/2)} - 1/4*e*b/c^{(3/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})$   
 $*a + 1/16*e*b^3/c^{(5/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)}) + 1/2*d*(c*x^2+b*x+a)^{(1/2)}*x + 1/4*d/c*(c*x^2+b*x+a)^{(1/2)}*b + 1/2*d/c^{(1/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})$   
 $*a - 1/8*d/c^{(3/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*b^2$

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details) Is 4\*a\*c-b^2 positive, negative or zero?

**mupad [B]** time = 3.91, size = 320, normalized size = 1.83

$$d\left(\frac{x}{2} + \frac{b}{4c}\right)\sqrt{cx^2 + bx + a} - \frac{af\left(\left(\frac{x}{2} + \frac{b}{4c}\right)\sqrt{cx^2 + bx + a} + \frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}} + \sqrt{cx^2 + bx + a}\right)\left(ac - \frac{b^2}{4}\right)}{2c^{3/2}}\right)}{4c} + \frac{d\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}} + \sqrt{cx^2 + bx + a}\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2),x)`

[Out]  $d*(x/2 + b/(4*c))*(a + b*x + c*x^2)^{(1/2)} - (a*f*((x/2 + b/(4*c))*(a + b*x + c*x^2)^{(1/2)} + (\log((b/2 + c*x)/c^{(1/2)} + (a + b*x + c*x^2)^{(1/2)))*(a*c - b^2/4))/(2*c^{(3/2)})))/(4*c) + (d*\log((b/2 + c*x)/c^{(1/2)} + (a + b*x + c*x^2)^{(1/2)))*(a*c - b^2/4))/(2*c^{(3/2)}) + (e*\log((b + 2*c*x)/c^{(1/2)} + 2*(a + b*x + c*x^2)^{(1/2)))*(b^3 - 4*a*b*c))/(16*c^{(5/2)}) - (5*b*f*((\log((b + 2*c*x)/c^{(1/2)} + 2*(a + b*x + c*x^2)^{(1/2)))*(b^3 - 4*a*b*c))/(16*c^{(5/2)}) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^{(1/2)))/(24*c^2)))/(8*c) + (e*(8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^{(1/2)))/(24*c^2) + (f*x*(a + b*x + c*x^2)^{(3/2)))/(4*c)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + bx + cx^2} (d + ex + fx^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+b*x+a)**(1/2)*(f*x**2+e*x+d),x)`

[Out] `Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2), x)`

$$3.102 \quad \int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2} dx$$

**Optimal.** Leaf size=431

$$\frac{\sqrt{f(2af - b(e - \sqrt{e^2 - 4df})) + c(-e\sqrt{e^2 - 4df} - 2df + e^2)} \tanh^{-1} \left( \frac{4af + 2x(bf - c(e - \sqrt{e^2 - 4df})) - b(e - \sqrt{e^2 - 4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf}} \right)}{\sqrt{2}f\sqrt{e^2 - 4df}}$$

[Out] arctanh(1/2\*(2\*c\*x+b)/c^(1/2)/(c\*x^2+b\*x+a)^(1/2))\*c^(1/2)/f-1/2\*arctanh(1/4\*(4\*a\*f+2\*x\*(b\*f-c\*(e-(-4\*d\*f+e^2)^(1/2))))-b\*(e-(-4\*d\*f+e^2)^(1/2)))\*2^(1/2)/(c\*x^2+b\*x+a)^(1/2)/(c\*e^2-2\*c\*d\*f-b\*e\*f+2\*a\*f^2-(-b\*f+c\*e)\*(-4\*d\*f+e^2)^(1/2))^(1/2))\*(f\*(2\*a\*f-b\*(e-(-4\*d\*f+e^2)^(1/2)))+(-(-4\*d\*f+e^2)^(1/2)\*e-2\*d\*f+e^2)\*c)^(1/2)/f\*2^(1/2)/(-4\*d\*f+e^2)^(1/2)+1/2\*arctanh(1/4\*(4\*a\*f-b\*(e+(-4\*d\*f+e^2)^(1/2))+2\*x\*(b\*f-c\*(e+(-4\*d\*f+e^2)^(1/2))))\*2^(1/2)/(c\*x^2+b\*x+a)^(1/2)/(c\*e^2-2\*c\*d\*f-b\*e\*f+2\*a\*f^2+(-b\*f+c\*e)\*(-4\*d\*f+e^2)^(1/2))^(1/2))\*((( -4\*d\*f+e^2)^(1/2)\*e-2\*d\*f+e^2)\*c+f\*(2\*a\*f-b\*(e+(-4\*d\*f+e^2)^(1/2))))^(1/2)/f\*2^(1/2)/(-4\*d\*f+e^2)^(1/2)

**Rubi [A]** time = 1.05, antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {989, 621, 206, 1032, 724}

$$\frac{\sqrt{f(2af - b(e - \sqrt{e^2 - 4df})) + c(-e\sqrt{e^2 - 4df} - 2df + e^2)} \tanh^{-1} \left( \frac{4af + 2x(bf - c(e - \sqrt{e^2 - 4df})) - b(e - \sqrt{e^2 - 4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf}} \right)}{\sqrt{2}f\sqrt{e^2 - 4df}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b\*x + c\*x^2]/(d + e\*x + f\*x^2), x]

[Out] (Sqrt[c]\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/f - (Sqrt[c\*(e^2 - 2\*d\*f - e\*Sqrt[e^2 - 4\*d\*f]) + f\*(2\*a\*f - b\*(e - Sqrt[e^2 - 4\*d\*f]))]\*ArcTanh[(4\*a\*f - b\*(e - Sqrt[e^2 - 4\*d\*f]) + 2\*(b\*f - c\*(e - Sqrt[e^2 - 4\*d\*f]))]\*x)/(2\*Sqrt[2]\*Sqrt[c\*e^2 - 2\*c\*d\*f - b\*e\*f + 2\*a\*f^2 - (c\*e - b\*f)\*Sqrt[e^2 - 4\*d\*f]]\*Sqrt[a + b\*x + c\*x^2]))/(Sqrt[2]\*f\*Sqrt[e^2 - 4\*d\*f]) + (Sqrt[c\*(e^2 - 2\*d\*f + e\*Sqrt[e^2 - 4\*d\*f]) + f\*(2\*a\*f - b\*(e + Sqrt[e^2 - 4\*d\*f]))]\*ArcTanh[(4\*a\*f - b\*(e + Sqrt[e^2 - 4\*d\*f]) + 2\*(b\*f - c\*(e + Sqrt[e^2 - 4\*d\*f]))]\*x)/(2\*Sqrt[2]\*Sqrt[c\*e^2 - 2\*c\*d\*f - b\*e\*f + 2\*a\*f^2 + (c\*e - b\*f)\*Sqrt[e^2 - 4\*d\*f]]\*Sqrt[a + b\*x + c\*x^2]))/(Sqrt[2]\*f\*Sqrt[e^2 - 4\*d\*f])

Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 621

Int[1/Sqrt[(a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 724

Int[1/(((d\_) + (e\_)\*(x\_))\*Sqrt[(a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

### Rule 989

Int[Sqrt[(a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2]/((d\_) + (e\_)\*(x\_) + (f\_)\*(x\_)^2), x\_Symbol] := Dist[c/f, Int[1/Sqrt[a + b\*x + c\*x^2], x], x] - Dist[1/f, Int[(c\*d - a\*f + (c\*e - b\*f)\*x)/(Sqrt[a + b\*x + c\*x^2]\*(d + e\*x + f\*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0]

### Rule 1032

Int[((g\_) + (h\_)\*(x\_))/(((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)\*Sqrt[(d\_) + (e\_)\*(x\_) + (f\_)\*(x\_)^2]), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(2\*c\*g - h\*(b - q))/q, Int[1/((b - q + 2\*c\*x)\*Sqrt[d + e\*x + f\*x^2]), x], x] - Dist[(2\*c\*g - h\*(b + q))/q, Int[1/((b + q + 2\*c\*x)\*Sqrt[d + e\*x + f\*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && PosQ[b^2 - 4\*a\*c]

### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2} dx &= -\frac{\int \frac{cd-af+(ce-bf)x}{\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx}{f} + \frac{c \int \frac{1}{\sqrt{a+bx+cx^2}} dx}{f} \\
&= \frac{(2c) \text{Subst}\left(\int \frac{1}{4c-x^2} dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f} - \frac{(2f(cd-af) - (ce-bf)(e - \sqrt{e^2-4df})) \int \frac{1}{(e-\sqrt{e^2-4df})}}{f\sqrt{e^2-4df}} \\
&= \frac{\sqrt{c} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f} + \frac{(2(2f(cd-af) - (ce-bf)(e - \sqrt{e^2-4df}))) \text{Subst}\left(\int \frac{1}{16a}\right)}{f\sqrt{e^2-4df}} \\
&= \frac{\sqrt{c} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f} - \frac{\sqrt{c}(e^2 - 2df - e\sqrt{e^2-4df}) + f(2af - b(e - \sqrt{e^2-4df}))}{\sqrt{2}f\sqrt{e^2-4df}}
\end{aligned}$$

**Mathematica [A]** time = 1.22, size = 417, normalized size = 0.97

$$\frac{\sqrt{f(2af - b(\sqrt{e^2-4df} + e)) + c(e\sqrt{e^2-4df} - 2df + e^2)} \tanh^{-1}\left(\frac{4af - b(\sqrt{e^2-4df} + e) - 2cx(\sqrt{e^2-4df} + e)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{f(2af - b(\sqrt{e^2-4df} + e)) + c(e\sqrt{e^2-4df} - 2df + e^2)}}\right)}{\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{f(2af - b(\sqrt{e^2-4df} + e)) + c(e\sqrt{e^2-4df} - 2df + e^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b\*x + c\*x^2]/(d + e\*x + f\*x^2), x]

[Out] (Sqrt[c]\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + x\*(b + c\*x)])])/f + (Sqrt[c\*(e^2 - 2\*d\*f + e\*Sqrt[e^2 - 4\*d\*f]) + f\*(2\*a\*f - b\*(e + Sqrt[e^2 - 4\*d\*f]))]\*ArcTanh[(4\*a\*f - 2\*c\*(e + Sqrt[e^2 - 4\*d\*f])\*x - b\*(e + Sqrt[e^2 - 4\*d\*f] - 2\*f\*x))/(2\*Sqrt[2]\*Sqrt[c\*(e^2 - 2\*d\*f + e\*Sqrt[e^2 - 4\*d\*f]) + f\*(2\*a\*f - b\*(e + Sqrt[e^2 - 4\*d\*f]))]\*Sqrt[a + x\*(b + c\*x)]] - Sqrt[f\*(-(b\*e) + 2\*a\*f + b\*Sqrt[e^2 - 4\*d\*f]) + c\*(e^2 - 2\*d\*f - e\*Sqrt[e^2 - 4\*d\*f])]\*ArcTanh[(4\*a\*f + 2\*c\*(-e + Sqrt[e^2 - 4\*d\*f])\*x + b\*(-e + Sqrt[e^2 - 4\*d\*f] + 2\*f\*x))/(2\*Sqrt[2]\*Sqrt[f\*(-(b\*e) + 2\*a\*f + b\*Sqrt[e^2 - 4\*d\*f]) + c\*(e^2 - 2\*d\*f - e\*Sqrt[e^2 - 4\*d\*f])]\*Sqrt[a + x\*(b + c\*x)]])/(Sqrt[2]\*f\*Sqrt[e^2 - 4\*d\*f])

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(1/2)/(f\*x^2+e\*x+d),x, algorithm="fricas")

[Out] Timed out

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(1/2)/(f\*x^2+e\*x+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP  
UT:sage2:=int(sage0,x)::OUTPUT:sym2poly/r2sym(const gen & e,const index\_m &  
i,const vecteur & l) Error: Bad Argument Value

**maple** [B] time = 0.05, size = 6019, normalized size = 13.97

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+b\*x+a)^(1/2)/(f\*x^2+e\*x+d),x)

[Out] result too large to display

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(1/2)/(f\*x^2+e\*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a  
dditional constraints; using the 'assume' command before evaluation \*may\* h  
elp (example of legal syntax is 'assume(4\*d\*f-e^2>0)', see `assume?` for mo  
re details)Is 4\*d\*f-e^2 positive, negative or zero?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{cx^2 + bx + a}}{fx^2 + ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x + c\*x^2)^(1/2)/(d + e\*x + f\*x^2),x)

```
[Out] int((a + b*x + c*x^2)^(1/2)/(d + e*x + f*x^2), x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{\sqrt{a + bx + cx^2}}{d + ex + fx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d), x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/(d + e*x + f*x**2), x)
```

$$3.103 \quad \int \frac{\sqrt{a+bx+cx^2}}{(d+ex+fx^2)^2} dx$$

Optimal. Leaf size=488

$$\frac{(f(be - 4af) - (e - \sqrt{e^2 - 4df})(ce - bf)) \tanh^{-1} \left( \frac{4af + 2x(bf - c(e - \sqrt{e^2 - 4df})) - b(e - \sqrt{e^2 - 4df})}{2\sqrt{2} \sqrt{a+bx+cx^2} \sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf + ce^2}} \right)}{\sqrt{2} (e^2 - 4df)^{3/2} \sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf + ce^2}} + \dots$$

[Out]  $-(2*f*x+e)*(c*x^2+b*x+a)^{(1/2)/(-4*d*f+e^2)/(f*x^2+e*x+d)-1/2*\operatorname{arctanh}(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^{(1/2)}))-b*(e-(-4*d*f+e^2)^{(1/2)}))*2^{(1/2)/(c*x^2+b*x+a)^{(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^{(1/2)})^{(1/2)}*(f*(-4*a*f+b*e)-(-b*f+c*e)*(e-(-4*d*f+e^2)^{(1/2)}))/(-4*d*f+e^2)^{(3/2)*2^{(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^{(1/2)})^{(1/2)+1/2*\operatorname{arctanh}(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^{(1/2)}+2*x*(b*f-c*(e+(-4*d*f+e^2)^{(1/2)})))})^{(1/2)/(c*x^2+b*x+a)^{(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^{(1/2)})^{(1/2)}*(f*(-4*a*f+b*e)-(-b*f+c*e)*(e+(-4*d*f+e^2)^{(1/2)}))/(-4*d*f+e^2)^{(3/2)*2^{(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^{(1/2)})^{(1/2)}}$

**Rubi [A]** time = 2.93, antiderivative size = 488, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$ , Rules used = {971, 1032, 724, 206}

$$\frac{(f(be - 4af) - (e - \sqrt{e^2 - 4df})(ce - bf)) \tanh^{-1} \left( \frac{4af + 2x(bf - c(e - \sqrt{e^2 - 4df})) - b(e - \sqrt{e^2 - 4df})}{2\sqrt{2} \sqrt{a+bx+cx^2} \sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf + ce^2}} \right)}{\sqrt{2} (e^2 - 4df)^{3/2} \sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf + ce^2}} + \dots$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b\*x + c\*x^2]/(d + e\*x + f\*x^2)^2,x]

[Out]  $-(((e + 2*f*x)*\operatorname{Sqrt}[a + b*x + c*x^2])/((e^2 - 4*d*f)*(d + e*x + f*x^2))) - ((f*(b*e - 4*a*f) - (c*e - b*f)*(e - \operatorname{Sqrt}[e^2 - 4*d*f]))*\operatorname{ArcTanh}[(4*a*f - b*(e - \operatorname{Sqrt}[e^2 - 4*d*f]) + 2*(b*f - c*(e - \operatorname{Sqrt}[e^2 - 4*d*f]))*x)/(2*\operatorname{Sqrt}[2]*\operatorname{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*\operatorname{Sqrt}[e^2 - 4*d*f]]*\operatorname{Sqrt}[a + b*x + c*x^2]])/(\operatorname{Sqrt}[2]*(e^2 - 4*d*f)^{(3/2)*\operatorname{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*\operatorname{Sqrt}[e^2 - 4*d*f]]} + ((f*(b*e - 4*a*f) - (c*e - b*f)*(e + \operatorname{Sqrt}[e^2 - 4*d*f]))*\operatorname{ArcTanh}[(4*a*f - b*(e + \operatorname{Sqrt}[e^2 - 4*d*f]) + 2*(b*f - c*(e + \operatorname{Sqrt}[e^2 - 4*d*f]))*x)/(2*\operatorname{Sqrt}[2]*\operatorname{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*\operatorname{Sqrt}[e^2 - 4*d*f]]*\operatorname{Sqrt}[a + b*x + c*x^2]))]$

)/(Sqrt[2]\*(e^2 - 4\*d\*f)^(3/2)\*Sqrt[c\*e^2 - 2\*c\*d\*f - b\*e\*f + 2\*a\*f^2 + (c\*e - b\*f)\*Sqrt[e^2 - 4\*d\*f]])

### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 724

Int[1/(((d\_) + (e\_)\*(x\_))\*Sqrt[(a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

### Rule 971

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_)\*((d\_) + (e\_)\*(x\_) + (f\_)\*(x\_)^2)^(q\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^q)/((b^2 - 4\*a\*c)\*(p + 1)), x] - Dist[1/((b^2 - 4\*a\*c)\*(p + 1)), Int[(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^(q - 1)\*Simp[2\*c\*d\*(2\*p + 3) + b\*e\*q + (2\*b\*f\*q + 2\*c\*e\*(2\*p + q + 3))\*x + 2\*c\*f\*(2\*p + 2\*q + 3)\*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && LtQ[p, -1] && GtQ[q, 0] && !IGtQ[q, 0]

### Rule 1032

Int[((g\_) + (h\_)\*(x\_))/(((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)\*Sqrt[(d\_) + (e\_)\*(x\_) + (f\_)\*(x\_)^2]), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(2\*c\*g - h\*(b - q))/q, Int[1/((b - q + 2\*c\*x)\*Sqrt[d + e\*x + f\*x^2]), x], x] - Dist[(2\*c\*g - h\*(b + q))/q, Int[1/((b + q + 2\*c\*x)\*Sqrt[d + e\*x + f\*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && PosQ[b^2 - 4\*a\*c]

### Rubi steps



$$\begin{aligned}
\int \frac{\sqrt{a+bx+cx^2}}{(d+ex+fx^2)^2} dx &= -\frac{(e+2fx)\sqrt{a+bx+cx^2}}{(e^2-4df)(d+ex+fx^2)} - \frac{\int \frac{\frac{1}{2}(be-4af)+(ce-bf)x}{\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx}{-e^2+4df} \\
&= -\frac{(e+2fx)\sqrt{a+bx+cx^2}}{(e^2-4df)(d+ex+fx^2)} - \frac{(ce(e-\sqrt{e^2-4df})+f(4af-b(2e-\sqrt{e^2-4df}))) \int}{(e^2-4df)^{3/2}} \\
&= -\frac{(e+2fx)\sqrt{a+bx+cx^2}}{(e^2-4df)(d+ex+fx^2)} + \frac{(2(ce(e-\sqrt{e^2-4df})+f(4af-b(2e-\sqrt{e^2-4df}))))}{(e^2-4df)^{3/2}} \\
&= -\frac{(e+2fx)\sqrt{a+bx+cx^2}}{(e^2-4df)(d+ex+fx^2)} + \frac{(ce(e-\sqrt{e^2-4df})+f(4af-b(2e-\sqrt{e^2-4df}))) \operatorname{tanh}^{-1}\left(\frac{\sqrt{a+bx+cx^2}}{\sqrt{e^2-4df}}\right)}{\sqrt{2}(e^2-4df)^{3/2}\sqrt{ce^2-2cdf-be}}
\end{aligned}$$

**Mathematica [A]** time = 5.09, size = 555, normalized size = 1.14

$$\frac{4f(e+2fx)\sqrt{a+x(b+cx)}}{(e^2-4df)(\sqrt{e^2-4df}-e-2fx)(\sqrt{e^2-4df}+e+2fx)} + \frac{(ce(\sqrt{e^2-4df}-e)-f(4af+b(\sqrt{e^2-4df}-2e)))}{\sqrt{2}(e^2-4df)^{3/2}\sqrt{f(2af+b(\sqrt{e^2-4df}-2e))}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b\*x + c\*x^2]/(d + e\*x + f\*x^2)^2,x]

[Out] 
$$\frac{(4f(e+2fx)\sqrt{a+x(b+cx)})/((e^2-4df)(-e+\sqrt{e^2-4df}-2fx)(e+\sqrt{e^2-4df}+2fx))+((ce(e-\sqrt{e^2-4df})-f(4af+b(\sqrt{e^2-4df}-2e))))\operatorname{ArcTanh}\left[\frac{(-4af+2c(e-\sqrt{e^2-4df}))x+b(e-\sqrt{e^2-4df}-2fx)}{2\sqrt{2}\sqrt{c(e^2-2df-e\sqrt{e^2-4df})+f(2af+b(-e+\sqrt{e^2-4df}))}}\right]}{(\sqrt{2}(e^2-4df)^{3/2}\sqrt{c(e^2-2df-e\sqrt{e^2-4df})+f(2af+b(-e+\sqrt{e^2-4df}))})-(ce(e+\sqrt{e^2-4df})+f(4af-b(2e+\sqrt{e^2-4df})))\operatorname{ArcTan}\left[\frac{(4af-2c(e+\sqrt{e^2-4df}))x-b(e+\sqrt{e^2-4df}-2fx)}{2\sqrt{2}\sqrt{c(e^2-2df+e\sqrt{e^2-4df})+f(2af-b(e+\sqrt{e^2-4df}))}}\right]}\sqrt{a+x(b+cx)}}{(\sqrt{2}(e^2-4df)^{3/2}\sqrt{c(e^2-2df+e\sqrt{e^2-4df})+f(2af-b(e+\sqrt{e^2-4df}))})}$$

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(1/2)/(f\*x^2+e\*x+d)^2,x, algorithm="fricas")

[Out] Timed out

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(1/2)/(f\*x^2+e\*x+d)^2,x, algorithm="giac")

[Out] Timed out

**maple** [B] time = 0.04, size = 22287, normalized size = 45.67

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+b\*x+a)^(1/2)/(f\*x^2+e\*x+d)^2,x)

[Out] result too large to display

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + bx + a}}{(fx^2 + ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(1/2)/(f\*x^2+e\*x+d)^2,x, algorithm="maxima")

[Out] integrate(sqrt(c\*x^2 + b\*x + a)/(f\*x^2 + e\*x + d)^2, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{cx^2 + bx + a}}{(fx^2 + ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)/(d + e*x + f*x^2)^2,x)
```

```
[Out] int((a + b*x + c*x^2)^(1/2)/(d + e*x + f*x^2)^2, x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d)**2,x)
```

```
[Out] Timed out
```

$$3.104 \quad \int (a + bx + cx^2)^{3/2} (d + ex + fx^2)^2 dx$$

**Optimal.** Leaf size=564

$$\frac{(b^2 - 4ac)^2 \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) (16c^2 (3a^2 f^2 + 24abef + 14b^2 (2df + e^2)) - 72b^2 cf(3af + 4be) - 128c^3 (a(2df + e^2)))}{32768c^{13/2}}$$

[Out] 1/6144\*(768\*c^4\*d^2+99\*b^4\*f^2-72\*b^2\*c\*f\*(3\*a\*f+4\*b\*e)-128\*c^3\*(6\*b\*d\*e+a\*(2\*d\*f+e^2))+16\*c^2\*(24\*a\*b\*e\*f+3\*a^2\*f^2+14\*b^2\*(2\*d\*f+e^2)))\*(2\*c\*x+b)\*(c\*x^2+b\*x+a)^(3/2)/c^5+1/13440\*(5376\*c^3\*d\*e-693\*b^3\*f^2+36\*b\*c\*f\*(31\*a\*f+56\*b\*e)-32\*c^2\*(48\*a\*e\*f+49\*b\*(2\*d\*f+e^2)))\*(c\*x^2+b\*x+a)^(5/2)/c^4+1/1344\*(9\*9\*b^2\*f^2-12\*c\*f\*(7\*a\*f+24\*b\*e)+224\*c^2\*(2\*d\*f+e^2))\*x\*(c\*x^2+b\*x+a)^(5/2)/c^3+1/112\*f\*(-11\*b\*f+32\*c\*e)\*x^2\*(c\*x^2+b\*x+a)^(5/2)/c^2+1/8\*f^2\*x^3\*(c\*x^2+b\*x+a)^(5/2)/c+1/32768\*(-4\*a\*c+b^2)^2\*(768\*c^4\*d^2+99\*b^4\*f^2-72\*b^2\*c\*f\*(3\*a\*f+4\*b\*e)-128\*c^3\*(6\*b\*d\*e+a\*(2\*d\*f+e^2))+16\*c^2\*(24\*a\*b\*e\*f+3\*a^2\*f^2+14\*b^2\*(2\*d\*f+e^2)))\*arctanh(1/2\*(2\*c\*x+b)/c^(1/2)/(c\*x^2+b\*x+a)^(1/2))/c^(13/2)-1/16384\*(-4\*a\*c+b^2)\*(768\*c^4\*d^2+99\*b^4\*f^2-72\*b^2\*c\*f\*(3\*a\*f+4\*b\*e)-128\*c^3\*(6\*b\*d\*e+a\*(2\*d\*f+e^2))+16\*c^2\*(24\*a\*b\*e\*f+3\*a^2\*f^2+14\*b^2\*(2\*d\*f+e^2)))\*(2\*c\*x+b)\*(c\*x^2+b\*x+a)^(1/2)/c^6

**Rubi [A]** time = 0.93, antiderivative size = 564, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {1661, 640, 612, 621, 206}

$$\frac{(b + 2cx)(a + bx + cx^2)^{3/2} (16c^2 (3a^2 f^2 + 24abef + 14b^2 (2df + e^2)) - 72b^2 cf(3af + 4be) - 128c^3 (a(2df + e^2)))}{6144c^5}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2)^2,x]

[Out] -((b^2 - 4\*a\*c)\*(768\*c^4\*d^2 + 99\*b^4\*f^2 - 72\*b^2\*c\*f\*(4\*b\*e + 3\*a\*f) - 128\*c^3\*(6\*b\*d\*e + a\*(e^2 + 2\*d\*f)) + 16\*c^2\*(24\*a\*b\*e\*f + 3\*a^2\*f^2 + 14\*b^2\*(e^2 + 2\*d\*f)))\*(b + 2\*c\*x)\*Sqrt[a + b\*x + c\*x^2])/(16384\*c^6) + ((768\*c^4\*d^2 + 99\*b^4\*f^2 - 72\*b^2\*c\*f\*(4\*b\*e + 3\*a\*f) - 128\*c^3\*(6\*b\*d\*e + a\*(e^2 + 2\*d\*f)) + 16\*c^2\*(24\*a\*b\*e\*f + 3\*a^2\*f^2 + 14\*b^2\*(e^2 + 2\*d\*f)))\*(b + 2\*c\*x)\*(a + b\*x + c\*x^2)^(3/2))/(6144\*c^5) + ((5376\*c^3\*d\*e - 693\*b^3\*f^2 + 36\*b\*c\*f\*(56\*b\*e + 31\*a\*f) - 32\*c^2\*(48\*a\*e\*f + 49\*b\*(e^2 + 2\*d\*f)))\*(a + b\*x + c\*x^2)^(5/2))/(13440\*c^4) + ((99\*b^2\*f^2 - 12\*c\*f\*(24\*b\*e + 7\*a\*f) + 224\*c^2\*(e^2 + 2\*d\*f))\*x\*(a + b\*x + c\*x^2)^(5/2))/(1344\*c^3) + (f\*(32\*c\*e - 11\*b\*f)\*x^2\*(a + b\*x + c\*x^2)^(5/2))/(112\*c^2) + (f^2\*x^3\*(a + b\*x + c\*x^2)^(5/2))/(8\*c) + ((b^2 - 4\*a\*c)^2\*(768\*c^4\*d^2 + 99\*b^4\*f^2 - 72\*b^2\*c\*f\*(4\*b

$$\frac{e + 3af - 128c^3(6bd e + a(e^2 + 2df)) + 16c^2(24abef + 3a^2f^2 + 14b^2(e^2 + 2df)) \operatorname{ArcTanh}\left[\frac{b + 2cx}{2\sqrt{c}\sqrt{a + bx + cx^2}}\right]}{(32768c^{13/2})}$$

### Rule 206

$$\operatorname{Int}\left[\frac{(a_.) + (b_.)x + (c_.)x^2}{(a + bx + cx^2)^p}, x\right] \rightarrow \operatorname{Simp}\left[\frac{1 \operatorname{ArcTanh}\left[\frac{\operatorname{Rt}[-b, 2]x}{\operatorname{Rt}[a, 2]}\right]}{\operatorname{Rt}[a, 2] \operatorname{Rt}[-b, 2]}\right], x \;/; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$$

### Rule 612

$$\operatorname{Int}\left[\frac{(a_.) + (b_.)x + (c_.)x^2}{(a + bx + cx^2)^p}, x\right] \rightarrow \operatorname{Simp}\left[\frac{(b + 2cx)(a + bx + cx^2)^p}{2c(2p + 1)}, x\right] - \operatorname{Dist}\left[\frac{p(b^2 - 4ac)}{2c(2p + 1)}, \operatorname{Int}\left[\frac{(a + bx + cx^2)^{p-1}}{x}\right], x\right] \;/; \operatorname{FreeQ}\{a, b, c\}, x \ \&\& \operatorname{NeQ}[b^2 - 4ac, 0] \ \&\& \operatorname{GtQ}[p, 0] \ \&\& \operatorname{IntegerQ}[4p]$$

### Rule 621

$$\operatorname{Int}\left[\frac{1}{\sqrt{(a_.) + (b_.)x + (c_.)x^2}}, x\right] \rightarrow \operatorname{Dist}\left[2, \operatorname{Subst}\left[\operatorname{Int}\left[\frac{1}{4c - x^2}\right], x\right], x, \frac{b + 2cx}{\sqrt{a + bx + cx^2}}\right], x \;/; \operatorname{FreeQ}\{a, b, c\}, x \ \&\& \operatorname{NeQ}[b^2 - 4ac, 0]$$

### Rule 640

$$\operatorname{Int}\left[\frac{(d_.) + (e_.)x}{(a_.) + (b_.)x + (c_.)x^2}, x\right] \rightarrow \operatorname{Simp}\left[\frac{e(a + bx + cx^2)^{p+1}}{2c(p+1)}, x\right] + \operatorname{Dist}\left[\frac{2cd - be}{2c}, \operatorname{Int}\left[\frac{(a + bx + cx^2)^p}{x}\right], x\right] \;/; \operatorname{FreeQ}\{a, b, c, d, e, p\}, x \ \&\& \operatorname{NeQ}[2cd - be, 0] \ \&\& \operatorname{NeQ}[p, -1]$$

### Rule 1661

$$\operatorname{Int}\left[\frac{Pq}{(a_.) + (b_.)x + (c_.)x^2}, x\right] \rightarrow \operatorname{With}\left[\{q = \operatorname{Expon}[Pq, x], e = \operatorname{Coeff}[Pq, x, \operatorname{Expon}[Pq, x]]\}, \operatorname{Simp}\left[\frac{e x^{q-1}(a + bx + cx^2)^{p+1}}{c(q + 2p + 1)}, x\right] + \operatorname{Dist}\left[\frac{1}{c(q + 2p + 1)}, \operatorname{Int}\left[\frac{(a + bx + cx^2)^p \operatorname{ExpandToSum}[c(q + 2p + 1)Pq - a e x^{q-1} x^{q-2} - b e (q + p) x^{q-1} - c e (q + 2p + 1) x^q]}{x}\right], x\right] \;/; \operatorname{FreeQ}\{a, b, c, p\}, x \ \&\& \operatorname{PolyQ}[Pq, x] \ \&\& \operatorname{NeQ}[b^2 - 4ac, 0] \ \&\& \operatorname{!LeQ}[p, -1]$$

### Rubi steps

$$\begin{aligned}
\int (a + bx + cx^2)^{3/2} (d + ex + fx^2)^2 dx &= \frac{f^2 x^3 (a + bx + cx^2)^{5/2}}{8c} + \frac{\int (a + bx + cx^2)^{3/2} (8cd^2 + 16cdex - (3af^2 - 8c^2d^2)) dx}{8c} \\
&= \frac{f(32ce - 11bf)x^2 (a + bx + cx^2)^{5/2}}{112c^2} + \frac{f^2 x^3 (a + bx + cx^2)^{5/2}}{8c} + \frac{\int (a + bx + cx^2)^{3/2} (8cd^2 + 16cdex - (3af^2 - 8c^2d^2)) dx}{8c} \\
&= \frac{(99b^2 f^2 - 12cf(24be + 7af) + 224c^2 (e^2 + 2df)) x (a + bx + cx^2)^{5/2}}{1344c^3} + \frac{\int (a + bx + cx^2)^{3/2} (8cd^2 + 16cdex - (3af^2 - 8c^2d^2)) dx}{8c} \\
&= \frac{(5376c^3 de - 693b^3 f^2 + 36bcf(56be + 31af) - 32c^2 (48aef + 49b(e^2 + 2df))) x (a + bx + cx^2)^{5/2}}{13440c^4} + \frac{\int (a + bx + cx^2)^{3/2} (8cd^2 + 16cdex - (3af^2 - 8c^2d^2)) dx}{8c} \\
&= \frac{(768c^4 d^2 + 99b^4 f^2 - 72b^2 cf(4be + 3af) - 128c^3 (6bde + a(e^2 + 2df))) x (a + bx + cx^2)^{5/2}}{61440c^5} + \frac{\int (a + bx + cx^2)^{3/2} (8cd^2 + 16cdex - (3af^2 - 8c^2d^2)) dx}{8c} \\
&= -\frac{(b^2 - 4ac)(768c^4 d^2 + 99b^4 f^2 - 72b^2 cf(4be + 3af) - 128c^3 (6bde + a(e^2 + 2df))) x (a + bx + cx^2)^{5/2}}{61440c^5} + \frac{\int (a + bx + cx^2)^{3/2} (8cd^2 + 16cdex - (3af^2 - 8c^2d^2)) dx}{8c} \\
&= -\frac{(b^2 - 4ac)(768c^4 d^2 + 99b^4 f^2 - 72b^2 cf(4be + 3af) - 128c^3 (6bde + a(e^2 + 2df))) x (a + bx + cx^2)^{5/2}}{61440c^5} + \frac{\int (a + bx + cx^2)^{3/2} (8cd^2 + 16cdex - (3af^2 - 8c^2d^2)) dx}{8c} \\
&= -\frac{(b^2 - 4ac)(768c^4 d^2 + 99b^4 f^2 - 72b^2 cf(4be + 3af) - 128c^3 (6bde + a(e^2 + 2df))) x (a + bx + cx^2)^{5/2}}{61440c^5} + \frac{\int (a + bx + cx^2)^{3/2} (8cd^2 + 16cdex - (3af^2 - 8c^2d^2)) dx}{8c}
\end{aligned}$$

**Mathematica [A]** time = 1.76, size = 829, normalized size = 1.47

$$\frac{430080f^2(a + x(b + cx))^{5/2}x^3 + 983040ef(a + x(b + cx))^{5/2}x^2 + 573440(e^2 + 2df)(a + x(b + cx))^{5/2}x + 1376256(a + x(b + cx))^{5/2}}{8c}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2)^2,x]

[Out] (430080\*d^2\*(b + 2\*c\*x)\*(a + x\*(b + c\*x))^(3/2) + 1376256\*d\*e\*(a + x\*(b + c\*x))^(5/2) + 573440\*(e^2 + 2\*d\*f)\*x\*(a + x\*(b + c\*x))^(5/2) + 983040\*e\*f\*x^2\*(a + x\*(b + c\*x))^(5/2) + 430080\*f^2\*x^3\*(a + x\*(b + c\*x))^(5/2) + (80640\*(b^2 - 4\*a\*c)\*d^2\*(-2\*sqrt[c]\*(b + 2\*c\*x)\*sqrt[a + x\*(b + c\*x)] + (b^2 - 4\*a\*c)\*ArcTanh[(b + 2\*c\*x)/(2\*sqrt[c]\*sqrt[a + x\*(b + c\*x)])]))/c^(3/2) - (2\*6880\*b\*d\*e\*(16\*c^(3/2)\*(b + 2\*c\*x)\*(a + x\*(b + c\*x))^(3/2) - 3\*(b^2 - 4\*a\*c)\*(2\*sqrt[c]\*(b + 2\*c\*x)\*sqrt[a + x\*(b + c\*x)] - (b^2 - 4\*a\*c)\*ArcTanh[(b + 2\*c\*x)/(2\*sqrt[c]\*sqrt[a + x\*(b + c\*x)])])))/c^(5/2) + (96\*e\*f\*(-256\*c^(5/2)

$$2)*(-21*b^2 + 16*a*c + 30*b*c*x)*(a + x*(b + c*x))^(5/2) - 35*b*(3*b^2 - 4*a*c)*(16*c^(3/2)*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4*a*c)*(2*sqrt[c]*(b + 2*c*x)*sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*sqrt[c]*sqrt[a + x*(b + c*x)])))/c^(9/2) - (224*(e^2 + 2*d*f)*(1792*b*c^(5/2)*(a + x*(b + c*x))^(5/2) - 5*(7*b^2 - 4*a*c)*(16*c^(3/2)*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4*a*c)*(2*sqrt[c]*(b + 2*c*x)*sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*sqrt[c]*sqrt[a + x*(b + c*x)])))/c^(7/2) - (3*f^2*(112640*b*c^(9/2)*x^2*(a + x*(b + c*x))^(5/2) + 256*c^(5/2)*(231*b^3 - 372*a*b*c - 330*b^2*c*x + 280*a*c^2*x)*(a + x*(b + c*x))^(5/2) - 35*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*(16*c^(3/2)*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4*a*c)*(2*sqrt[c]*(b + 2*c*x)*sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*sqrt[c]*sqrt[a + x*(b + c*x)])))/c^(11/2))/(3440640*c)$$

**fricas [B]** time = 3.79, size = 2179, normalized size = 3.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(3/2)\*(f\*x^2+e\*x+d)^2,x, algorithm="fricas")

[Out] [1/6881280\*(105\*(768\*(b^4\*c^4 - 8\*a\*b^2\*c^5 + 16\*a^2\*c^6)\*d^2 - 768\*(b^5\*c^3 - 8\*a\*b^3\*c^4 + 16\*a^2\*b\*c^5)\*d\*e + 32\*(7\*b^6\*c^2 - 60\*a\*b^4\*c^3 + 144\*a^2\*b^2\*c^4 - 64\*a^3\*c^5)\*e^2 + 3\*(33\*b^8 - 336\*a\*b^6\*c + 1120\*a^2\*b^4\*c^2 - 1280\*a^3\*b^2\*c^3 + 256\*a^4\*c^4)\*f^2 + 32\*(2\*(7\*b^6\*c^2 - 60\*a\*b^4\*c^3 + 144\*a^2\*b^2\*c^4 - 64\*a^3\*c^5)\*d - 3\*(3\*b^7\*c - 28\*a\*b^5\*c^2 + 80\*a^2\*b^3\*c^3 - 64\*a^3\*b\*c^4)\*e)\*f)\*sqrt(c)\*log(-8\*c^2\*x^2 - 8\*b\*c\*x - b^2 - 4\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(c) - 4\*a\*c) + 4\*(215040\*c^8\*f^2\*x^7 + 15360\*(32\*c^8\*e\*f + 17\*b\*c^7\*f^2)\*x^6 + 1280\*(224\*c^8\*e^2 + 3\*(b^2\*c^6 + 84\*a\*c^7)\*f^2 + 32\*(14\*c^8\*d + 15\*b\*c^7\*e)\*f)\*x^5 + 128\*(5376\*c^8\*d\*e + 2912\*b\*c^7\*e^2 - 3\*(11\*b^3\*c^5 - 52\*a\*b\*c^6)\*f^2 + 32\*(182\*b\*c^7\*d + 3\*(b^2\*c^6 + 64\*a\*c^7)\*e)\*f)\*x^4 + 16\*(26880\*c^8\*d^2 + 59136\*b\*c^7\*d\*e + 224\*(3\*b^2\*c^6 + 140\*a\*c^7)\*e^2 + 3\*(99\*b^4\*c^4 - 568\*a\*b^2\*c^5 + 560\*a^2\*c^6)\*f^2 + 32\*(14\*(3\*b^2\*c^6 + 140\*a\*c^7)\*d - 3\*(9\*b^3\*c^5 - 44\*a\*b\*c^6)\*e)\*f)\*x^3 - 26880\*(3\*b^3\*c^5 - 20\*a\*b\*c^6)\*d^2 + 5376\*(15\*b^4\*c^4 - 100\*a\*b^2\*c^5 + 128\*a^2\*c^6)\*d\*e - 224\*(105\*b^5\*c^3 - 760\*a\*b^3\*c^4 + 1296\*a^2\*b\*c^5)\*e^2 - 3\*(3465\*b^7\*c - 30660\*a\*b^5\*c^2 + 81648\*a^2\*b^3\*c^3 - 58816\*a^3\*b\*c^4)\*f^2 + 8\*(80640\*b\*c^7\*d^2 + 5376\*(b^2\*c^6 + 32\*a\*c^7)\*d\*e - 224\*(7\*b^3\*c^5 - 36\*a\*b\*c^6)\*e^2 - 3\*(231\*b^5\*c^3 - 1560\*a\*b^3\*c^4 + 2416\*a^2\*b\*c^5)\*f^2 - 32\*(14\*(7\*b^3\*c^5 - 36\*a\*b\*c^6)\*d - 3\*(21\*b^4\*c^4 - 124\*a\*b^2\*c^5 + 128\*a^2\*c^6)\*e)\*f)\*x^2 - 32\*(14\*(105\*b^5\*c^3 - 760\*a\*b^3\*c^4 + 1296\*a^2\*b\*c^5)\*d - 3\*(315\*b^6\*c^2 - 2520\*a\*b^4\*c^3 + 5488\*a^2\*b^2\*c^4 - 2048\*a^3\*c^5)\*e)\*f + 2\*(26880\*(b^2\*c^6 + 20\*a\*c^7)\*d^2 - 5376\*(5\*b^3\*c^5 - 28\*a\*b\*c^6)\*d\*e + 224\*(35\*b^4\*c^4 - 216\*a\*b^2\*c^5 + 240\*a^2\*c^6)\*e^2 + 3\*(1155\*b^6\*c^2 - 8988\*a\*b^4\*c^3 + 18896\*a^2\*b^2\*c^4 - 6720\*a^3\*c^5)\*f^2 + 32\*(14\*(35\*b^4\*c^4 - 216\*a\*b^2\*c^5 + 240\*a^2

```

*c^6)*d - 3*(105*b^5*c^3 - 728*a*b^3*c^4 + 1168*a^2*b*c^5)*e)*f)*x)*sqrt(c*x^2 + b*x + a))/c^7, -1/3440640*(105*(768*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*d^2 - 768*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*d*e + 32*(7*b^6*c^2 - 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*e^2 + 3*(33*b^8 - 336*a*b^6*c + 1120*a^2*b^4*c^2 - 1280*a^3*b^2*c^3 + 256*a^4*c^4)*f^2 + 32*(2*(7*b^6*c^2 - 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*d - 3*(3*b^7*c - 28*a*b^5*c^2 + 80*a^2*b^3*c^3 - 64*a^3*b*c^4)*e)*f)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(215040*c^8*f^2*x^7 + 15360*(32*c^8*e*f + 17*b*c^7*f^2)*x^6 + 1280*(224*c^8*e^2 + 3*(b^2*c^6 + 84*a*c^7)*f^2 + 32*(14*c^8*d + 15*b*c^7*e)*f)*x^5 + 128*(5376*c^8*d*e + 2912*b*c^7*e^2 - 3*(11*b^3*c^5 - 52*a*b*c^6)*f^2 + 32*(182*b*c^7*d + 3*(b^2*c^6 + 64*a*c^7)*e)*f)*x^4 + 16*(26880*c^8*d^2 + 59136*b*c^7*d*e + 224*(3*b^2*c^6 + 140*a*c^7)*e^2 + 3*(99*b^4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*f^2 + 32*(14*(3*b^2*c^6 + 140*a*c^7)*d - 3*(9*b^3*c^5 - 44*a*b*c^6)*e)*f)*x^3 - 26880*(3*b^3*c^5 - 20*a*b*c^6)*d^2 + 5376*(15*b^4*c^4 - 100*a*b^2*c^5 + 128*a^2*c^6)*d*e - 224*(105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5)*e^2 - 3*(3465*b^7*c - 30660*a*b^5*c^2 + 81648*a^2*b^3*c^3 - 58816*a^3*b*c^4)*f^2 + 8*(80640*b*c^7*d^2 + 5376*(b^2*c^6 + 32*a*c^7)*d*e - 224*(7*b^3*c^5 - 36*a*b*c^6)*e^2 - 3*(231*b^5*c^3 - 1560*a*b^3*c^4 + 2416*a^2*b*c^5)*f^2 - 32*(14*(7*b^3*c^5 - 36*a*b*c^6)*d - 3*(21*b^4*c^4 - 124*a*b^2*c^5 + 128*a^2*c^6)*e)*f)*x^2 - 32*(14*(105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5)*d - 3*(315*b^6*c^2 - 2520*a*b^4*c^3 + 5488*a^2*b^2*c^4 - 2048*a^3*c^5)*e)*f + 2*(26880*(b^2*c^6 + 20*a*c^7)*d^2 - 5376*(5*b^3*c^5 - 28*a*b*c^6)*d*e + 224*(35*b^4*c^4 - 216*a*b^2*c^5 + 240*a^2*c^6)*e^2 + 3*(1155*b^6*c^2 - 8988*a*b^4*c^3 + 18896*a^2*b^2*c^4 - 6720*a^3*c^5)*f^2 + 32*(14*(35*b^4*c^4 - 216*a*b^2*c^5 + 240*a^2*c^6)*d - 3*(105*b^5*c^3 - 728*a*b^3*c^4 + 1168*a^2*b*c^5)*e)*f)*x)*sqrt(c*x^2 + b*x + a))/c^7]

```

**giac [B]** time = 0.60, size = 1150, normalized size = 2.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(3/2)\*(f\*x^2+e\*x+d)^2,x, algorithm="giac")

```

[Out] 1/1720320*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(12*(14*c*f^2*x + (17*b*c^7*f^2 + 32*c^8*f*e))/c^7)*x + (448*c^8*d*f + 3*b^2*c^6*f^2 + 252*a*c^7*f^2 + 480*b*c^7*f*e + 224*c^8*e^2)/c^7)*x + (5824*b*c^7*d*f - 33*b^3*c^5*f^2 + 156*a*b*c^6*f^2 + 5376*c^8*d*e + 96*b^2*c^6*f*e + 6144*a*c^7*f*e + 2912*b*c^7*e^2)/c^7)*x + (26880*c^8*d^2 + 1344*b^2*c^6*d*f + 62720*a*c^7*d*f + 297*b^4*c^4*f^2 - 1704*a*b^2*c^5*f^2 + 1680*a^2*c^6*f^2 + 59136*b*c^7*d*e - 864*b^3*c^5*f*e + 4224*a*b*c^6*f*e + 672*b^2*c^6*e^2 + 31360*a*c^7*e^2)/c^7)*x + (80640*b*c^7*d^2 - 3136*b^3*c^5*d*f + 16128*a*b*c^6*d*f - 693*b^5*c^3*f^2 + 4680*a*b^3*c^4*f^2 - 7248*a^2*b*c^5*f^2 + 5376*b^2*c^6*d*e + 172032*a*c^7*d*e + 2016*b^4*c^4*f*e - 11904*a*b^2*c^5*f*e + 12288*a^2*c^6*f*e - 1568*b^

```



$$\begin{aligned}
& 3c^5e^2 + 8064ab^2c^6e^2)/c^7)*x + (26880b^2c^6d^2 + 537600a^2c^7d^2 \\
& + 15680b^4c^4d^2f - 96768a^2b^2c^5d^2f + 107520a^2c^6d^2f + 3465b^6 \\
& c^2f^2 - 26964a^2b^4c^3f^2 + 56688a^2b^2c^4f^2 - 20160a^3c^5f^2 \\
& - 26880b^3c^5d^2e + 150528a^2b^2c^6d^2e - 10080b^5c^3f^2e + 69888a^2b^3c^4f^2e \\
& - 112128a^2b^2c^5f^2e + 7840b^4c^4e^2 - 48384a^2b^2c^5e^2 + 5 \\
& 3760a^2c^6e^2)/c^7)*x - (80640b^3c^5d^2 - 537600ab^2c^6d^2 + 47040b^5c^3d^2f \\
& - 340480a^2b^3c^4d^2f + 580608a^2b^2c^5d^2f + 10395b^7c^2f^2 \\
& - 91980a^2b^5c^2f^2 + 244944a^2b^3c^3f^2 - 176448a^3b^2c^4f^2 - 80 \\
& 640b^4c^4d^2e + 537600a^2b^2c^5d^2e - 688128a^2c^6d^2e - 30240b^6c^2 \\
& f^2e + 241920a^2b^4c^3f^2e - 526848a^2b^2c^4f^2e + 196608a^3c^5f^2e + \\
& 23520b^5c^3e^2 - 170240a^2b^3c^4e^2 + 290304a^2b^2c^5e^2)/c^7) - 1/ \\
& 32768*(768b^4c^4d^2 - 6144a^2b^2c^5d^2 + 12288a^2c^6d^2 + 448b^6c^2 \\
& d^2f - 3840a^2b^4c^3d^2f + 9216a^2b^2c^4d^2f - 4096a^3c^5d^2f + 99b^8f^2 \\
& - 1008a^2b^6c^2f^2 + 3360a^2b^4c^2f^2 - 3840a^3b^2c^3f^2 + 768a^4c^4f^2 \\
& - 768b^5c^3d^2e + 6144a^2b^3c^4d^2e - 12288a^2b^2c^5d^2e - 288b^7c^2f^2e \\
& + 2688a^2b^5c^2f^2e - 7680a^2b^3c^3f^2e + 6144a^3b^2c^4f^2e + 224b^6c^2e^2 \\
& - 1920a^2b^4c^3e^2 + 4608a^2b^2c^4e^2 - 2048a^3c^5e^2)*\log(\text{abs}(-2*\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})*\sqrt{c} - b)) \\
& /c^{(13/2)}
\end{aligned}$$

**maple [B]** time = 0.02, size = 2458, normalized size = 4.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((c*x^2+b*x+a)^{(3/2)}*(f*x^2+e*x+d)^2,x)$

[Out]  $\begin{aligned}
& 1/8*f^2*x^3*(c*x^2+b*x+a)^{(5/2)}/c+1/8*d^2/c*(c*x^2+b*x+a)^{(3/2)}*b+3/8*d^2*( \\
& c*x^2+b*x+a)^{(1/2)}*x*a-3/64*d^2/c^2*(c*x^2+b*x+a)^{(1/2)}*b^3+3/8*d^2/c^{(1/2)} \\
& *ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a^2+3/128*d^2/c^{(5/2)}*ln((c*x+ \\
& 1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*b^4+2/5*d^2e*(c*x^2+b*x+a)^{(5/2)}/c+1/6*x \\
& *(c*x^2+b*x+a)^{(5/2)}/c*e^2-7/60*b/c^2*(c*x^2+b*x+a)^{(5/2)}*e^2+7/192*b^3/c^3 \\
& *(c*x^2+b*x+a)^{(3/2)}*e^2-7/512*b^5/c^4*(c*x^2+b*x+a)^{(1/2)}*e^2+7/1024*b^6/c \\
& ^{(9/2)}*ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*e^2-1/16*a^3/c^{(3/2)}*ln( \\
& (c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*e^2+3/128*f^2*a^4/c^{(5/2)}*ln((c*x+ \\
& 1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})+99/32768*f^2*b^8/c^{(13/2)}*ln((c*x+1/2*b) \\
& )/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})-33/640*f^2*b^3/c^4*(c*x^2+b*x+a)^{(5/2)}+33/20 \\
& 48*f^2*b^5/c^5*(c*x^2+b*x+a)^{(3/2)}-99/16384*f^2*b^7/c^6*(c*x^2+b*x+a)^{(1/2)} \\
& -3/16*e*f*b^3/c^3*(c*x^2+b*x+a)^{(1/2)}*x*a+1/8*e*f*b/c^2*a*x*(c*x^2+b*x+a)^{( \\
& 3/2)}+3/16*e*f*b/c^2*a^2*(c*x^2+b*x+a)^{(1/2)}*x+1/4*b^2/c^2*(c*x^2+b*x+a)^{(1/ \\
& 2)}*x*a*d^2f-3/8*d^2e*b/c*(c*x^2+b*x+a)^{(1/2)}*x*a+3/16*d^2e*b^3/c^{(5/2)}*ln((c*x \\
& +1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a+1/16*e*f*b^2/c^3*a*(c*x^2+b*x+a)^{(3/ \\
& 2)}+3/32*e*f*b^2/c^3*a^2*(c*x^2+b*x+a)^{(1/2)}-3/14*e*f*b/c^2*x*(c*x^2+b*x+a)^ \\
& (5/2)-3/32*e*f*b^3/c^3*x*(c*x^2+b*x+a)^{(3/2)}+9/256*e*f*b^5/c^4*(c*x^2+b*x+a) \\
& )^{(1/2)}*x-3/32*e*f*b^4/c^4*(c*x^2+b*x+a)^{(1/2)}*a-15/64*e*f*b^3/c^{(7/2)}*ln((
\end{aligned}$

$$\begin{aligned}
& c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a^2+21/256*e*f*b^5/c^{(9/2)}*\ln((c*x+ \\
& 1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a+3/16*e*f*b/c^{(5/2)}*a^3*\ln((c*x+1/2*b) \\
& /c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})+7/48*b^2/c^2*x*(c*x^2+b*x+a)^{(3/2)}*d*f+1/8*b^ \\
& 2/c^2*(c*x^2+b*x+a)^{(1/2)}*x*a*e^2-7/128*b^4/c^3*(c*x^2+b*x+a)^{(1/2)}*x*d*f-5 \\
& 7/512*f^2*b^2/c^3*a^2*(c*x^2+b*x+a)^{(1/2)}*x+153/2048*f^2*b^4/c^4*(c*x^2+b*x \\
& +a)^{(1/2)}*x*a-9/128*f^2*b^2/c^3*a*x*(c*x^2+b*x+a)^{(3/2)}+1/8*b^3/c^3*(c*x^2+ \\
& b*x+a)^{(1/2)}*a*d*f+9/32*b^2/c^{(5/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1 \\
& /2)})*a^2*d*f-15/128*b^4/c^{(7/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)}) \\
& *a*d*f-1/12*a/c*x*(c*x^2+b*x+a)^{(3/2)}*d*f-1/24*a/c^2*(c*x^2+b*x+a)^{(3/2)}*b* \\
& d*f-1/8*a^2/c*(c*x^2+b*x+a)^{(1/2)}*x*d*f-1/16*a^2/c^2*(c*x^2+b*x+a)^{(1/2)}*b* \\
& d*f-1/4*d*e*b/c*x*(c*x^2+b*x+a)^{(3/2)}+3/32*d*e*b^3/c^2*(c*x^2+b*x+a)^{(1/2)}* \\
& x-3/16*d*e*b^2/c^2*(c*x^2+b*x+a)^{(1/2)}*a-3/8*d*e*b/c^{(3/2)}*\ln((c*x+1/2*b)/c \\
& ^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a^2+1/4*d^2*x*(c*x^2+b*x+a)^{(3/2)}+153/4096*f^2* \\
& b^5/c^5*(c*x^2+b*x+a)^{(1/2)}*a+93/1120*f^2*b/c^3*a*(c*x^2+b*x+a)^{(5/2)}-11/11 \\
& 2*f^2*b/c^2*x^2*(c*x^2+b*x+a)^{(5/2)}+105/1024*f^2*b^4/c^{(9/2)}*\ln((c*x+1/2*b) \\
& /c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a^2-1/8*a^3/c^{(3/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c \\
& *x^2+b*x+a)^{(1/2)})*d*f-7/30*b/c^2*(c*x^2+b*x+a)^{(5/2)}*d*f+7/96*b^2/c^2*x*(c \\
& *x^2+b*x+a)^{(3/2)}*e^2+7/96*b^3/c^3*(c*x^2+b*x+a)^{(3/2)}*d*f-7/256*b^4/c^3*(c \\
& *x^2+b*x+a)^{(1/2)}*x*e^2+1/16*b^3/c^3*(c*x^2+b*x+a)^{(1/2)}*a*e^2-9/1024*e*f*b \\
& ^7/c^{(11/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})-4/35*e*f*a/c^2*(c*x \\
& ^2+b*x+a)^{(5/2)}+3/20*e*f*b^2/c^3*(c*x^2+b*x+a)^{(5/2)}-3/64*e*f*b^4/c^4*(c*x^ \\
& 2+b*x+a)^{(3/2)}+9/512*e*f*b^6/c^5*(c*x^2+b*x+a)^{(1/2)}+2/7*e*f*x^2*(c*x^2+b*x \\
& +a)^{(5/2)}/c-9/256*f^2*b^3/c^4*a*(c*x^2+b*x+a)^{(3/2)}-57/1024*f^2*b^3/c^4*a^2 \\
& *(c*x^2+b*x+a)^{(1/2)}+33/448*f^2*b^2/c^3*x*(c*x^2+b*x+a)^{(5/2)}-63/2048*f^2*b \\
& ^6/c^{(11/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a-15/128*f^2*b^2/c^ \\
& (7/2)*a^3*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})+1/64*f^2*a^2/c^2*x*(c \\
& *x^2+b*x+a)^{(3/2)}+1/128*f^2*a^2/c^3*(c*x^2+b*x+a)^{(3/2)}*b+3/128*f^2*a^3/c^2 \\
& *(c*x^2+b*x+a)^{(1/2)}*x+3/256*f^2*a^3/c^3*(c*x^2+b*x+a)^{(1/2)}*b-1/16*f^2*a/c \\
& ^2*x*(c*x^2+b*x+a)^{(5/2)}+33/1024*f^2*b^4/c^4*x*(c*x^2+b*x+a)^{(3/2)}-99/8192* \\
& f^2*b^6/c^5*(c*x^2+b*x+a)^{(1/2)}*x-15/256*b^4/c^{(7/2)}*\ln((c*x+1/2*b)/c^{(1/2)} \\
& +(c*x^2+b*x+a)^{(1/2)})*a*e^2+7/512*b^6/c^{(9/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2 \\
& +b*x+a)^{(1/2)})*d*f-1/24*a/c*x*(c*x^2+b*x+a)^{(3/2)}*e^2-1/48*a/c^2*(c*x^2+b*x \\
& +a)^{(3/2)}*b*e^2-1/16*a^2/c*(c*x^2+b*x+a)^{(1/2)}*x*e^2+1/3*x*(c*x^2+b*x+a)^{(5 \\
& /2)}/c*d*f-1/8*d*e*b^2/c^2*(c*x^2+b*x+a)^{(3/2)}+3/64*d*e*b^4/c^3*(c*x^2+b*x+a \\
& )^{(1/2)}-3/128*d*e*b^5/c^{(7/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})-3 \\
& /32*d^2/c*(c*x^2+b*x+a)^{(1/2)}*x*b^2+3/16*d^2/c*(c*x^2+b*x+a)^{(1/2)}*b*a-3/16 \\
& *d^2/c^{(3/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*b^2*a-1/32*a^2/c^2 \\
& *(c*x^2+b*x+a)^{(1/2)}*b*e^2-7/256*b^5/c^4*(c*x^2+b*x+a)^{(1/2)}*d*f+9/64*b^2/c \\
& ^{(5/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a^2*e^2
\end{aligned}$$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(3/2)\*(f\*x^2+e\*x+d)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details)Is 4\*a\*c-b^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (cx^2 + bx + a)^{3/2} (fx^2 + ex + d)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2)^2,x)

[Out] int((a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2)^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + bx + cx^2)^{3/2} (d + ex + fx^2)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+b\*x+a)\*\*(3/2)\*(f\*x\*\*2+e\*x+d)\*\*2,x)

[Out] Integral((a + b\*x + c\*x\*\*2)\*\*(3/2)\*(d + e\*x + f\*x\*\*2)\*\*2, x)

### 3.105 $\int (a + bx + cx^2)^{3/2} (d + ex + fx^2) dx$

**Optimal.** Leaf size=236

$$\frac{(b^2 - 4ac)^2 \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) (-4c(af + 3be) + 7b^2f + 24c^2d)}{1024c^{9/2}} - \frac{(b^2 - 4ac)(b + 2cx)\sqrt{a + bx + cx^2} (-4c(af + 3be) + 7b^2f + 24c^2d)}{512c^4}$$

[Out]  $\frac{1}{192}(-4ac^2f + 7b^2f - 12bce + 24c^2d)(2cx + b)(cx^2 + bx + a)^{3/2}/c^3 + \frac{1}{60}(-7bf + 12ce)(cx^2 + bx + a)^{5/2}/c^2 + \frac{1}{6}fx^2(cx^2 + bx + a)^{5/2}/c + \frac{1}{1024}(-4ac + b^2)^2(24c^2d + 7b^2f - 4c(af + 3be))\operatorname{arctanh}\left(\frac{1}{2}(2cx + b)/c\right)/(cx^2 + bx + a)^{1/2}/c^{9/2} - \frac{1}{512}(-4ac + b^2)(24c^2d + 7b^2f - 4c(af + 3be))(2cx + b)(cx^2 + bx + a)^{1/2}/c^4$

**Rubi [A]** time = 0.23, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {1661, 640, 612, 621, 206}

$$\frac{(b + 2cx)(a + bx + cx^2)^{3/2} (-4acf + 7b^2f - 12bce + 24c^2d)}{192c^3} - \frac{(b^2 - 4ac)(b + 2cx)\sqrt{a + bx + cx^2} (-4c(af + 3be) + 7b^2f + 24c^2d)}{512c^4}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + bx + cx^2)^{3/2}(d + ex + fx^2), x]$

[Out]  $-\frac{(b^2 - 4ac)(24c^2d + 7b^2f - 4c(3be + af))(b + 2cx)\operatorname{Sqrt}[a + bx + cx^2]}{512c^4} + \frac{(24c^2d - 12bce + 7b^2f - 4ac^2f)(b + 2cx)(a + bx + cx^2)^{3/2}}{192c^3} + \frac{(12ce - 7bf)(a + bx + cx^2)^{5/2}}{60c^2} + \frac{f(a + bx + cx^2)^{5/2}}{6c} + \frac{(b^2 - 4ac)^2(24c^2d + 7b^2f - 4c(3be + af))\operatorname{ArcTanh}\left[\frac{b + 2cx}{2\operatorname{Sqrt}[a + bx + cx^2]}\right]}{1024c^{9/2}}$

#### Rule 206

$\operatorname{Int}[(a + (b + cx^2)^{-1}), x\_Symbol] \rightarrow \operatorname{Simp}\left[\frac{\operatorname{ArcTanh}\left[\frac{Rt[-b, 2]x}{Rt[a, 2]}\right]}{Rt[a, 2]Rt[-b, 2]}, x\right] /; \operatorname{FreeQ}\{a, b, x\} \ \&\& \operatorname{NegQ}[a/b] \ \&\& \operatorname{GtQ}[a, 0] \ \|\ \operatorname{LtQ}[b, 0]$

#### Rule 612

$\operatorname{Int}[(a + (b + cx^2)^p), x\_Symbol] \rightarrow \operatorname{Simp}\left[\frac{(b + 2cx)(a + bx + cx^2)^p}{2c(2p + 1)}, x\right] - \operatorname{Dist}\left[\frac{p(b^2 - 4ac)}{2c(2p + 1)}, \operatorname{Int}[(a + bx + cx^2)^{p-1}, x], x\right] /; \operatorname{FreeQ}\{a, b, c, x\} \ \&\& \operatorname{NeQ}[b^2 - 4ac, 0] \ \&\& \operatorname{GtQ}[p, 0] \ \&\& \operatorname{IntegerQ}[4p]$

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a + b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int (a + bx + cx^2)^{3/2} (d + ex + fx^2) dx &= \frac{fx(a + bx + cx^2)^{5/2}}{6c} + \frac{\int (6cd - af + \frac{1}{2}(12ce - 7bf)x)(a + bx + cx^2)^3}{6c} \\
&= \frac{(12ce - 7bf)(a + bx + cx^2)^{5/2}}{60c^2} + \frac{fx(a + bx + cx^2)^{5/2}}{6c} + \frac{(2c(6cd - af))}{6c} \\
&= \frac{(24c^2d - 12bce + 7b^2f - 4acf)(b + 2cx)(a + bx + cx^2)^{3/2}}{192c^3} + \frac{(12ce - 7bf)(a + bx + cx^2)^{5/2}}{6c} \\
&= -\frac{(b^2 - 4ac)(24c^2d + 7b^2f - 4c(3be + af))(b + 2cx)\sqrt{a + bx + cx^2}}{512c^4} + \frac{(12ce - 7bf)(a + bx + cx^2)^{5/2}}{6c} \\
&= -\frac{(b^2 - 4ac)(24c^2d + 7b^2f - 4c(3be + af))(b + 2cx)\sqrt{a + bx + cx^2}}{512c^4} + \frac{(12ce - 7bf)(a + bx + cx^2)^{5/2}}{6c} \\
&= -\frac{(b^2 - 4ac)(24c^2d + 7b^2f - 4c(3be + af))(b + 2cx)\sqrt{a + bx + cx^2}}{512c^4} + \frac{(12ce - 7bf)(a + bx + cx^2)^{5/2}}{6c}
\end{aligned}$$

**Mathematica [A]** time = 0.61, size = 392, normalized size = 1.66

$$\frac{360d(b^2-4ac)\left((b^2-4ac)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)-2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}\right)}{c^{3/2}} - 60be\left(\frac{3(b^2-4ac)\left((b^2-4ac)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)-2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}\right)}{c^{5/2}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2), x]

[Out] (1920\*d\*(b + 2\*c\*x)\*(a + x\*(b + c\*x))^(3/2) + 3072\*e\*(a + x\*(b + c\*x))^(5/2) + 2560\*f\*x\*(a + x\*(b + c\*x))^(5/2) + (360\*(b^2 - 4\*a\*c)\*d\*(-2\*Sqrt[c]\*(b + 2\*c\*x)\*Sqrt[a + x\*(b + c\*x)] + (b^2 - 4\*a\*c)\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + x\*(b + c\*x)])]))/c^(3/2) - 60\*b\*e\*((16\*(b + 2\*c\*x)\*(a + x\*(b + c\*x))^(3/2))/c + (3\*(b^2 - 4\*a\*c)\*(-2\*Sqrt[c]\*(b + 2\*c\*x)\*Sqrt[a + x\*(b + c\*x)] + (b^2 - 4\*a\*c)\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + x\*(b + c\*x)])]))/c^(5/2)) + (f\*(-1792\*b\*(a + x\*(b + c\*x))^(5/2) + 5\*(7\*b^2 - 4\*a\*c)\*((16\*(b + 2\*c\*x)\*(a + x\*(b + c\*x))^(3/2))/c + (3\*(b^2 - 4\*a\*c)\*(-2\*Sqrt[c]\*(b + 2\*c\*x)\*Sqrt[a + x\*(b + c\*x)] + (b^2 - 4\*a\*c)\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + x\*(b + c\*x)])]))/c^(5/2))))/c/(15360\*c)

**fricas [A]** time = 1.27, size = 839, normalized size = 3.56

$$\left[ \frac{15(24(b^4c^2 - 8ab^2c^3 + 16a^2c^4)d - 12(b^5c - 8ab^3c^2 + 16a^2bc^3)e + (7b^6 - 60ab^4c + 144a^2b^2c^2 - 64a^3c^3)f)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(3/2)\*(f\*x^2+e\*x+d), x, algorithm="fricas")

[Out] [-1/30720\*(15\*(24\*(b^4\*c^2 - 8\*a\*b^2\*c^3 + 16\*a^2\*c^4)\*d - 12\*(b^5\*c - 8\*a\*b^3\*c^2 + 16\*a^2\*b\*c^3)\*e + (7\*b^6 - 60\*a\*b^4\*c + 144\*a^2\*b^2\*c^2 - 64\*a^3\*c^3)\*f)\*sqrt(c)\*log(-8\*c^2\*x^2 - 8\*b\*c\*x - b^2 + 4\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(c) - 4\*a\*c) - 4\*(1280\*c^6\*f\*x^5 + 128\*(12\*c^6\*e + 13\*b\*c^5\*f)\*x^4 + 16\*(120\*c^6\*d + 132\*b\*c^5\*e + (3\*b^2\*c^4 + 140\*a\*c^5)\*f)\*x^3 + 8\*(360\*b\*c^5\*d + 12\*(b^2\*c^4 + 32\*a\*c^5)\*e - (7\*b^3\*c^3 - 36\*a\*b\*c^4)\*f)\*x^2 - 120\*(3\*b^3\*c^3 - 20\*a\*b\*c^4)\*d + 12\*(15\*b^4\*c^2 - 100\*a\*b^2\*c^3 + 128\*a^2\*c^4)\*e - (105\*b^5\*c - 760\*a\*b^3\*c^2 + 1296\*a^2\*b\*c^3)\*f + 2\*(120\*(b^2\*c^4 + 20\*a\*c^5)\*d - 12\*(5\*b^3\*c^3 - 28\*a\*b\*c^4)\*e + (35\*b^4\*c^2 - 216\*a\*b^2\*c^3 + 240\*a^2\*c^4)\*f)\*x)\*sqrt(c\*x^2 + b\*x + a))/c^5, -1/15360\*(15\*(24\*(b^4\*c^2 - 8\*a\*b^2\*c^3 + 16\*a^2\*c^4)\*d - 12\*(b^5\*c - 8\*a\*b^3\*c^2 + 16\*a^2\*b\*c^3)\*e +

```
(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*f)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(1280*c^6*f*x^5 + 128*(12*c^6*e + 13*b*c^5*f)*x^4 + 16*(120*c^6*d + 132*b*c^5*e + (3*b^2*c^4 + 140*a*c^5)*f)*x^3 + 8*(360*b*c^5*d + 12*(b^2*c^4 + 32*a*c^5)*e - (7*b^3*c^3 - 36*a*b*c^4)*f)*x^2 - 120*(3*b^3*c^3 - 20*a*b*c^4)*d + 12*(15*b^4*c^2 - 100*a*b^2*c^3 + 128*a^2*c^4)*e - (105*b^5*c - 760*a*b^3*c^2 + 1296*a^2*b*c^3)*f + 2*(120*(b^2*c^4 + 20*a*c^5)*d - 12*(5*b^3*c^3 - 28*a*b*c^4)*e + (35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*f)*x)*sqrt(c*x^2 + b*x + a))/c^5]
```

**giac** [A] time = 0.61, size = 417, normalized size = 1.77

$$\frac{1}{7680} \sqrt{cx^2 + bx + a} \left( 2 \left( 4 \left( 2 \left( 8 \left( 10 c f x + \frac{13 b c^5 f + 12 c^6 e}{c^5} \right) x + \frac{120 c^6 d + 3 b^2 c^4 f + 140 a c^5 f + 132 b c^5 e}{c^5} \right) x + \frac{360 b c^5 d - 7 b^3 c^3 f + 36 a b c^4 f + 12 b^2 c^4 e + 384 a c^5 e}{c^5} \right) x + (120 b^2 c^4 d + 2400 a c^5 d + 35 b^4 c^2 f - 216 a b^2 c^3 f + 240 a^2 c^4 f - 60 b^3 c^3 e + 336 a b c^4 e) / c^5 \right) x - (360 b^3 c^3 d - 2400 a b c^4 d + 105 b^5 c f - 760 a b^3 c^2 f + 1296 a^2 b c^3 f - 180 b^4 c^2 e + 1200 a b^2 c^3 e - 1536 a^2 c^4 e) / c^5 - 1/1024 * (24 b^4 c^2 d - 192 a b^2 c^3 d + 384 a^2 c^4 d + 7 b^6 f - 60 a b^4 c f + 144 a^2 b^2 c^2 f - 64 a^3 c^3 f - 12 b^5 c e + 96 a b^3 c^2 e - 192 a^2 b c^3 e) * \log(\text{abs}(-2 * (\text{sqrt}(c) * x - \text{sqrt}(c * x^2 + b * x + a)) * \text{sqrt}(c) - b)) / c^{9/2}) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(3/2)\*(f\*x^2+e\*x+d),x, algorithm="giac")

```
[Out] 1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*c*f*x + (13*b*c^5*f + 12*c^6*e)/c^5)*x + (120*c^6*d + 3*b^2*c^4*f + 140*a*c^5*f + 132*b*c^5*e)/c^5)*x + (360*b*c^5*d - 7*b^3*c^3*f + 36*a*b*c^4*f + 12*b^2*c^4*e + 384*a*c^5*e)/c^5)*x + (120*b^2*c^4*d + 2400*a*c^5*d + 35*b^4*c^2*f - 216*a*b^2*c^3*f + 240*a^2*c^4*f - 60*b^3*c^3*e + 336*a*b*c^4*e)/c^5)*x - (360*b^3*c^3*d - 2400*a*b*c^4*d + 105*b^5*c*f - 760*a*b^3*c^2*f + 1296*a^2*b*c^3*f - 180*b^4*c^2*e + 1200*a*b^2*c^3*e - 1536*a^2*c^4*e)/c^5 - 1/1024*(24*b^4*c^2*d - 192*a*b^2*c^3*d + 384*a^2*c^4*d + 7*b^6*f - 60*a*b^4*c*f + 144*a^2*b^2*c^2*f - 64*a^3*c^3*f - 12*b^5*c*e + 96*a*b^3*c^2*e - 192*a^2*b*c^3*e)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(9/2)
```

**maple** [B] time = 0.01, size = 862, normalized size = 3.65

$$\frac{a^3 f \ln\left(\frac{cx + \frac{b}{2}}{\sqrt{c}} + \sqrt{cx^2 + bx + a}\right)}{16c^{\frac{3}{2}}} + \frac{9a^2 b^2 f \ln\left(\frac{cx + \frac{b}{2}}{\sqrt{c}} + \sqrt{cx^2 + bx + a}\right)}{64c^{\frac{5}{2}}} - \frac{3a^2 b e \ln\left(\frac{cx + \frac{b}{2}}{\sqrt{c}} + \sqrt{cx^2 + bx + a}\right)}{16c^{\frac{3}{2}}} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+b\*x+a)^(3/2)\*(f\*x^2+e\*x+d),x)

```
[Out] 1/8*f*b^2/c^2*(c*x^2+b*x+a)^(1/2)*x*a-3/16*e*b/c*(c*x^2+b*x+a)^(1/2)*x*a+1/4*d*(c*x^2+b*x+a)^(3/2)*x+1/5*e*(c*x^2+b*x+a)^(5/2)/c+1/6*f*x*(c*x^2+b*x+a)^(5/2)/c-3/32*d/c*(c*x^2+b*x+a)^(1/2)*x*b^2-3/16*e*b/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2+3/128*d/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^4+1/8*d/c*(c*x^2+b*x+a)^(3/2)*b-1/16*e*b^2/c^2*(c*x^2+
```

```

b*x+a)^(3/2)+3/128*e*b^4/c^3*(c*x^2+b*x+a)^(1/2)-3/256*e*b^5/c^(7/2)*ln((c*
x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-7/60*f*b/c^2*(c*x^2+b*x+a)^(5/2)+7/19
2*f*b^3/c^3*(c*x^2+b*x+a)^(3/2)-7/512*f*b^5/c^4*(c*x^2+b*x+a)^(1/2)+7/1024*
f*b^6/c^(9/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/16*f*a^3/c^(3/2
)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+3/8*d/c^(1/2)*ln((c*x+1/2*b)/
c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2+3/8*d*(c*x^2+b*x+a)^(1/2)*x*a-3/64*d/c^2*(
c*x^2+b*x+a)^(1/2)*b^3-1/24*f*a/c*(c*x^2+b*x+a)^(3/2)*x-1/48*f*a/c^2*(c*x^2
+b*x+a)^(3/2)*b-1/16*f*a^2/c*(c*x^2+b*x+a)^(1/2)*x+7/96*f*b^2/c^2*(c*x^2+b*
x+a)^(3/2)*x-7/256*f*b^4/c^3*(c*x^2+b*x+a)^(1/2)*x+1/16*f*b^3/c^3*(c*x^2+b*
x+a)^(1/2)*a+9/64*f*b^2/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
*a^2-15/256*f*b^4/c^(7/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-3/1
6*d/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2*a+3/16*d/c*(c*x
^2+b*x+a)^(1/2)*b*a+3/64*e*b^3/c^2*(c*x^2+b*x+a)^(1/2)*x-3/32*e*b^2/c^2*(c*
x^2+b*x+a)^(1/2)*a-1/8*e*b/c*(c*x^2+b*x+a)^(3/2)*x-1/32*f*a^2/c^2*(c*x^2+b*
x+a)^(1/2)*b+3/32*e*b^3/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
*a

```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (cx^2 + bx + a)^{3/2} (fx^2 + ex + d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
```

```
[Out] int((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + bx + cx^2)^{3/2} (d + ex + fx^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2), x)
```

$$3.106 \quad \int \frac{(a+bx+cx^2)^{3/2}}{d+ex+fx^2} dx$$

**Optimal.** Leaf size=679

$$\left( (e - \sqrt{e^2 - 4df}) (ce - bf) (f(be - 2af) - c(e^2 - 2df)) - 2f(-f^2(b^2d - a^2f) + 2cdf(be - af) + c^2(-d)(e^2 - df) \right.$$

$$\left. \sqrt{2} f^3 \sqrt{e^2 - 4df} \sqrt{2af^2 - \sqrt{e^2 - 4df} (ce - bf) - bef - 2cdf} \right)$$

[Out]  $\frac{1}{8} (3b^2f^2 - 12c*f*(-af+be) + 8c^2*(-df+e^2)) * \operatorname{arctanh}\left(\frac{1}{2}(2cx+b)/c^{1/2}\right) / (cx^2+bx+a)^{1/2} / f^3 / c^{1/2} - \frac{1}{4} (-2cfx-5bf+4ce) * (cx^2+bx+a)^{1/2} / f^2 + \frac{1}{2} \operatorname{arctanh}\left(\frac{1}{4}(4af+2x*(bf-c*(e-(-4df+e^2)^{1/2}))) - b*(e-(-4df+e^2)^{1/2})\right) * 2^{1/2} / (cx^2+bx+a)^{1/2} / (ce^2-2c*df-b*ef+2a*f^2 - (-bf+ce)*(-4df+e^2)^{1/2})^{1/2} * (-2f*(2c*df*(-af+be) - f^2*(-a^2f+b^2d) - c^2*d*(-df+e^2)) + (-bf+ce)*(f*(-2af+be) - c*(-2df+e^2)) * (e-(-4df+e^2)^{1/2})) / f^3 * 2^{1/2} / (-4df+e^2)^{1/2} / (ce^2-2c*df-b*ef+2a*f^2 - (-bf+ce)*(-4df+e^2)^{1/2})^{1/2} - \frac{1}{2} \operatorname{arctanh}\left(\frac{1}{4}(4af-b*(e+(-4df+e^2)^{1/2})+2x*(bf-c*(e+(-4df+e^2)^{1/2})))\right) * 2^{1/2} / (cx^2+bx+a)^{1/2} / (ce^2-2c*df-b*ef+2a*f^2+(-bf+ce)*(-4df+e^2)^{1/2})^{1/2} * (-2f*(2c*df*(-af+be) - f^2*(-a^2f+b^2d) - c^2*d*(-df+e^2)) + (-bf+ce)*(f*(-2af+be) - c*(-2df+e^2)) * (e+(-4df+e^2)^{1/2})) / f^3 * 2^{1/2} / (-4df+e^2)^{1/2} / (ce^2-2c*df-b*ef+2a*f^2+(-bf+ce)*(-4df+e^2)^{1/2})^{1/2}$

**Rubi [A]** time = 11.03, antiderivative size = 678, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {977, 1076, 621, 206, 1032, 724}

$$\left( (e - \sqrt{e^2 - 4df}) (ce - bf) (f(be - 2af) - c(e^2 - 2df)) - 2f(-f^2(b^2d - a^2f) + 2cdf(be - af) + c^2(-d)(e^2 - df) \right.$$

$$\left. \sqrt{2} f^3 \sqrt{e^2 - 4df} \sqrt{2af^2 - \sqrt{e^2 - 4df} (ce - bf) - bef - 2cdf} \right)$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + bx + cx^2)^{3/2} / (d + ex + fx^2), x]$

[Out]  $-\left(\frac{(4ce - 5bf - 2cfx) \operatorname{Sqrt}[a + bx + cx^2]}{(4f^2)} + \frac{((3b^2f^2 - 12c*f*(be - af) + 8c^2*(e^2 - df)) \operatorname{ArcTanh}[(b + 2cx)/(2\operatorname{Sqrt}[c] \operatorname{Sqrt}[a + bx + cx^2])])}{(8\operatorname{Sqrt}[c] f^3)} + \frac{((c*e - bf)*(e - \operatorname{Sqrt}[e^2 - 4df]) * (f*(be - 2af) - c*(e^2 - 2df)) - 2f*(2c*df*(be - af) - f^2*(b^2d - a^2f) - c^2*d*(e^2 - df)) \operatorname{ArcTanh}[(4af - b*(e - \operatorname{Sqrt}[e^2 - 4df])) + 2*(bf - c*(e - \operatorname{Sqrt}[e^2 - 4df])) * x]}{(2\operatorname{Sqrt}[2] \operatorname{Sqrt}[ce^2 - 2c*df])}\right)$

$$\frac{-b*ef + 2*a*f^2 - (c*e - b*f)*\sqrt{e^2 - 4*d*f}*\sqrt{a + b*x + c*x^2}}{(\sqrt{2}*f^3*\sqrt{e^2 - 4*d*f}*\sqrt{c*e^2 - 2*c*d*f - b*ef + 2*a*f^2 - (c*e - b*f)*\sqrt{e^2 - 4*d*f}}) + ((4*c*d*f^2*(b*e - a*f) - 2*f^3*(b^2*d - a^2*f) - 2*c^2*d*f*(e^2 - d*f) - (c*e - b*f)*(e + \sqrt{e^2 - 4*d*f})*(f*(b*e - 2*a*f) - c*(e^2 - 2*d*f))) * \text{ArcTanh}[(4*a*f - b*(e + \sqrt{e^2 - 4*d*f}) + 2*(b*f - c*(e + \sqrt{e^2 - 4*d*f})) * x] / (2*\sqrt{2}*\sqrt{c*e^2 - 2*c*d*f - b*ef + 2*a*f^2 + (c*e - b*f)*\sqrt{e^2 - 4*d*f}*\sqrt{a + b*x + c*x^2}})}{(\sqrt{2}*f^3*\sqrt{e^2 - 4*d*f}*\sqrt{c*e^2 - 2*c*d*f - b*ef + 2*a*f^2 + (c*e - b*f)*\sqrt{e^2 - 4*d*f}})}$$

### Rule 206

$$\text{Int}[(a_) + (b_)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x] / \text{Rt}[a, 2]) / (\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$$

### Rule 621

$$\text{Int}[1/\sqrt{(a_) + (b_)*(x_) + (c_)*(x_)^2}, x\_Symbol] \rightarrow \text{Dist}[2, \text{Subst}[\text{Int}[1/(4*c - x^2), x], x, (b + 2*c*x)/\sqrt{a + b*x + c*x^2}], x] /; \text{FreeQ}\{a, b, c\}, x \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$$

### Rule 724

$$\text{Int}[1/(((d_) + (e_)*(x_))*\sqrt{(a_) + (b_)*(x_) + (c_)*(x_)^2}), x\_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/\sqrt{a + b*x + c*x^2}], x] /; \text{FreeQ}\{a, b, c, d, e\}, x \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0]$$

### Rule 977

$$\text{Int}[(a_) + (b_)*(x_) + (c_)*(x_)^2)^{(p_)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^{(q_)}, x\_Symbol] \rightarrow \text{Simp}[(b*f*(3*p + 2*q) - c*e*(2*p + q) + 2*c*f*(p + q)*x)*(a + b*x + c*x^2)^{(p-1)}*(d + e*x + f*x^2)^{(q+1)} / (2*f^2*(p + q)*(2*p + 2*q + 1)), x] - \text{Dist}[1/(2*f^2*(p + q)*(2*p + 2*q + 1)), \text{Int}[(a + b*x + c*x^2)^{(p-2)}*(d + e*x + f*x^2)^q * \text{Simp}[(b*d - a*e)*(c*e - b*f)*(1 - p)*(2*p + q) - (p + q)*(b^2*d*f*(1 - p) - a*(f*(b*e - 2*a*f)*(2*p + 2*q + 1) + c*(2*d*f - e^2*(2*p + q)))] + (2*(c*d - a*f)*(c*e - b*f)*(1 - p)*(2*p + q) - (p + q)*((b^2 - 4*a*c)*e*f*(1 - p) + b*(c*(e^2 - 4*d*f)*(2*p + q) + f*(2*c*d - b*e + 2*a*f)*(2*p + 2*q + 1)))] * x + ((c*e - b*f)^2*(1 - p)*p + c*(p + q)*(f*(b*e - 2*a*f)*(4*p + 2*q - 1) - c*(2*d*f*(1 - 2*p) + e^2*(3*p + q - 1)))] * x^2, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, q\}, x \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[e^2 - 4*d*f, 0] \ \&\& \ \text{GtQ}[p, 1] \ \&\& \ \text{NeQ}[p + q, 0] \ \&\& \ \text{NeQ}[2*p + 2*q + 1, 0] \ \&\& \ !\text{IGtQ}[p, 0] \ \&\& \ !\text{IGtQ}[q, 0]$$

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{(a + bx + cx^2)^{3/2}}{d + ex + fx^2} dx = -\frac{(4ce - 5bf - 2cfx)\sqrt{a + bx + cx^2}}{4f^2} - \frac{\int \frac{\frac{1}{4}(-4bcde + 5b^2df + 4af(cd - 2af)) - \frac{1}{4}(8c^2de - 4acef - bf(5be - 16c^2))}{\sqrt{a + bx + cx^2}} dx}{2}$$

$$= -\frac{(4ce - 5bf - 2cfx)\sqrt{a + bx + cx^2}}{4f^2} - \frac{\int \frac{\frac{1}{4}f(-4bcde + 5b^2df + 4af(cd - 2af)) - \frac{1}{4}d(-3b^2f^2 + 12cf(be - af))}{\sqrt{a + bx + cx^2}} dx}{2}$$

$$= -\frac{(4ce - 5bf - 2cfx)\sqrt{a + bx + cx^2}}{4f^2} + \frac{(3b^2f^2 - 12cf(be - af) + 8c^2(e^2 - df)) \text{Subst}\left(\int \frac{1}{\sqrt{a + bx + cx^2}} dx\right)}{4f^3}$$

$$= -\frac{(4ce - 5bf - 2cfx)\sqrt{a + bx + cx^2}}{4f^2} + \frac{(3b^2f^2 - 12cf(be - af) + 8c^2(e^2 - df)) \tanh^{-1}\left(\frac{2cx + b}{\sqrt{a + bx + cx^2}}\right)}{8\sqrt{c}f^3}$$

$$= -\frac{(4ce - 5bf - 2cfx)\sqrt{a + bx + cx^2}}{4f^2} + \frac{(3b^2f^2 - 12cf(be - af) + 8c^2(e^2 - df)) \tanh^{-1}\left(\frac{2cx + b}{\sqrt{a + bx + cx^2}}\right)}{8\sqrt{c}f^3}$$

**Mathematica [A]** time = 4.58, size = 1232, normalized size = 1.81

$$\sqrt{a + x(b + cx)} \left( -4 \left( e + \sqrt{e^2 - 4df} \right)^2 c^2 + 4f \left( e + \sqrt{e^2 - 4df} \right) xc^2 - 16af^2c + 10bf \left( e + \sqrt{e^2 - 4df} \right) c - 4bf^2xc \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x + c\*x^2)^(3/2)/(d + e\*x + f\*x^2), x]

[Out] 
$$\frac{\left( (-2*b^2*f^2 - 16*a*c*f^2 + 10*b*c*f*(e + \sqrt{e^2 - 4*d*f})) - 4*c^2*(e + \sqrt{e^2 - 4*d*f})^2 - 4*b*c*f^2*x + 4*c^2*f*(e + \sqrt{e^2 - 4*d*f})*x \right) * \sqrt{a + x*(b + c*x)} + 2*\sqrt{a + x*(b + c*x)}*(b^2*f^2 - 2*c^2*(-2*e^2 + 4*d*f + 2*e*\sqrt{e^2 - 4*d*f} + e*f*x - f*\sqrt{e^2 - 4*d*f}*x) + c*f*(8*a*f + b*(-5*e + 5*\sqrt{e^2 - 4*d*f} + 2*f*x))) - ((b*f + c*(-e + \sqrt{e^2 - 4*d*f}))*(b^2*f^2 + 4*c^2*(-e^2 + 2*d*f + e*\sqrt{e^2 - 4*d*f})) - 4*c*f*(3*a*f + b*(-e + \sqrt{e^2 - 4*d*f}))) * \text{ArcTanh}[(b + 2*c*x)/(2*\sqrt{c}*\sqrt{a + x*(b + c*x)})]} \right) / (\sqrt{c}*f) + ((-(b*f) + c*(e + \sqrt{e^2 - 4*d*f})) * (-(b^2*f^2) + 4*c^2*(e^2 - 2*d*f + e*\sqrt{e^2 - 4*d*f})) - 4*c*f*(-3*a*f + b*(e + \sqrt{e^2 - 4*d*f}))) * \text{ArcTanh}[(b + 2*c*x)/(2*\sqrt{c}*\sqrt{a + x*(b + c*x)})]} \right) / (\sqrt{c}*f) + (8*\sqrt{2}*c*(c^2*(e^4 - 4*d*e^2*f + 2*d^2*f^2 + e^3*\sqrt{e^2 - 4*d*f}) - 2*d*e*f*\sqrt{e^2 - 4*d*f}) + f^2*(2*a^2*f^2 - 2*a*b*f*(e + \sqrt{e^2 - 4*d*f}) + b^2*(e^2 - 2*d*f + e*\sqrt{e^2 - 4*d*f})) + 2*c*f*(a*f*(e^2 - 2*d*f + e*\sqrt{e^2 - 4*d*f}) - b*(e^3 - 3*d*e*f + e^2*\sqrt{e^2 - 4*d*f} - d*f*\sqrt{e^2 - 4*d*f}))) * \text{ArcTanh}[(4*a*f - 2*c*(e + \sqrt{e^2 - 4*d*f}))*x - b*(e + \sqrt{e^2 - 4*d*f} - 2*f*x)] / (2*\sqrt{2}*\sqrt{c*(e^2 - 2*d*f + e*\sqrt{e^2 - 4*d*f}) + f*(2*a*f - b*(e + \sqrt{e^2 - 4*d*f}))}) * \sqrt{a + x*(b + c*x)} \right) / (f*\sqrt{c*(e^2 - 2*d*f + e*\sqrt{e^2 - 4*d*f}) + f*(2*a*f - b*(e + \sqrt{e^2 - 4*d*f}))}) + (8*\sqrt{2}*c*(c^2*(-e^4 + 4*d*e^2*f - 2*d^2*f^2 + e^3*\sqrt{e^2 - 4*d*f}) - 2*d*e*f*\sqrt{e^2 - 4*d*f}) + f^2*(-2*a^2*f^2 + 2*a*b*f*(e - \sqrt{e^2 - 4*d*f}) + b^2*(-e^2 + 2*d*f + e*\sqrt{e^2 - 4*d*f})) + 2*c*f*(a*f*(-e^2 + 2*d*f + e*\sqrt{e^2 - 4*d*f}) + b*(e^3 - 3*d*e*f - e^2*\sqrt{e^2 - 4*d*f} + d*f*\sqrt{e^2 - 4*d*f}))) * \text{ArcTanh}[(4*a*f + 2*c*(-e + \sqrt{e^2 - 4*d*f}))*x + b*(-e + \sqrt{e^2 - 4*d*f} + 2*f*x)] / (2*\sqrt{2}*\sqrt{c*(e^2 - 2*d*f - e*\sqrt{e^2 - 4*d*f}) + f*(2*a*f + b*(-e + \sqrt{e^2 - 4*d*f}))}) * \sqrt{a + x*(b + c*x)} \right) / (f*\sqrt{c*(e^2 - 2*d*f - e*\sqrt{e^2 - 4*d*f}) + f*(2*a*f + b*(-e + \sqrt{e^2 - 4*d*f}))}) \right) / (16*c*f^2*\sqrt{e^2 - 4*d*f})$$

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

```
giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
i,const vecteur & l) Error: Bad Argument Value
```

```
maple [B] time = 0.03, size = 22523, normalized size = 33.17
```

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

```
maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

```
mupad [F] time = 0.00, size = -1, normalized size = -0.00
```

$$\int \frac{(cx^2 + bx + a)^{3/2}}{fx^2 + ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2),x)
```

```
[Out] int((a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d), x)
```

```
[Out] Timed out
```

$$3.107 \quad \int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^2} dx$$

**Optimal.** Leaf size=704

$$\left( (e - \sqrt{e^2 - 4df}) (ce - bf) (f(be - 2af) + 2c(e^2 - 5df)) - 2f(f(-be(3af + cd) + 4af(af + cd) + 2b^2df) + 2c$$

$$2\sqrt{2} f^2 (e^2 - 4df)^{3/2} \sqrt{2af^2 - \sqrt{e^2 - 4df} (ce - bf) - be} \right)$$

[Out]  $-(2fx+e)(cx^2+bx+a)^{3/2}/(-4d*fx+e^2)/(fx^2+ex+d)+c^{3/2}*\operatorname{arctanh}(1/2*(2cx+b)/c^{1/2}/(cx^2+bx+a)^{1/2})/f^2-(-2c*fx-2b*fx+e)(cx^2+bx+a)^{1/2}/f/(-4d*fx+e^2)-1/4*\operatorname{arctanh}(1/4*(4a*fx+2*x*(b*fx-c*(e-(-4d*fx+e^2)^{1/2}))) - b*(e-(-4d*fx+e^2)^{1/2}))*2^{1/2}/(cx^2+bx+a)^{1/2}/(c*e^2-2c*d*fx-b*e*fx+2*a*fx^2-(-b*fx+c*e)*(-4d*fx+e^2)^{1/2}))^{1/2}*(-2f*(2*c^2*d*(-4d*fx+e^2)+f*(2*b^2*d*fx+4*a*fx*(a*fx+c*d)-b*e*(3*a*fx+c*d)))+(-b*fx+c*e)*(f*(-2*a*fx+b*e)+2*c*(-5*d*fx+e^2))*(e-(-4d*fx+e^2)^{1/2}))/f^2/(-4d*fx+e^2)^{3/2}*2^{1/2}/(c*e^2-2c*d*fx-b*e*fx+2*a*fx^2-(-b*fx+c*e)*(-4d*fx+e^2)^{1/2}))^{1/2}+1/4*\operatorname{arctanh}(1/4*(4a*fx-b*(e+(-4d*fx+e^2)^{1/2}))+2*x*(b*fx-c*(e+(-4d*fx+e^2)^{1/2}))))*2^{1/2}/(cx^2+bx+a)^{1/2}/(c*e^2-2c*d*fx-b*e*fx+2*a*fx^2+(-b*fx+c*e)*(-4d*fx+e^2)^{1/2}))^{1/2}*(-2f*(2*c^2*d*(-4d*fx+e^2)+f*(2*b^2*d*fx+4*a*fx*(a*fx+c*d)-b*e*(3*a*fx+c*d)))+(-b*fx+c*e)*(f*(-2*a*fx+b*e)+2*c*(-5*d*fx+e^2))*(e+(-4d*fx+e^2)^{1/2}))/f^2/(-4d*fx+e^2)^{3/2}*2^{1/2}/(c*e^2-2c*d*fx-b*e*fx+2*a*fx^2+(-b*fx+c*e)*(-4d*fx+e^2)^{1/2}))^{1/2}$

**Rubi [A]** time = 11.95, antiderivative size = 704, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$ , Rules used = {971, 1066, 1076, 621, 206, 1032, 724}

$$\left( (e - \sqrt{e^2 - 4df}) (ce - bf) (f(be - 2af) + 2c(e^2 - 5df)) - 2f(f(-be(3af + cd) + 4af(af + cd) + 2b^2df) + 2c$$

$$2\sqrt{2} f^2 (e^2 - 4df)^{3/2} \sqrt{2af^2 - \sqrt{e^2 - 4df} (ce - bf) - be} \right)$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b*x + c*x^2)^{3/2}/(d + e*x + f*x^2)^2, x]$

[Out]  $-(((c*e - 2*b*fx - 2*c*fx)*\operatorname{Sqrt}[a + b*x + c*x^2])/(f*(e^2 - 4*d*fx))) - ((e + 2*fx)*(a + b*x + c*x^2)^{3/2})/((e^2 - 4*d*fx)*(d + e*x + f*x^2)) + (c^{3/2}*\operatorname{ArcTanh}[(b + 2*c*x)/(2*\operatorname{Sqrt}[c]*\operatorname{Sqrt}[a + b*x + c*x^2])])/f^2 - (((c*e - b*fx)*(f*(b*e - 2*a*fx) + 2*c*(e^2 - 5*d*fx))*(e - \operatorname{Sqrt}[e^2 - 4*d*fx]) - 2*fx*(2*c^2*d*(e^2 - 4*d*fx) + f*(2*b^2*d*fx + 4*a*fx*(c*d + a*fx) - b*e*(c*d + 3*a*fx)$



$$\left. \right) \text{ArcTanh}[(4*af - b*(e - \sqrt{e^2 - 4*df})) + 2*(bf - c*(e - \sqrt{e^2 - 4*df}))]*x) / (2*\sqrt{2}*\sqrt{c*e^2 - 2*c*df - b*ef + 2*a*f^2 - (c*e - b*f)*\sqrt{e^2 - 4*df}}) * \sqrt{e^2 - 4*df}] * \sqrt{a + b*x + c*x^2}) / (2*\sqrt{2}*f^2*(e^2 - 4*df)^{(3/2)}*\sqrt{c*e^2 - 2*c*df - b*ef + 2*a*f^2 - (c*e - b*f)*\sqrt{e^2 - 4*df}}) + (((c*e - b*f)*(f*(b*e - 2*af) + 2*c*(e^2 - 5*df))*(e + \sqrt{e^2 - 4*df}) - 2*f*(2*c^2*d*(e^2 - 4*df) + f*(2*b^2*df + 4*af*(c*d + af) - b*e*(c*d + 3*af)))) * \text{ArcTanh}[(4*af - b*(e + \sqrt{e^2 - 4*df})) + 2*(bf - c*(e + \sqrt{e^2 - 4*df}))]*x) / (2*\sqrt{2}*\sqrt{c*e^2 - 2*c*df - b*ef + 2*a*f^2 + (c*e - b*f)*\sqrt{e^2 - 4*df}}) * \sqrt{a + b*x + c*x^2}) / (2*\sqrt{2}*f^2*(e^2 - 4*df)^{(3/2)}*\sqrt{c*e^2 - 2*c*df - b*ef + 2*a*f^2 + (c*e - b*f)*\sqrt{e^2 - 4*df}})$$

### Rule 206

$$\text{Int}[(a + (b \cdot x)^{-1}), x\_Symbol] \rightarrow \text{Simp}[(1 * \text{ArcTanh}[\text{Rt}[-b, 2]*x] / \text{Rt}[a, 2]) / (\text{Rt}[a, 2] * \text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$$

### Rule 621

$$\text{Int}[1/\sqrt{(a + (b \cdot x) + (c \cdot x)^2)}, x\_Symbol] \rightarrow \text{Dist}[2, \text{Subst}[\text{Int}[1/(4*c - x^2), x], x, (b + 2*c*x)/\sqrt{a + b*x + c*x^2}], x] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$$

### Rule 724

$$\text{Int}[1/(((d \cdot x) + (e \cdot x)) * \sqrt{(a \cdot x) + (b \cdot x) + (c \cdot x)^2}), x\_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/\sqrt{a + b*x + c*x^2}], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[2*c*d - b*e, 0]$$

### Rule 971

$$\text{Int}[(a + (b \cdot x) + (c \cdot x)^2)^{(p)} * ((d \cdot x) + (e \cdot x) + (f \cdot x)^2)^{(q)}, x\_Symbol] \rightarrow \text{Simp}[(b + 2*c*x) * (a + b*x + c*x^2)^{(p+1)} * (d + e*x + f*x^2)^q / ((b^2 - 4*a*c) * (p+1)), x] - \text{Dist}[1/((b^2 - 4*a*c) * (p+1)), \text{Int}[(a + b*x + c*x^2)^{(p+1)} * (d + e*x + f*x^2)^{(q-1)} * \text{Simp}[2*c*d*(2*p+3) + b*e*q + (2*b*f*q + 2*c*e*(2*p+q+3))*x + 2*c*f*(2*p+2*q+3)*x^2, x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[e^2 - 4*d*f, 0] \&\& \text{LtQ}[p, -1] \&\& \text{GtQ}[q, 0] \&\& !\text{IGtQ}[q, 0]$$

### Rule 1032

$$\text{Int}[(g + (h \cdot x)) / ((a + (b \cdot x) + (c \cdot x)^2) * \sqrt{(d \cdot x) + (e \cdot x) + (f \cdot x)^2}), x\_Symbol] \rightarrow \text{With}\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[(2*c*g - h*(b - q))/q, \text{Int}[1/((b - q + 2*c*x) * \sqrt{d + e*x + f*x^2}), x],$$

```
x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

### Rule 1066

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((A_) + (B_)*(x_) + (C_)*(x_)
^2)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[((B*c*f*(2*
p + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a +
b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x
^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) -
c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f
- e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3)))] + (2*p*(c*d - a*f)
*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e
*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e +
2*A*f)*(2*p + 2*q + 3)))]*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*
e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^
2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3)))]*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]
```

### Rule 1076

```
Int[((A_) + (B_)*(x_) + (C_)*(x_)^2)/(((a_) + (b_)*(x_) + (c_)*(x_)^2)
*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^2} dx &= \frac{(e+2fx)(a+bx+cx^2)^{3/2}}{(e^2-4df)(d+ex+fx^2)} - \frac{\int \frac{\sqrt{a+bx+cx^2} \left( \frac{1}{2}(3be-4af) + (3ce+bf)x + 4cfx^2 \right)}{d+ex+fx^2} dx}{-e^2+4df} \\
&= \frac{(ce-2bf-2cfx)\sqrt{a+bx+cx^2}}{f(e^2-4df)} - \frac{(e+2fx)(a+bx+cx^2)^{3/2}}{(e^2-4df)(d+ex+fx^2)} - \frac{\int \frac{cf(2b^2df+4af(cd+a))}{\sqrt{a+bx+cx^2}} dx}{f^2} \\
&= \frac{(ce-2bf-2cfx)\sqrt{a+bx+cx^2}}{f(e^2-4df)} - \frac{(e+2fx)(a+bx+cx^2)^{3/2}}{(e^2-4df)(d+ex+fx^2)} + \frac{c^2 \int \frac{1}{\sqrt{a+bx+cx^2}} dx}{f^2} \\
&= \frac{(ce-2bf-2cfx)\sqrt{a+bx+cx^2}}{f(e^2-4df)} - \frac{(e+2fx)(a+bx+cx^2)^{3/2}}{(e^2-4df)(d+ex+fx^2)} + \frac{(2c^2) \text{Subst} \left( \int \frac{1}{4c-\sqrt{a+bx+cx^2}} dx \right)}{f^2} \\
&= \frac{(ce-2bf-2cfx)\sqrt{a+bx+cx^2}}{f(e^2-4df)} - \frac{(e+2fx)(a+bx+cx^2)^{3/2}}{(e^2-4df)(d+ex+fx^2)} + \frac{c^{3/2} \tanh^{-1} \left( \frac{b}{2\sqrt{c}\sqrt{a+bx+cx^2}} \right)}{f^2} \\
&= \frac{(ce-2bf-2cfx)\sqrt{a+bx+cx^2}}{f(e^2-4df)} - \frac{(e+2fx)(a+bx+cx^2)^{3/2}}{(e^2-4df)(d+ex+fx^2)} + \frac{c^{3/2} \tanh^{-1} \left( \frac{b}{2\sqrt{c}\sqrt{a+bx+cx^2}} \right)}{f^2}
\end{aligned}$$

**Mathematica [B]** time = 6.83, size = 2843, normalized size = 4.04

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x + c\*x^2)^(3/2)/(d + e\*x + f\*x^2)^2,x]

[Out] 
$$\begin{aligned}
&(-2*f*(a + x*(b + c*x))^(3/2))/((e^2 - 4*d*f)*(e - \text{Sqrt}[e^2 - 4*d*f] + 2*f*x)) - (2*f*(a + x*(b + c*x))^(3/2))/((e^2 - 4*d*f)*(e + \text{Sqrt}[e^2 - 4*d*f] + 2*f*x)) \\
&- (3*f*(a + x*(b + c*x))^(3/2)*((( -4*b*c*f - 2*c*(b*f + 2*c*(-e + \text{Sqrt}[e^2 - 4*d*f])) - 4*c^2*f*x)*\text{Sqrt}[a + b*x + c*x^2]))/(8*c*f^2) - ((2*\text{Sqrt}[c]*(b^2*f^2 + 4*c^2*(e^2 - 2*d*f - e*\text{Sqrt}[e^2 - 4*d*f]) + 4*c*f*(a*f - b*(e - \text{Sqrt}[e^2 - 4*d*f])))*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])))/f \\
&+ (2*\text{Sqrt}[2]*\text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*\text{Sqrt}[e^2 - 4*d*f] + b*f*\text{Sqrt}[e^2 - 4*d*f]]*(4*c*f*(8*a*b*f^2 - 3*b^2*f*(e - \text{Sqrt}[e^2 - 4*d*f]) - 4*a*c*f*(e - \text{Sqrt}[e^2 - 4*d*f]) + 4*b*c*(e^2 - 2*d*f - e*\text{Sqrt}[e^2 - 4*d*f])
\end{aligned}$$



$$\begin{aligned} & *f + e*\text{Sqrt}[e^2 - 4*d*f]) + 4*c*f*(a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f])) + 4*c* \\ & f*(3*b^2*f*(e + \text{Sqrt}[e^2 - 4*d*f]) + 4*a*c*f*(e + \text{Sqrt}[e^2 - 4*d*f]) - 4*b* \\ & (2*a*f^2 + c*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]))) * \text{ArcTanh}[(4*a*f - b*(e + \\ & \text{Sqrt}[e^2 - 4*d*f]) - (-2*b*f + 2*c*(e + \text{Sqrt}[e^2 - 4*d*f]))*x)/(2*\text{Sqrt}[2]* \\ & \text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*\text{Sqrt}[e^2 - 4*d*f] - b*f*\text{Sqrt}[e \\ & ^2 - 4*d*f])* \text{Sqrt}[a + b*x + c*x^2]])/(f*(16*a*f^2 - 8*b*f*(e + \text{Sqrt}[e^2 - \\ & 4*d*f]) + 4*c*(e + \text{Sqrt}[e^2 - 4*d*f])^2))/(16*c*f^2))/((e^2 - 4*d*f)*(a + \\ & b*x + c*x^2)^(3/2)) \end{aligned}$$

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(3/2)/(f\*x^2+e\*x+d)^2,x, algorithm="fricas")

[Out] Timed out

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(3/2)/(f\*x^2+e\*x+d)^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP  
 UT:sage2:=int(sage0,x);OUTPUT:Unable to divide, perhaps due to rounding er  
 ror%%{-1,0]:[1,0,%%{-1,[1]%%}]%%},[8,4,8,0,0,0]%%}+%%{-16,[1]%%},0]:[1,0,%%{-1,[1]%%}]%%},[8,4,6,0,1,0]%%}+%%{-96,[2]%%},0]:[1,0,%%{-1,[1]%%}]%%},[8,4,4,0,2,0]%%}+%%{256,[3]%%},0]:[1,0,%%{-1,[1]%%}]%%},[8,4,2,0,3,0]%%}+%%{-256,[4]%%},0]:[1,0,%%{-1,[1]%%}]%%},[8,4,0,0,4,0]%%}+%%{4,[1]%%},[7,3,8,1,0,0]%%}+%%{-64,[2]%%},[7,3,6,1,1,0]%%}+%%{384,[3]%%},[7,3,4,1,2,0]%%}+%%{-1024,[4]%%},[7,3,2,1,3,0]%%}+%%{1024,[5]%%},[7,3,0,1,4,0]%%}+%%{4,0]:[1,0,%%{-1,[1]%%}]%%},[6,4,8,0,1,0]%%}+%%{-64,[1]%%},0]:[1,0,%%{-1,[1]%%}]%%},[6,4,6,0,2,0]%%}+%%{384,[2]%%},0]:[1,0,%%{-1,[1]%%}]%%},[6,4,4,0,3,0]%%}+%%{-1024,[3]%%},0]:[1,0,%%{-1,[1]%%}]%%},[6,4,2,0,4,0]%%}+%%{1024,[4]%%},0]:[1,0,%%{-1,[1]%%}]%%},[6,4,0,0,5,0]%%}+%%{-2,0]:[1,0,%%{-1,[1]%%}]%%},[6,3,9,1,0,0]%%}+%%{-8,[1]%%},0]:[1,0,%%{-1,[1]%%}]%%},[6,3,8,0,0,1]%%}+%%{32,[1]%%},0]:[1,0,%%{-1,[1]%%}]%%},[6,3,7,1,1,0]%%}+%%{128,[2]%%},0]:[1,0,%%{-1,[1]%%}]%%},[6,3,6,0,1,1]%%}+%%{-192,[2]%%},0]:[1,0,%%{-1,[1]%%}]%%},[6,3,5,1,2,0]%%}+%%{-768,[3]%%},0]:[1,0,%%{-1,[1]%%}]%%},[6,3,4,0,2,1]%%}+%%{512,[3]%%},0]:[1,0,%%{-1,[1]%%}]%%},[6,3,3,1,3,0]%%}+%%{2048,[4]

$\{\}, 0\} : [1, 0, \{\{-1, [1]\}\}] \{\}, [6, 3, 2, 0, 3, 1]\{\} + \{\{\{-512, [4]\}\}\},$   
 $0\} : [1, 0, \{\{-1, [1]\}\}] \{\}, [6, 3, 1, 1, 4, 0]\{\} + \{\{\{-2048, [5]\}\}\}, 0\} : [$   
 $1, 0, \{\{-1, [1]\}\}] \{\}, [6, 3, 0, 0, 4, 1]\{\} + \{\{\{-4, [1]\}\}\}, 0\} : [1, 0, \{\{-1, [1]\}\}] \{\}, [6, 2, 8, 2, 0, 0]\{\} + \{\{\{64, [2]\}\}\}, 0\} : [1, 0, \{\{-1, [1]\}\}] \{\}, [6, 2, 6, 2, 1, 0]\{\} + \{\{\{-384, [3]\}\}\}, 0\} : [1, 0, \{\{-1, [1]\}\}] \{\}, [6, 2, 4, 2, 2, 0]\{\} + \{\{\{1024, [4]\}\}\}, 0\} : [1, 0, \{\{-1, [1]\}\}] \{\}, [6, 2, 2, 2, 3, 0]\{\} + \{\{\{-1024, [5]\}\}\}, 0\} : [1, 0, \{\{-1, [1]\}\}] \{\}, [6, 2, 0, 2, 4, 0]\{\} + \{\{\{8, [1]\}\}\}, [5, 3, 9, 0, 0, 1]\{\} + \{\{\{-12, [1]\}\}\}, [5, 3, 8, 1, 1, 0]\{\} + \{\{\{-128, [2]\}\}\}, [5, 3, 7, 0, 1, 1]\{\} + \{\{\{192, [2]\}\}\}, [5, 3, 6, 1, 2, 0]\{\} + \{\{\{768, [3]\}\}\}, [5, 3, 5, 0, 2, 1]\{\} + \{\{\{-1152, [3]\}\}\}, [5, 3, 4, 1, 3, 0]\{\} + \{\{\{-2048, [4]\}\}\}, [5, 3, 3, 0, 3, 1]\{\} + \{\{\{3072, [4]\}\}\}, [5, 3, 2, 1, 4, 0]\{\} + \{\{\{2048, [5]\}\}\}, [5, 3, 1, 0, 4, 1]\{\} + \{\{\{-3072, [5]\}\}\}, [5, 3, 0, 1, 5, 0]\{\} + \{\{\{4, [1]\}\}\}, [5, 2, 9, 2, 0, 0]\{\} + \{\{\{16, [2]\}\}\}, [5, 2, 8, 1, 0, 1]\{\} + \{\{\{-64, [2]\}\}\}, [5, 2, 7, 2, 1, 0]\{\} + \{\{\{-256, [3]\}\}\}, [5, 2, 6, 1, 1, 1]\{\} + \{\{\{384, [3]\}\}\}, [5, 2, 5, 2, 2, 0]\{\} + \{\{\{1536, [4]\}\}\}, [5, 2, 4, 1, 2, 1]\{\} + \{\{\{-1024, [4]\}\}\}, [5, 2, 3, 2, 3, 0]\{\} + \{\{\{-4096, [5]\}\}\}, [5, 2, 2, 1, 3, 1]\{\} + \{\{\{1024, [5]\}\}\}, [5, 2, 1, 2, 4, 0]\{\} + \{\{\{4096, [6]\}\}\}, [5, 2, 0, 1, 4, 1]\{\} + \{\{\{-6, 0\}\} : [1, 0, \{\{-1, [1]\}\}] \{\}, [4, 4, 8, 0, 2, 0]\{\} + \{\{\{96, [1]\}\}\}, 0\} : [1, 0, \{\{-1, [1]\}\}] \{\}, [4, 4, 6, 0, 3, 0]\{\} + \{\{\{-576, [2]\}\}\}, 0\} : [1, 0, \{\{-1, [1]\}\}] \{\}, [4, 4, 4, 0, 4, 0]\{\} + \{\{\{1536, [3]\}\}\}, 0\} : [1, 0, \{\{-1, [1]\}\}] \{\}, [4, 4, 2, 0, 5, 0]\{\} + \{\{\{-1536, [4]\}\}\}, 0\} : [1, 0, \{\{-1, [1]\}\}] \{\}, [4, 4, 0, 0, 6, 0]\{\} + \{\{\{-2, 0\}\} : [1, 0, \{\{-1, [1]\}\}] \{\}, [4, 3, 10, 0, 0, 1]\{\} + \{\{\{6, 0\}\} : [1, 0, \{\{-1, [1]\}\}] \{\}, [4, 3, 9, 1, 1, 0]\{\} + \{\{\{48, [1]\}\}\}, 0\} : [1, 0, \{\{-1, [1]\}\}] \{\}, [4, 3, 8, 0, 1, 1]\{\} + \{\{\{-96, [1]\}\}\}, 0\} : [1, 0, \{\{-1, [1]\}\}] \{\}, [4, 3, 7, 1, 2, 0]\{\} + \{\{\{-448, [2]\}\}\}, 0\} : [1, 0, \{\{-1, [1]\}\}] \{\}, [4, 3, 6, 0, 2, 1]\{\} + \{\{\{576, [2]\}\}\}, 0\} : [1, 0, \{\{-1, [1]\}\}] \{\}, [4, 3, 5, 1, 3, 0]\{\} + \{\{\{2048, [3]\}\}\}, 0\} : [1, 0, \{\{-1, [1]\}\}] \{\}, [4, 3, 4, 0, 3, 1]\{\} + \{\{\{-1536, [3]\}\}\}, 0\} : [1, 0, \{\{-1, [1]\}\}] \{\}, [4, 3, 3, 1, 4, 0]\{\} + \{\{\{-4608, [4]\}\}\}, 0\} : [1, 0, \{\{-1, [1]\}\}] \{\}, [4, 3, 2, 0, 4, 1]\{\} + \{\{\{1536, [4]\}\}\}, 0\} : [1, 0, \{\{-1, [1]\}\}] \{\}, [4, 3, 1, 1, 5, 0]\{\} + \{\{\{4096, [5]\}\}\}, 0\} : [1, 0, \{\{-1, [1]\}\}] \{\}, [4, 3, 0, 0, 5, 1]\{\} + \{\{\{-1, 0\}\} : [1, 0, \{\{-1, [1]\}\}] \{\}, [4, 2, 10, 2, 0, 0]\{\} + \{\{\{-24, [1]\}\}\}, 0\} : [1, 0, \{\{-1, [1]\}\}] \{\}, [4, 2, 9, 1, 0, 1]\{\} + \{\{\{24, [1]\}\}\}, 0\} : [1, 0, \{\{-1, [1]\}\}] \{\}, [4, 2, 8, 2, 1, 0]\{\} + \{\{\{-16, [2]\}\}\}, 0\} : [1, 0, \{\{-1, [1]\}\}] \{\}, [4, 2, 8, 0, 0, 2]\{\} + \{\{\{384, [2]\}\}\}, 0\} : [1, 0, \{\{-1, [1]\}\}] \{\}, [4, 2, 7, 1, 1, 1]\{\} + \{\{\{-224, [2]\}\}\}, 0\} : [1, 0, \{\{-1, [1]\}\}] \{\}, [4, 2, 6, 2, 2, 0]\{\} + \{\{\{256, [3]\}\}\}, 0\} : [1, 0, \{\{-1, [1]\}\}] \{\}, [4, 2, 6, 0, 1, 2]\{\} + \{\{\{-2304, [3]\}\}\}, 0\} : [1, 0, \{\{-1, [1]\}\}] \{\}, [4, 2, 5, 1, 2, 1]\{\} + \{\{\{1024, [3]\}\}\}, 0\} : [1, 0, \{\{-1, [1]\}\}] \{\}, [4, 2, 4, 2, 3, 0]\{\} + \{\{\{-1536, [4]\}\}\}, 0\} : [1, 0, \{\{-1, [1]\}\}] \{\}, [4, 2, 4, 0, 2, 2]\{\} + \{\{\{6144, [4]\}\}\}, 0\} : [1, 0, \{\{-1, [1]\}\}] \{\}, [4, 2, 3, 1, 3, 1]\{\} + \{\{\{-2304, [4]\}\}\}, 0\} : [1, 0, \{\{-1, [1]\}\}] \{\}, [4, 2, 2, 2, 4, 0]\{\} + \{\{\{4096, [5]\}\}\}, 0\} : [1, 0, \{\{-1, [1]\}\}] \{\}, [4, 2, 2, 0, 3, 2]\{\} + \{\{\{-6144, [5]\}\}\}, 0\} : [1, 0, \{\{-1, [1]\}\}] \{\}, [4, 2, 1, 1, 4, 1]\{\} + \{\{\{2048, [5]\}\}\},$

0] : [1, 0, %%%{-1, [1]%%}]%}, [4, 2, 0, 2, 5, 0]%%}+%%{-%%{[-4096, [6]%%}, 0] : [1, 0, %%%{-1, [1]%%}]%}, [4, 2, 0, 0, 4, 2]%%}+%%{-%%{-16, [1]%%}, [3, 3, 9, 0, 1, 1]%%}+%%{-%%{-12, [1]%%}, [3, 3, 8, 1, 2, 0]%%}+%%{-%%{-256, [2]%%}, [3, 3, 7, 0, 2, 1]%%}+%%{-%%{-192, [2]%%}, [3, 3, 6, 1, 3, 0]%%}+%%{-%%{-1536, [3]%%}, [3, 3, 5, 0, 3, 1]%%}+%%{-%%{-1152, [3]%%}, [3, 3, 4, 1, 4, 0]%%}+%%{-%%{-4096, [4]%%}, [3, 3, 3, 0, 4, 1]%%}+%%{-%%{-3072, [4]%%}, [3, 3, 2, 1, 5, 0]%%}+%%{-%%{-4096, [5]%%}, [3, 3, 1, 0, 5, 1]%%}+%%{-%%{-3072, [5]%%}, [3, 3, 0, 1, 6, 0]%%}+%%{-%%{-12, [1]%%}, [3, 2, 10, 1, 0, 1]%%}+%%{-%%{-8, [1]%%}, [3, 2, 9, 2, 1, 0]%%}+%%{-%%{-32, [2]%%}, [3, 2, 9, 0, 0, 2]%%}+%%{-%%{-208, [2]%%}, [3, 2, 8, 1, 1, 1]%%}+%%{-%%{-128, [2]%%}, [3, 2, 7, 2, 2, 0]%%}+%%{-%%{-512, [3]%%}, [3, 2, 7, 0, 1, 2]%%}+%%{-%%{-1408, [3]%%}, [3, 2, 6, 1, 2, 1]%%}+%%{-%%{-768, [3]%%}, [3, 2, 5, 2, 3, 0]%%}+%%{-%%{-3072, [4]%%}, [3, 2, 5, 0, 2, 2]%%}+%%{-%%{-4608, [4]%%}, [3, 2, 4, 1, 3, 1]%%}+%%{-%%{-2048, [4]%%}, [3, 2, 3, 2, 4, 0]%%}+%%{-%%{-8192, [5]%%}, [3, 2, 3, 0, 3, 2]%%}+%%{-%%{-7168, [5]%%}, [3, 2, 2, 1, 4, 1]%%}+%%{-%%{-2048, [5]%%}, [3, 2, 1, 2, 5, 0]%%}+%%{-%%{-8192, [6]%%}, [3, 2, 1, 0, 4, 2]%%}+%%{-%%{-4096, [6]%%}, [3, 2, 0, 1, 5, 1]%%}+%%{-%%{-4, 0] : [1, 0, %%%{-1, [1]%%}]%}, [2, 4, 8, 0, 3, 0]%%}+%%{-%%{-64, [1]%%}, 0] : [1, 0, %%%{-1, [1]%%}]%}, [2, 4, 6, 0, 4, 0]%%}+%%{-%%{-384, [2]%%}, 0] : [1, 0, %%%{-1, [1]%%}]%}, [2, 4, 4, 0, 5, 0]%%}+%%{-%%{-1024, [3]%%}, 0] : [1, 0, %%%{-1, [1]%%}]%}, [2, 4, 2, 0, 6, 0]%%}+%%{-%%{-1024, [4]%%}, 0] : [1, 0, %%%{-1, [1]%%}]%}, [2, 4, 0, 0, 7, 0]%%}+%%{-%%{-4, 0] : [1, 0, %%%{-1, [1]%%}]%}, [2, 3, 10, 0, 1, 1]%%}+%%{-%%{-6, 0] : [1, 0, %%%{-1, [1]%%}]%}, [2, 3, 9, 1, 2, 0]%%}+%%{-%%{-72, [1]%%}, 0] : [1, 0, %%%{-1, [1]%%}]%}, [2, 3, 8, 0, 2, 1]%%}+%%{-%%{-96, [1]%%}, 0] : [1, 0, %%%{-1, [1]%%}]%}, [2, 3, 7, 1, 3, 0]%%}+%%{-%%{-512, [2]%%}, 0] : [1, 0, %%%{-1, [1]%%}]%}, [2, 3, 6, 0, 3, 1]%%}+%%{-%%{-576, [2]%%}, 0] : [1, 0, %%%{-1, [1]%%}]%}, [2, 3, 5, 1, 4, 0]%%}+%%{-%%{-1792, [3]%%}, 0] : [1, 0, %%%{-1, [1]%%}]%}, [2, 3, 4, 0, 4, 1]%%}+%%{-%%{-1536, [3]%%}, 0] : [1, 0, %%%{-1, [1]%%}]%}, [2, 3, 3, 1, 5, 0]%%}+%%{-%%{-3072, [4]%%}, 0] : [1, 0, %%%{-1, [1]%%}]%}, [2, 3, 2, 0, 5, 1]%%}+%%{-%%{-1536, [4]%%}, 0] : [1, 0, %%%{-1, [1]%%}]%}, [2, 3, 1, 1, 6, 0]%%}+%%{-%%{-2048, [5]%%}, 0] : [1, 0, %%%{-1, [1]%%}]%}, [2, 3, 0, 0, 6, 1]%%}+%%{-%%{-2, 0] : [1, 0, %%%{-1, [1]%%}]%}, [2, 2, 11, 1, 0, 1]%%}+%%{-%%{-2, 0] : [1, 0, %%%{-1, [1]%%}]%}, [2, 2, 10, 2, 1, 0]%%}+%%{-%%{-24, [1]%%}, 0] : [1, 0, %%%{-1, [1]%%}]%}, [2, 2, 10, 0, 0, 2]%%}+%%{-%%{-56, [1]%%}, 0] : [1, 0, %%%{-1, [1]%%}]%}, [2, 2, 9, 1, 1, 1]%%}+%%{-%%{-36, [1]%%}, 0] : [1, 0, %%%{-1, [1]%%}]%}, [2, 2, 8, 2, 2, 0]%%}+%%{-%%{-384, [2]%%}, 0] : [1, 0, %%%{-1, [1]%%}]%}, [2, 2, 8, 0, 1, 2]%%}+%%{-%%{-576, [2]%%}, 0] : [1, 0, %%%{-1, [1]%%}]%}, [2, 2, 7, 1, 2, 1]%%}+%%{-%%{-256, [2]%%}, 0] : [1, 0, %%%{-1, [1]%%}]%}, [2, 2, 6, 2, 3, 0]%%}+%%{-%%{-2304, [3]%%}, 0] : [1, 0, %%%{-1, [1]%%}]%}, [2, 2, 6, 0, 2, 2]%%}+%%{-%%{-2816, [3]%%}, 0] : [1, 0, %%%{-1, [1]%%}]%}, [2, 2, 5, 1, 3, 1]%%}+%%{-%%{-896, [3]%%}, 0] : [1, 0, %%%{-1, [1]%%}]%}, [2, 2, 4, 2, 4, 0]%%}+%%{-%%{-6144, [4]%%}, 0] : [1, 0, %%%{-1, [1]%%}]%}, [2, 2, 4, 0, 3, 2]%%}+%%{-%%{-6656, [4]%%}, 0] : [1, 0, %%%{-1, [1]%%}]%}, [2, 2, 3, 1, 4, 1]%%}+%%{-%%{-1536, [4]%%}, 0] : [1, 0, %%%{-1, [1]%%}]%}, [2, 2, 2, 2, 5, 0]%%}+%%{-%%{-6144, [5]%%}, 0] : [1, 0, %%%{-1, [1]%%}]%}, [2, 2, 2, 0, 4, 2]%%}+%%{-%%{-6144, [5]%%}, 0] : [1, 0, %%%{-1, [1]%%}]%}, [2, 2, 1, 1, 5, 1]%%}+%%{-%%{-1024, [5]%%}, 0] : [1, 0, %%%{-1, [1]%%}]%}, [2, 2, 0, 2, 6, 0]%%}+%%{-%%{-8, [

$[1]^{1000}\}, [1, 3, 9, 0, 2, 1]^{1000}\} + \{ -4, [1]^{1000}\}, [1, 3, 8, 1, 3, 0]^{1000}\} + \{ -128, [2]^{1000}\}, [1, 3, 7, 0, 3, 1]^{1000}\} + \{ 64, [2]^{1000}\}, [1, 3, 6, 1, 4, 0]^{1000}\} + \{ 768, [3]^{1000}\}, [1, 3, 5, 0, 4, 1]^{1000}\} + \{ -384, [3]^{1000}\}, [1, 3, 4, 1, 5, 0]^{1000}\} + \{ -2048, [4]^{1000}\}, [1, 3, 3, 0, 5, 1]^{1000}\} + \{ 1024, [4]^{1000}\}, [1, 3, 2, 1, 6, 0]^{1000}\} + \{ 2048, [5]^{1000}\}, [1, 3, 1, 0, 6, 1]^{1000}\} + \{ -1024, [5]^{1000}\}, [1, 3, 0, 1, 7, 0]^{1000}\} + \{ 8, [1]^{1000}\}, [1, 2, 11, 0, 0, 2]^{1000}\} + \{ -12, [1]^{1000}\}, [1, 2, 10, 1, 1, 1]^{1000}\} + \{ 4, [1]^{1000}\}, [1, 2, 9, 2, 2, 0]^{1000}\} + \{ -128, [2]^{1000}\}, [1, 2, 9, 0, 1, 2]^{1000}\} + \{ 192, [2]^{1000}\}, [1, 2, 8, 1, 2, 1]^{1000}\} + \{ -64, [2]^{1000}\}, [1, 2, 7, 2, 3, 0]^{1000}\} + \{ 768, [3]^{1000}\}, [1, 2, 7, 0, 2, 2]^{1000}\} + \{ -1152, [3]^{1000}\}, [1, 2, 6, 1, 3, 1]^{1000}\} + \{ 384, [3]^{1000}\}, [1, 2, 5, 2, 4, 0]^{1000}\} + \{ -2048, [4]^{1000}\}, [1, 2, 5, 0, 3, 2]^{1000}\} + \{ 3072, [4]^{1000}\}, [1, 2, 4, 1, 4, 1]^{1000}\} + \{ -1024, [4]^{1000}\}, [1, 2, 3, 2, 5, 0]^{1000}\} + \{ 2048, [5]^{1000}\}, [1, 2, 3, 0, 4, 2]^{1000}\} + \{ -3072, [5]^{1000}\}, [1, 2, 2, 1, 5, 1]^{1000}\} + \{ 1024, [5]^{1000}\}, [1, 2, 1, 2, 6, 0]^{1000}\} + \{ -1, 0 : [1, 0, \{ -1, [1]^{1000}\}]\}^{1000}\}, [0, 4, 8, 0, 4, 0]^{1000}\} + \{ \{ 16, [1]^{1000}\}, 0 : [1, 0, \{ -1, [1]^{1000}\}]\}^{1000}\}, [0, 4, 6, 0, 5, 0]^{1000}\} + \{ \{ -96, [2]^{1000}\}, 0 : [1, 0, \{ -1, [1]^{1000}\}]\}^{1000}\}, [0, 4, 4, 0, 6, 0]^{1000}\} + \{ \{ 256, [3]^{1000}\}, 0 : [1, 0, \{ -1, [1]^{1000}\}]\}^{1000}\}, [0, 4, 2, 0, 7, 0]^{1000}\} + \{ \{ -256, [4]^{1000}\}, 0 : [1, 0, \{ -1, [1]^{1000}\}]\}^{1000}\}, [0, 4, 0, 0, 8, 0]^{1000}\} + \{ \{ -2, 0 : [1, 0, \{ -1, [1]^{1000}\}]\}^{1000}\}, [0, 3, 10, 0, 2, 1]^{1000}\} + \{ \{ [2, 0 : [1, 0, \{ -1, [1]^{1000}\}]\}^{1000}\}, [0, 3, 9, 1, 3, 0]^{1000}\} + \{ \{ [32, [1]^{1000}\}, 0 : [1, 0, \{ -1, [1]^{1000}\}]\}^{1000}\}, [0, 3, 8, 0, 3, 1]^{1000}\} + \{ \{ [ -32, [1]^{1000}\}, 0 : [1, 0, \{ -1, [1]^{1000}\}]\}^{1000}\}, [0, 3, 7, 1, 4, 0]^{1000}\} + \{ \{ [ -192, [2]^{1000}\}, 0 : [1, 0, \{ -1, [1]^{1000}\}]\}^{1000}\}, [0, 3, 6, 0, 4, 1]^{1000}\} + \{ \{ [192, [2]^{1000}\}, 0 : [1, 0, \{ -1, [1]^{1000}\}]\}^{1000}\}, [0, 3, 5, 1, 5, 0]^{1000}\} + \{ \{ [512, [3]^{1000}\}, 0 : [1, 0, \{ -1, [1]^{1000}\}]\}^{1000}\}, [0, 3, 4, 0, 5, 1]^{1000}\} + \{ \{ [ -512, [3]^{1000}\}, 0 : [1, 0, \{ -1, [1]^{1000}\}]\}^{1000}\}, [0, 3, 3, 1, 6, 0]^{1000}\} + \{ \{ [ -512, [4]^{1000}\}, 0 : [1, 0, \{ -1, [1]^{1000}\}]\}^{1000}\}, [0, 3, 2, 0, 6, 1]^{1000}\} + \{ \{ [512, [4]^{1000}\}, 0 : [1, 0, \{ -1, [1]^{1000}\}]\}^{1000}\}, [0, 3, 1, 1, 7, 0]^{1000}\} + \{ \{ [ -1, 0 : [1, 0, \{ -1, [1]^{1000}\}]\}^{1000}\}, [0, 2, 12, 0, 0, 2]^{1000}\} + \{ \{ [2, 0 : [1, 0, \{ -1, [1]^{1000}\}]\}^{1000}\}, [0, 2, 11, 1, 1, 1]^{1000}\} + \{ \{ [ -1, 0 : [1, 0, \{ -1, [1]^{1000}\}]\}^{1000}\}, [0, 2, 10, 2, 2, 0]^{1000}\} + \{ \{ [16, [1]^{1000}\}, 0 : [1, 0, \{ -1, [1]^{1000}\}]\}^{1000}\}, [0, 2, 10, 0, 1, 2]^{1000}\} + \{ \{ [ -32, [1]^{1000}\}, 0 : [1, 0, \{ -1, [1]^{1000}\}]\}^{1000}\}, [0, 2, 9, 1, 2, 1]^{1000}\} + \{ \{ [16, [1]^{1000}\}, 0 : [1, 0, \{ -1, [1]^{1000}\}]\}^{1000}\}, [0, 2, 8, 2, 3, 0]^{1000}\} + \{ \{ [ -96, [2]^{1000}\}, 0 : [1, 0, \{ -1, [1]^{1000}\}]\}^{1000}\}, [0, 2, 8, 0, 2, 2]^{1000}\} + \{ \{ [192, [2]^{1000}\}, 0 : [1, 0, \{ -1, [1]^{1000}\}]\}^{1000}\}, [0, 2, 7, 1, 3, 1]^{1000}\} + \{ \{ [ -96, [2]^{1000}\}, 0 : [1, 0, \{ -1, [1]^{1000}\}]\}^{1000}\}, [0, 2, 6, 2, 4, 0]^{1000}\} + \{ \{ [256, [3]^{1000}\}, 0 : [1, 0, \{ -1, [1]^{1000}\}]\}^{1000}\}, [0, 2, 6, 0, 3, 2]^{1000}\} + \{ \{ [ -512, [3]^{1000}\}, 0 : [1, 0, \{ -1, [1]^{1000}\}]\}^{1000}\}, [0, 2, 5, 1, 4, 1]^{1000}\} + \{ \{ [256, [3]^{1000}\}, 0 : [1, 0, \{ -1, [1]^{1000}\}]\}^{1000}\}, [0, 2, 4, 2, 5, 0]^{1000}\} + \{ \{ [ -256, [4]^{1000}\}, 0 : [1, 0, \{ -1, [1]^{1000}\}]\}^{1000}\}, [0, 2, 4, 0, 4, 2]^{1000}\} + \{ \{ [512, [4]^{1000}\}, 0 : [1, 0, \{ -1, [1]^{1000}\}]\}^{1000}\}, [0, 2, 3, 1, 5, 1]^{1000}\} + \{ \{ [ -256, [4]^{1000}\}, 0 : [1, 0, \{ -1, [1]^{1000}\}]\}^{1000}\}, [0, 2, 2, 2, 6, 0]^{1000}\} / \{ 1, [1]^{1000}\}, [8, 2, 0, 0, 0, 0]^{1000}\} + \{ \{ poly1 [ -4, [1]^{1000}\}, 0 : [1, 0, \{ -1, [1]^{1000}\}]\}^{1000}\}, [7, 1, 0, 1, 0, 0]^{1000}\} + \{ \{ -4, [1]^{1000}\}, [6, 2, 0, 0, 1, 0]^{1000}\} + \{ \{ 2, [1]^{1000}\}, [6, 1, 1, 1, 0, 0]^{1000}\} + \{ \{ 8, [2]^{1000}\}, [6, 1, 0, 0, 0, 1]^{1000}\} + \{ \{ 4, [2]^{1000}\}, [6, 0, 0, 2, 0, 0]^{1000}\} + \{ \{ poly1 [ -8, [1]^{1000}\}, 0 : [1, 0, \{ -1, [1]^{1000}\}]\}^{1000}\}, [5, 1, 1, 0, 0, 1]^{1000}\} + \{ \{ poly1 [12, [1]^{1000}\}, 0 : [1, 0, \{ -1, [1]^{1000}\}]\}^{1000}\}, [5, 1, 0, 1, 1, 0]^{1000}\} + \{ \{ poly1 [ -4, [1]^{1000}\}, 0 : [1, 0, \{ -1, [1]^{1000}\}]\}^{1000}\}$



```

]%%},0]:[1,0,%%{-1,[1]%%}]%%},[5,0,1,2,0,0]%%}+%%{%%{poly1[%%{-16,[2]
%%},0]:[1,0,%%{-1,[1]%%}]%%},[5,0,0,1,0,1]%%}+%%{%%{6,[1]%%},[4,2,0,
0,2,0]%%}+%%{%%{2,[1]%%},[4,1,2,0,0,1]%%}+%%{%%{-6,[1]%%},[4,1,1,1,
1,0]%%}+%%{%%{-16,[2]%%},[4,1,0,0,1,1]%%}+%%{%%{1,[1]%%},[4,0,2,2,0
,0]%%}+%%{%%{24,[2]%%},[4,0,1,1,0,1]%%}+%%{%%{-8,[2]%%},[4,0,0,2,1,
0]%%}+%%{%%{16,[3]%%},[4,0,0,0,0,2]%%}+%%{%%{poly1[%%{16,[1]%%},0]:
[1,0,%%{-1,[1]%%}]%%},[3,1,1,0,1,1]%%}+%%{%%{poly1[%%{-12,[1]%%},0]:[
1,0,%%{-1,[1]%%}]%%},[3,1,0,1,2,0]%%}+%%{%%{poly1[%%{-12,[1]%%},0]:[1
,0,%%{-1,[1]%%}]%%},[3,0,2,1,0,1]%%}+%%{%%{poly1[%%{8,[1]%%},0]:[1,0,
%%{-1,[1]%%}]%%},[3,0,1,2,1,0]%%}+%%{%%{poly1[%%{-32,[2]%%},0]:[1,0,%%
%%{-1,[1]%%}]%%},[3,0,1,0,0,2]%%}+%%{%%{poly1[%%{16,[2]%%},0]:[1,0,%%
%%{-1,[1]%%}]%%},[3,0,0,1,1,1]%%}+%%{%%{-4,[1]%%},[2,2,0,0,3,0]%%}+%%{
%%{-4,[1]%%},[2,1,2,0,1,1]%%}+%%{%%{6,[1]%%},[2,1,1,1,2,0]%%}+%%{%%
%%{8,[2]%%},[2,1,0,0,2,1]%%}+%%{%%{2,[1]%%},[2,0,3,1,0,1]%%}+%%{%%{-
2,[1]%%},[2,0,2,2,1,0]%%}+%%{%%{24,[2]%%},[2,0,2,0,0,2]%%}+%%{%%{-2
4,[2]%%},[2,0,1,1,1,1]%%}+%%{%%{4,[2]%%},[2,0,0,2,2,0]%%}+%%{%%{poly
1[%%{-8,[1]%%},0]:[1,0,%%{-1,[1]%%}]%%},[1,1,1,0,2,1]%%}+%%{%%{poly1[
%%{4,[1]%%},0]:[1,0,%%{-1,[1]%%}]%%},[1,1,0,1,3,0]%%}+%%{%%{poly1[%%
{-8,[1]%%},0]:[1,0,%%{-1,[1]%%}]%%},[1,0,3,0,0,2]%%}+%%{%%{poly1[%%{1
2,[1]%%},0]:[1,0,%%{-1,[1]%%}]%%},[1,0,2,1,1,1]%%}+%%{%%{poly1[%%{-4,
[1]%%},0]:[1,0,%%{-1,[1]%%}]%%},[1,0,1,2,2,0]%%}+%%{%%{1,[1]%%},[0,2
,0,0,4,0]%%}+%%{%%{2,[1]%%},[0,1,2,0,2,1]%%}+%%{%%{-2,[1]%%},[0,1,1
,1,3,0]%%}+%%{%%{1,[1]%%},[0,0,4,0,0,2]%%}+%%{%%{-2,[1]%%},[0,0,3,1
,1,1]%%}+%%{%%{1,[1]%%},[0,0,2,2,2,0]%%} Error: Bad Argument Value

```

**maple [B]** time = 0.04, size = 72576, normalized size = 103.09

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^2,x)
```

```
[Out] result too large to display
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2 + bx + a)^{\frac{3}{2}}}{(fx^2 + ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(3/2)/(f*x^2 + e*x + d)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(cx^2 + bx + a)^{3/2}}{(fx^2 + ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x + c\*x^2)^(3/2)/(d + e\*x + f\*x^2)^2,x)

[Out] int((a + b\*x + c\*x^2)^(3/2)/(d + e\*x + f\*x^2)^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+b\*x+a)\*\*(3/2)/(f\*x\*\*2+e\*x+d)\*\*2,x)

[Out] Timed out

$$3.108 \quad \int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^3} dx$$

**Optimal.** Leaf size=671

$$3 \left( 2 \left( e - \sqrt{e^2 - 4df} \right) (ce - bf)(2af - be + 2cd) - f \left( 4be(3af + cd) - 4a \left( 4af^2 + ce^2 \right) - \left( b^2 \left( 4df + e^2 \right) \right) \right) \right) \tanh$$

$$\frac{4\sqrt{2} \left( e^2 - 4df \right)^{5/2} \sqrt{2af^2 - \sqrt{e^2 - 4df} (ce - bf) - bef - 2cdf} +$$

[Out]  $-1/2*(2*f*x+e)*(c*x^2+b*x+a)^{(3/2)/(-4*d*f+e^2)/(f*x^2+e*x+d)^2+3/4*(4*c*d*e+4*a*e*f-b*(4*d*f+e^2)+2*(4*a*f^2-2*b*e*f+c*e^2)*x)*(c*x^2+b*x+a)^{(1/2)/(-4*d*f+e^2)^2/(f*x^2+e*x+d)-3/8*\arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^{(1/2)}))-b*(e-(-4*d*f+e^2)^{(1/2)}))*2^{(1/2)/(c*x^2+b*x+a)^{(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^{(1/2)})^{(1/2)}*(-f*(4*b*e*(3*a*f+c*d)-b^2*(4*d*f+e^2)-4*a*(4*a*f^2+c*e^2))+2*(2*a*f-b*e+2*c*d)*(-b*f+c*e)*(e-(-4*d*f+e^2)^{(1/2)}))/(-4*d*f+e^2)^{(5/2)*2^{(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^{(1/2)})^{(1/2)}+3/8*\arctanh(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^{(1/2)}))+2*x*(b*f-c*(e+(-4*d*f+e^2)^{(1/2)})))*)2^{(1/2)/(c*x^2+b*x+a)^{(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^{(1/2)})^{(1/2)}*(-f*(4*b*e*(3*a*f+c*d)-b^2*(4*d*f+e^2)-4*a*(4*a*f^2+c*e^2))+2*(2*a*f-b*e+2*c*d)*(-b*f+c*e)*(e+(-4*d*f+e^2)^{(1/2)}))/(-4*d*f+e^2)^{(5/2)*2^{(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^{(1/2)})^{(1/2)}}$

**Rubi [A]** time = 11.60, antiderivative size = 669, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {971, 1013, 1032, 724, 206}

$$3 \left( -2 \left( e - \sqrt{e^2 - 4df} \right) (ce - bf)(2af - be + 2cd) + 4bef(3af + cd) - 4af \left( 4af^2 + ce^2 \right) + b^2(-f) \left( 4df + e^2 \right) \right) \tan$$

$$\frac{4\sqrt{2} \left( e^2 - 4df \right)^{5/2} \sqrt{2af^2 - \sqrt{e^2 - 4df} (ce - bf) - bef - 2cdf} +$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x + c\*x^2)^(3/2)/(d + e\*x + f\*x^2)^3,x]

[Out]  $-((e + 2*f*x)*(a + b*x + c*x^2)^{(3/2))/(2*(e^2 - 4*d*f)*(d + e*x + f*x^2)^2) + (3*(4*c*d*e + 4*a*e*f - b*(e^2 + 4*d*f) + 2*(c*e^2 - 2*b*e*f + 4*a*f^2)*x)*\text{Sqrt}[a + b*x + c*x^2])/(4*(e^2 - 4*d*f)^2*(d + e*x + f*x^2)) + (3*(4*b*e*f*(c*d + 3*a*f) - b^2*f*(e^2 + 4*d*f) - 4*a*f*(c*e^2 + 4*a*f^2) - 2*(2*c*d - b*e + 2*a*f)*(c*e - b*f)*(e - \text{Sqrt}[e^2 - 4*d*f]))*\text{ArcTanh}[(4*a*f - b*(e - \text{Sqrt}[e^2 - 4*d*f]) + 2*(b*f - c*(e - \text{Sqrt}[e^2 - 4*d*f]))*x)/(2*\text{Sqrt}[2]*S$

```

qrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt
[a + b*x + c*x^2]])/(4*Sqrt[2]*(e^2 - 4*d*f)^(5/2)*Sqrt[c*e^2 - 2*c*d*f -
b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) - (3*(4*b*e*f*(c*d + 3*a*
f) - b^2*f*(e^2 + 4*d*f) - 4*a*f*(c*e^2 + 4*a*f^2) - 2*(2*c*d - b*e + 2*a*f
)*(c*e - b*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4
*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x]/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c
*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^
2]])]/(4*Sqrt[2]*(e^2 - 4*d*f)^(5/2)*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2
+ (c*e - b*f)*Sqrt[e^2 - 4*d*f]])

```

### Rule 206

```

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/
Rt[a, 2])]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

```

### Rule 724

```

Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Sym
bol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

```

### Rule 971

```

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((d_) + (e_)*(x_) + (f_)*(x
_)^2)^(q_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*(d + e
*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1))
, Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p +
3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] && !IGtQ[q, 0]

```

### Rule 1013

```

Int[((g_) + (h_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((d_) + (e
_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[((g*b - 2*a*h - (b*h - 2*g
*c)*x)*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1
)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d +
e*x + f*x^2)^(q - 1)*Simp[e*q*(g*b - 2*a*h) - d*(b*h - 2*g*c)*(2*p + 3) +
(2*f*q*(g*b - 2*a*h) - e*(b*h - 2*g*c)*(2*p + q + 3))*x - f*(b*h - 2*g*c)*(
2*p + 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0]

```

### Rule 1032

```
Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

### Rubi steps

$$\int \frac{(a + bx + cx^2)^{3/2}}{(d + ex + fx^2)^3} dx = -\frac{(e + 2fx)(a + bx + cx^2)^{3/2}}{2(e^2 - 4df)(d + ex + fx^2)^2} + \frac{\int \frac{\left(\frac{3}{2}(be - 4af) + 3(ce - bf)x\right)\sqrt{a + bx + cx^2}}{(d + ex + fx^2)^2} dx}{2(e^2 - 4df)}$$

$$= -\frac{(e + 2fx)(a + bx + cx^2)^{3/2}}{2(e^2 - 4df)(d + ex + fx^2)^2} + \frac{3(4cde + 4aef - b(e^2 + 4df) + 2(ce^2 - 2bef + 4af))}{4(e^2 - 4df)^2(d + ex + fx^2)}$$

$$= -\frac{(e + 2fx)(a + bx + cx^2)^{3/2}}{2(e^2 - 4df)(d + ex + fx^2)^2} + \frac{3(4cde + 4aef - b(e^2 + 4df) + 2(ce^2 - 2bef + 4af))}{4(e^2 - 4df)^2(d + ex + fx^2)}$$

$$= -\frac{(e + 2fx)(a + bx + cx^2)^{3/2}}{2(e^2 - 4df)(d + ex + fx^2)^2} + \frac{3(4cde + 4aef - b(e^2 + 4df) + 2(ce^2 - 2bef + 4af))}{4(e^2 - 4df)^2(d + ex + fx^2)}$$

$$= -\frac{(e + 2fx)(a + bx + cx^2)^{3/2}}{2(e^2 - 4df)(d + ex + fx^2)^2} + \frac{3(4cde + 4aef - b(e^2 + 4df) + 2(ce^2 - 2bef + 4af))}{4(e^2 - 4df)^2(d + ex + fx^2)}$$

**Mathematica [B]** time = 7.23, size = 4727, normalized size = 7.04

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x + c\*x^2)^(3/2)/(d + e\*x + f\*x^2)^3,x]

[Out] (-2\*f^2\*(a + x\*(b + c\*x))^(3/2))/((e^2 - 4\*d\*f)^(3/2)\*(e - Sqrt[e^2 - 4\*d\*f] + 2\*f\*x)^2) + (6\*f^2\*(a + x\*(b + c\*x))^(3/2))/((e^2 - 4\*d\*f)^2\*(e - Sqrt[e^2 - 4\*d\*f] + 2\*f\*x)) + (2\*f^2\*(a + x\*(b + c\*x))^(3/2))/((e^2 - 4\*d\*f)^(3/2)

$$\begin{aligned}
& 2) * (e + \sqrt{e^2 - 4*d*f}) + 2*f*x)^2) + (6*f^2*(a + x*(b + c*x))^{(3/2)}) / ((e \\
& ^2 - 4*d*f)^2*(e + \sqrt{e^2 - 4*d*f}) + 2*f*x)) + (9*f^2*(a + x*(b + c*x))^{( \\
& 3/2)} * (((-4*b*c*f - 2*c*(b*f + 2*c*(-e + \sqrt{e^2 - 4*d*f}))) - 4*c^2*f*x)*\sqrt{ \\
& a + b*x + c*x^2}) / (8*c*f^2) - ((2*\sqrt{c}*(b^2*f^2 + 4*c^2*(e^2 - 2*d*f \\
& - e*\sqrt{e^2 - 4*d*f})) + 4*c*f*(a*f - b*(e - \sqrt{e^2 - 4*d*f})))) * \text{ArcTanh}[( \\
& b + 2*c*x) / (2*\sqrt{c}*\sqrt{a + b*x + c*x^2})]) / f + (2*\sqrt{2}*\sqrt{c*e^2 - \\
& 2*c*d*f - b*e*f + 2*a*f^2 - c*e*\sqrt{e^2 - 4*d*f}} + b*f*\sqrt{e^2 - 4*d*f}) * \\
& (4*c*f*(8*a*b*f^2 - 3*b^2*f*(e - \sqrt{e^2 - 4*d*f})) - 4*a*c*f*(e - \sqrt{e^2 \\
& - 4*d*f})) + 4*b*c*(e^2 - 2*d*f - e*\sqrt{e^2 - 4*d*f})) + 4*c*(-e + \sqrt{e^ \\
& 2 - 4*d*f})*(b^2*f^2 + 4*c^2*(e^2 - 2*d*f - e*\sqrt{e^2 - 4*d*f})) + 4*c*f*(a \\
& *f - b*(e - \sqrt{e^2 - 4*d*f})))) * \text{ArcTanh}[(-4*a*f - b*(-e + \sqrt{e^2 - 4*d* \\
& f})) - (2*b*f + 2*c*(-e + \sqrt{e^2 - 4*d*f})) * x) / (2*\sqrt{2}*\sqrt{c*e^2 - 2*c \\
& *d*f - b*e*f + 2*a*f^2 - c*e*\sqrt{e^2 - 4*d*f}} + b*f*\sqrt{e^2 - 4*d*f}) * \sqrt{ \\
& a + b*x + c*x^2})] / (f*(16*a*f^2 + 8*b*f*(-e + \sqrt{e^2 - 4*d*f})) + 4*c*( \\
& -e + \sqrt{e^2 - 4*d*f})^2)) / (16*c*f^2)) / ((e^2 - 4*d*f)^2*(a + b*x + c*x^2 \\
& )^{(3/2)}) - (3*f^2*(a + x*(b + c*x))^{(3/2)} * (((-4*c*f*(4*a*f - b*(e - \sqrt{e^ \\
& 2 - 4*d*f})) - 2*(b*f - c*(e - \sqrt{e^2 - 4*d*f}))*(b*f + 2*c*(-e + \sqrt{e^ \\
& 2 - 4*d*f})) - 4*c*f*(b*f - c*(e - \sqrt{e^2 - 4*d*f})) * x) * \sqrt{a + b*x + c \\
& *x^2}) / (8*c*f^2) - ((-2*(b*f - c*(e - \sqrt{e^2 - 4*d*f}))*(b^2*f^2 - 4*c^2*( \\
& e^2 - 2*d*f - e*\sqrt{e^2 - 4*d*f})) - 4*c*f*(3*a*f - b*(e - \sqrt{e^2 - 4*d*f} \\
& )))) * \text{ArcTanh}[(b + 2*c*x) / (2*\sqrt{c}*\sqrt{a + b*x + c*x^2})]) / (\sqrt{c}*f) + \\
& (2*\sqrt{2}*\sqrt{c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*\sqrt{e^2 - 4*d*f}} + \\
& b*f*\sqrt{e^2 - 4*d*f}) * (-4*(-e + \sqrt{e^2 - 4*d*f})*(b*f - c*(e - \sqrt{e^2 \\
& - 4*d*f})) * (b^2*f^2 - 4*c^2*(e^2 - 2*d*f - e*\sqrt{e^2 - 4*d*f})) - 4*c*f*(3 \\
& *a*f - b*(e - \sqrt{e^2 - 4*d*f})))) + 4*f*(2*c*f*(4*a*f - b*(e - \sqrt{e^2 - \\
& 4*d*f}))^2 - (e - \sqrt{e^2 - 4*d*f})*(b*f - c*(e - \sqrt{e^2 - 4*d*f})) * (b^2 \\
& *f + 4*a*c*f - 2*b*c*(e - \sqrt{e^2 - 4*d*f})))) * \text{ArcTanh}[(-4*a*f - b*(-e + \sqrt{ \\
& e^2 - 4*d*f})) - (2*b*f + 2*c*(-e + \sqrt{e^2 - 4*d*f})) * x) / (2*\sqrt{2}*\sqrt{ \\
& c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*\sqrt{e^2 - 4*d*f}} + b*f*\sqrt{e^2 \\
& - 4*d*f}) * \sqrt{a + b*x + c*x^2})] / (f*(16*a*f^2 + 8*b*f*(-e + \sqrt{e^2 - 4 \\
& *d*f}) + 4*c*(-e + \sqrt{e^2 - 4*d*f})^2)) / (16*c*f^2)) / ((e^2 - 4*d*f)^{(5/2)} \\
& ) * (a + b*x + c*x^2)^{(3/2)}) + (3*f^2*(a + x*(b + c*x))^{(3/2)} * (((-2*b*f - 2*c \\
& *(-e + \sqrt{e^2 - 4*d*f})) * (a + b*x + c*x^2)^{(3/2)}) / ((-4*a*f^2 - 2*b*f*(-e \\
& + \sqrt{e^2 - 4*d*f}) - c*(-e + \sqrt{e^2 - 4*d*f})^2) * (-e + \sqrt{e^2 - 4*d*f} \\
& ] - 2*f*x)) + (((-4*c*f*(b^2*f + 4*a*c*f - 2*b*c*(e - \sqrt{e^2 - 4*d*f}))) - \\
& 4*c*(b*f - c*(e - \sqrt{e^2 - 4*d*f})) * (b*f + 2*c*(-e + \sqrt{e^2 - 4*d*f})) \\
& - 8*c^2*f*(b*f - c*(e - \sqrt{e^2 - 4*d*f})) * x) * \sqrt{a + b*x + c*x^2}) / (8*c \\
& *f^2) - ((16*c^{(3/2)}*(b*f - c*(e - \sqrt{e^2 - 4*d*f})) * (c*(e^2 - 2*d*f - e \\
& *\sqrt{e^2 - 4*d*f}) + f*(2*a*f - b*(e - \sqrt{e^2 - 4*d*f})))) * \text{ArcTanh}[(b + 2* \\
& c*x) / (2*\sqrt{c}*\sqrt{a + b*x + c*x^2})]) / f + (2*\sqrt{2}*\sqrt{c*e^2 - 2*c*d* \\
& f - b*e*f + 2*a*f^2 - c*e*\sqrt{e^2 - 4*d*f}} + b*f*\sqrt{e^2 - 4*d*f}) * (32*c^ \\
& 2*(-e + \sqrt{e^2 - 4*d*f})*(b*f - c*(e - \sqrt{e^2 - 4*d*f})) * (c*(e^2 - 2*d* \\
& f - e*\sqrt{e^2 - 4*d*f}) + f*(2*a*f - b*(e - \sqrt{e^2 - 4*d*f})))) + 16*c*f* \\
& (b^2*f + 4*a*c*f - 2*b*c*(e - \sqrt{e^2 - 4*d*f})) * (c*(e^2 - 2*d*f - e*\sqrt{ \\
& e^2 - 4*d*f}) + f*(2*a*f - b*(e - \sqrt{e^2 - 4*d*f})))) * \text{ArcTanh}[(-4*a*f - b
\end{aligned}$$

$$\begin{aligned}
& *(-e + \text{Sqrt}[e^2 - 4*d*f]) - (2*b*f + 2*c*(-e + \text{Sqrt}[e^2 - 4*d*f]))*x)/(2*\text{Sqrt}[2]*\text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*\text{Sqrt}[e^2 - 4*d*f] + b*f*\text{Sqrt}[e^2 - 4*d*f]]*\text{Sqrt}[a + b*x + c*x^2])]/(f*(16*a*f^2 + 8*b*f*(-e + \text{Sqrt}[e^2 - 4*d*f]) + 4*c*(-e + \text{Sqrt}[e^2 - 4*d*f])^2)))/(16*c*f^2)/(-4*a*f^2 - 2*b*f*(-e + \text{Sqrt}[e^2 - 4*d*f]) - c*(-e + \text{Sqrt}[e^2 - 4*d*f])^2))/((e^2 - 4*d*f)^(3/2)*(a + b*x + c*x^2)^(3/2)) + (3*f^2*(a + x*(b + c*x))^(3/2)*(((4*c*f*(-4*a*f + b*(e + \text{Sqrt}[e^2 - 4*d*f])) + 2*(b*f - c*(e + \text{Sqrt}[e^2 - 4*d*f]))*(-(b*f) + 2*c*(e + \text{Sqrt}[e^2 - 4*d*f])) - 4*c*f*(b*f - c*(e + \text{Sqrt}[e^2 - 4*d*f])))*x*\text{Sqrt}[a + b*x + c*x^2])/(8*c*f^2) - ((-2*(b*f - c*(e + \text{Sqrt}[e^2 - 4*d*f]))*(b^2*f^2 - 4*c^2*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]) - 4*c*f*(3*a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f])))*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])/( \text{Sqrt}[c]*f) - (2*\text{Sqrt}[2]*\text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*\text{Sqrt}[e^2 - 4*d*f] - b*f*\text{Sqrt}[e^2 - 4*d*f]]*(4*(e + \text{Sqrt}[e^2 - 4*d*f]))*(b*f - c*(e + \text{Sqrt}[e^2 - 4*d*f]))*(b^2*f^2 - 4*c^2*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]) - 4*c*f*(3*a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f])))) + 4*f*(2*c*f*(4*a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f]))^2 - (e + \text{Sqrt}[e^2 - 4*d*f])*(b*f - c*(e + \text{Sqrt}[e^2 - 4*d*f]))*(b^2*f + 4*a*c*f - 2*b*c*(e + \text{Sqrt}[e^2 - 4*d*f]))))*\text{ArcTanh}[(4*a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f]) - (-2*b*f + 2*c*(e + \text{Sqrt}[e^2 - 4*d*f])))*x)/(2*\text{Sqrt}[2]*\text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*\text{Sqrt}[e^2 - 4*d*f] - b*f*\text{Sqrt}[e^2 - 4*d*f]]*\text{Sqrt}[a + b*x + c*x^2])]/(f*(16*a*f^2 - 8*b*f*(e + \text{Sqrt}[e^2 - 4*d*f]) + 4*c*(e + \text{Sqrt}[e^2 - 4*d*f])^2)))/(16*c*f^2)))/((e^2 - 4*d*f)^(5/2)*(a + b*x + c*x^2)^(3/2)) - (9*f^2*(a + x*(b + c*x))^(3/2)*(((4*b*c*f - 2*c*(-(b*f) + 2*c*(e + \text{Sqrt}[e^2 - 4*d*f])) + 4*c^2*f*x)*\text{Sqrt}[a + b*x + c*x^2])/(8*c*f^2) - ((-2*\text{Sqrt}[c]*(b^2*f^2 + 4*c^2*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]) + 4*c*f*(a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f]))))*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])/f - (2*\text{Sqrt}[2]*\text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*\text{Sqrt}[e^2 - 4*d*f] - b*f*\text{Sqrt}[e^2 - 4*d*f]]*(4*c*(e + \text{Sqrt}[e^2 - 4*d*f]))*(b^2*f^2 + 4*c^2*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]) + 4*c*f*(a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f])) + 4*c*f*(3*b^2*f*(e + \text{Sqrt}[e^2 - 4*d*f]) + 4*a*c*f*(e + \text{Sqrt}[e^2 - 4*d*f]) - 4*b*(2*a*f^2 + c*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]))))*\text{ArcTanh}[(4*a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f]) - (-2*b*f + 2*c*(e + \text{Sqrt}[e^2 - 4*d*f])))*x)/(2*\text{Sqrt}[2]*\text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*\text{Sqrt}[e^2 - 4*d*f] - b*f*\text{Sqrt}[e^2 - 4*d*f]]*\text{Sqrt}[a + b*x + c*x^2])]/(f*(16*a*f^2 - 8*b*f*(e + \text{Sqrt}[e^2 - 4*d*f]) + 4*c*(e + \text{Sqrt}[e^2 - 4*d*f])^2)))/(16*c*f^2)))/((e^2 - 4*d*f)^2*(a + b*x + c*x^2)^(3/2)) - (3*f^2*(a + x*(b + c*x))^(3/2)*(((2*b*f - 2*c*(e + \text{Sqrt}[e^2 - 4*d*f]))*(a + b*x + c*x^2)^(3/2))/((-4*a*f^2 + 2*b*f*(e + \text{Sqrt}[e^2 - 4*d*f]) - c*(e + \text{Sqrt}[e^2 - 4*d*f])^2)*(e + \text{Sqrt}[e^2 - 4*d*f] + 2*f*x)) + (((4*c*(b*f - c*(e + \text{Sqrt}[e^2 - 4*d*f]))*(-(b*f) + 2*c*(e + \text{Sqrt}[e^2 - 4*d*f])) + 4*c*f*(-(b^2*f) - 4*a*c*f + 2*b*c*(e + \text{Sqrt}[e^2 - 4*d*f])) - 8*c^2*f*(b*f - c*(e + \text{Sqrt}[e^2 - 4*d*f])))*x*\text{Sqrt}[a + b*x + c*x^2])/(8*c*f^2) - ((16*c^(3/2))*(b*f - c*(e + \text{Sqrt}[e^2 - 4*d*f]))*(c*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]) + f*(2*a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f]))))*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])/f - (2*\text{Sqrt}[2]*\text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*\text{Sqrt}[e^2 - 4*d*f] - b*f*\text{Sqrt}[e^2 - 4*d*f]]*(-32*c^2*(e + \text{Sqrt}[e^2 -
\end{aligned}$$

$$4*d*f))*(b*f - c*(e + \text{Sqrt}[e^2 - 4*d*f]))*(c*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]) + f*(2*a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f]))) + 16*c*f*(b^2*f + 4*a*c*f - 2*b*c*(e + \text{Sqrt}[e^2 - 4*d*f]))*(c*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]) + f*(2*a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f]))) * \text{ArcTanh}[(4*a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f]) - (-2*b*f + 2*c*(e + \text{Sqrt}[e^2 - 4*d*f]))*x)/(2*\text{Sqrt}[2]*\text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*\text{Sqrt}[e^2 - 4*d*f] - b*f*\text{Sqrt}[e^2 - 4*d*f]])*\text{Sqrt}[a + b*x + c*x^2]])/(f*(16*a*f^2 - 8*b*f*(e + \text{Sqrt}[e^2 - 4*d*f]) + 4*c*(e + \text{Sqrt}[e^2 - 4*d*f])^2))/(16*c*f^2)/(-4*a*f^2 + 2*b*f*(e + \text{Sqrt}[e^2 - 4*d*f]) - c*(e + \text{Sqrt}[e^2 - 4*d*f])^2))/((e^2 - 4*d*f)^(3/2)*(a + b*x + c*x^2)^(3/2))$$

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(3/2)/(f\*x^2+e\*x+d)^3,x, algorithm="fricas")

[Out] Timed out

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(3/2)/(f\*x^2+e\*x+d)^3,x, algorithm="giac")

[Out] Timed out

**maple** [B] time = 0.07, size = 178044, normalized size = 265.34

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+b\*x+a)^(3/2)/(f\*x^2+e\*x+d)^3,x)

[Out] result too large to display

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2 + bx + a)^{\frac{3}{2}}}{(fx^2 + ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((c\*x^2+b\*x+a)^(3/2)/(f\*x^2+e\*x+d)^3,x, algorithm="maxima")

[Out] integrate((c\*x^2 + b\*x + a)^(3/2)/(f\*x^2 + e\*x + d)^3, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(cx^2 + bx + a)^{3/2}}{(fx^2 + ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x + c\*x^2)^(3/2)/(d + e\*x + f\*x^2)^3,x)

[Out] int((a + b\*x + c\*x^2)^(3/2)/(d + e\*x + f\*x^2)^3, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+b\*x+a)\*\*(3/2)/(f\*x\*\*2+e\*x+d)\*\*3,x)

[Out] Timed out

$$3.109 \quad \int \frac{(d+ex+fx^2)^3}{\sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=717

$$\frac{x\sqrt{a+bx+cx^2} (24c^2f(50a^2f^2+322abef+175b^2(df+e^2)) - 252b^2cf^2(14af+15be) - 160c^3(27af(df+e^2)))}{3840c^5}$$

[Out] 1/1024\*(1024\*c^6\*d^3+231\*b^6\*f^3-252\*b^4\*c\*f^2\*(5\*a\*f+3\*b\*e)-1536\*c^5\*d\*(b\*d\*e+a\*(d\*f+e^2))+840\*b^2\*c^2\*f\*(4\*a\*b\*e\*f+2\*a^2\*f^2+b^2\*(d\*f+e^2))+384\*c^4\*(3\*b^2\*d\*(d\*f+e^2)+3\*a^2\*f\*(d\*f+e^2)+2\*a\*b\*e\*(6\*d\*f+e^2))-320\*c^3\*(9\*a^2\*b\*e\*f^2+a^3\*f^3+9\*a\*b^2\*f\*(d\*f+e^2)+b^3\*(6\*d\*e\*f+e^3))\*arctanh(1/2\*(2\*c\*x+b)/c^(1/2)/(c\*x^2+b\*x+a)^(1/2))/c^(13/2)+1/7680\*(23040\*c^5\*d^2\*e-3465\*b^5\*f^3+420\*b^3\*c\*f^2\*(34\*a\*f+27\*b\*e)-504\*b\*c^2\*f\*(70\*a\*b\*e\*f+22\*a^2\*f^2+25\*b^2\*(d\*f+e^2))-640\*c^4\*(27\*b\*d\*(d\*f+e^2)+8\*a\*e\*(6\*d\*f+e^2))+96\*c^3\*(128\*a^2\*e\*f^2+275\*a\*b\*f\*(d\*f+e^2)+50\*b^2\*(6\*d\*e\*f+e^3)))\*(c\*x^2+b\*x+a)^(1/2)/c^6+1/3840\*(1155\*b^4\*f^3-252\*b^2\*c\*f^2\*(14\*a\*f+15\*b\*e)+5760\*c^4\*d\*(d\*f+e^2)+24\*c^2\*f\*(322\*a\*b\*e\*f+50\*a^2\*f^2+175\*b^2\*(d\*f+e^2))-160\*c^3\*(27\*a\*f\*(d\*f+e^2)+10\*b\*(6\*d\*e\*f+e^3)))\*x\*(c\*x^2+b\*x+a)^(1/2)/c^5-1/960\*(231\*b^3\*f^3-36\*b\*c\*f^2\*(13\*a\*f+21\*b\*e)-320\*c^3\*(6\*d\*e\*f+e^3)+24\*c^2\*f\*(32\*a\*e\*f+35\*b\*(d\*f+e^2)))\*x^2\*(c\*x^2+b\*x+a)^(1/2)/c^4+1/480\*f\*(99\*b^2\*f^2-4\*c\*f\*(25\*a\*f+81\*b\*e)+360\*c^2\*(d\*f+e^2))\*x^3\*(c\*x^2+b\*x+a)^(1/2)/c^3+1/60\*f^2\*(-11\*b\*f+36\*c\*e)\*x^4\*(c\*x^2+b\*x+a)^(1/2)/c^2+1/6\*f^3\*x^5\*(c\*x^2+b\*x+a)^(1/2)/c

**Rubi [A]** time = 2.71, antiderivative size = 717, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$ , Rules used = {1661, 640, 621, 206}

$$\frac{\sqrt{a+bx+cx^2} (-504bc^2f(22a^2f^2+70abef+25b^2(df+e^2)) + 96c^3(128a^2ef^2+275abf(df+e^2)) + 50b^2(6d^2e^2+3a^2f^3))}{7680c^6}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x + f\*x^2)^3/Sqrt[a + b\*x + c\*x^2], x]

[Out] ((23040\*c^5\*d^2\*e - 3465\*b^5\*f^3 + 420\*b^3\*c\*f^2\*(27\*b\*e + 34\*a\*f) - 504\*b\*c^2\*f\*(70\*a\*b\*e\*f + 22\*a^2\*f^2 + 25\*b^2\*(e^2 + d\*f)) - 640\*c^4\*(27\*b\*d\*(e^2 + d\*f) + 8\*a\*e\*(e^2 + 6\*d\*f)) + 96\*c^3\*(128\*a^2\*e\*f^2 + 275\*a\*b\*f\*(e^2 + d\*f) + 50\*b^2\*(e^3 + 6\*d\*e\*f)))\*Sqrt[a + b\*x + c\*x^2])/(7680\*c^6) + ((1155\*b^4\*f^3 - 252\*b^2\*c\*f^2\*(15\*b\*e + 14\*a\*f) + 5760\*c^4\*d\*(e^2 + d\*f) + 24\*c^2\*f\*(322\*a\*b\*e\*f + 50\*a^2\*f^2 + 175\*b^2\*(e^2 + d\*f)) - 160\*c^3\*(27\*a\*f\*(e^2 + d\*f) + 10\*b\*(e^3 + 6\*d\*e\*f)))\*x\*Sqrt[a + b\*x + c\*x^2])/(3840\*c^5) - ((231\*

$$b^3 f^3 - 36 b c f^2 (21 b e + 13 a f) - 320 c^3 (e^3 + 6 d e f) + 24 c^2 f (32 a e f + 35 b (e^2 + d f)) x^2 \sqrt{a + b x + c x^2} / (960 c^4) + (f (99 b^2 f^2 - 4 c f (81 b e + 25 a f) + 360 c^2 (e^2 + d f)) x^3 \sqrt{a + b x + c x^2}) / (480 c^3) + (f^2 (36 c e - 11 b f) x^4 \sqrt{a + b x + c x^2}) / (60 c^2) + (f^3 x^5 \sqrt{a + b x + c x^2}) / (6 c) + ((1024 c^6 d^3 + 231 b^6 f^3 - 252 b^4 c f^2 (3 b e + 5 a f) - 1536 c^5 d (b d e + a (e^2 + d f)) + 840 b^2 c^2 f (4 a b e f + 2 a^2 f^2 + b^2 (e^2 + d f)) + 384 c^4 (3 b^2 d (e^2 + d f) + 3 a^2 f (e^2 + d f) + 2 a b e (e^2 + 6 d f)) - 320 c^3 (9 a^2 b e f^2 + a^3 f^3 + 9 a b^2 f (e^2 + d f) + b^3 (e^3 + 6 d e f))) \operatorname{ArcTanh}[(b + 2 c x) / (2 \sqrt{c} \sqrt{a + b x + c x^2})]) / (1024 c^{13/2})$$

### Rule 206

$\operatorname{Int}[(a + (b \cdot x)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(1 \cdot \operatorname{ArcTanh}[(Rt[-b, 2] \cdot x) / Rt[a, 2]]) / (Rt[a, 2] \cdot Rt[-b, 2]), x] /;$   $\operatorname{FreeQ}\{a, b, x\} \ \&\& \ \operatorname{NegQ}[a/b] \ \&\& \ (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

### Rule 621

$\operatorname{Int}[1/\sqrt{(a + (b \cdot x) + (c \cdot x)^2)}, x\_Symbol] \rightarrow \operatorname{Dist}[2, \operatorname{Subst}[\operatorname{Int}[1/(4c - x^2), x], x, (b + 2cx)/\sqrt{a + bx + cx^2}], x] /;$   $\operatorname{FreeQ}\{a, b, c, x\} \ \&\& \ \operatorname{NeQ}[b^2 - 4ac, 0]$

### Rule 640

$\operatorname{Int}[(d + (e \cdot x)) \cdot ((a + (b \cdot x) + (c \cdot x)^2)^{p}), x\_Symbol] \rightarrow \operatorname{Simp}[(e \cdot (a + b x + c x^2)^{p+1}) / (2c \cdot (p+1)), x] + \operatorname{Dist}[(2cd - b \cdot e) / (2c), \operatorname{Int}[(a + b x + c x^2)^p, x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, e, p, x\} \ \&\& \ \operatorname{NeQ}[2cd - b \cdot e, 0] \ \&\& \ \operatorname{NeQ}[p, -1]$

### Rule 1661

$\operatorname{Int}[(Pq) \cdot ((a + (b \cdot x) + (c \cdot x)^2)^{p}), x\_Symbol] \rightarrow \operatorname{With}\{q = \operatorname{Expon}[Pq, x], e = \operatorname{Coeff}[Pq, x, \operatorname{Expon}[Pq, x]]\}, \operatorname{Simp}[(e \cdot x^{q-1}) \cdot (a + b x + c x^2)^{p+1}) / (c \cdot (q + 2p + 1)), x] + \operatorname{Dist}[1 / (c \cdot (q + 2p + 1)), \operatorname{Int}[(a + b x + c x^2)^p \cdot \operatorname{ExpandToSum}[c \cdot (q + 2p + 1) \cdot Pq - a \cdot e \cdot (q - 1) \cdot x^{q-2} - b \cdot e \cdot (q + p) \cdot x^{q-1} - c \cdot e \cdot (q + 2p + 1) \cdot x^q, x], x] /;$   $\operatorname{FreeQ}\{a, b, c, p, x\} \ \&\& \ \operatorname{PolyQ}[Pq, x] \ \&\& \ \operatorname{NeQ}[b^2 - 4ac, 0] \ \&\& \ !\operatorname{LeQ}[p, -1]$

### Rubi steps



$$61*e*f*x + f*(275*d + 39*f*x^2)) - 64*c^4*(a*(80*e^3 + 135*e^2*f*x + 96*e*f*(5*d + f*x^2) + 5*f^2*x*(27*d + 5*f*x^2)) + b*(270*d^2*f + 15*d*(18*e^2 + 20*e*f*x + 7*f^2*x^2) + x*(50*e^3 + 105*e^2*f*x + 81*e*f^2*x^2 + 22*f^3*x^3))))/(7680*c^6) + ((1024*c^6*d^3 + 231*b^6*f^3 - 252*b^4*c*f^2*(3*b*e + 5*a*f) - 1536*c^5*d*(b*d*e + a*(e^2 + d*f)) + 840*b^2*c^2*f*(4*a*b*e*f + 2*a^2*f^2 + b^2*(e^2 + d*f)) + 384*c^4*(3*b^2*d*(e^2 + d*f) + 3*a^2*f*(e^2 + d*f) + 2*a*b*e*(e^2 + 6*d*f)) - 320*c^3*(9*a^2*b*e*f^2 + a^3*f^3 + 9*a*b^2*f*(e^2 + d*f) + b^3*(e^3 + 6*d*e*f)))*ArcTanh[(b + 2*c*x)/(2*sqrt[c]*sqrt[a + x*(b + c*x)])]/(1024*c^(13/2))$$

**fricas** [A] time = 3.20, size = 1583, normalized size = 2.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)^3/(c\*x^2+b\*x+a)^(1/2),x, algorithm="fricas")

[Out] [-1/30720\*(15\*(1024\*c^6\*d^3 - 1536\*b\*c^5\*d^2\*e + 384\*(3\*b^2\*c^4 - 4\*a\*c^5)\*d\*e^2 - 64\*(5\*b^3\*c^3 - 12\*a\*b\*c^4)\*e^3 + (231\*b^6 - 1260\*a\*b^4\*c + 1680\*a^2\*b^2\*c^2 - 320\*a^3\*c^3)\*f^3 + 12\*(2\*(35\*b^4\*c^2 - 120\*a\*b^2\*c^3 + 48\*a^2\*c^4)\*d - (63\*b^5\*c - 280\*a\*b^3\*c^2 + 240\*a^2\*b\*c^3)\*e)\*f^2 + 24\*(16\*(3\*b^2\*c^4 - 4\*a\*c^5)\*d^2 - 16\*(5\*b^3\*c^3 - 12\*a\*b\*c^4)\*d\*e + (35\*b^4\*c^2 - 120\*a\*b^2\*c^3 + 48\*a^2\*c^4)\*e^2)\*f)\*sqrt(c)\*log(-8\*c^2\*x^2 - 8\*b\*c\*x - b^2 + 4\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(c) - 4\*a\*c) - 4\*(1280\*c^6\*f^3\*x^5 + 23040\*c^6\*d^2\*e - 17280\*b\*c^5\*d\*e^2 + 128\*(36\*c^6\*e\*f^2 - 11\*b\*c^5\*f^3)\*x^4 + 320\*(15\*b^2\*c^4 - 16\*a\*c^5)\*e^3 - 21\*(165\*b^5\*c - 680\*a\*b^3\*c^2 + 528\*a^2\*b\*c^3)\*f^3 + 16\*(360\*c^6\*e^2\*f + (99\*b^2\*c^4 - 100\*a\*c^5)\*f^3 + 36\*(10\*c^6\*d - 9\*b\*c^5\*e)\*f^2)\*x^3 - 12\*(50\*(21\*b^3\*c^3 - 44\*a\*b\*c^4)\*d - (945\*b^4\*c^2 - 2940\*a\*b^2\*c^3 + 1024\*a^2\*c^4)\*e)\*f^2 + 8\*(320\*c^6\*e^3 - 3\*(77\*b^3\*c^3 - 156\*a\*b\*c^4)\*f^3 - 12\*(70\*b\*c^5\*d - (63\*b^2\*c^4 - 64\*a\*c^5)\*e)\*f^2 + 120\*(16\*c^6\*d\*e - 7\*b\*c^5\*e^2)\*f)\*x^2 - 120\*(144\*b\*c^5\*d^2 - 16\*(15\*b^2\*c^4 - 16\*a\*c^5)\*d\*e + 5\*(21\*b^3\*c^3 - 44\*a\*b\*c^4)\*e^2)\*f + 2\*(5760\*c^6\*d\*e^2 - 1600\*b\*c^5\*e^3 + 3\*(385\*b^4\*c^2 - 1176\*a\*b^2\*c^3 + 400\*a^2\*c^4)\*f^3 + 12\*(10\*(35\*b^2\*c^4 - 36\*a\*c^5)\*d - 7\*(45\*b^3\*c^3 - 92\*a\*b\*c^4)\*e)\*f^2 + 120\*(48\*c^6\*d^2 - 80\*b\*c^5\*d\*e + (35\*b^2\*c^4 - 36\*a\*c^5)\*e^2)\*f)\*x)\*sqrt(c\*x^2 + b\*x + a))/c^7, -1/15360\*(15\*(1024\*c^6\*d^3 - 1536\*b\*c^5\*d^2\*e + 384\*(3\*b^2\*c^4 - 4\*a\*c^5)\*d\*e^2 - 64\*(5\*b^3\*c^3 - 12\*a\*b\*c^4)\*e^3 + (231\*b^6 - 1260\*a\*b^4\*c + 1680\*a^2\*b^2\*c^2 - 320\*a^3\*c^3)\*f^3 + 12\*(2\*(35\*b^4\*c^2 - 120\*a\*b^2\*c^3 + 48\*a^2\*c^4)\*d - (63\*b^5\*c - 280\*a\*b^3\*c^2 + 240\*a^2\*b\*c^3)\*e)\*f^2 + 24\*(16\*(3\*b^2\*c^4 - 4\*a\*c^5)\*d^2 - 16\*(5\*b^3\*c^3 - 12\*a\*b\*c^4)\*d\*e + (35\*b^4\*c^2 - 120\*a\*b^2\*c^3 + 48\*a^2\*c^4)\*e^2)\*f)\*sqrt(-c)\*arctan(1/2\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(-c)/(c^2\*x^2 + b\*c\*x + a\*c)) - 2\*(1280\*c^6\*f^3\*x^5 + 23040\*c^6\*d^2\*e - 17280\*b\*c^5\*d\*e^2 + 128\*(36\*c^6\*e\*f^2 - 11\*b\*c^5\*f^3)\*x^4 + 320\*(15\*b^2\*c^4 - 16\*a\*c^5)\*e^3 - 21\*(165\*b^5\*c - 680\*a\*b^3\*c^2 + 528\*a^2\*b\*c^3)\*f^3 + 16\*(360\*c^6\*e^2\*f + (99\*b^2\*c^4 - 100\*a\*c^5)\*f^3 + 36\*(10\*c^6\*d

$$d - 9*b*c^5*e)*f^2)*x^3 - 12*(50*(21*b^3*c^3 - 44*a*b*c^4)*d - (945*b^4*c^2 - 2940*a*b^2*c^3 + 1024*a^2*c^4)*e)*f^2 + 8*(320*c^6*e^3 - 3*(77*b^3*c^3 - 156*a*b*c^4)*f^3 - 12*(70*b*c^5*d - (63*b^2*c^4 - 64*a*c^5)*e)*f^2 + 120*(16*c^6*d*e - 7*b*c^5*e^2)*f)*x^2 - 120*(144*b*c^5*d^2 - 16*(15*b^2*c^4 - 16*a*c^5)*d*e + 5*(21*b^3*c^3 - 44*a*b*c^4)*e^2)*f + 2*(5760*c^6*d*e^2 - 1600*b*c^5*e^3 + 3*(385*b^4*c^2 - 1176*a*b^2*c^3 + 400*a^2*c^4)*f^3 + 12*(10*(35*b^2*c^4 - 36*a*c^5)*d - 7*(45*b^3*c^3 - 92*a*b*c^4)*e)*f^2 + 120*(48*c^6*d^2 - 80*b*c^5*d*e + (35*b^2*c^4 - 36*a*c^5)*e^2)*f)*x)*sqrt(c*x^2 + b*x + a))/c^7]$$

**giac** [A] time = 0.53, size = 824, normalized size = 1.15

$$\frac{1}{7680} \sqrt{cx^2 + bx + a} \left( 2 \left( 4 \left( 2 \left( 8 \left( \frac{10 f^3 x}{c} - \frac{11 bc^4 f^3 - 36 c^5 f^2 e}{c^6} \right) x + \frac{360 c^5 d f^2 + 99 b^2 c^3 f^3 - 100 ac^4 f^3 - 324 bc^4 f^3}{c^6} \right) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)^3/(c\*x^2+b\*x+a)^(1/2),x, algorithm="giac")

[Out] 1/7680\*sqrt(c\*x^2 + b\*x + a)\*(2\*(4\*(2\*(8\*(10\*f^3\*x/c - (11\*b\*c^4\*f^3 - 36\*c^5\*f^2\*e)/c^6)\*x + (360\*c^5\*d\*f^2 + 99\*b^2\*c^3\*f^3 - 100\*a\*c^4\*f^3 - 324\*b\*c^4\*f^2\*e + 360\*c^5\*f\*e^2)/c^6)\*x - (840\*b\*c^4\*d\*f^2 + 231\*b^3\*c^2\*f^3 - 468\*a\*b\*c^3\*f^3 - 1920\*c^5\*d\*f\*e - 756\*b^2\*c^3\*f^2\*e + 768\*a\*c^4\*f^2\*e + 840\*b\*c^4\*f\*e^2 - 320\*c^5\*e^3)/c^6)\*x + (5760\*c^5\*d^2\*f + 4200\*b^2\*c^3\*d\*f^2 - 4320\*a\*c^4\*d\*f^2 + 1155\*b^4\*c\*f^3 - 3528\*a\*b^2\*c^2\*f^3 + 1200\*a^2\*c^3\*f^3 - 9600\*b\*c^4\*d\*f\*e - 3780\*b^3\*c^2\*f^2\*e + 7728\*a\*b\*c^3\*f^2\*e + 5760\*c^5\*d\*e^2 + 4200\*b^2\*c^3\*f\*e^2 - 4320\*a\*c^4\*f\*e^2 - 1600\*b\*c^4\*e^3)/c^6)\*x - (17280\*b\*c^4\*d^2\*f + 12600\*b^3\*c^2\*d\*f^2 - 26400\*a\*b\*c^3\*d\*f^2 + 3465\*b^5\*f^3 - 14280\*a\*b^3\*c\*f^3 + 11088\*a^2\*b\*c^2\*f^3 - 23040\*c^5\*d^2\*e - 28800\*b^2\*c^3\*d\*f\*e + 30720\*a\*c^4\*d\*f\*e - 11340\*b^4\*c\*f^2\*e + 35280\*a\*b^2\*c^2\*f^2\*e - 12288\*a^2\*c^3\*f^2\*e + 17280\*b\*c^4\*d\*e^2 + 12600\*b^3\*c^2\*f\*e^2 - 26400\*a\*b\*c^3\*f\*e^2 - 4800\*b^2\*c^3\*e^3 + 5120\*a\*c^4\*e^3)/c^6) - 1/1024\*(1024\*c^6\*d^3 + 1152\*b^2\*c^4\*d^2\*f - 1536\*a\*c^5\*d^2\*f + 840\*b^4\*c^2\*d\*f^2 - 2880\*a\*b^2\*c^3\*d\*f^2 + 1152\*a^2\*c^4\*d\*f^2 + 231\*b^6\*f^3 - 1260\*a\*b^4\*c\*f^3 + 1680\*a^2\*b^2\*c^2\*f^3 - 320\*a^3\*c^3\*f^3 - 1536\*b\*c^5\*d^2\*e - 1920\*b^3\*c^3\*d\*f\*e + 4608\*a\*b\*c^4\*d\*f\*e - 756\*b^5\*c\*f^2\*e + 3360\*a\*b^3\*c^2\*f^2\*e - 2880\*a^2\*b\*c^3\*f^2\*e + 1152\*b^2\*c^4\*d\*e^2 - 1536\*a\*c^5\*d\*e^2 + 840\*b^4\*c^2\*f\*e^2 - 2880\*a\*b^2\*c^3\*f\*e^2 + 1152\*a^2\*c^4\*f\*e^2 - 320\*b^3\*c^3\*e^3 + 768\*a\*b\*c^4\*e^3)\*log(abs(-2\*(sqrt(c)\*x - sqrt(c\*x^2 + b\*x + a))\*sqrt(c) - b))/c^(13/2)

**maple** [B] time = 0.03, size = 1930, normalized size = 2.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^{(1/2)}, x)$

[Out]  $3*d^2*e/c*(c*x^2+b*x+a)^{(1/2)}+1/3*x^2/c*(c*x^2+b*x+a)^{(1/2)}*e^3+5/8*b^2/c^3*(c*x^2+b*x+a)^{(1/2)}*e^3-5/16*b^3/c^{(7/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*e^3-2/3*a/c^2*(c*x^2+b*x+a)^{(1/2)}*e^3-231/512*f^3*b^5/c^6*(c*x^2+b*x+a)^{(1/2)}+231/1024*f^3*b^6/c^{(13/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})-5/16*f^3*a^3/c^{(7/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})+d^3*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})/c^{(1/2)}+161/80*e*f^2*b/c^3*a*x*(c*x^2+b*x+a)^{(1/2)}-5/2*b/c^2*x*(c*x^2+b*x+a)^{(1/2)}*d*e*f+9/2*b/c^{(5/2)}*a*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*d*e*f-45/16*b^2/c^{(7/2)}*a*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*e^2*f+55/16*b/c^3*a*(c*x^2+b*x+a)^{(1/2)}*d*f^2+55/16*b/c^3*a*(c*x^2+b*x+a)^{(1/2)}*e^2*f-9/8*a/c^2*x*(c*x^2+b*x+a)^{(1/2)}*d*f^2-147/160*f^3*b^2/c^4*a*x*(c*x^2+b*x+a)^{(1/2)}+39/80*f^3*b/c^3*a*x^2*(c*x^2+b*x+a)^{(1/2)}-9/8*a/c^2*x*(c*x^2+b*x+a)^{(1/2)}*e^2*f+2*x^2/c*(c*x^2+b*x+a)^{(1/2)}*d*e*f+15/4*b^2/c^3*(c*x^2+b*x+a)^{(1/2)}*d*e*f-15/8*b^3/c^{(7/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*d*e*f-4*a/c^2*(c*x^2+b*x+a)^{(1/2)}*d*e*f-27/40*e*f^2*b/c^2*x^3*(c*x^2+b*x+a)^{(1/2)}+63/80*e*f^2*b^2/c^3*x^2*(c*x^2+b*x+a)^{(1/2)}-63/64*e*f^2*b^3/c^4*x*(c*x^2+b*x+a)^{(1/2)}+105/32*e*f^2*b^3/c^{(9/2)}*a*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})-147/32*e*f^2*b^2/c^4*a*(c*x^2+b*x+a)^{(1/2)}-45/16*e*f^2*b/c^{(7/2)}*a^2*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})-4/5*e*f^2*a/c^2*x^2*(c*x^2+b*x+a)^{(1/2)}-7/8*b/c^2*x^2*(c*x^2+b*x+a)^{(1/2)}*d*f^2-7/8*b/c^2*x^2*(c*x^2+b*x+a)^{(1/2)}*e^2*f+35/32*b^2/c^3*x*(c*x^2+b*x+a)^{(1/2)}*d*f^2+35/32*b^2/c^3*x*(c*x^2+b*x+a)^{(1/2)}*e^2*f-45/16*b^2/c^{(7/2)}*a*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*d*f^2+1/6*f^3*x^5*(c*x^2+b*x+a)^{(1/2)}/c+9/8*a^2/c^{(5/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*e^2*f-5/12*b/c^2*x*(c*x^2+b*x+a)^{(1/2)}*e^3+3/4*b/c^{(5/2)}*a*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*e^3-105/64*b^3/c^4*(c*x^2+b*x+a)^{(1/2)}*e^2*f+105/128*b^4/c^{(9/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*d*f^2+105/128*b^4/c^{(9/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*e^2*f+9/8*a^2/c^{(5/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*d*f^2+3/4*x^3/c*(c*x^2+b*x+a)^{(1/2)}*d*f^2+3/4*x^3/c*(c*x^2+b*x+a)^{(1/2)}*e^2*f-105/64*b^3/c^4*(c*x^2+b*x+a)^{(1/2)}*d*f^2+3/5*e*f^2*x^4/c*(c*x^2+b*x+a)^{(1/2)}+189/128*e*f^2*b^4/c^5*(c*x^2+b*x+a)^{(1/2)}-189/256*e*f^2*b^5/c^{(11/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})+8/5*e*f^2*a^2/c^3*(c*x^2+b*x+a)^{(1/2)}-315/256*f^3*b^4/c^{(11/2)}*a*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})+119/64*f^3*b^3/c^5*a*(c*x^2+b*x+a)^{(1/2)}+105/64*f^3*b^2/c^{(9/2)}*a^2*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})-231/160*f^3*b/c^4*a^2*(c*x^2+b*x+a)^{(1/2)}-5/24*f^3*a/c^2*x^3*(c*x^2+b*x+a)^{(1/2)}+5/16*f^3*a^2/c^3*x*(c*x^2+b*x+a)^{(1/2)}-11/60*f^3*b/c^2*x^4*(c*x^2+b*x+a)^{(1/2)}+33/160*f^3*b^2/c^3*x^3*(c*x^2+b*x+a)^{(1/2)}-77/320*f^3*b^3/c^4*x^2*(c*x^2+b*x+a)^{(1/2)}+77/256*f^3*b^4/c^5*x*(c*x^2+b*x+a)^{(1/2)}-9/4*b/c^2*(c*x^2+b*x+a)^{(1/2)}*f*d^2-9/4*b/c^2*(c*x^2+b*x+a)^{(1/2)}*e^2*d+9/8*b^2/c^{(5/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*f*d^2+9/8*b^2/c^{(5/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*e^2*d-3/2*a/c^{(3/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*f*d^2-3/2*a/c^{(3/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*e^2*d-3/2*d^2*e*b/c^{(3/2)}*\ln((c$

$x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)}+3/2*x/c*(c*x^2+b*x+a)^{(1/2)}*f*d^2+3/2*x/c*(c*x^2+b*x+a)^{(1/2)}*e^2*d$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)^3/(c\*x^2+b\*x+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details)Is 4\*a\*c-b^2 zero or nonzero?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f x^2 + e x + d)^3}{\sqrt{c x^2 + b x + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x + f\*x^2)^3/(a + b\*x + c\*x^2)^(1/2),x)

[Out] int((d + e\*x + f\*x^2)^3/(a + b\*x + c\*x^2)^(1/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + e x + f x^2)^3}{\sqrt{a + b x + c x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x\*\*2+e\*x+d)\*\*3/(c\*x\*\*2+b\*x+a)\*\*(1/2),x)

[Out] Integral((d + e\*x + f\*x\*\*2)\*\*3/sqrt(a + b\*x + c\*x\*\*2), x)



$$3.110 \quad \int \frac{(d+ex+fx^2)^2}{\sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=316

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2\left(a^2f^2+4abef+b^2(2df+e^2)\right)-40b^2cf(3af+2be)-64c^3\left(a(2df+e^2)+2bde\right)\right)}{128c^{9/2}}$$

[Out] 1/128\*(128\*c^4\*d^2+35\*b^4\*f^2-40\*b^2\*c\*f\*(3\*a\*f+2\*b\*e)-64\*c^3\*(2\*b\*d\*e+a\*(2\*d\*f+e^2))+48\*c^2\*(4\*a\*b\*e\*f+a^2\*f^2+b^2\*(2\*d\*f+e^2)))\*arctanh(1/2\*(2\*c\*x+b)/c^(1/2)/(c\*x^2+b\*x+a)^(1/2))/c^(9/2)+1/192\*(384\*c^3\*d\*e-105\*b^3\*f^2+20\*b\*c\*f\*(11\*a\*f+12\*b\*e)-16\*c^2\*(16\*a\*e\*f+9\*b\*(2\*d\*f+e^2)))\*(c\*x^2+b\*x+a)^(1/2)/c^4+1/96\*(35\*b^2\*f^2-4\*c\*f\*(9\*a\*f+20\*b\*e)+48\*c^2\*(2\*d\*f+e^2))\*x\*(c\*x^2+b\*x+a)^(1/2)/c^3+1/24\*f\*(-7\*b\*f+16\*c\*e)\*x^2\*(c\*x^2+b\*x+a)^(1/2)/c^2+1/4\*f^2\*x^3\*(c\*x^2+b\*x+a)^(1/2)/c

**Rubi [A]** time = 0.63, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$ , Rules used = {1661, 640, 621, 206}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2\left(a^2f^2+4abef+b^2(2df+e^2)\right)-40b^2cf(3af+2be)-64c^3\left(a(2df+e^2)+2bde\right)\right)}{128c^{9/2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x + f\*x^2)^2/Sqrt[a + b\*x + c\*x^2], x]

[Out] ((384\*c^3\*d\*e - 105\*b^3\*f^2 + 20\*b\*c\*f\*(12\*b\*e + 11\*a\*f) - 16\*c^2\*(16\*a\*e\*f + 9\*b\*(e^2 + 2\*d\*f)))\*Sqrt[a + b\*x + c\*x^2])/(192\*c^4) + ((35\*b^2\*f^2 - 4\*c\*f\*(20\*b\*e + 9\*a\*f) + 48\*c^2\*(e^2 + 2\*d\*f))\*x\*Sqrt[a + b\*x + c\*x^2])/(96\*c^3) + (f\*(16\*c\*e - 7\*b\*f)\*x^2\*Sqrt[a + b\*x + c\*x^2])/(24\*c^2) + (f^2\*x^3\*Sqrt[a + b\*x + c\*x^2])/(4\*c) + ((128\*c^4\*d^2 + 35\*b^4\*f^2 - 40\*b^2\*c\*f\*(2\*b\*e + 3\*a\*f) - 64\*c^3\*(2\*b\*d\*e + a\*(e^2 + 2\*d\*f)) + 48\*c^2\*(4\*a\*b\*e\*f + a^2\*f^2 + b^2\*(e^2 + 2\*d\*f)))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(128\*c^(9/2))

**Rule 206**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

**Rule 621**

```
Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 640

```
Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

### Rule 1661

```
Int[(Pq_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a + b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

### Rubi steps

$$\begin{aligned}
 \int \frac{(d + ex + fx^2)^2}{\sqrt{a + bx + cx^2}} dx &= \frac{f^2 x^3 \sqrt{a + bx + cx^2}}{4c} + \frac{\int \frac{4cd^2 + 8cdex - (3af^2 - 4c(e^2 + 2df))x^2 + \frac{1}{2}f(16ce - 7bf)x^3}{\sqrt{a + bx + cx^2}} dx}{4c} \\
 &= \frac{f(16ce - 7bf)x^2 \sqrt{a + bx + cx^2}}{24c^2} + \frac{f^2 x^3 \sqrt{a + bx + cx^2}}{4c} + \frac{\int \frac{12c^2 d^2 + (24c^2 de - 16acef + 7abf^2)x + \frac{1}{4}}{\sqrt{a + bx + cx^2}} dx}{\sqrt{a + bx + cx^2}} \\
 &= \frac{(35b^2 f^2 - 4cf(20be + 9af) + 48c^2(e^2 + 2df))x \sqrt{a + bx + cx^2}}{96c^3} + \frac{f(16ce - 7bf)x^2 \sqrt{a + bx + cx^2}}{24c^2} \\
 &= \frac{(384c^3 de - 105b^3 f^2 + 20bcf(12be + 11af) - 16c^2(16aef + 9b(e^2 + 2df))) \sqrt{a + bx + cx^2}}{192c^4} \\
 &= \frac{(384c^3 de - 105b^3 f^2 + 20bcf(12be + 11af) - 16c^2(16aef + 9b(e^2 + 2df))) \sqrt{a + bx + cx^2}}{192c^4} \\
 &= \frac{(384c^3 de - 105b^3 f^2 + 20bcf(12be + 11af) - 16c^2(16aef + 9b(e^2 + 2df))) \sqrt{a + bx + cx^2}}{192c^4}
 \end{aligned}$$

**Mathematica [A]** time = 0.50, size = 251, normalized size = 0.79

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\left(48c^2(a^2f^2+4abef+b^2(2df+e^2))-40b^2cf(3af+2be)-64c^3(a(2df+e^2)+2bde)\right)}{128c^{9/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x + f\*x^2)^2/Sqrt[a + b\*x + c\*x^2], x]

[Out] (Sqrt[a + x\*(b + c\*x)]\*(-105\*b^3\*f^2 + 10\*b\*c\*f\*(24\*b\*e + 22\*a\*f + 7\*b\*f\*x) + 16\*c^3\*(12\*d\*(2\*e + f\*x) + x\*(6\*e^2 + 8\*e\*f\*x + 3\*f^2\*x^2)) - 8\*c^2\*(a\*f\*(32\*e + 9\*f\*x) + b\*(18\*e^2 + 36\*d\*f + 20\*e\*f\*x + 7\*f^2\*x^2)))/(192\*c^4) + ((128\*c^4\*d^2 + 35\*b^4\*f^2 - 40\*b^2\*c\*f\*(2\*b\*e + 3\*a\*f) - 64\*c^3\*(2\*b\*d\*e + a\*(e^2 + 2\*d\*f)) + 48\*c^2\*(4\*a\*b\*e\*f + a^2\*f^2 + b^2\*(e^2 + 2\*d\*f)))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + x\*(b + c\*x)])])/(128\*c^(9/2))

**fricas [A]** time = 1.44, size = 637, normalized size = 2.02

$$\left[ \frac{3(128c^4d^2 - 128bc^3de + 16(3b^2c^2 - 4ac^3)e^2 + (35b^4 - 120ab^2c + 48a^2c^2)f^2 + 16(2(3b^2c^2 - 4ac^3)d - (5b^3c - 12ab^2c^2)e)*f)\sqrt{c}\log(-8c^2x^2 - 8b^2cx - b^2 - 4\sqrt{c}(cx^2 + bx + a)(2cx + b)\sqrt{c} - 4ac) + 4(48c^4f^2x^3 + 384c^4de - 144b^3c^3e^2 - 5(21b^3c - 44ab^2c^2)f^2 + 8(16c^4ef - 7b^3c^3f^2)x^2 - 16(18b^3cd - (15b^2c^2 - 16ac^3)e)*f + 2(48c^4e^2 + (35b^2c^2 - 36ac^3)f^2 + 16(6c^4d - 5b^3c^3e)*f)*x)\sqrt{c}x^2 + b^2x + a)/c^5, -1/384(3(128c^4d^2 - 128b^3c^3de + 16(3b^2c^2 - 4ac^3)e^2 + (35b^4 - 120ab^2c + 48a^2c^2)f^2 + 16(2(3b^2c^2 - 4ac^3)d - (5b^3c - 12ab^2c^2)e)*f)\sqrt{-c})\arctan(1/2\sqrt{c}x^2 + b^2x + a)(2cx + b)\sqrt{-c}/(c^2x^2 + b^2cx + ac)) - 2(48c^4f^2x^3 + 384c^4de - 144b^3c^3e^2 - 5(21b^3c - 44ab^2c^2)f^2 + 8(16c^4ef - 7b^3c^3f^2)x^2 - 16(18b^3cd - (15b^2c^2 - 16ac^3)e)*f + 2(48c^4e^2 + (35b^2c^2 - 36ac^3)f^2 + 16(6c^4d - 5b^3c^3e)*f)*x)\sqrt{c}x^2 + b^2x + a)/c^5 \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)^2/(c\*x^2+b\*x+a)^(1/2), x, algorithm="fricas")

[Out] [1/768\*(3\*(128\*c^4\*d^2 - 128\*b^3\*c^3\*d\*e + 16\*(3\*b^2\*c^2 - 4\*a\*c^3)\*e^2 + (35\*b^4 - 120\*a\*b^2\*c + 48\*a^2\*c^2)\*f^2 + 16\*(2\*(3\*b^2\*c^2 - 4\*a\*c^3)\*d - (5\*b^3\*c - 12\*a\*b^2\*c^2)\*e)\*f)\*sqrt(c)\*log(-8\*c^2\*x^2 - 8\*b^2\*c\*x - b^2 - 4\*sqrt(c)\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(c) - 4\*a\*c) + 4\*(48\*c^4\*f^2\*x^3 + 384\*c^4\*d\*e - 144\*b^3\*c^3\*e^2 - 5\*(21\*b^3\*c - 44\*a\*b^2\*c^2)\*f^2 + 8\*(16\*c^4\*e\*f - 7\*b^3\*c^3\*f^2)\*x^2 - 16\*(18\*b^3\*c\*d - (15\*b^2\*c^2 - 16\*a\*c^3)\*e)\*f + 2\*(48\*c^4\*e^2 + (35\*b^2\*c^2 - 36\*a\*c^3)\*f^2 + 16\*(6\*c^4\*d - 5\*b^3\*c^3\*e)\*f)\*x)\*sqrt(c\*x^2 + b\*x + a))/c^5, -1/384\*(3\*(128\*c^4\*d^2 - 128\*b^3\*c^3\*d\*e + 16\*(3\*b^2\*c^2 - 4\*a\*c^3)\*e^2 + (35\*b^4 - 120\*a\*b^2\*c + 48\*a^2\*c^2)\*f^2 + 16\*(2\*(3\*b^2\*c^2 - 4\*a\*c^3)\*d - (5\*b^3\*c - 12\*a\*b^2\*c^2)\*e)\*f)\*sqrt(-c)\*arctan(1/2\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(-c)/(c^2\*x^2 + b\*c\*x + a\*c)) - 2\*(48\*c^4\*f^2\*x^3 + 384\*c^4\*d\*e - 144\*b^3\*c^3\*e^2 - 5\*(21\*b^3\*c - 44\*a\*b^2\*c^2)\*f^2 + 8\*(16\*c^4\*e\*f - 7\*b^3\*c^3\*f^2)\*x^2 - 16\*(18\*b^3\*c\*d - (15\*b^2\*c^2 - 16\*a\*c^3)\*e)\*f + 2\*(48\*c^4\*e^2 + (35\*b^2\*c^2 - 36\*a\*c^3)\*f^2 + 16\*(6\*c^4\*d - 5\*b^3\*c^3\*e)\*f)\*x)\*sqrt(c\*x^2 + b\*x + a))/c^5]

**giac [A]** time = 0.63, size = 304, normalized size = 0.96

$$\frac{1}{192} \sqrt{cx^2 + bx + a} \left( 2 \left( 4 \left( \frac{6f^2x}{c} - \frac{7bc^2f^2 - 16c^3fe}{c^4} \right) x + \frac{96c^3df + 35b^2cf^2 - 36ac^2f^2 - 80bc^2fe + 48c^3e^2}{c^4} \right) \right) x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)^2/(c\*x^2+b\*x+a)^(1/2),x, algorithm="giac")

[Out]  $\frac{1}{192}\sqrt{c^2x^2 + bx + a} \cdot (2 \cdot (4 \cdot (6f^2x/c - (7bc^2f^2 - 16c^3fe)/c^4)x + (96c^3df + 35b^2cf^2 - 36ac^2f^2 - 80b^2cf^2e + 48c^3e^2)/c^4)x - (288b^2c^2df + 105b^3f^2 - 220abcf^2 - 384c^3de - 240b^2c^2f^2e + 256a^2c^2f^2e + 144b^2c^2e^2)/c^4) - \frac{1}{128} \cdot (128c^4d^2 + 96b^2c^2d^2f - 128a^2c^3d^2f + 35b^4f^2 - 120ab^2c^2f^2 + 48a^2c^2f^2 - 128b^2c^3de - 80b^3c^2f^2e + 192ab^2c^2f^2e + 48b^2c^2e^2 - 64a^2c^3e^2) \cdot \log(\text{abs}(-2 \cdot (\sqrt{c}x - \sqrt{c^2x^2 + bx + a}) \cdot \sqrt{c} - b)) / c^{9/2}$

**maple** [B] time = 0.01, size = 706, normalized size = 2.23

$$\frac{\sqrt{cx^2 + bx + a} f^2 x^3}{4c} - \frac{7\sqrt{cx^2 + bx + a} b f^2 x^2}{24c^2} + \frac{2\sqrt{cx^2 + bx + a} e f x^2}{3c} + \frac{3a^2 f^2 \ln\left(\frac{cx + \frac{b}{2}}{\sqrt{c}} + \sqrt{cx^2 + bx + a}\right)}{8c^{\frac{5}{2}}} - \frac{15ab^2}{8c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x^2+e\*x+d)^2/(c\*x^2+b\*x+a)^(1/2),x)

[Out]  $2de/c \cdot (cx^2 + bx + a)^{1/2} + 1/2x/c \cdot (cx^2 + bx + a)^{1/2} \cdot e^2 - 3/4b/c^2 \cdot (cx^2 + bx + a)^{1/2} \cdot e^2 + 3/8b^2/c^{5/2} \cdot \ln((cx + 1/2b)/c^{1/2} + (cx^2 + bx + a)^{1/2}) \cdot e^2 - 1/2a/c^{3/2} \cdot \ln((cx + 1/2b)/c^{1/2} + (cx^2 + bx + a)^{1/2}) \cdot e^2 - 35/64 \cdot f^2 b^3/c^4 \cdot (cx^2 + bx + a)^{1/2} + 35/128 \cdot f^2 b^4/c^{9/2} \cdot \ln((cx + 1/2b)/c^{1/2} + (cx^2 + bx + a)^{1/2}) + 3/8 \cdot f^2 a^2/c^{5/2} \cdot \ln((cx + 1/2b)/c^{1/2} + (cx^2 + bx + a)^{1/2}) - 5/6 \cdot e \cdot f \cdot b/c^2 \cdot x \cdot (cx^2 + bx + a)^{1/2} + 1/4 \cdot f^2 \cdot x^3 \cdot (cx^2 + bx + a)^{1/2}/c + 3/2 \cdot e \cdot f \cdot b/c^{5/2} \cdot a \cdot \ln((cx + 1/2b)/c^{1/2} + (cx^2 + bx + a)^{1/2}) + d^2 \cdot \ln((cx + 1/2b)/c^{1/2} + (cx^2 + bx + a)^{1/2})/c^{1/2} - 5/8 \cdot e \cdot f \cdot b^3/c^{7/2} \cdot \ln((cx + 1/2b)/c^{1/2} + (cx^2 + bx + a)^{1/2}) - 4/3 \cdot e \cdot f \cdot a/c^2 \cdot (cx^2 + bx + a)^{1/2} + x/c \cdot (cx^2 + bx + a)^{1/2} \cdot d \cdot f - 3/2 \cdot b/c^2 \cdot (cx^2 + bx + a)^{1/2} \cdot d \cdot f + 3/4 \cdot b^2/c^{5/2} \cdot \ln((cx + 1/2b)/c^{1/2} + (cx^2 + bx + a)^{1/2}) \cdot d \cdot f - a/c^{3/2} \cdot \ln((cx + 1/2b)/c^{1/2} + (cx^2 + bx + a)^{1/2}) \cdot d \cdot f - d \cdot e \cdot b/c^{3/2} \cdot \ln((cx + 1/2b)/c^{1/2} + (cx^2 + bx + a)^{1/2}) - 3/8 \cdot f^2 \cdot a/c^2 \cdot x \cdot (cx^2 + bx + a)^{1/2} + 55/48 \cdot f^2 \cdot b/c^3 \cdot a \cdot (cx^2 + bx + a)^{1/2} - 7/24 \cdot f^2 \cdot b/c^2 \cdot x^2 \cdot (cx^2 + bx + a)^{1/2} + 35/96 \cdot f^2 \cdot b^2/c^3 \cdot x \cdot (cx^2 + bx + a)^{1/2} - 15/16 \cdot f^2 \cdot b^2/c^{7/2} \cdot a \cdot \ln((cx + 1/2b)/c^{1/2} + (cx^2 + bx + a)^{1/2}) + 2/3 \cdot e \cdot f \cdot x^2/c \cdot (cx^2 + bx + a)^{1/2} + 5/4 \cdot e \cdot f \cdot b^2/c^3 \cdot (cx^2 + bx + a)^{1/2}$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)^2/(c\*x^2+b\*x+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details)Is 4\*a\*c-b^2 zero or nonzero?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(fx^2 + ex + d)^2}{\sqrt{cx^2 + bx + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x + f\*x^2)^2/(a + b\*x + c\*x^2)^(1/2),x)

[Out] int((d + e\*x + f\*x^2)^2/(a + b\*x + c\*x^2)^(1/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex + fx^2)^2}{\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x\*\*2+e\*x+d)\*\*2/(c\*x\*\*2+b\*x+a)\*\*(1/2),x)

[Out] Integral((d + e\*x + f\*x\*\*2)\*\*2/sqrt(a + b\*x + c\*x\*\*2), x)

$$3.111 \quad \int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=116

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-4c(af+be)+3b^2f+8c^2d)}{8c^{5/2}} + \frac{\sqrt{a+bx+cx^2}(4ce-3bf)}{4c^2} + \frac{fx\sqrt{a+bx+cx^2}}{2c}$$

[Out] 1/8\*(8\*c^2\*d+3\*b^2\*f-4\*c\*(a\*f+b\*e))\*arctanh(1/2\*(2\*c\*x+b)/c^(1/2)/(c\*x^2+b\*x+a)^(1/2))/c^(5/2)+1/4\*(-3\*b\*f+4\*c\*e)\*(c\*x^2+b\*x+a)^(1/2)/c^2+1/2\*f\*x\*(c\*x^2+b\*x+a)^(1/2)/c

**Rubi [A]** time = 0.11, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$ , Rules used = {1661, 640, 621, 206}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-4c(af+be)+3b^2f+8c^2d)}{8c^{5/2}} + \frac{\sqrt{a+bx+cx^2}(4ce-3bf)}{4c^2} + \frac{fx\sqrt{a+bx+cx^2}}{2c}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x + f\*x^2)/Sqrt[a + b\*x + c\*x^2], x]

[Out] ((4\*c\*e - 3\*b\*f)\*Sqrt[a + b\*x + c\*x^2])/(4\*c^2) + (f\*x\*Sqrt[a + b\*x + c\*x^2])/(2\*c) + ((8\*c^2\*d + 3\*b^2\*f - 4\*c\*(b\*e + a\*f))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(8\*c^(5/2))

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 621

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] :> Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]

&& NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

### Rule 1661

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(q + 2\*p + 1)), x] + Dist[1/(c\*(q + 2\*p + 1)), Int[(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*(q + 2\*p + 1)\*Pq - a\*e\*(q - 1)\*x^(q - 2) - b\*e\*(q + p)\*x^(q - 1) - c\*e\*(q + 2\*p + 1)\*x^q, x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

### Rubi steps

$$\begin{aligned} \int \frac{d + ex + fx^2}{\sqrt{a + bx + cx^2}} dx &= \frac{fx\sqrt{a + bx + cx^2}}{2c} + \frac{\int \frac{2cd - af + \frac{1}{2}(4ce - 3bf)x}{\sqrt{a + bx + cx^2}} dx}{2c} \\ &= \frac{(4ce - 3bf)\sqrt{a + bx + cx^2}}{4c^2} + \frac{fx\sqrt{a + bx + cx^2}}{2c} + \frac{\left(2c(2cd - af) - \frac{1}{2}b(4ce - 3bf)\right) \int \frac{1}{\sqrt{a + bx + cx^2}} dx}{4c^2} \\ &= \frac{(4ce - 3bf)\sqrt{a + bx + cx^2}}{4c^2} + \frac{fx\sqrt{a + bx + cx^2}}{2c} + \frac{\left(2c(2cd - af) - \frac{1}{2}b(4ce - 3bf)\right) \operatorname{Subst}\left(\int \frac{1}{\sqrt{u}} du, u, a + bx + cx^2\right)}{2c^2} \\ &= \frac{(4ce - 3bf)\sqrt{a + bx + cx^2}}{4c^2} + \frac{fx\sqrt{a + bx + cx^2}}{2c} + \frac{(8c^2d + 3b^2f - 4c(be + af)) \tanh^{-1}\left(\frac{2cx + b}{\sqrt{a + bx + cx^2}}\right)}{8c^{5/2}} \end{aligned}$$

**Mathematica [A]** time = 0.15, size = 96, normalized size = 0.83

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\left(-4c(af+be)+3b^2f+8c^2d\right)}{8c^{5/2}} + \frac{\sqrt{a+x(b+cx)}(-3bf+4ce+2cfx)}{4c^2}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x + f\*x^2)/Sqrt[a + b\*x + c\*x^2], x]

[Out] ((4\*c\*e - 3\*b\*f + 2\*c\*f\*x)\*Sqrt[a + x\*(b + c\*x)]/(4\*c^2) + ((8\*c^2\*d + 3\*b^2\*f - 4\*c\*(b\*e + a\*f))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + x\*(b + c\*x)])])/(8\*c^(5/2))

**fricas** [A] time = 1.45, size = 227, normalized size = 1.96

$$\left[ \frac{(8c^2d - 4bce + (3b^2 - 4ac)f)\sqrt{c} \log\left(-8c^2x^2 - 8bcx - b^2 + 4\sqrt{cx^2 + bx + a}(2cx + b)\sqrt{c} - 4ac\right) - 4(2c^2fx - 4c^2d + 4bce - (3b^2 - 4ac)f)\sqrt{c}}{16c^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(1/2),x, algorithm="fricas")

[Out] [-1/16\*((8\*c^2\*d - 4\*b\*c\*e + (3\*b^2 - 4\*a\*c)\*f)\*sqrt(c)\*log(-8\*c^2\*x^2 - 8\*b\*c\*x - b^2 + 4\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(c) - 4\*a\*c) - 4\*(2\*c^2\*f\*x + 4\*c^2\*e - 3\*b\*c\*f)\*sqrt(c\*x^2 + b\*x + a))/c^3, -1/8\*((8\*c^2\*d - 4\*b\*c\*e + (3\*b^2 - 4\*a\*c)\*f)\*sqrt(-c)\*arctan(1/2\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(-c)/(c^2\*x^2 + b\*c\*x + a\*c)) - 2\*(2\*c^2\*f\*x + 4\*c^2\*e - 3\*b\*c\*f)\*sqrt(c\*x^2 + b\*x + a))/c^3]

**giac** [A] time = 0.50, size = 98, normalized size = 0.84

$$\frac{1}{4} \sqrt{cx^2 + bx + a} \left( \frac{2fx}{c} - \frac{3bf - 4ce}{c^2} \right) - \frac{(8c^2d + 3b^2f - 4acf - 4bce) \log\left(\left| -2\left(\sqrt{c}x - \sqrt{cx^2 + bx + a}\right)\sqrt{c} - b \right|\right)}{8c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(1/2),x, algorithm="giac")

[Out] 1/4\*sqrt(c\*x^2 + b\*x + a)\*(2\*f\*x/c - (3\*b\*f - 4\*c\*e)/c^2) - 1/8\*(8\*c^2\*d + 3\*b^2\*f - 4\*a\*c\*f - 4\*b\*c\*e)\*log(abs(-2\*(sqrt(c)\*x - sqrt(c\*x^2 + b\*x + a))\*sqrt(c) - b))/c^(5/2)

**maple** [A] time = 0.01, size = 185, normalized size = 1.59

$$-\frac{af \ln\left(\frac{cx + \frac{b}{2}}{\sqrt{c}} + \sqrt{cx^2 + bx + a}\right)}{2c^{\frac{3}{2}}} + \frac{3b^2f \ln\left(\frac{cx + \frac{b}{2}}{\sqrt{c}} + \sqrt{cx^2 + bx + a}\right)}{8c^{\frac{5}{2}}} - \frac{be \ln\left(\frac{cx + \frac{b}{2}}{\sqrt{c}} + \sqrt{cx^2 + bx + a}\right)}{2c^{\frac{3}{2}}} + \frac{d \ln\left(\frac{cx + \frac{b}{2}}{\sqrt{c}} + \sqrt{cx^2 + bx + a}\right)}{2c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(1/2),x)

[Out] 1/2\*f\*x\*(c\*x^2+b\*x+a)^(1/2)/c-3/4\*f\*b/c^2\*(c\*x^2+b\*x+a)^(1/2)+3/8\*f\*b^2/c^(5/2)\*ln((c\*x+1/2\*b)/c^(1/2)+(c\*x^2+b\*x+a)^(1/2))-1/2\*f\*a/c^(3/2)\*ln((c\*x+1/2\*b)/c^(1/2)+(c\*x^2+b\*x+a)^(1/2))+e/c\*(c\*x^2+b\*x+a)^(1/2)-1/2\*e\*b/c^(3/2)\*1



$n((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+d*\ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details)Is 4\*a\*c-b^2 zero or nonzero?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{f x^2 + e x + d}{\sqrt{c x^2 + b x + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x + f\*x^2)/(a + b\*x + c\*x^2)^(1/2),x)

[Out] int((d + e\*x + f\*x^2)/(a + b\*x + c\*x^2)^(1/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + e x + f x^2}{\sqrt{a + b x + c x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x\*\*2+e\*x+d)/(c\*x\*\*2+b\*x+a)\*\*(1/2),x)

[Out] Integral((d + e\*x + f\*x\*\*2)/sqrt(a + b\*x + c\*x\*\*2), x)

$$3.112 \quad \int \frac{1}{\sqrt{a+bx+cx^2} (d+ex+fx^2)} dx$$

**Optimal.** Leaf size=374

$$\frac{\sqrt{2} f \tanh^{-1} \left( \frac{4af+2x(bf-c(\sqrt{e^2-4df}+e))-b(\sqrt{e^2-4df}+e)}{2\sqrt{2} \sqrt{a+bx+cx^2} \sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} \right)}{\sqrt{e^2-4df} \sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} - \frac{\sqrt{2} f \tanh^{-1} \left( \frac{4af+2x(bf-c(e-\sqrt{e^2-4df}))-b(e-\sqrt{e^2-4df})}{2\sqrt{2} \sqrt{a+bx+cx^2} \sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} \right)}{\sqrt{e^2-4df} \sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

[Out]  $-f \operatorname{arctanh} \left( \frac{1}{4} (4af+2x(bf-c(\sqrt{e^2-4df}+e))-b(\sqrt{e^2-4df}+e)) \right) - b(e-\sqrt{e^2-4df}) \operatorname{arctanh} \left( \frac{1}{4} (4af+2x(bf-c(e-\sqrt{e^2-4df}))-b(e-\sqrt{e^2-4df})) \right) \sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2} / \sqrt{e^2-4df} \sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2} - \left( \frac{1}{4} (4af+2x(bf-c(\sqrt{e^2-4df}+e))-b(\sqrt{e^2-4df}+e)) \right) \sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2} / \sqrt{e^2-4df} \sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2} + b(e-\sqrt{e^2-4df}) \operatorname{arctanh} \left( \frac{1}{4} (4af+2x(bf-c(e-\sqrt{e^2-4df}))-b(e-\sqrt{e^2-4df})) \right) \sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2} / \sqrt{e^2-4df} \sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}$

**Rubi [A]** time = 0.58, antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {983, 724, 206}

$$\frac{\sqrt{2} f \tanh^{-1} \left( \frac{4af+2x(bf-c(\sqrt{e^2-4df}+e))-b(\sqrt{e^2-4df}+e)}{2\sqrt{2} \sqrt{a+bx+cx^2} \sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} \right)}{\sqrt{e^2-4df} \sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} - \frac{\sqrt{2} f \tanh^{-1} \left( \frac{4af+2x(bf-c(e-\sqrt{e^2-4df}))-b(e-\sqrt{e^2-4df})}{2\sqrt{2} \sqrt{a+bx+cx^2} \sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} \right)}{\sqrt{e^2-4df} \sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + b\*x + c\*x^2]\*(d + e\*x + f\*x^2)),x]

[Out]  $-\left( \frac{\operatorname{ArcTanh} \left[ \frac{4af+2x(bf-c(\sqrt{e^2-4df}+e))-b(\sqrt{e^2-4df}+e)}{2\sqrt{2} \sqrt{a+bx+cx^2} \sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} \right] + \frac{2(bf-c(e-\sqrt{e^2-4df}))}{\sqrt{e^2-4df}} \operatorname{ArcTanh} \left[ \frac{4af+2x(bf-c(e-\sqrt{e^2-4df}))-b(e-\sqrt{e^2-4df})}{2\sqrt{2} \sqrt{a+bx+cx^2} \sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} \right]}{\sqrt{e^2-4df} \sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} - \left( \frac{\operatorname{ArcTanh} \left[ \frac{4af+2x(bf-c(\sqrt{e^2-4df}+e))-b(\sqrt{e^2-4df}+e)}{2\sqrt{2} \sqrt{a+bx+cx^2} \sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} \right] + \frac{2(bf-c(e-\sqrt{e^2-4df}))}{\sqrt{e^2-4df}} \operatorname{ArcTanh} \left[ \frac{4af+2x(bf-c(e-\sqrt{e^2-4df}))-b(e-\sqrt{e^2-4df})}{2\sqrt{2} \sqrt{a+bx+cx^2} \sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} \right]}{\sqrt{e^2-4df} \sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} + \frac{2(bf-c(e-\sqrt{e^2-4df}))}{\sqrt{e^2-4df}} \operatorname{ArcTanh} \left[ \frac{4af+2x(bf-c(e-\sqrt{e^2-4df}))-b(e-\sqrt{e^2-4df})}{2\sqrt{2} \sqrt{a+bx+cx^2} \sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} \right]}{\sqrt{e^2-4df} \sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} \right)$

**Rule 206**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && Gt

Q[a, 0] || LtQ[b, 0])

### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_.))\*Sqrt[(a\_.) + (b\_.)\*(x\_.) + (c\_.)\*(x\_.)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

### Rule 983

Int[1/(((a\_.) + (b\_.)\*(x\_.) + (c\_.)\*(x\_.)^2)\*Sqrt[(d\_.) + (e\_.)\*(x\_.) + (f\_.)\*(x\_.)^2]), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(2\*c)/q, Int[1/((b - q + 2\*c\*x)\*Sqrt[d + e\*x + f\*x^2]), x], x] - Dist[(2\*c)/q, Int[1/((b + q + 2\*c\*x)\*Sqrt[d + e\*x + f\*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && NeQ[c\*e - b\*f, 0] && PosQ[b^2 - 4\*a\*c]

### Rubi steps

$$\int \frac{1}{\sqrt{a+bx+cx^2} (d+ex+fx^2)} dx = \frac{(2f) \int \frac{1}{(e-\sqrt{e^2-4df}+2fx)\sqrt{a+bx+cx^2}} dx}{\sqrt{e^2-4df}} - \frac{(2f) \int \frac{1}{(e+\sqrt{e^2-4df}+2fx)\sqrt{a+bx+cx^2}} dx}{\sqrt{e^2-4df}}$$

$$= \frac{(4f) \text{Subst} \left( \int \frac{1}{16af^2-8bf(e-\sqrt{e^2-4df})+4c(e-\sqrt{e^2-4df})^2-x^2} dx, x, \frac{4af-b(e-\sqrt{e^2-4df})}{\sqrt{e^2-4df}} \right)}{\sqrt{e^2-4df}}$$

$$= \frac{\sqrt{2} f \tanh^{-1} \left( \frac{4af-b(e-\sqrt{e^2-4df})+2(bf-c(e-\sqrt{e^2-4df}))x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)}\sqrt{e^2-4df}\sqrt{a+bx+cx^2}} \right)}{\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)}\sqrt{e^2-4df}} + \frac{\sqrt{2} f \tanh^{-1} \left( \frac{4af-b(e+\sqrt{e^2-4df})+2(bf-c(e+\sqrt{e^2-4df}))x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)}\sqrt{e^2-4df}\sqrt{a+bx+cx^2}} \right)}{\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)}\sqrt{e^2-4df}}$$

**Mathematica [A]** time = 1.45, size = 376, normalized size = 1.01

$$\sqrt{2} f \left[ \frac{\tanh^{-1} \left( \frac{4af-b(\sqrt{e^2-4df}+e-2fx)-2cx(\sqrt{e^2-4df}+e)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f(2af-b(\sqrt{e^2-4df}+e))+c(e\sqrt{e^2-4df}-2df+e^2))}} \right)}{\sqrt{f(2af-b(\sqrt{e^2-4df}+e))+c(e\sqrt{e^2-4df}-2df+e^2)}} - \frac{\tanh^{-1} \left( \frac{4af+b(\sqrt{e^2-4df}-e+2fx)+2cx(\sqrt{e^2-4df}-e)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f(2af+b\sqrt{e^2-4df}+b(-e))+c(-e\sqrt{e^2-4df}-2df+e^2)}} \right)}{\sqrt{f(2af+b\sqrt{e^2-4df}+b(-e))+c(-e\sqrt{e^2-4df}-2df+e^2)}} \right] \frac{1}{\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[a + b\*x + c\*x^2]\*(d + e\*x + f\*x^2)),x]

[Out] (Sqrt[2]\*f\*(ArcTanh[(4\*a\*f - 2\*c\*(e + Sqrt[e^2 - 4\*d\*f])\*x - b\*(e + Sqrt[e^2 - 4\*d\*f] - 2\*f\*x))/(2\*Sqrt[2]\*Sqrt[c\*(e^2 - 2\*d\*f + e\*Sqrt[e^2 - 4\*d\*f]) + f\*(2\*a\*f - b\*(e + Sqrt[e^2 - 4\*d\*f]))])\*Sqrt[a + x\*(b + c\*x)])/Sqrt[c\*(e^2 - 2\*d\*f + e\*Sqrt[e^2 - 4\*d\*f]) + f\*(2\*a\*f - b\*(e + Sqrt[e^2 - 4\*d\*f]))] - ArcTanh[(4\*a\*f + 2\*c\*(-e + Sqrt[e^2 - 4\*d\*f])\*x + b\*(-e + Sqrt[e^2 - 4\*d\*f] + 2\*f\*x))/(2\*Sqrt[2]\*Sqrt[f\*(-(b\*e) + 2\*a\*f + b\*Sqrt[e^2 - 4\*d\*f]) + c\*(e^2 - 2\*d\*f - e\*Sqrt[e^2 - 4\*d\*f])])\*Sqrt[a + x\*(b + c\*x)])/Sqrt[f\*(-(b\*e) + 2\*a\*f + b\*Sqrt[e^2 - 4\*d\*f]) + c\*(e^2 - 2\*d\*f - e\*Sqrt[e^2 - 4\*d\*f])]))/Sqrt[e^2 - 4\*d\*f]

**fricas** [B] time = 14.92, size = 11287, normalized size = 30.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x^2+b\*x+a)^(1/2)/(f\*x^2+e\*x+d),x, algorithm="fricas")

[Out]  $\frac{1}{4} \sqrt{2} \sqrt{(c e^2 + 2 a f^2 - (2 c d + b e) f + (c^2 d^2 e^2 - b c d e^3 + a c e^4 - 4 a^2 d f^3 + (4 a b d e + a^2 e^2 - 4 (b^2 - 2 a c) d^2) f^2 - (4 c^2 d^3 - 4 b c d^2 e + a b e^3 - (b^2 - 6 a c) d e^2) f) \sqrt{(c^2 e^2 - 2 b c e f + b^2 f^2) / (c^4 d^4 e^2 - 2 b c^3 d^3 e^3 - 2 a b c^2 d e^5 + a^2 c^2 e^6 - 4 a^4 d f^5 + (b^2 c^2 + 2 a c^3) d^2 e^4 + (8 a^3 b d e + a^4 e^2 - 8 (a^2 b^2 - 2 a^3 c) d^2) f^4 - 2 (a^3 b e^3 + 2 (b^4 - 4 a b^2 c + 6 a^2 c^2) d^3 - 4 (a b^3 - a^2 b c) d^2 e + (a^2 b^2 + 6 a^3 c) d e^2) f^3 - (8 (b^2 c^2 - 2 a c^3) d^4 - 8 (b^3 c - a b c^2) d^3 e - (b^4 - 20 a b^2 c + 22 a^2 c^2) d^2 e^2 + 2 (a b^3 - 5 a^2 b c) d e^3 - (a^2 b^2 + 2 a^3 c) e^4) f^2 - 2 (2 c^4 d^5 - 4 b c^3 d^4 e + a^2 b c e^5 + (b^2 c^2 + 6 a c^3) d^3 e^2 + (b^3 c - 5 a b c^2) d^2 e^3 - 2 (a b^2 c - 2 a^2 c^2) d e^4) f) / (c^2 d^2 e^2 - b c d e^3 + a c e^4 - 4 a^2 d f^3 + (4 a b d e + a^2 e^2 - 4 (b^2 - 2 a c) d^2) f^2 - (4 c^2 d^3 - 4 b c d^2 e + a b e^3 - (b^2 - 6 a c) d e^2) f) \log((2 (b^2 d - a b e) f^2 + \sqrt{2} (c^2 d e^3 - 4 a b d f^3 + (4 b c d^2 + 4 a c d e + a b e^2) f^2 - (4 c^2 d^2 e + b c d e^2 + a c e^3) f - (c^3 d^3 e^3 - b c^2 d^2 e^4 + a c^2 d e^5 + 4 (2 a^2 b d^2 - a^3 d e) f^4 + (2 a^2 b d e^2 + a^3 e^3 + 8 (b^3 - 2 a b c) d^3 - 4 (3 a b^2 - a^2 c) d^2 e) f^3 + (8 b c^2 d^4 - a^2 b e^4 - 4 (3 b^2 c - a c^2) d^3 e - 2 (b^3 - 10 a b c) d^2 e^2 + (3 a b^2 - 5 a^2 c) d e^3) f^2 - (4 c^3 d^4 e - 2 b c^2 d^3 e^2 + 4 a b c d e^4 - a^2 c e^5 - (3 b^2 c - 5 a c^2) d^2 e^3) f) \sqrt{(c^2 e^2 - 2 b c e f + b^2 f^2) / (c^4 d^4 e^2 - 2 b c^3 d^3 e^3 - 2 a b c^2 d e^5 + a^2 c^2 e^6 - 4 a^4 d f^5 + (b^2 c^2 + 2 a c^3) d^2 e^4 + (8 a^3 b d e + a^4 e^2 - 8 (a^2 b^2 - 2 a^3 c) d^2) f^4 - 2 (a^3 b e^3 + 2 (b^4 - 4 a b^2 c + 6 a^2 c^2) d^3 - 4 (a b^3 - a^2 b c) d^2 e + (a^2 b^2 + 6 a^3 c) d e^2) f^3 - (8 (b^2 c^2 - 2 a c^3) d^4 - 8 (b^3 c - a b c^2) d^3 e - (b^4 - 20 a b^2 c + 22 a^2 c^2) d^2 e^2 + 2 (a b^3 - 5 a^2 b c) d e^3 - (a^2 b^2 + 2 a^3 c) e^4) f^2 - 2 (2 c^4 d^5 - 4 b c^3 d^4 e + a^2 b c e^5 + (b^2 c^2 + 6 a c^3) d^3 e^2 + (b^3 c - 5 a b c^2) d^2 e^3 - 2 (a b^2 c - 2 a^2 c^2) d e^4) f)}}$

$$\begin{aligned}
&^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c*x^2 + b*x + a)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d + b*e)*f + (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)) - 2*(b*c*d*e - a*c*e^2)*f + ((4*b*c*d - b^2*e)*f^2 - (4*c^2*d*e - b*c*e^2)*f)*x - (8*a^3*d*f^4 - 2*(4*a^2*b*d*e + a^3*e^2 - 4*(a*b^2 - 2*a^2*c)*d^2)*f^3 + 2*(4*a*c^2*d^3 - 4*a*b*c*d^2*e + a^2*b*e^3 - (a*b^2 - 6*a^2*c)*d*e^2)*f^2 - 2*(a*c^2*d^2*e^2 - a*b*c*d*e^3 + a^2*c*e^4)*f + (4*a^2*b*d*f^4 - (4*a*b^2*d*e + a^2*b*e^2 - 4*(b^3 - 2*a*b*c)*d^2)*f^3 + (4*b*c^2*d^3 - 4*b^2*c*d^2*e + a*b^2*e^3 - (b^3 - 6*a*b*c)*d*e^2)*f^2 - (b*c^2*d^2*e^2 - b^2*c*d*e^3 + a*b*c*e^4)*f)*x)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/x) - 1/4*sqrt(2)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d + b*e)*f + (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*
\end{aligned}$$

$$\begin{aligned}
& c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)) * \log((2*(b^2*d - \\
& a*b*e)*f^2 - \sqrt{2}*(c^2*d*e^3 - 4*a*b*d*f^3 + (4*b*c*d^2 + 4*a*c*d*e + a \\
& b*e^2)*f^2 - (4*c^2*d^2*e + b*c*d*e^2 + a*c*e^3)*f - (c^3*d^3*e^3 - b*c^2*d \\
& ^2*e^4 + a*c^2*d*e^5 + 4*(2*a^2*b*d^2 - a^3*d*e)*f^4 + (2*a^2*b*d*e^2 + a^3 \\
& *e^3 + 8*(b^3 - 2*a*b*c)*d^3 - 4*(3*a*b^2 - a^2*c)*d^2*e)*f^3 + (8*b*c^2*d^ \\
& 4 - a^2*b*e^4 - 4*(3*b^2*c - a*c^2)*d^3*e - 2*(b^3 - 10*a*b*c)*d^2*e^2 + (3 \\
& *a*b^2 - 5*a^2*c)*d*e^3)*f^2 - (4*c^3*d^4*e - 2*b*c^2*d^3*e^2 + 4*a*b*c*d*e \\
& ^4 - a^2*c*e^5 - (3*b^2*c - 5*a*c^2)*d^2*e^3)*f) * \sqrt{(c^2*e^2 - 2*b*c*e*f \\
& + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - \\
& 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^ \\
& 2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)* \\
& d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c \\
& ^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2 \\
& *c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 \\
& - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 \\
& + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)) * \sqrt{c*x \\
& ^2 + b*x + a} * \sqrt{(c*e^2 + 2*a*f^2 - (2*c*d + b*e)*f + (c^2*d^2*e^2 - b*c \\
& *d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2 \\
& )*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f) * \sqrt{(( \\
& c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d \\
& *e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d \\
& *e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a \\
& *b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d \\
& *e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - \\
& 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 \\
& + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 \\
& + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2) \\
& *d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e \\
& + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - \\
& (b^2 - 6*a*c)*d*e^2)*f) - 2*(b*c*d*e - a*c*e^2)*f + ((4*b*c*d - b^2*e)*f^ \\
& 2 - (4*c^2*d*e - b*c*e^2)*f)*x - (8*a^3*d*f^4 - 2*(4*a^2*b*d*e + a^3*e^2 - \\
& 4*(a*b^2 - 2*a^2*c)*d^2)*f^3 + 2*(4*a*c^2*d^3 - 4*a*b*c*d^2*e + a^2*b*e^3 - \\
& (a*b^2 - 6*a^2*c)*d*e^2)*f^2 - 2*(a*c^2*d^2*e^2 - a*b*c*d*e^3 + a^2*c*e^4) \\
& *f + (4*a^2*b*d*f^4 - (4*a*b^2*d*e + a^2*b*e^2 - 4*(b^3 - 2*a*b*c)*d^2)*f^3 \\
& + (4*b*c^2*d^3 - 4*b^2*c*d^2*e + a*b^2*e^3 - (b^3 - 6*a*b*c)*d*e^2)*f^2 - \\
& (b*c^2*d^2*e^2 - b^2*c*d*e^3 + a*b*c*e^4)*f)*x) * \sqrt{(c^2*e^2 - 2*b*c*e*f + \\
& b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - \\
& 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2 \\
& *b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d \\
& ^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c \\
& ^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2* \\
& c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - \\
& 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + \\
& (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/x) + 1/4 \\
& * \sqrt{2} * \sqrt{(c*e^2 + 2*a*f^2 - (2*c*d + b*e)*f - (c^2*d^2*e^2 - b*c*d*e^3
\end{aligned}$$

$$\begin{aligned}
& + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 \\
& - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*\sqrt{(c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + \\
& a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c \\
& + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)* \\
& f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a* \\
& b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a \\
& *c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4 \\
& )*f))/((c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2* \\
& e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 \\
& - 6*a*c)*d*e^2)*f))*\log((2*(b^2*d - a*b*e)*f^2 + \sqrt{2}*(c^2*d*e^3 - 4*a*b \\
& *d*f^3 + (4*b*c*d^2 + 4*a*c*d*e + a*b*e^2)*f^2 - (4*c^2*d^2*e + b*c*d*e^2 + \\
& a*c*e^3)*f + (c^3*d^3*e^3 - b*c^2*d^2*e^4 + a*c^2*d*e^5 + 4*(2*a^2*b*d^2 - \\
& a^3*d*e)*f^4 + (2*a^2*b*d*e^2 + a^3*e^3 + 8*(b^3 - 2*a*b*c)*d^3 - 4*(3*a*b \\
& ^2 - a^2*c)*d^2*e)*f^3 + (8*b*c^2*d^4 - a^2*b*e^4 - 4*(3*b^2*c - a*c^2)*d^3 \\
& *e - 2*(b^3 - 10*a*b*c)*d^2*e^2 + (3*a*b^2 - 5*a^2*c)*d*e^3)*f^2 - (4*c^3*d \\
& ^4*e - 2*b*c^2*d^3*e^2 + 4*a*b*c*d*e^4 - a^2*c*e^5 - (3*b^2*c - 5*a*c^2)*d^ \\
& 2*e^3)*f)*\sqrt{(c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e \\
& ^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2* \\
& e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 \\
& + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2* \\
& b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2 \\
& )*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d \\
& *e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b* \\
& c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^ \\
& 2*c - 2*a^2*c^2)*d*e^4)*f))*\sqrt{c*x^2 + b*x + a)*\sqrt{(c*e^2 + 2*a*f^2 - \\
& (2*c*d + b*e)*f - (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b \\
& *d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b* \\
& e^3 - (b^2 - 6*a*c)*d*e^2)*f)*\sqrt{(c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4 \\
& *e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2 \\
& *c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^ \\
& 2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^ \\
& 2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 \\
& - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*( \\
& a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4* \\
& b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^ \\
& 2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f))/((c^2*d^2*e^2 - b*c*d*e^3 + \\
& a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - \\
& (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)) - 2*(b*c*d*e \\
& - a*c*e^2)*f + ((4*b*c*d - b^2*e)*f^2 - (4*c^2*d*e - b*c*e^2)*f)*x + (8*a^3 \\
& *d*f^4 - 2*(4*a^2*b*d*e + a^3*e^2 - 4*(a*b^2 - 2*a^2*c)*d^2)*f^3 + 2*(4*a*c \\
& ^2*d^3 - 4*a*b*c*d^2*e + a^2*b*e^3 - (a*b^2 - 6*a^2*c)*d*e^2)*f^2 - 2*(a*c^ \\
& 2*d^2*e^2 - a*b*c*d*e^3 + a^2*c*e^4)*f + (4*a^2*b*d*f^4 - (4*a*b^2*d*e + a^
\end{aligned}$$





$$\frac{2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f) - 2*(b*c*d*e - a*c*e^2)*f + ((4*b*c*d - b^2*e)*f^2 - (4*c^2*d*e - b*c*e^2)*f)*x + (8*a^3*d*f^4 - 2*(4*a^2*b*d*e + a^3*e^2 - 4*(a*b^2 - 2*a^2*c)*d^2)*f^3 + 2*(4*a*c^2*d^3 - 4*a*b*c*d^2*e + a^2*b*e^3 - (a*b^2 - 6*a^2*c)*d*e^2)*f^2 - 2*(a*c^2*d^2*e^2 - a*b*c*d*e^3 + a^2*c*e^4)*f + (4*a^2*b*d*f^4 - (4*a*b^2*d*e + a^2*b*e^2 - 4*(b^3 - 2*a*b*c)*d^2)*f^3 + (4*b*c^2*d^3 - 4*b^2*c*d^2*e + a*b^2*e^3 - (b^3 - 6*a*b*c)*d*e^2)*f^2 - (b*c^2*d^2*e^2 - b^2*c*d*e^3 + a*b*c*e^4)*f)*x)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2))/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/x)$$

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x^2+b\*x+a)^(1/2)/(f\*x^2+e\*x+d),x, algorithm="giac")

[Out] Timed out

**maple** [B] time = 0.02, size = 761, normalized size = 2.03

$$\sqrt{2} \ln \left( \frac{\left( \frac{bf-ce-\sqrt{-4df+e^2}}{f} c \right) \left( x + \frac{e+\sqrt{-4df+e^2}}{2f} \right) + \frac{2af^2-bef-2cdf+ce^2-\sqrt{-4df+e^2}bf+\sqrt{-4df+e^2}ce}{f^2}}{\frac{2af^2-bef-2cdf+ce^2-\sqrt{-4df+e^2}bf+\sqrt{-4df+e^2}ce}{f^2}} + \frac{\sqrt{2} \sqrt{\frac{2af^2-bef-2cdf+ce^2-\sqrt{-4df+e^2}bf+\sqrt{-4df+e^2}ce}{f^2}}}{x + \frac{e+\sqrt{-4df+e^2}}{2f}} \right) \sqrt{-4df+e^2} \sqrt{\frac{2af^2-bef-2cdf+ce^2-\sqrt{-4df+e^2}bf+\sqrt{-4df+e^2}ce}{f^2}}$$



sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + bx + cx^2} (d + ex + fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x\*\*2+b\*x+a)\*\*(1/2)/(f\*x\*\*2+e\*x+d), x)

[Out] Integral(1/(sqrt(a + b\*x + c\*x\*\*2)\*(d + e\*x + f\*x\*\*2)), x)

$$3.113 \quad \int \frac{1}{\sqrt{a+bx+cx^2} (d+ex+fx^2)^2} dx$$

**Optimal.** Leaf size=789

$$\frac{(f(e - \sqrt{e^2 - 4df})(ce - bf)(2af - be + 2cd) - 2f(f(-4a^2f^2 + 3abef + b^2(e^2 - 6df))) - c(4af(e^2 - 3df) + b^2(e^2 - 4df)))}{2\sqrt{2}(e^2 - 4df)^{3/2}((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{2af^2 - \sqrt{e^2 - 4df}}}$$

[Out] (f\*(-a\*e\*f-2\*b\*d\*f+b\*e^2)-c\*(-3\*d\*e\*f+e^3)+f\*(f\*(-2\*a\*f+b\*e)-c\*(-2\*d\*f+e^2))\*x)\*(c\*x^2+b\*x+a)^(1/2)/(-4\*d\*f+e^2)/((-a\*f+c\*d)^2-(-a\*e+b\*d)\*(-b\*f+c\*e))/(f\*x^2+e\*x+d)+1/4\*arctanh(1/4\*(4\*a\*f+2\*x\*(b\*f-c\*(e-(-4\*d\*f+e^2)^(1/2))))-b\*(e-(-4\*d\*f+e^2)^(1/2)))\*2^(1/2)/(c\*x^2+b\*x+a)^(1/2)/(c\*e^2-2\*c\*d\*f-b\*e\*f+2\*a\*f^2-(-b\*f+c\*e)\*(-4\*d\*f+e^2)^(1/2))^(1/2))\*(-2\*f\*(2\*c^2\*d\*(-4\*d\*f+e^2)+f\*(3\*a\*b\*e\*f-4\*a^2\*f^2+b^2\*(-6\*d\*f+e^2))-c\*(4\*a\*f\*(-3\*d\*f+e^2)+b\*(-5\*d\*e\*f+e^3)))+f\*(2\*a\*f-b\*e+2\*c\*d)\*(-b\*f+c\*e)\*(e-(-4\*d\*f+e^2)^(1/2)))/(-4\*d\*f+e^2)^(3/2)/((-a\*f+c\*d)^2-(-a\*e+b\*d)\*(-b\*f+c\*e))\*2^(1/2)/(c\*e^2-2\*c\*d\*f-b\*e\*f+2\*a\*f^2-(-b\*f+c\*e)\*(-4\*d\*f+e^2)^(1/2))^(1/2)-1/4\*arctanh(1/4\*(4\*a\*f-b\*(e+(-4\*d\*f+e^2)^(1/2))+2\*x\*(b\*f-c\*(e+(-4\*d\*f+e^2)^(1/2)))))\*2^(1/2)/(c\*x^2+b\*x+a)^(1/2)/(c\*e^2-2\*c\*d\*f-b\*e\*f+2\*a\*f^2+(-b\*f+c\*e)\*(-4\*d\*f+e^2)^(1/2))^(1/2))\*(-2\*f\*(2\*c^2\*d\*(-4\*d\*f+e^2)+f\*(3\*a\*b\*e\*f-4\*a^2\*f^2+b^2\*(-6\*d\*f+e^2))-c\*(4\*a\*f\*(-3\*d\*f+e^2)+b\*(-5\*d\*e\*f+e^3)))+f\*(2\*a\*f-b\*e+2\*c\*d)\*(-b\*f+c\*e)\*(e+(-4\*d\*f+e^2)^(1/2)))/(-4\*d\*f+e^2)^(3/2)/((-a\*f+c\*d)^2-(-a\*e+b\*d)\*(-b\*f+c\*e))\*2^(1/2)/(c\*e^2-2\*c\*d\*f-b\*e\*f+2\*a\*f^2+(-b\*f+c\*e)\*(-4\*d\*f+e^2)^(1/2))^(1/2)

**Rubi [A]** time = 8.21, antiderivative size = 787, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$ , Rules used = {974, 1032, 724, 206}

$$\frac{(f(e - \sqrt{e^2 - 4df})(ce - bf)(2af - be + 2cd) - 2f(-4a^2f^3 + 3abef^2 - 4acf(e^2 - 3df) + b^2f(e^2 - 6df) - bc(e^3 - 3dfe^2)))}{2\sqrt{2}(e^2 - 4df)^{3/2}((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{2af^2 - \sqrt{e^2 - 4df}}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + b\*x + c\*x^2]\*(d + e\*x + f\*x^2)^2), x]

[Out] ((f\*(b\*e^2 - 2\*b\*d\*f - a\*e\*f) - c\*(e^3 - 3\*d\*e\*f) + f\*(f\*(b\*e - 2\*a\*f) - c\*(e^2 - 2\*d\*f))\*x)\*Sqrt[a + b\*x + c\*x^2])/((e^2 - 4\*d\*f)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(d + e\*x + f\*x^2)) + ((f\*(2\*c\*d - b\*e + 2\*a\*f)\*(c\*e - b\*f)\*(e - Sqrt[e^2 - 4\*d\*f]) - 2\*f\*(3\*a\*b\*e\*f^2 - 4\*a^2\*f^3 + b^2\*f\*(e^2 - 6\*d\*f) + 2\*c^2\*d\*(e^2 - 4\*d\*f) - 4\*a\*c\*f\*(e^2 - 3\*d\*f) - b\*c\*(e^3 - 5\*d\*e

$$\begin{aligned} & f)) * \text{ArcTanh}[(4*a*f - b*(e - \sqrt{e^2 - 4*d*f}) + 2*(b*f - c*(e - \sqrt{e^2 - 4*d*f}))) * x] / (2*\sqrt{2}*\sqrt{c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*\sqrt{e^2 - 4*d*f}}*\sqrt{a + b*x + c*x^2})) / (2*\sqrt{2}*(e^2 - 4*d*f)^{(3/2)}*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*\sqrt{c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*\sqrt{e^2 - 4*d*f}}) - ((f*(2*c*d - b*e + 2*a*f)*(c*e - b*f)*(e + \sqrt{e^2 - 4*d*f}) - 2*f*(3*a*b*e*f^2 - 4*a^2*f^3 + b^2*f*(e^2 - 6*d*f) + 2*c^2*d*(e^2 - 4*d*f) - 4*a*c*f*(e^2 - 3*d*f) - b*c*(e^3 - 5*d*e*f))) * \text{ArcTanh}[(4*a*f - b*(e + \sqrt{e^2 - 4*d*f}) + 2*(b*f - c*(e + \sqrt{e^2 - 4*d*f}))) * x] / (2*\sqrt{2}*\sqrt{c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*\sqrt{e^2 - 4*d*f}}*\sqrt{a + b*x + c*x^2})) / (2*\sqrt{2}*(e^2 - 4*d*f)^{(3/2)}*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*\sqrt{c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*\sqrt{e^2 - 4*d*f}}) \end{aligned}$$

### Rule 206

$$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x\_Symbol] := \text{Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x] / \text{Rt}[a, 2]) / (\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$$

### Rule 724

$$\text{Int}[1/(((d_.) + (e_)*(x_))*\sqrt{(a_.) + (b_)*(x_) + (c_)*(x_)^2}), x\_Symbol] := \text{Dist}[-2, \text{Subst}[\text{Int}[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/\sqrt{a + b*x + c*x^2}], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0]$$

### Rule 974

$$\begin{aligned} & \text{Int}[(a_ + (b_)*(x_) + (c_)*(x_)^2)^{(p_)}*((d_.) + (e_)*(x_) + (f_)*(x_)^2)^{(q_)}, x\_Symbol] := \text{Simp}[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x]*(a + b*x + c*x^2)^{(p+1)}*(d + e*x + f*x^2)^{(q+1)} / ((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))^{(p+1)}), x] - \text{Dist}[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))^{(p+1)}), \text{Int}[(a + b*x + c*x^2)^{(p+1)}*(d + e*x + f*x^2)^q * \text{Simp}[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))^{(p+1)} - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(a*f*(p+1) - c*d*(p+2)) - e*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f))^{(p+q+2)} + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f))^{(p+q+2)} - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*(p+1) - c*e*(2*p+q+4))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))^{(2*p+2*q+5)*x^2}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, q\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[e^2 - 4*d*f, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{NeQ}[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] \ \&\& \ !( \ !\text{IntegerQ}[p] \ \&\& \ \text{ILtQ}[q, -1]) \ \&\& \ !\text{IGtQ}[q, 0] \end{aligned}$$

### Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

### Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{a+bx+cx^2} (d+ex+fx^2)^2} dx &= \frac{(f(be^2 - 2bdf - aef) - c(e^3 - 3def) + f(f(be - 2af) - c(e^2 - 2df))x)}{(e^2 - 4df)((cd - af)^2 - (bd - ae)(ce - bf))(d + ex + fx^2)} \\ &= \frac{(f(be^2 - 2bdf - aef) - c(e^3 - 3def) + f(f(be - 2af) - c(e^2 - 2df))x)}{(e^2 - 4df)((cd - af)^2 - (bd - ae)(ce - bf))(d + ex + fx^2)} \\ &= \frac{(f(be^2 - 2bdf - aef) - c(e^3 - 3def) + f(f(be - 2af) - c(e^2 - 2df))x)}{(e^2 - 4df)((cd - af)^2 - (bd - ae)(ce - bf))(d + ex + fx^2)} \\ &= \frac{(f(be^2 - 2bdf - aef) - c(e^3 - 3def) + f(f(be - 2af) - c(e^2 - 2df))x)}{(e^2 - 4df)((cd - af)^2 - (bd - ae)(ce - bf))(d + ex + fx^2)} \end{aligned}$$

**Mathematica [A]** time = 6.69, size = 1377, normalized size = 1.75

$$\frac{8(cx^2 + bx + a)f^3}{(e^2 - 4df)\left(4af^2 - 2b(e - \sqrt{e^2 - 4df})f + c(e - \sqrt{e^2 - 4df})^2\right)(e + 2fx - \sqrt{e^2 - 4df})\sqrt{a + x(b + cx)}} \quad (e^2 - 4df)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)^2),x]
```

```
[Out] (-8*f^3*(a + b*x + c*x^2))/((e^2 - 4*d*f)*(4*a*f^2 - 2*b*f*(e - Sqrt[e^2 - 4*d*f]) + c*(e - Sqrt[e^2 - 4*d*f])^2)*(e - Sqrt[e^2 - 4*d*f] + 2*f*x)*Sqrt[a + x*(b + c*x)]) - (8*f^3*(a + b*x + c*x^2))/((e^2 - 4*d*f)*(4*a*f^2 - 2*b*f*(e + Sqrt[e^2 - 4*d*f]) + c*(e + Sqrt[e^2 - 4*d*f])^2)*(e + Sqrt[e^2 - 4*d*f] + 2*f*x)*Sqrt[a + x*(b + c*x)])
```

$$\begin{aligned}
& 4*d*f] + 2*f*x)*\text{Sqrt}[a + x*(b + c*x))] + (2*\text{Sqrt}[2]*f^2*\text{Sqrt}[a + b*x + c*x^2] \\
& *\text{ArcTanh}[(4*a*f - b*(e - \text{Sqrt}[e^2 - 4*d*f]) + 2*(b*f - c*(e - \text{Sqrt}[e^2 - 4*d*f]))*x) / (2*\text{Sqrt}[2]*\text{Sqrt}[c*(e^2 - 2*d*f - e*\text{Sqrt}[e^2 - 4*d*f]) + f*(2*a*f - b*(e - \text{Sqrt}[e^2 - 4*d*f]))]*\text{Sqrt}[a + b*x + c*x^2])]) / ((e^2 - 4*d*f)^(3/2) * \text{Sqrt}[c*(e^2 - 2*d*f - e*\text{Sqrt}[e^2 - 4*d*f]) + f*(2*a*f - b*(e - \text{Sqrt}[e^2 - 4*d*f]))]*\text{Sqrt}[a + x*(b + c*x)]) - (8*\text{Sqrt}[2]*f^2*\text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*\text{Sqrt}[e^2 - 4*d*f] + b*f*\text{Sqrt}[e^2 - 4*d*f]]*(2*b*f + 2*c*(-e + \text{Sqrt}[e^2 - 4*d*f]))*\text{Sqrt}[a + b*x + c*x^2]*\text{ArcTanh}[(-4*a*f - b*(-e + \text{Sqrt}[e^2 - 4*d*f]) - (2*b*f + 2*c*(-e + \text{Sqrt}[e^2 - 4*d*f]))*x) / (2*\text{Sqrt}[2]*\text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*\text{Sqrt}[e^2 - 4*d*f] + b*f*\text{Sqrt}[e^2 - 4*d*f]]*\text{Sqrt}[a + b*x + c*x^2])]) / ((e^2 - 4*d*f)*(4*a*f^2 + 2*b*f*(-e + \text{Sqrt}[e^2 - 4*d*f]) + c*(-e + \text{Sqrt}[e^2 - 4*d*f])^2)*(16*a*f^2 + 8*b*f*(-e + \text{Sqrt}[e^2 - 4*d*f]) + 4*c*(-e + \text{Sqrt}[e^2 - 4*d*f])^2)*\text{Sqrt}[a + x*(b + c*x)]) - (2*\text{Sqrt}[2]*f^2*\text{Sqrt}[a + b*x + c*x^2]*\text{ArcTanh}[(4*a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f]) + 2*(b*f - c*(e + \text{Sqrt}[e^2 - 4*d*f]))*x) / (2*\text{Sqrt}[2]*\text{Sqrt}[c*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]) + f*(2*a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f]))]*\text{Sqrt}[a + b*x + c*x^2])]) / ((e^2 - 4*d*f)^(3/2) * \text{Sqrt}[c*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]) + f*(2*a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f]))]*\text{Sqrt}[a + x*(b + c*x)]) - (8*\text{Sqrt}[2]*f^2*\text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*\text{Sqrt}[e^2 - 4*d*f] - b*f*\text{Sqrt}[e^2 - 4*d*f]]*(-2*b*f + 2*c*(e + \text{Sqrt}[e^2 - 4*d*f]))*\text{Sqrt}[a + b*x + c*x^2]*\text{ArcTanh}[(4*a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f]) - (-2*b*f + 2*c*(e + \text{Sqrt}[e^2 - 4*d*f]))*x) / (2*\text{Sqrt}[2]*\text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*\text{Sqrt}[e^2 - 4*d*f] - b*f*\text{Sqrt}[e^2 - 4*d*f]]*\text{Sqrt}[a + b*x + c*x^2])]) / ((e^2 - 4*d*f)*(4*a*f^2 - 2*b*f*(e + \text{Sqrt}[e^2 - 4*d*f]) + c*(e + \text{Sqrt}[e^2 - 4*d*f])^2)*(16*a*f^2 - 8*b*f*(e + \text{Sqrt}[e^2 - 4*d*f]) + 4*c*(e + \text{Sqrt}[e^2 - 4*d*f])^2)*\text{Sqrt}[a + x*(b + c*x)])
\end{aligned}$$

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x^2+b\*x+a)^(1/2)/(f\*x^2+e\*x+d)^2,x, algorithm="fricas")

[Out] Timed out

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x^2+b\*x+a)^(1/2)/(f\*x^2+e\*x+d)^2,x, algorithm="giac")

[Out] Timed out





$$\begin{aligned}
& e^{-2})^{(1/2)})/f)^2*c+4*(b*f-c*e+(-4*d*f+e^2)^{(1/2)}*c)*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)})/(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f))*c*e+2/(4*d*f-e^2)/(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e)*f^2/(x+1/2/f*(-4*d*f+e^2)^{(1/2)}+1/2*e/f)*((x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)^2*c+(b*f-c*e-(-4*d*f+e^2)^{(1/2)}*c)*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}+1/(4*d*f-e^2)*f/(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e)*2^{(1/2)})/((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*ln(((b*f-c*e-(-4*d*f+e^2)^{(1/2)}*c)*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2+1/2*2^{(1/2)}*((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*(4*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)^2*c+4*(b*f-c*e-(-4*d*f+e^2)^{(1/2)}*c)*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)})/(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f))*c*(-4*d*f+e^2)^{(1/2)}-1/(4*d*f-e^2)*f^2/(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e)*2^{(1/2)})/((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*ln(((b*f-c*e-(-4*d*f+e^2)^{(1/2)}*c)*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2+1/2*2^{(1/2)}*((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*(4*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)^2*c+4*(b*f-c*e-(-4*d*f+e^2)^{(1/2)}*c)*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)})/(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f))*b+1/(4*d*f-e^2)*f/(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e)*2^{(1/2)})/((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*ln(((b*f-c*e-(-4*d*f+e^2)^{(1/2)}*c)*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2+1/2*2^{(1/2)}*((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*(4*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)^2*c+4*(b*f-c*e-(-4*d*f+e^2)^{(1/2)}*c)*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)})/(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f))*c*e-2*f/(4*d*f-e^2)/(-4*d*f+e^2)^{(1/2)}*2^{(1/2)})/((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*ln(((b*f-c*e+(-4*d*f+e^2)^{(1/2)}*c)*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2+1/2*2^{(1/2)}*((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*(4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)^2*c+4*(b*f-c*e+(-4*d*f+e^2)^{(1/2)}*c)*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)})/(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f))
\end{aligned}$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + bx + a} (fx^2 + ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x^2+b\*x+a)^(1/2)/(f\*x^2+e\*x+d)^2,x, algorithm="maxima")

[Out] integrate(1/(sqrt(c\*x^2 + b\*x + a)\*(f\*x^2 + e\*x + d)^2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{cx^2 + bx + a} (fx^2 + ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x + c\*x^2)^(1/2)\*(d + e\*x + f\*x^2)^2),x)

[Out] int(1/((a + b\*x + c\*x^2)^(1/2)\*(d + e\*x + f\*x^2)^2), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x\*\*2+b\*x+a)\*\*(1/2)/(f\*x\*\*2+e\*x+d)\*\*2,x)

[Out] Timed out

$$3.114 \quad \int \frac{(d+ex+fx^2)^3}{(a+bx+cx^2)^{3/2}} dx$$

**Optimal.** Leaf size=649

$$\frac{3 \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) \left(80c^2f(a^2f^2 + 6abef + 3b^2(df + e^2)) - 280b^2cf^2(af + be) - 64c^3(3af(df + e^2) + b^2e^2)\right)}{128c^{11/2}}$$

[Out]  $3/128*(105*b^4*f^3-280*b^2*c*f^2*(a*f+b*e)+128*c^4*d*(d*f+e^2)+80*c^2*f*(6*a*b*e*f+a^2*f^2+3*b^2*(d*f+e^2))-64*c^3*(3*a*f*(d*f+e^2)+b*(6*d*e*f+e^3)))*\arctanh(1/2*(2*c*x+b)/c^{(1/2)}/(c*x^2+b*x+a)^{(1/2)})/c^{(11/2)}+2*(3*a*b^4*c*e*f^2-a*b^5*f^3+a*b^3*c*f*(5*a*f^2-3*c*(d*f+e^2))-b*c^2*(c^3*d^3+5*a^3*f^3+3*a*c^2*d*(d*f+e^2)-9*a^2*c*f*(d*f+e^2))-a*b^2*c^2*e*(12*a*f^2-c*(6*d*f+e^2))+2*a*c^3*e*(3*c^2*d^2+3*a^2*f^2-a*c*(6*d*f+e^2))-(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*(a^2*c^2*f^2-4*a*b^2*c*f^2+7*a*b*c^2*e*f-2*a*c^3*d*f-3*a*c^3*e^2+b^4*f^2-2*b^3*c*e*f+b^2*c^2*d*f+b^2*c^2*e^2-b*c^3*d*e+c^4*d^2)*x)/c^5/(-4*a*c+b^2)/(c*x^2+b*x+a)^{(1/2)}-1/64*(187*b^3*f^3-4*b*c*f^2*(73*a*f+114*b*e)-64*c^3*(6*d*e*f+e^3)+16*c^2*f*(20*a*e*f+21*b*(d*f+e^2)))*(c*x^2+b*x+a)^{(1/2)}/c^5+1/32*f*(41*b^2*f^2-4*c*f*(7*a*f+22*b*e)+48*c^2*(d*f+e^2))*x*(c*x^2+b*x+a)^{(1/2)}/c^4+1/8*f^2*(-5*b*f+8*c*e)*x^2*(c*x^2+b*x+a)^{(1/2)}/c^3+1/4*f^3*x^3*(c*x^2+b*x+a)^{(1/2)}/c^2$

**Rubi [A]** time = 2.11, antiderivative size = 649, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {1660, 1661, 640, 621, 206}

$$2(-x(-2acf + b^2f - bce + 2c^2d)(a^2c^2f^2 - 4ab^2cf^2 + 7abc^2ef - 2ac^3df - 3ac^3e^2 + b^2c^2df + b^2c^2e^2 - 2b^3cef -$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x + f\*x^2)^3/(a + b\*x + c\*x^2)^(3/2), x]

[Out]  $(2*(3*a*b^4*c*e*f^2 - a*b^5*f^3 + a*b^3*c*f*(5*a*f^2 - 3*c*(e^2 + d*f)) - b*c^2*(c^3*d^3 + 5*a^3*f^3 + 3*a*c^2*d*(e^2 + d*f) - 9*a^2*c*f*(e^2 + d*f)) - a*b^2*c^2*e*(12*a*f^2 - c*(e^2 + 6*d*f)) + 2*a*c^3*e*(3*c^2*d^2 + 3*a^2*f^2 - a*c*(e^2 + 6*d*f)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*(c^4*d^2 - b*c^3*d*e + b^2*c^2*e^2 - 3*a*c^3*e^2 + b^2*c^2*d*f - 2*a*c^3*d*f - 2*b^3*c*e*f + 7*a*b*c^2*e*f + b^4*f^2 - 4*a*b^2*c*f^2 + a^2*c^2*f^2)*x)/c^5*(b^2 - 4*a*c)*\text{Sqrt}[a + b*x + c*x^2] - ((187*b^3*f^3 - 4*b*c*f^2*(114*b*e + 73*a*f) - 64*c^3*(e^3 + 6*d*e*f) + 16*c^2*f*(20*a*e*f + 21*b*(e^2 + d*f)))*\text{Sqrt}[$

$$\frac{a + b*x + c*x^2}{(64*c^5) + (f*(41*b^2*f^2 - 4*c*f*(22*b*e + 7*a*f) + 48*c^2*(e^2 + d*f))*x*\sqrt{a + b*x + c*x^2})/(32*c^4) + (f^2*(8*c*e - 5*b*f))*x^2*\sqrt{a + b*x + c*x^2})/(8*c^3) + (f^3*x^3*\sqrt{a + b*x + c*x^2})/(4*c^2) + (3*(105*b^4*f^3 - 280*b^2*c*f^2*(b*e + a*f) + 128*c^4*d*(e^2 + d*f) + 80*c^2*f*(6*a*b*e*f + a^2*f^2 + 3*b^2*(e^2 + d*f)) - 64*c^3*(3*a*f*(e^2 + d*f) + b*(e^3 + 6*d*e*f)))*\text{ArcTanh}[(b + 2*c*x)/(2*\sqrt{c}*\sqrt{a + b*x + c*x^2})])/(128*c^{(11/2)})$$

### Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

### Rule 621

```
Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 640

```
Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

### Rule 1660

```
Int[(Pq_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

### Rule 1661

```
Int[(Pq_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a + b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
```



```
[Out] (315*b^6*f^3*x + 105*b^5*f^2*(3*a*f + c*x*(-8*e + f*x)) - 2*b^4*c*f*(105*a*
f*(4*e + 9*f*x) + c*x*(-360*e^2 - 360*d*f + 140*e*f*x + 21*f^2*x^2)) - 8*b^
3*c*(210*a^2*f^3 - c^2*x*(-24*e^3 + 30*e^2*f*x + 3*f^2*x*(10*d + f*x^2) + 2
*e*f*(-72*d + 7*f*x^2)) + a*c*f*(-90*e^2 - 530*e*f*x + f*(-90*d + 77*f*x^2)
)) - 16*b^2*c^2*(-(a^2*f^2*(230*e + 169*f*x)) + a*c*(12*e^3 + 186*e^2*f*x +
2*e*f*(36*d - 43*f*x^2) + f^2*x*(186*d - 13*f*x^2)) + c^2*x*(-24*d^2*f + 6
*d*(-4*e^2 + 4*e*f*x + f^2*x^2) + x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*
x^3))) + 32*c^3*(8*c^3*d^3*x - a^3*f^2*(64*e + 15*f*x) + a^2*c*(16*e^3 + 36
*e^2*f*x + f^2*x*(36*d - 5*f*x^2) - 32*e*f*(-3*d + f*x^2)) + 2*a*c^2*(-12*d
^2*(e + f*x) + 6*d*x*(-2*e^2 + 4*e*f*x + f^2*x^2) + x^2*(4*e^3 + 6*e^2*f*x
+ 4*e*f^2*x^2 + f^3*x^3))) + 16*b*c^2*(113*a^3*f^3 + 8*c^3*d^2*(d - 3*e*x)
+ a^2*c*f*(-156*e^2 - 244*e*f*x + f*(-156*d + 49*f*x^2)) + 2*a*c^2*(12*d^2*
f + 6*d*(2*e^2 + 20*e*f*x - 5*f^2*x^2) - x*(-20*e^3 + 30*e^2*f*x + 14*e*f^2
*x^2 + 3*f^3*x^3)))/(64*c^5*(-b^2 + 4*a*c)*Sqrt[a + x*(b + c*x)]) + (3*(10
5*b^4*f^3 - 280*b^2*c*f^2*(b*e + a*f) + 128*c^4*d*(e^2 + d*f) + 80*c^2*f*(6
*a*b*e*f + a^2*f^2 + 3*b^2*(e^2 + d*f)) - 64*c^3*(3*a*f*(e^2 + d*f) + b*(e^
3 + 6*d*e*f)))*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]]/(128*c^(11
/2))
```

**fricas [B]** time = 5.60, size = 3143, normalized size = 4.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/256*(3*(128*(a*b^2*c^4 - 4*a^2*c^5)*d*e^2 - 64*(a*b^3*c^3 - 4*a^2*b*c^4)
*e^3 + 5*(21*a*b^6 - 140*a^2*b^4*c + 240*a^3*b^2*c^2 - 64*a^4*c^3)*f^3 + 8*
(6*(5*a*b^4*c^2 - 24*a^2*b^2*c^3 + 16*a^3*c^4)*d - 5*(7*a*b^5*c - 40*a^2*b^
3*c^2 + 48*a^3*b*c^3)*e)*f^2 + (128*(b^2*c^5 - 4*a*c^6)*d*e^2 - 64*(b^3*c^4
- 4*a*b*c^5)*e^3 + 5*(21*b^6*c - 140*a*b^4*c^2 + 240*a^2*b^2*c^3 - 64*a^3*
c^4)*f^3 + 8*(6*(5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*c^5)*d - 5*(7*b^5*c^2 -
40*a*b^3*c^3 + 48*a^2*b*c^4)*e)*f^2 + 16*(8*(b^2*c^5 - 4*a*c^6)*d^2 - 24*(b
^3*c^4 - 4*a*b*c^5)*d*e + 3*(5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*c^5)*e^2)*f)
*x^2 + 16*(8*(a*b^2*c^4 - 4*a^2*c^5)*d^2 - 24*(a*b^3*c^3 - 4*a^2*b*c^4)*d*e
+ 3*(5*a*b^4*c^2 - 24*a^2*b^2*c^3 + 16*a^3*c^4)*e^2)*f + (128*(b^3*c^4 - 4
*a*b*c^5)*d*e^2 - 64*(b^4*c^3 - 4*a*b^2*c^4)*e^3 + 5*(21*b^7 - 140*a*b^5*c
+ 240*a^2*b^3*c^2 - 64*a^3*b*c^3)*f^3 + 8*(6*(5*b^5*c^2 - 24*a*b^3*c^3 + 16
*a^2*b*c^4)*d - 5*(7*b^6*c - 40*a*b^4*c^2 + 48*a^2*b^2*c^3)*e)*f^2 + 16*(8*
(b^3*c^4 - 4*a*b*c^5)*d^2 - 24*(b^4*c^3 - 4*a*b^2*c^4)*d*e + 3*(5*b^5*c^2 -
24*a*b^3*c^3 + 16*a^2*b*c^4)*e^2)*f)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x -
b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(128*b*c^6*
d^3 - 768*a*c^6*d^2*e + 384*a*b*c^5*d*e^2 - 16*(b^2*c^5 - 4*a*c^6)*f^3*x^5
- 8*(8*(b^2*c^5 - 4*a*c^6)*e*f^2 - 3*(b^3*c^4 - 4*a*b*c^5)*f^3)*x^4 - 64*(3
*a*b^2*c^4 - 8*a^2*c^5)*e^3 + (315*a*b^5*c - 1680*a^2*b^3*c^2 + 1808*a^3*b*
```

$$\begin{aligned}
& c^3) * f^3 - 2 * (48 * (b^2 * c^5 - 4 * a * c^6) * e^2 * f + (21 * b^4 * c^3 - 104 * a * b^2 * c^4 + \\
& 80 * a^2 * c^5) * f^3 + 8 * (6 * (b^2 * c^5 - 4 * a * c^6) * d - 7 * (b^3 * c^4 - 4 * a * b * c^5) * e) * f \\
& ^2) * x^3 + 8 * (6 * (15 * a * b^3 * c^3 - 52 * a^2 * b * c^4) * d - (105 * a * b^4 * c^2 - 460 * a^2 * b \\
& ^2 * c^3 + 256 * a^3 * c^4) * e) * f^2 - (64 * (b^2 * c^5 - 4 * a * c^6) * e^3 - 7 * (15 * b^5 * c^2 \\
& - 88 * a * b^3 * c^3 + 112 * a^2 * b * c^4) * f^3 - 8 * (30 * (b^3 * c^4 - 4 * a * b * c^5) * d - (35 * b \\
& ^4 * c^3 - 172 * a * b^2 * c^4 + 128 * a^2 * c^5) * e) * f^2 + 48 * (8 * (b^2 * c^5 - 4 * a * c^6) * d * \\
& e - 5 * (b^3 * c^4 - 4 * a * b * c^5) * e^2) * f) * x^2 + 48 * (8 * a * b * c^5 * d^2 - 8 * (3 * a * b^2 * c^ \\
& 4 - 8 * a^2 * c^5) * d * e + (15 * a * b^3 * c^3 - 52 * a^2 * b * c^4) * e^2) * f + (256 * c^7 * d^3 - \\
& 384 * b * c^6 * d^2 * e + 384 * (b^2 * c^5 - 2 * a * c^6) * d * e^2 - 64 * (3 * b^3 * c^4 - 10 * a * b * c^ \\
& 5) * e^3 + (315 * b^6 * c - 1890 * a * b^4 * c^2 + 2704 * a^2 * b^2 * c^3 - 480 * a^3 * c^4) * f^3 \\
& + 8 * (6 * (15 * b^4 * c^3 - 62 * a * b^2 * c^4 + 24 * a^2 * c^5) * d - (105 * b^5 * c^2 - 530 * a * b^ \\
& 3 * c^3 + 488 * a^2 * b * c^4) * e) * f^2 + 48 * (8 * (b^2 * c^5 - 2 * a * c^6) * d^2 - 8 * (3 * b^3 * c^ \\
& 4 - 10 * a * b * c^5) * d * e + (15 * b^4 * c^3 - 62 * a * b^2 * c^4 + 24 * a^2 * c^5) * e^2) * f) * x) * s \\
& \text{qrt}(c * x^2 + b * x + a) / (a * b^2 * c^6 - 4 * a^2 * c^7 + (b^2 * c^7 - 4 * a * c^8) * x^2 + (b \\
& ^3 * c^6 - 4 * a * b * c^7) * x), -1/128 * (3 * (128 * (a * b^2 * c^4 - 4 * a^2 * c^5) * d * e^2 - 64 * ( \\
& a * b^3 * c^3 - 4 * a^2 * b * c^4) * e^3 + 5 * (21 * a * b^6 - 140 * a^2 * b^4 * c + 240 * a^3 * b^2 * c^ \\
& 2 - 64 * a^4 * c^3) * f^3 + 8 * (6 * (5 * a * b^4 * c^2 - 24 * a^2 * b^2 * c^3 + 16 * a^3 * c^4) * d - \\
& 5 * (7 * a * b^5 * c - 40 * a^2 * b^3 * c^2 + 48 * a^3 * b * c^3) * e) * f^2 + (128 * (b^2 * c^5 - 4 * a * \\
& c^6) * d * e^2 - 64 * (b^3 * c^4 - 4 * a * b * c^5) * e^3 + 5 * (21 * b^6 * c - 140 * a * b^4 * c^2 + 2 \\
& 40 * a^2 * b^2 * c^3 - 64 * a^3 * c^4) * f^3 + 8 * (6 * (5 * b^4 * c^3 - 24 * a * b^2 * c^4 + 16 * a^2 * \\
& c^5) * d - 5 * (7 * b^5 * c^2 - 40 * a * b^3 * c^3 + 48 * a^2 * b * c^4) * e) * f^2 + 16 * (8 * (b^2 * c^ \\
& 5 - 4 * a * c^6) * d^2 - 24 * (b^3 * c^4 - 4 * a * b * c^5) * d * e + 3 * (5 * b^4 * c^3 - 24 * a * b^2 * c \\
& ^4 + 16 * a^2 * c^5) * e^2) * f) * x^2 + 16 * (8 * (a * b^2 * c^4 - 4 * a^2 * c^5) * d^2 - 24 * (a * b^ \\
& 3 * c^3 - 4 * a^2 * b * c^4) * d * e + 3 * (5 * a * b^4 * c^2 - 24 * a^2 * b^2 * c^3 + 16 * a^3 * c^4) * e^ \\
& 2) * f + (128 * (b^3 * c^4 - 4 * a * b * c^5) * d * e^2 - 64 * (b^4 * c^3 - 4 * a * b^2 * c^4) * e^3 + \\
& 5 * (21 * b^7 - 140 * a * b^5 * c + 240 * a^2 * b^3 * c^2 - 64 * a^3 * b * c^3) * f^3 + 8 * (6 * (5 * b^5 \\
& * c^2 - 24 * a * b^3 * c^3 + 16 * a^2 * b * c^4) * d - 5 * (7 * b^6 * c - 40 * a * b^4 * c^2 + 48 * a^2 * \\
& b^2 * c^3) * e) * f^2 + 16 * (8 * (b^3 * c^4 - 4 * a * b * c^5) * d^2 - 24 * (b^4 * c^3 - 4 * a * b^2 * c \\
& ^4) * d * e + 3 * (5 * b^5 * c^2 - 24 * a * b^3 * c^3 + 16 * a^2 * b * c^4) * e^2) * f) * x) * \text{sqrt}(-c) * a \\
& \text{rctan}(1/2 * \text{sqrt}(c * x^2 + b * x + a) * (2 * c * x + b) * \text{sqrt}(-c) / (c^2 * x^2 + b * c * x + a * c \\
& )) + 2 * (128 * b * c^6 * d^3 - 768 * a * c^6 * d^2 * e + 384 * a * b * c^5 * d * e^2 - 16 * (b^2 * c^5 - \\
& 4 * a * c^6) * f^3 * x^5 - 8 * (8 * (b^2 * c^5 - 4 * a * c^6) * e * f^2 - 3 * (b^3 * c^4 - 4 * a * b * c^5 \\
& ) * f^3) * x^4 - 64 * (3 * a * b^2 * c^4 - 8 * a^2 * c^5) * e^3 + (315 * a * b^5 * c - 1680 * a^2 * b^3 \\
& * c^2 + 1808 * a^3 * b * c^3) * f^3 - 2 * (48 * (b^2 * c^5 - 4 * a * c^6) * e^2 * f + (21 * b^4 * c^3 \\
& - 104 * a * b^2 * c^4 + 80 * a^2 * c^5) * f^3 + 8 * (6 * (b^2 * c^5 - 4 * a * c^6) * d - 7 * (b^3 * c^4 \\
& - 4 * a * b * c^5) * e) * f^2) * x^3 + 8 * (6 * (15 * a * b^3 * c^3 - 52 * a^2 * b * c^4) * d - (105 * a * b \\
& ^4 * c^2 - 460 * a^2 * b^2 * c^3 + 256 * a^3 * c^4) * e) * f^2 - (64 * (b^2 * c^5 - 4 * a * c^6) * e^ \\
& 3 - 7 * (15 * b^5 * c^2 - 88 * a * b^3 * c^3 + 112 * a^2 * b * c^4) * f^3 - 8 * (30 * (b^3 * c^4 - 4 * \\
& a * b * c^5) * d - (35 * b^4 * c^3 - 172 * a * b^2 * c^4 + 128 * a^2 * c^5) * e) * f^2 + 48 * (8 * (b^2 \\
& * c^5 - 4 * a * c^6) * d * e - 5 * (b^3 * c^4 - 4 * a * b * c^5) * e^2) * f) * x^2 + 48 * (8 * a * b * c^5 * d \\
& ^2 - 8 * (3 * a * b^2 * c^4 - 8 * a^2 * c^5) * d * e + (15 * a * b^3 * c^3 - 52 * a^2 * b * c^4) * e^2) * f \\
& + (256 * c^7 * d^3 - 384 * b * c^6 * d^2 * e + 384 * (b^2 * c^5 - 2 * a * c^6) * d * e^2 - 64 * (3 * b \\
& ^3 * c^4 - 10 * a * b * c^5) * e^3 + (315 * b^6 * c - 1890 * a * b^4 * c^2 + 2704 * a^2 * b^2 * c^3 - \\
& 480 * a^3 * c^4) * f^3 + 8 * (6 * (15 * b^4 * c^3 - 62 * a * b^2 * c^4 + 24 * a^2 * c^5) * d - (105 * \\
& b^5 * c^2 - 530 * a * b^3 * c^3 + 488 * a^2 * b * c^4) * e) * f^2 + 48 * (8 * (b^2 * c^5 - 2 * a * c^6)
\end{aligned}$$

$$*d^2 - 8*(3*b^3*c^4 - 10*a*b*c^5)*d*e + (15*b^4*c^3 - 62*a*b^2*c^4 + 24*a^2*c^5)*e^2)*f)*x)*\sqrt{c*x^2 + b*x + a})/(a*b^2*c^6 - 4*a^2*c^7 + (b^2*c^7 - 4*a*c^8)*x^2 + (b^3*c^6 - 4*a*b*c^7)*x)]$$

**giac** [A] time = 0.42, size = 1099, normalized size = 1.69

$$\left( \left( 2 \left( 4 \left( \frac{2(b^2c^4f^3 - 4ac^5f^3)x}{b^2c^5 - 4ac^6} - \frac{3b^3c^3f^3 - 12abc^4f^3 - 8b^2c^4f^2e + 32ac^5f^2e}{b^2c^5 - 4ac^6} \right) \right) x + \frac{48b^2c^4df^2 - 192ac^5df^2 + 21b^4c^2f^3 - 104ab^2c^3f^3 + 80a^2c^4f^3 - 56b^3c^3f^3}{b^2c^5 - 4ac^6} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)^3/(c\*x^2+b\*x+a)^(3/2),x, algorithm="giac")

[Out] 1/64\*(((2\*(4\*(2\*(b^2\*c^4\*f^3 - 4\*a\*c^5\*f^3)\*x/(b^2\*c^5 - 4\*a\*c^6) - (3\*b^3\*c^3\*f^3 - 12\*a\*b\*c^4\*f^3 - 8\*b^2\*c^4\*f^2\*e + 32\*a\*c^5\*f^2\*e)/(b^2\*c^5 - 4\*a\*c^6))\*x + (48\*b^2\*c^4\*d\*f^2 - 192\*a\*c^5\*d\*f^2 + 21\*b^4\*c^2\*f^3 - 104\*a\*b^2\*c^3\*f^3 + 80\*a^2\*c^4\*f^3 - 56\*b^3\*c^3\*f^2\*e + 224\*a\*b\*c^4\*f^2\*e + 48\*b^2\*c^4\*f\*e^2 - 192\*a\*c^5\*f\*e^2)/(b^2\*c^5 - 4\*a\*c^6))\*x - (240\*b^3\*c^3\*d\*f^2 - 960\*a\*b\*c^4\*d\*f^2 + 105\*b^5\*c\*f^3 - 616\*a\*b^3\*c^2\*f^3 + 784\*a^2\*b\*c^3\*f^3 - 384\*b^2\*c^4\*d\*f\*e + 1536\*a\*c^5\*d\*f\*e - 280\*b^4\*c^2\*f^2\*e + 1376\*a\*b^2\*c^3\*f^2\*e - 1024\*a^2\*c^4\*f^2\*e + 240\*b^3\*c^3\*f\*e^2 - 960\*a\*b\*c^4\*f\*e^2 - 64\*b^2\*c^4\*e^3 + 256\*a\*c^5\*e^3)/(b^2\*c^5 - 4\*a\*c^6))\*x - (256\*c^6\*d^3 + 384\*b^2\*c^4\*d^2\*f - 768\*a\*c^5\*d^2\*f + 720\*b^4\*c^2\*d\*f^2 - 2976\*a\*b^2\*c^3\*d\*f^2 + 1152\*a^2\*c^4\*d\*f^2 + 315\*b^6\*f^3 - 1890\*a\*b^4\*c\*f^3 + 2704\*a^2\*b^2\*c^2\*f^3 - 480\*a^3\*c^3\*f^3 - 384\*b\*c^5\*d^2\*e - 1152\*b^3\*c^3\*d\*f\*e + 3840\*a\*b\*c^4\*d\*f\*e - 840\*b^5\*c\*f^2\*e + 4240\*a\*b^3\*c^2\*f^2\*e - 3904\*a^2\*b\*c^3\*f^2\*e + 384\*b^2\*c^4\*d\*e^2 - 768\*a\*c^5\*d\*e^2 + 720\*b^4\*c^2\*f\*e^2 - 2976\*a\*b^2\*c^3\*f\*e^2 + 1152\*a^2\*c^4\*f\*e^2 - 192\*b^3\*c^3\*e^3 + 640\*a\*b\*c^4\*e^3)/(b^2\*c^5 - 4\*a\*c^6))\*x - (128\*b\*c^5\*d^3 + 384\*a\*b\*c^4\*d^2\*f + 720\*a\*b^3\*c^2\*d\*f^2 - 2496\*a^2\*b\*c^3\*d\*f^2 + 315\*a\*b^5\*f^3 - 1680\*a^2\*b^3\*c\*f^3 + 1808\*a^3\*b\*c^2\*f^3 - 768\*a\*c^5\*d^2\*e - 1152\*a\*b^2\*c^3\*d\*f\*e + 3072\*a^2\*c^4\*d\*f\*e - 840\*a\*b^4\*c\*f^2\*e + 3680\*a^2\*b^2\*c^2\*f^2\*e - 2048\*a^3\*c^3\*f^2\*e + 384\*a\*b\*c^4\*d\*e^2 + 720\*a\*b^3\*c^2\*f\*e^2 - 2496\*a^2\*b\*c^3\*f\*e^2 - 192\*a\*b^2\*c^3\*e^3 + 512\*a^2\*c^4\*e^3)/(b^2\*c^5 - 4\*a\*c^6))/sqrt(c\*x^2 + b\*x + a) - 3/128\*(128\*c^4\*d^2\*f + 240\*b^2\*c^2\*d\*f^2 - 192\*a\*c^3\*d\*f^2 + 105\*b^4\*f^3 - 280\*a\*b^2\*c\*f^3 + 80\*a^2\*c^2\*f^3 - 384\*b\*c^3\*d\*f\*e - 280\*b^3\*c\*f^2\*e + 480\*a\*b\*c^2\*f^2\*e + 128\*c^4\*d\*e^2 + 240\*b^2\*c^2\*f\*e^2 - 192\*a\*c^3\*f\*e^2 - 64\*b\*c^3\*e^3)\*log(abs(-2\*(sqrt(c)\*x - sqrt(c\*x^2 + b\*x + a))\*sqrt(c) - b))/c^(11/2)

**maple** [B] time = 0.03, size = 2827, normalized size = 4.36

output too large to display

Verification of antiderivative is not currently implemented for this CAS.



[In]  $\text{int}((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^{(3/2)}, x)$

[Out]  $2*a/c^2/(c*x^2+b*x+a)^{(1/2)}*e^3+2*d^3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}+315/128*f^3*b^4/c^{(11/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})+15/8*f^3*a^2/c^{(7/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})+315/256*f^3*b^5/c^6/(c*x^2+b*x+a)^{(1/2)}+1/4*f^3*x^5/c/(c*x^2+b*x+a)^{(1/2)}+3/c^{(3/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*f*d^2+3/c^{(3/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*e^2*d-3*d^2*e/c/(c*x^2+b*x+a)^{(1/2)}+x^2/c/(c*x^2+b*x+a)^{(1/2)}*e^3-3/4*b^2/c^3/(c*x^2+b*x+a)^{(1/2)}*e^3-3/2*b/c^{(5/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*e^3+115/4*e*f^2*b^3/c^3*a/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x-9*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x*d*e*f+12*a/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*d*e*f-39/2*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x*d*f^2-39/2*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x*e^2*f-16*e*f^2*a^2/c^2*b/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x+21/32*f^3*b^2/c^3*x^3/(c*x^2+b*x+a)^{(1/2)}-105/64*f^3*b^3/c^4*x^2/(c*x^2+b*x+a)^{(1/2)}-315/128*f^3*b^4/c^5*x/(c*x^2+b*x+a)^{(1/2)}+315/256*f^3*b^7/c^6/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}-105/16*f^3*b^2/c^{(9/2)}*a*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})-5/8*f^3*a/c^2*x^3/(c*x^2+b*x+a)^{(1/2)}-15/8*f^3*a^2/c^3*x/(c*x^2+b*x+a)^{(1/2)}-3*x/c/(c*x^2+b*x+a)^{(1/2)}*f*d^2-3*x/c/(c*x^2+b*x+a)^{(1/2)}*e^2*d+3/2*x^3/c/(c*x^2+b*x+a)^{(1/2)}*d*f^2+3/2*x^3/c/(c*x^2+b*x+a)^{(1/2)}*e^2*f+45/16*b^3/c^4/(c*x^2+b*x+a)^{(1/2)}*d*f^2+45/16*b^3/c^4/(c*x^2+b*x+a)^{(1/2)}*e^2*f+45/8*b^2/c^{(7/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*d*f^2+45/8*b^2/c^{(7/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*e^2*f-9/2*a/c^{(5/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*d*f^2-9/2*a/c^{(5/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*e^2*f+3/2*b/c^2*x/(c*x^2+b*x+a)^{(1/2)}*e^3-3/4*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*e^3+e*f^2*x^4/c/(c*x^2+b*x+a)^{(1/2)}-105/32*e*f^2*b^4/c^5/(c*x^2+b*x+a)^{(1/2)}-105/16*e*f^2*b^3/c^{(9/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})-8*e*f^2*a^2/c^3/(c*x^2+b*x+a)^{(1/2)}+3/2*b/c^2/(c*x^2+b*x+a)^{(1/2)}*f*d^2+3/2*b/c^2/(c*x^2+b*x+a)^{(1/2)}*e^2*d-105/16*f^3*b^3/c^5*a/(c*x^2+b*x+a)^{(1/2)}+113/16*f^3*b/c^4*a^2/(c*x^2+b*x+a)^{(1/2)}+24*a/c*b/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x*d*e*f+9*b/c^2*x/(c*x^2+b*x+a)^{(1/2)}*d*e*f-9/2*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*d*e*f+4*a/c*b/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x*e^3+3*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x*f*d^2+3*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x*e^2*d-8*e*f^2*a^2/c^3*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}+45/8*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x*d*f^2+45/8*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x*e^2*f-39/4*b^3/c^3*a/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*d*f^2-39/4*b^3/c^3*a/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*e^2*f-105/16*e*f^2*b^5/c^4/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x+115/8*e*f^2*b^4/c^4*a/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}-45/4*e*f^2*b/c^3*a*x/(c*x^2+b*x+a)^{(1/2)}+113/8*f^3*b^2/c^3*a^2/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x-105/8*f^3*b^4/c^4*a/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x+105/16*e*f^2*b^3/c^4*x/(c*x^2+b*x+a)^{(1/2)}-105/32*e*f^2*b^6/c^5/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}+115/8*e*f^2*b^2/c^4*a/(c*x^2+b*x+a)^{(1/2)}+45/4*e*f^2*b/c^{(7/2)}*a*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})-4*e*f^2*a/c^2*x^2/(c*x^2+b*x+a)^{(1/2)}+315/128*f^3*b^6/c^5/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x-105/16*f^3*b^5/c^5*a/($

$$\begin{aligned} & 4ac-b^2)/(c^2+bx+a)^{1/2}+105/16f^3b^2/c^4ax/(c^2+bx+a)^{1/2}+ \\ & 9/16f^3b/c^3ax^2/(c^2+bx+a)^{1/2}+113/16f^3b^3/c^4a^2/(4ac-b^2) \\ & / (c^2+bx+a)^{1/2}+6x^2/c/(c^2+bx+a)^{1/2}*d*ef-9/2b^2/c^3/(c^2+bx+a)^{1/2} \\ & *d*ef-3/2b^3/c^2/(4ac-b^2)/(c^2+bx+a)^{1/2}*x*e^3-9b/c^( \\ & 5/2)*\ln((c*x+1/2*b)/c^(1/2)+(c^2+bx+a)^{1/2})*d*ef+12*a/c^2/(c^2+bx+a)^{1/2} \\ & *d*ef+2*a/c^2*b^2/(4ac-b^2)/(c^2+bx+a)^{1/2}*e^3+3/2*b^3/c^2/ \\ & (4ac-b^2)/(c^2+bx+a)^{1/2}*f*d^2+3/2*b^3/c^2/(4ac-b^2)/(c^2+bx+a)^{1/2} \\ & *e^2*d-6*d^2*e*b/(4ac-b^2)/(c^2+bx+a)^{1/2}*x-3*d^2*e*b^2/c/(4ac-b^2) \\ & / (c^2+bx+a)^{1/2}-15/4*b/c^2*x^2/(c^2+bx+a)^{1/2}*d*f^2-15/4*b \\ & /c^2*x^2/(c^2+bx+a)^{1/2}*e^2*f-45/8*b^2/c^3*x/(c^2+bx+a)^{1/2}*d*f^2 \\ & -45/8*b^2/c^3*x/(c^2+bx+a)^{1/2}*e^2*f-7/4*e*f^2*b/c^2*x^3/(c^2+bx+a)^{1/2} \\ & +35/8*e*f^2*b^2/c^3*x^2/(c^2+bx+a)^{1/2}+45/16*b^5/c^4/(4ac-b^2) \\ & / (c^2+bx+a)^{1/2}*e^2*f-39/4*b/c^3*a/(c^2+bx+a)^{1/2}*d*f^2-39/4*b/c^ \\ & 3*a/(c^2+bx+a)^{1/2}*e^2*f+9/2*a/c^2*x/(c^2+bx+a)^{1/2}*d*f^2+9/2*a/c^ \\ & 2*x/(c^2+bx+a)^{1/2}*e^2*f-3/8*f^3*b/c^2*x^4/(c^2+bx+a)^{1/2}+45/16*b^ \\ & 5/c^4/(4ac-b^2)/(c^2+bx+a)^{1/2}*d*f^2 \end{aligned}$$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)^3/(c\*x^2+b\*x+a)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details)Is 4\*a\*c-b^2 zero or nonzero?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(fx^2 + ex + d)^3}{(cx^2 + bx + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x + f\*x^2)^3/(a + b\*x + c\*x^2)^(3/2),x)

[Out] int((d + e\*x + f\*x^2)^3/(a + b\*x + c\*x^2)^(3/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex + fx^2)^3}{(a + bx + cx^2)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)**3/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)**3/(a + b*x + c*x**2)**(3/2), x)
```

$$3.115 \quad \int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{3/2}} dx$$

**Optimal.** Leaf size=309

$$\frac{2(-x(c^2(2a^2f^2 + 6abef + b^2(2df + e^2)) - 2b^2cf(2af + be) - 2c^3(a(2df + e^2) + bde) + b^4f^2 + 2c^4d^2) - bc(-3c^3(b^2 - 4ac)\sqrt{a + bx + cx^2})}{c^3(b^2 - 4ac)\sqrt{a + bx + cx^2}}$$

[Out]  $\frac{1}{8}*(15*b^2*f^2 - 12*c*f*(a*f + 2*b*e) + 8*c^2*(2*d*f + e^2))*\operatorname{arctanh}\left(\frac{1}{2}*(2*c*x + b)/c^{1/2}/(c*x^2 + b*x + a)^{1/2}\right)/c^{7/2} + 2*(2*a*b^2*c*e*f - a*b^3*f^2 + 4*a*c^2*e*(-a*f + c*d) - b*c*(c^2*d^2 - 3*a^2*f^2 + a*c*(2*d*f + e^2)) - (2*c^4*d^2 + b^4*f^2 - 2*b^2*c*f*(2*a*f + b*e) - 2*c^3*(b*d*e + a*(2*d*f + e^2)) + c^2*(6*a*b*e*f + 2*a^2*f^2 + b^2*(2*d*f + e^2)))*x)/c^3/(-4*a*c + b^2)/(c*x^2 + b*x + a)^{1/2} + 1/4*f*(-7*b*f + 8*c*e)*(c*x^2 + b*x + a)^{1/2}/c^3 + 1/2*f^2*x*(c*x^2 + b*x + a)^{1/2}/c^2$

**Rubi [A]** time = 0.45, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {1660, 1661, 640, 621, 206}

$$\frac{2(-x(c^2(2a^2f^2 + 6abef + b^2(2df + e^2)) - 2b^2cf(2af + be) - 2c^3(a(2df + e^2) + bde) + b^4f^2 + 2c^4d^2) - bc(-3c^3(b^2 - 4ac)\sqrt{a + bx + cx^2})}{c^3(b^2 - 4ac)\sqrt{a + bx + cx^2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x + f\*x^2)^2/(a + b\*x + c\*x^2)^(3/2), x]

[Out]  $\frac{(2*(2*a*b^2*c*e*f - a*b^3*f^2 + 4*a*c^2*e*(c*d - a*f) - b*c*(c^2*d^2 - 3*a^2*f^2 + a*c*(e^2 + 2*d*f)) - (2*c^4*d^2 + b^4*f^2 - 2*b^2*c*f*(b*e + 2*a*f) - 2*c^3*(b*d*e + a*(e^2 + 2*d*f)) + c^2*(6*a*b*e*f + 2*a^2*f^2 + b^2*(e^2 + 2*d*f)))*x)/(c^3*(b^2 - 4*a*c)*\operatorname{Sqrt}[a + b*x + c*x^2]) + (f*(8*c*e - 7*b*f)*\operatorname{Sqrt}[a + b*x + c*x^2])/(4*c^3) + (f^2*x*\operatorname{Sqrt}[a + b*x + c*x^2])/(2*c^2) + ((15*b^2*f^2 - 12*c*f*(2*b*e + a*f) + 8*c^2*(e^2 + 2*d*f))*\operatorname{ArcTanh}[(b + 2*c*x)/(2*\operatorname{Sqrt}[c]*\operatorname{Sqrt}[a + b*x + c*x^2])])/(8*c^{7/2})$

**Rule 206**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

**Rule 621**

```
Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 640

```
Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

### Rule 1660

```
Int[(Pq_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

### Rule 1661

```
Int[(Pq_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a + b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(d + ex + fx^2)^2}{(a + bx + cx^2)^{3/2}} dx &= \frac{2(2ab^2cef - ab^3f^2 + 4ac^2e(cd - af) - bc(c^2d^2 - 3a^2f^2 + ac(e^2 + 2df)) - (2c^4d^2 + b^4))}{c^3(b^2 - 4ac)} \\
&= \frac{2(2ab^2cef - ab^3f^2 + 4ac^2e(cd - af) - bc(c^2d^2 - 3a^2f^2 + ac(e^2 + 2df)) - (2c^4d^2 + b^4))}{c^3(b^2 - 4ac)} \\
&= \frac{2(2ab^2cef - ab^3f^2 + 4ac^2e(cd - af) - bc(c^2d^2 - 3a^2f^2 + ac(e^2 + 2df)) - (2c^4d^2 + b^4))}{c^3(b^2 - 4ac)} \\
&= \frac{2(2ab^2cef - ab^3f^2 + 4ac^2e(cd - af) - bc(c^2d^2 - 3a^2f^2 + ac(e^2 + 2df)) - (2c^4d^2 + b^4))}{c^3(b^2 - 4ac)} \\
&= \frac{2(2ab^2cef - ab^3f^2 + 4ac^2e(cd - af) - bc(c^2d^2 - 3a^2f^2 + ac(e^2 + 2df)) - (2c^4d^2 + b^4))}{c^3(b^2 - 4ac)}
\end{aligned}$$

**Mathematica [A]** time = 0.73, size = 288, normalized size = 0.93

$$\frac{4bc(-13a^2f^2 + ac(4df + 2e^2 + 20efx - 5f^2x^2)) + 2c^2d(d - 2ex) + 8c^2(a^2f(8e + 3fx) + ac(x(-2e^2 + 4efx + f^2)))}{4c^3(4a^2 - b^2)}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x + f\*x^2)^2/(a + b\*x + c\*x^2)^(3/2), x]

[Out] (15\*b^4\*f^2\*x + b^3\*f\*(15\*a\*f + c\*x\*(-24\*e + 5\*f\*x)) + 4\*b\*c\*(-13\*a^2\*f^2 + 2\*c^2\*d\*(d - 2\*e\*x) + a\*c\*(2\*e^2 + 4\*d\*f + 20\*e\*f\*x - 5\*f^2\*x^2)) - 2\*b^2\*c\*(a\*f\*(12\*e + 31\*f\*x) + c\*x\*(-4\*e^2 - 8\*d\*f + 4\*e\*f\*x + f^2\*x^2)) + 8\*c^2\*(2\*c^2\*d^2\*x + a^2\*f\*(8\*e + 3\*f\*x) + a\*c\*(-4\*d\*(e + f\*x) + x\*(-2\*e^2 + 4\*e\*f\*x + f^2\*x^2)))/(4\*c^3\*(-b^2 + 4\*a\*c)\*Sqrt[a + x\*(b + c\*x)]) + ((15\*b^2\*f^2 - 12\*c\*f\*(2\*b\*e + a\*f) + 8\*c^2\*(e^2 + 2\*d\*f))\*Log[b + 2\*c\*x + 2\*Sqrt[c]\*Sqrt[a + x\*(b + c\*x)]])/(8\*c^(7/2))

**fricas [B]** time = 3.26, size = 1305, normalized size = 4.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)^2/(c\*x^2+b\*x+a)^(3/2),x, algorithm="fricas")

[Out] [-1/16\*((8\*(a\*b^2\*c^2 - 4\*a^2\*c^3)\*e^2 + 3\*(5\*a\*b^4 - 24\*a^2\*b^2\*c + 16\*a^3\*c^2)\*f^2 + (8\*(b^2\*c^3 - 4\*a\*c^4)\*e^2 + 3\*(5\*b^4\*c - 24\*a\*b^2\*c^2 + 16\*a^2\*c^3)\*f^2 + 8\*(2\*(b^2\*c^3 - 4\*a\*c^4)\*d - 3\*(b^3\*c^2 - 4\*a\*b\*c^3)\*e)\*f)\*x^2 + 8\*(2\*(a\*b^2\*c^2 - 4\*a^2\*c^3)\*d - 3\*(a\*b^3\*c - 4\*a^2\*b\*c^2)\*e)\*f + (8\*(b^3\*c^2 - 4\*a\*b\*c^3)\*e^2 + 3\*(5\*b^5 - 24\*a\*b^3\*c + 16\*a^2\*b\*c^2)\*f^2 + 8\*(2\*(b^3\*c^2 - 4\*a\*b\*c^3)\*d - 3\*(b^4\*c - 4\*a\*b^2\*c^2)\*e)\*f)\*x)\*sqrt(c)\*log(-8\*c^2\*x^2 - 8\*b\*c\*x - b^2 + 4\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(c) - 4\*a\*c) + 4\*(8\*b\*c^4\*d^2 - 32\*a\*c^4\*d\*e + 8\*a\*b\*c^3\*e^2 - 2\*(b^2\*c^3 - 4\*a\*c^4)\*f^2\*x^3 + (15\*a\*b^3\*c - 52\*a^2\*b\*c^2)\*f^2 - (8\*(b^2\*c^3 - 4\*a\*c^4)\*e\*f - 5\*(b^3\*c^2 - 4\*a\*b\*c^3)\*f^2)\*x^2 + 8\*(2\*a\*b\*c^3\*d - (3\*a\*b^2\*c^2 - 8\*a^2\*c^3)\*e)\*f + (16\*c^5\*d^2 - 16\*b\*c^4\*d\*e + 8\*(b^2\*c^3 - 2\*a\*c^4)\*e^2 + (15\*b^4\*c - 62\*a\*b^2\*c^2 + 24\*a^2\*c^3)\*f^2 + 8\*(2\*(b^2\*c^3 - 2\*a\*c^4)\*d - (3\*b^3\*c^2 - 10\*a\*b\*c^3)\*e)\*f)\*x)\*sqrt(c\*x^2 + b\*x + a))/(a\*b^2\*c^4 - 4\*a^2\*c^5 + (b^2\*c^5 - 4\*a\*c^6)\*x^2 + (b^3\*c^4 - 4\*a\*b\*c^5)\*x), -1/8\*((8\*(a\*b^2\*c^2 - 4\*a^2\*c^3)\*e^2 + 3\*(5\*a\*b^4 - 24\*a^2\*b^2\*c + 16\*a^3\*c^2)\*f^2 + (8\*(b^2\*c^3 - 4\*a\*c^4)\*e^2 + 3\*(5\*b^4\*c - 24\*a\*b^2\*c^2 + 16\*a^2\*c^3)\*f^2 + 8\*(2\*(b^2\*c^3 - 4\*a\*c^4)\*d - 3\*(b^3\*c^2 - 4\*a\*b\*c^3)\*e)\*f)\*x^2 + 8\*(2\*(a\*b^2\*c^2 - 4\*a^2\*c^3)\*d - 3\*(a\*b^3\*c - 4\*a^2\*b\*c^2)\*e)\*f + (8\*(b^3\*c^2 - 4\*a\*b\*c^3)\*e^2 + 3\*(5\*b^5 - 24\*a\*b^3\*c + 16\*a^2\*b\*c^2)\*f^2 + 8\*(2\*(b^3\*c^2 - 4\*a\*b\*c^3)\*d - 3\*(b^4\*c - 4\*a\*b^2\*c^2)\*e)\*f)\*x)\*sqrt(-c)\*arctan(1/2\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(-c)/(c^2\*x^2 + b\*c\*x + a\*c)) + 2\*(8\*b\*c^4\*d^2 - 32\*a\*c^4\*d\*e + 8\*a\*b\*c^3\*e^2 - 2\*(b^2\*c^3 - 4\*a\*c^4)\*f^2\*x^3 + (15\*a\*b^3\*c - 52\*a^2\*b\*c^2)\*f^2 - (8\*(b^2\*c^3 - 4\*a\*c^4)\*e\*f - 5\*(b^3\*c^2 - 4\*a\*b\*c^3)\*f^2)\*x^2 + 8\*(2\*a\*b\*c^3\*d - (3\*a\*b^2\*c^2 - 8\*a^2\*c^3)\*e)\*f + (16\*c^5\*d^2 - 16\*b\*c^4\*d\*e + 8\*(b^2\*c^3 - 2\*a\*c^4)\*e^2 + (15\*b^4\*c - 62\*a\*b^2\*c^2 + 24\*a^2\*c^3)\*f^2 + 8\*(2\*(b^2\*c^3 - 2\*a\*c^4)\*d - (3\*b^3\*c^2 - 10\*a\*b\*c^3)\*e)\*f)\*x)\*sqrt(c\*x^2 + b\*x + a))/(a\*b^2\*c^4 - 4\*a^2\*c^5 + (b^2\*c^5 - 4\*a\*c^6)\*x^2 + (b^3\*c^4 - 4\*a\*b\*c^5)\*x)]

**giac** [A] time = 0.35, size = 407, normalized size = 1.32

$$\left( \left( \frac{2(b^2c^2f^2 - 4ac^3f^2)x}{b^2c^3 - 4ac^4} - \frac{5b^3cf^2 - 20abc^2f^2 - 8b^2c^2fe + 32ac^3fe}{b^2c^3 - 4ac^4} \right) x - \frac{16c^4d^2 + 16b^2c^2df - 32ac^3df + 15b^4f^2 - 62ab^2cf^2 + 24a^2c^2f^2 - 16bc^3de - 24a^2c^2f^2}{b^2c^3 - 4ac^4} \right) / (4\sqrt{cx^2 + bx + a})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)^2/(c\*x^2+b\*x+a)^(3/2),x, algorithm="giac")

[Out] 1/4\*(((2\*(b^2\*c^2\*f^2 - 4\*a\*c^3\*f^2)\*x/(b^2\*c^3 - 4\*a\*c^4) - (5\*b^3\*c\*f^2 - 20\*a\*b\*c^2\*f^2 - 8\*b^2\*c^2\*f\*e + 32\*a\*c^3\*f\*e)/(b^2\*c^3 - 4\*a\*c^4))\*x - (16\*c^4\*d^2 + 16\*b^2\*c^2\*d\*f - 32\*a\*c^3\*d\*f + 15\*b^4\*f^2 - 62\*a\*b^2\*c\*f^2 + 24\*a^2\*c^2\*f^2 - 16\*b\*c^3\*d\*e - 24\*b^3\*c\*f\*e + 80\*a\*b\*c^2\*f\*e + 8\*b^2\*c^2\*e^2 - 16\*a\*c^3\*e^2)/(b^2\*c^3 - 4\*a\*c^4))\*x - (8\*b\*c^3\*d^2 + 16\*a\*b\*c^2\*d\*f +

$$15ab^3f^2 - 52a^2b^2c^2f^2 - 32a^2c^3d^2e - 24ab^2c^2f^2e + 64a^2c^2f^2e + 8ab^2c^2e^2)/(b^2c^3 - 4a^2c^4)/\sqrt{cx^2 + bx + a} - 1/8(16c^2d^2f + 15b^2f^2 - 12a^2c^2f^2 - 24b^2c^2f^2e + 8c^2e^2)*\log(\text{abs}(-2(\sqrt{cx^2 + bx + a})*\sqrt{c} - b))/c^{7/2}$$

**maple** [B] time = 0.02, size = 1011, normalized size = 3.27

$$\frac{13ab^2f^2x}{2(4ac - b^2)\sqrt{cx^2 + bx + a}c^2} + \frac{8abefx}{(4ac - b^2)\sqrt{cx^2 + bx + a}c} + \frac{15b^4f^2x}{8(4ac - b^2)\sqrt{cx^2 + bx + a}c^3} - \frac{3b^3efx}{(4ac - b^2)\sqrt{cx^2 + bx + a}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(3/2),x)`

[Out] 
$$\begin{aligned} & -2d^2e/c/(c*x^2+b*x+a)^{(1/2)} - x/c/(c*x^2+b*x+a)^{(1/2)} * e^2 + 1/2*b/c^2/(c*x^2+b*x+a)^{(1/2)} * e^2 + 2/c^3 * \ln((c*x+1/2*b)/c^{(1/2)} + (c*x^2+b*x+a)^{(1/2)}) * d*f + 1/2*f^2*x^3/c/(c*x^2+b*x+a)^{(1/2)} + 15/16*f^2*b^3/c^4/(c*x^2+b*x+a)^{(1/2)} + 15/8*f^2*b^2/c^7 * \ln((c*x+1/2*b)/c^{(1/2)} + (c*x^2+b*x+a)^{(1/2)}) - 3/2*f^2*a/c^5 * \ln((c*x+1/2*b)/c^{(1/2)} + (c*x^2+b*x+a)^{(1/2)}) + 2*d^2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)} + 4*e*f*a/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)} + 2*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)} * x*d*f - 13/2*f^2*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)} * x - 3*e*f*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)} * x + 8*e*f*a/c*b/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)} * x - 13/4*f^2*b^3/c^3*a/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)} - 4*d^2*b/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)} * x + 15/8*f^2*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)} * x - 2*d^2*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)} + 3*e*f*b/c^2*x/(c*x^2+b*x+a)^{(1/2)} - 3/2*e*f*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)} + b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)} * x * e^2 + b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)} * d*f + 1/c^3 * \ln((c*x+1/2*b)/c^{(1/2)} + (c*x^2+b*x+a)^{(1/2)}) * e^2 + 1/2*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)} * e^2 + 2*e*f*x^2/c/(c*x^2+b*x+a)^{(1/2)} - 3/2*e*f*b^2/c^3/(c*x^2+b*x+a)^{(1/2)} - 3*e*f*b/c^5 * \ln((c*x+1/2*b)/c^{(1/2)} + (c*x^2+b*x+a)^{(1/2)}) + 4*e*f*a/c^2/(c*x^2+b*x+a)^{(1/2)} - 5/4*f^2*b/c^2*x^2/(c*x^2+b*x+a)^{(1/2)} - 15/8*f^2*b^2/c^3*x/(c*x^2+b*x+a)^{(1/2)} + 15/16*f^2*b^5/c^4/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)} - 13/4*f^2*b/c^3*a/(c*x^2+b*x+a)^{(1/2)} + 3/2*f^2*a/c^2*x/(c*x^2+b*x+a)^{(1/2)} - 2*x/c/(c*x^2+b*x+a)^{(1/2)} * d*f + b/c^2/(c*x^2+b*x+a)^{(1/2)} * d*f \end{aligned}$$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* h



elp (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details) Is  $4ac - b^2$  zero or nonzero?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(fx^2 + ex + d)^2}{(cx^2 + bx + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x + f\*x^2)^2/(a + b\*x + c\*x^2)^(3/2), x)

[Out] int((d + e\*x + f\*x^2)^2/(a + b\*x + c\*x^2)^(3/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex + fx^2)^2}{(a + bx + cx^2)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x\*\*2+e\*x+d)\*\*2/(c\*x\*\*2+b\*x+a)\*\*(3/2), x)

[Out] Integral((d + e\*x + f\*x\*\*2)\*\*2/(a + b\*x + c\*x\*\*2)\*\*(3/2), x)

$$3.116 \quad \int \frac{d+ex+fx^2}{(a+bx+cx^2)^{3/2}} dx$$

**Optimal.** Leaf size=111

$$\frac{2\left(c\left(2ae - b\left(\frac{af}{c} + d\right)\right) - x\left(-2acf + b^2f - bce + 2c^2d\right)\right)}{c\left(b^2 - 4ac\right)\sqrt{a + bx + cx^2}} + \frac{f \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}$$

[Out] f\*arctanh(1/2\*(2\*c\*x+b)/c^(1/2)/(c\*x^2+b\*x+a)^(1/2))/c^(3/2)+2\*(c\*(2\*a\*e-b\*(d+a\*f/c))-(-2\*a\*c\*f+b^2\*f-b\*c\*e+2\*c^2\*d)\*x)/c/(-4\*a\*c+b^2)/(c\*x^2+b\*x+a)^(1/2)

**Rubi [A]** time = 0.08, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$ , Rules used = {1660, 12, 621, 206}

$$\frac{2\left(c\left(2ae - b\left(\frac{af}{c} + d\right)\right) - x\left(-2acf + b^2f - bce + 2c^2d\right)\right)}{c\left(b^2 - 4ac\right)\sqrt{a + bx + cx^2}} + \frac{f \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x + f\*x^2)/(a + b\*x + c\*x^2)^(3/2), x]

[Out] (2\*(c\*(2\*a\*e - b\*(d + (a\*f)/c)) - (2\*c^2\*d - b\*c\*e + b^2\*f - 2\*a\*c\*f)\*x))/(c\*(b^2 - 4\*a\*c)\*Sqrt[a + b\*x + c\*x^2]) + (f\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/c^(3/2)

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 621

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a,

b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 1660

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 1]}, Simp[((b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] + Dist[1/((p + 1)\*(b^2 - 4\*a\*c)), Int[(a + b\*x + c\*x^2)^(p + 1)\*ExpandToSum[(p + 1)\*(b^2 - 4\*a\*c)\*Q - (2\*p + 3)\*(2\*c\*f - b\*g), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && LtQ[p, -1]

### Rubi steps

$$\begin{aligned} \int \frac{d + ex + fx^2}{(a + bx + cx^2)^{3/2}} dx &= \frac{2 \left( c \left( 2ae - b \left( d + \frac{af}{c} \right) \right) - (2c^2d - bce + b^2f - 2acf) x \right)}{c(b^2 - 4ac) \sqrt{a + bx + cx^2}} - \frac{2 \int -\frac{(b^2 - 4ac)f}{2c\sqrt{a + bx + cx^2}} dx}{b^2 - 4ac} \\ &= \frac{2 \left( c \left( 2ae - b \left( d + \frac{af}{c} \right) \right) - (2c^2d - bce + b^2f - 2acf) x \right)}{c(b^2 - 4ac) \sqrt{a + bx + cx^2}} + \frac{f \int \frac{1}{\sqrt{a + bx + cx^2}} dx}{c} \\ &= \frac{2 \left( c \left( 2ae - b \left( d + \frac{af}{c} \right) \right) - (2c^2d - bce + b^2f - 2acf) x \right)}{c(b^2 - 4ac) \sqrt{a + bx + cx^2}} + \frac{(2f) \text{Subst} \left( \int \frac{1}{4c - x^2} dx, x, \frac{b + 2cx}{\sqrt{a + bx + cx^2}} \right)}{c} \\ &= \frac{2 \left( c \left( 2ae - b \left( d + \frac{af}{c} \right) \right) - (2c^2d - bce + b^2f - 2acf) x \right)}{c(b^2 - 4ac) \sqrt{a + bx + cx^2}} + \frac{f \tanh^{-1} \left( \frac{b + 2cx}{2\sqrt{c} \sqrt{a + bx + cx^2}} \right)}{c^{3/2}} \end{aligned}$$

**Mathematica [A]** time = 0.31, size = 113, normalized size = 1.02

$$\frac{2\sqrt{c}(abf - 2ac(e + fx) + b^2fx + bc(d - ex) + 2c^2dx)}{\sqrt{a + x(b + cx)}} - \frac{f(b^2 - 4ac) \log(2\sqrt{c} \sqrt{a + x(b + cx)} + b + 2cx)}{c^{3/2}(4ac - b^2)}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x + f\*x^2)/(a + b\*x + c\*x^2)^(3/2), x]

[Out] ((2\*Sqrt[c]\*(a\*b\*f + 2\*c^2\*d\*x + b^2\*f\*x + b\*c\*(d - e\*x) - 2\*a\*c\*(e + f\*x))/Sqrt[a + x\*(b + c\*x)] - (b^2 - 4\*a\*c)\*f\*Log[b + 2\*c\*x + 2\*Sqrt[c]\*Sqrt[a + x\*(b + c\*x)]])/(c^(3/2)\*(-b^2 + 4\*a\*c))

**fricas** [B] time = 1.74, size = 429, normalized size = 3.86

$$\left[ \frac{((b^2c - 4ac^2)fx^2 + (b^3 - 4abc)fx + (ab^2 - 4a^2c)f)\sqrt{c} \log\left(-8c^2x^2 - 8bcx - b^2 - 4\sqrt{cx^2 + bx + a}(2cx + b)\sqrt{c}\right)}{2(ab^2c^2 - 4a^2c^3 + (b^2c^3 - 4ac^4)x^2 + (b^3c^2 - 4a^2bc^3)x)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(3/2),x, algorithm="fricas")

[Out] [1/2\*((b^2\*c - 4\*a\*c^2)\*f\*x^2 + (b^3 - 4\*a\*b\*c)\*f\*x + (a\*b^2 - 4\*a^2\*c)\*f)\*sqrt(c)\*log(-8\*c^2\*x^2 - 8\*b\*c\*x - b^2 - 4\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(c) - 4\*a\*c) - 4\*(b\*c^2\*d - 2\*a\*c^2\*e + a\*b\*c\*f + (2\*c^3\*d - b\*c^2\*e + (b^2\*c - 2\*a\*c^2)\*f)\*x)\*sqrt(c\*x^2 + b\*x + a))/(a\*b^2\*c^2 - 4\*a^2\*c^3 + (b^2\*c^3 - 4\*a\*c^4)\*x^2 + (b^3\*c^2 - 4\*a\*b\*c^3)\*x), -(((b^2\*c - 4\*a\*c^2)\*f\*x^2 + (b^3 - 4\*a\*b\*c)\*f\*x + (a\*b^2 - 4\*a^2\*c)\*f)\*sqrt(-c)\*arctan(1/2\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(-c)/(c^2\*x^2 + b\*c\*x + a\*c)) + 2\*(b\*c^2\*d - 2\*a\*c^2\*e + a\*b\*c\*f + (2\*c^3\*d - b\*c^2\*e + (b^2\*c - 2\*a\*c^2)\*f)\*x)\*sqrt(c\*x^2 + b\*x + a))/(a\*b^2\*c^2 - 4\*a^2\*c^3 + (b^2\*c^3 - 4\*a\*c^4)\*x^2 + (b^3\*c^2 - 4\*a\*b\*c^3)\*x)]

**giac** [A] time = 0.31, size = 122, normalized size = 1.10

$$\frac{2\left(\frac{(2c^2d+b^2f-2acf-bce)x}{b^2c-4ac^2} + \frac{bcd+abf-2ace}{b^2c-4ac^2}\right)}{\sqrt{cx^2+bx+a}} - \frac{f \log\left(\left|-2\left(\sqrt{c}x - \sqrt{cx^2+bx+a}\right)\sqrt{c} - b\right|\right)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(3/2),x, algorithm="giac")

[Out] -2\*((2\*c^2\*d + b^2\*f - 2\*a\*c\*f - b\*c\*e)\*x/(b^2\*c - 4\*a\*c^2) + (b\*c\*d + a\*b\*f - 2\*a\*c\*e)/(b^2\*c - 4\*a\*c^2))/sqrt(c\*x^2 + b\*x + a) - f\*log(abs(-2\*(sqrt(c)\*x - sqrt(c\*x^2 + b\*x + a))\*sqrt(c) - b))/c^(3/2)

**maple** [B] time = 0.01, size = 249, normalized size = 2.24

$$\frac{b^2fx}{(4ac - b^2)\sqrt{cx^2 + bx + a}c} - \frac{2bex}{(4ac - b^2)\sqrt{cx^2 + bx + a}} + \frac{b^3f}{2(4ac - b^2)\sqrt{cx^2 + bx + a}c^2} - \frac{b^2e}{(4ac - b^2)\sqrt{cx^2 + bx + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(3/2),x)

```
[Out] -f*x/c/(c*x^2+b*x+a)^(1/2)+1/2*f*b/c^2/(c*x^2+b*x+a)^(1/2)+f*b^2/c/(4*a*c-b
^2)/(c*x^2+b*x+a)^(1/2)*x+1/2*f*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+f/c
^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-e/c/(c*x^2+b*x+a)^(1/2)-
2*e*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x-e*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(
1/2)+2*d*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

**mupad** [B] time = 3.73, size = 143, normalized size = 1.29

$$\frac{f \ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}} + \sqrt{cx^2+bx+a}\right)}{c^{3/2}} - \frac{e(4a+2bx)}{(4ac-b^2)\sqrt{cx^2+bx+a}} + \frac{d\left(\frac{b}{2}+cx\right)}{\left(ac-\frac{b^2}{4}\right)\sqrt{cx^2+bx+a}} + \frac{f\left(\frac{ab}{2}-x\left(ac-\frac{b^2}{4}\right)\right)}{c\left(ac-\frac{b^2}{4}\right)\sqrt{cx^2+bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/(a + b*x + c*x^2)^(3/2),x)
```

```
[Out] (f*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2)))/c^(3/2) - (e*(4*a +
2*b*x))/((4*a*c - b^2)*(a + b*x + c*x^2)^(1/2)) + (d*(b/2 + c*x))/((a*c - b
^2/4)*(a + b*x + c*x^2)^(1/2)) + (f*((a*b)/2 - x*(a*c - b^2/2)))/(c*(a*c -
b^2/4)*(a + b*x + c*x^2)^(1/2))
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex + fx^2}{(a + bx + cx^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/(a + b*x + c*x**2)**(3/2), x)
```

$$3.117 \quad \int \frac{1}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)} dx$$

Optimal. Leaf size=666

$$\frac{2(-cx(-2acf + b^2f - bce + 2c^2d) - bc(cd - 3af) - 2ac^2e + b^3(-f) + b^2ce)}{(b^2 - 4ac)\sqrt{a + bx + cx^2}((cd - af)^2 - (bd - ae)(ce - bf))} \frac{f(f(2af - b(\sqrt{e^2 - 4df} + e)) + c)}{\sqrt{2}\sqrt{e^2 - 4df}((cd - af)^2 - (bd - ae)(ce - bf))}$$

[Out]  $2*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f) - c*(2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x)/((-4*a*c + b^2)/((-a*f + c*d)^2 - (-a*e + b*d)*(-b*f + c*e))/(c*x^2 + b*x + a)^{(1/2)} - 1/2*f*arctanh(1/4*(4*a*f + 2*x*(b*f - c*(e - (-4*d*f + e^2)^{(1/2)})) - b*(e - (-4*d*f + e^2)^{(1/2)})))*2^{(1/2)}/(c*x^2 + b*x + a)^{(1/2)}/(c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (-b*f + c*e)*(-4*d*f + e^2)^{(1/2)})^{(1/2)}*(((-4*d*f + e^2)^{(1/2)}*e - 2*d*f + e^2)*c + f*(2*a*f - b*(e + (-4*d*f + e^2)^{(1/2)})))/((-a*f + c*d)^2 - (-a*e + b*d)*(-b*f + c*e))*2^{(1/2)}/(-4*d*f + e^2)^{(1/2)}/(f*(2*a*f - b*(e - (-4*d*f + e^2)^{(1/2)})) + (-4*d*f + e^2)^{(1/2)}*e - 2*d*f + e^2)*c)^{(1/2)} + 1/2*f*arctanh(1/4*(4*a*f - b*(e + (-4*d*f + e^2)^{(1/2)})))*2^{(1/2)}/(c*x^2 + b*x + a)^{(1/2)}/(c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (-b*f + c*e)*(-4*d*f + e^2)^{(1/2)})^{(1/2)}*(f*(2*a*f - b*(e - (-4*d*f + e^2)^{(1/2)})) + (-4*d*f + e^2)^{(1/2)}*e - 2*d*f + e^2)*c)/((-a*f + c*d)^2 - (-a*e + b*d)*(-b*f + c*e))*2^{(1/2)}/(-4*d*f + e^2)^{(1/2)}/(((4*d*f + e^2)^{(1/2)}*e - 2*d*f + e^2)*c + f*(2*a*f - b*(e + (-4*d*f + e^2)^{(1/2)})))^{(1/2)}$

**Rubi [A]** time = 1.83, antiderivative size = 666, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$ , Rules used = {974, 1032, 724, 206}

$$\frac{2(-cx(-2acf + b^2f - bce + 2c^2d) - bc(cd - 3af) - 2ac^2e + b^2ce + b^3(-f))}{(b^2 - 4ac)\sqrt{a + bx + cx^2}((cd - af)^2 - (bd - ae)(ce - bf))} \frac{f(f(2af - b(\sqrt{e^2 - 4df} + e)) + c)}{\sqrt{2}\sqrt{e^2 - 4df}((cd - af)^2 - (bd - ae)(ce - bf))}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2)),x]

[Out]  $(2*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f) - c*(2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)))*Sqrt[a + b*x + c*x^2] - (f*(c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))) * ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]])*Sqrt[a + b*x + c*x^2])]/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqr$

$$t[c*(e^2 - 2*d*f - e*\sqrt{e^2 - 4*d*f}) + f*(2*a*f - b*(e - \sqrt{e^2 - 4*d*f}))] + (f*(c*(e^2 - 2*d*f - e*\sqrt{e^2 - 4*d*f}) + f*(2*a*f - b*(e - \sqrt{e^2 - 4*d*f}))) * \text{ArcTanh}[(4*a*f - b*(e + \sqrt{e^2 - 4*d*f}) + 2*(b*f - c*(e + \sqrt{e^2 - 4*d*f})) * x] / (2*\sqrt{2}*\sqrt{c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*\sqrt{e^2 - 4*d*f}}*\sqrt{a + b*x + c*x^2}]) / (\sqrt{2}*\sqrt{e^2 - 4*d*f}*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*\sqrt{c*(e^2 - 2*d*f + e*\sqrt{e^2 - 4*d*f}) + f*(2*a*f - b*(e + \sqrt{e^2 - 4*d*f}))})$$

### Rule 206

$$\text{Int}[(a_ + (b_ )*(x_ )^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x] / \text{Rt}[a, 2]) / (\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$$

### Rule 724

$$\text{Int}[1/(((d_ ) + (e_ )*(x_ ))*\sqrt{(a_ ) + (b_ )*(x_ ) + (c_ )*(x_ )^2}), x\_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/\sqrt{a + b*x + c*x^2}], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[2*c*d - b*e, 0]$$

### Rule 974

$$\text{Int}[(a_ + (b_ )*(x_ ) + (c_ )*(x_ )^2)^{(p_ )}*((d_ ) + (e_ )*(x_ ) + (f_ )*(x_ )^2)^{(q_ )}, x\_Symbol] \rightarrow \text{Simp}[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x]*(a + b*x + c*x^2)^{(p + 1)}*(d + e*x + f*x^2)^{(q + 1)} / ((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)) * (p + 1)), x] - \text{Dist}[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)) * (p + 1)), \text{Int}[(a + b*x + c*x^2)^{(p + 1)}*(d + e*x + f*x^2)^q * \text{Simp}[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)) * (p + 1) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f)) * (a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f)) * (p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f)) * (p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f)) * (b*f*(p + 1) - c*e*(2*p + q + 4)) * x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) * (2*p + 2*q + 5)*x^2, x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, q\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[e^2 - 4*d*f, 0] \&\& \text{LtQ}[p, -1] \&\& \text{NeQ}[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] \&\& !( !\text{IntegerQ}[p] \&\& \text{ILtQ}[q, -1]) \&\& !\text{IGtQ}[q, 0]$$

### Rule 1032

$$\text{Int}[(g_ + (h_ )*(x_ )) / (((a_ ) + (b_ )*(x_ ) + (c_ )*(x_ )^2)*\sqrt{(d_ ) + (e_ )*(x_ ) + (f_ )*(x_ )^2}), x\_Symbol] \rightarrow \text{With}\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[(2*c*g - h*(b - q))/q, \text{Int}[1/((b - q + 2*c*x)*\sqrt{d + e*x + f*x^2}), x], x] - \text{Dist}[(2*c*g - h*(b + q))/q, \text{Int}[1/((b + q + 2*c*x)*\sqrt{d + e*x + f*x^2}), x], x]] /; \text{FreeQ}\{a, b, c, d, e, f, g, h\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$$

&& NeQ[e^2 - 4\*d\*f, 0] && PosQ[b^2 - 4\*a\*c]

### Rubi steps

$$\begin{aligned} \int \frac{1}{(a + bx + cx^2)^{3/2} (d + ex + fx^2)} dx &= \frac{2(b^2ce - 2ac^2e - b^3f - bc(cd - 3af) - c(2c^2d - bce + b^2f - 2acf)x)}{(b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{a + bx + cx^2}} + \\ &= \frac{2(b^2ce - 2ac^2e - b^3f - bc(cd - 3af) - c(2c^2d - bce + b^2f - 2acf)x)}{(b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{a + bx + cx^2}} + \\ &= \frac{2(b^2ce - 2ac^2e - b^3f - bc(cd - 3af) - c(2c^2d - bce + b^2f - 2acf)x)}{(b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{a + bx + cx^2}} + \\ &= \frac{2(b^2ce - 2ac^2e - b^3f - bc(cd - 3af) - c(2c^2d - bce + b^2f - 2acf)x)}{(b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{a + bx + cx^2}} \end{aligned}$$

**Mathematica** [A] time = 5.34, size = 700, normalized size = 1.05

$$2f \left( \frac{2(-2c(2af + cx(\sqrt{e^2 - 4df} + e)) + 2b^2f - bc(\sqrt{e^2 - 4df} + e - 2fx))}{(b^2 - 4ac)\sqrt{a + x(b + cx)}(4af^2 - 2bf(\sqrt{e^2 - 4df} + e) + c(\sqrt{e^2 - 4df} + e)^2)} + \frac{2c(cx(\sqrt{e^2 - 4df} - e) - 2af) + 2b^2f + bc(\sqrt{e^2 - 4df} - e + 2fx)}{(b^2 - 4ac)\sqrt{a + x(b + cx)}(f(2af + b(\sqrt{e^2 - 4df} - e)) + c(-e\sqrt{e^2 - 4df} - 2df + e^2))} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2)), x]

[Out] (2\*f\*((2\*b^2\*f + b\*c\*(-e + Sqrt[e^2 - 4\*d\*f] + 2\*f\*x) + 2\*c\*(-2\*a\*f + c\*(-e + Sqrt[e^2 - 4\*d\*f])\*x))/((b^2 - 4\*a\*c)\*(c\*(e^2 - 2\*d\*f - e\*Sqrt[e^2 - 4\*d\*f]) + f\*(2\*a\*f + b\*(-e + Sqrt[e^2 - 4\*d\*f])))\*Sqrt[a + x\*(b + c\*x)]) - (2\*(2\*b^2\*f - b\*c\*(e + Sqrt[e^2 - 4\*d\*f] - 2\*f\*x) - 2\*c\*(2\*a\*f + c\*(e + Sqrt[e^2 - 4\*d\*f])\*x)))/((b^2 - 4\*a\*c)\*(4\*a\*f^2 - 2\*b\*f\*(e + Sqrt[e^2 - 4\*d\*f]) + c\*(e + Sqrt[e^2 - 4\*d\*f])^2)\*Sqrt[a + x\*(b + c\*x)]) + (Sqrt[2]\*f^2\*ArcTanh



$$\frac{[(4*af - 2*c*(e + \sqrt{e^2 - 4*df}))*x - b*(e + \sqrt{e^2 - 4*df} - 2*f*x)] / (2*\sqrt{2}*\sqrt{c*(e^2 - 2*df + e*\sqrt{e^2 - 4*df}) + f*(2*af - b*(e + \sqrt{e^2 - 4*df}))}) * \sqrt{a + x*(b + c*x)}}{(c*(e^2 - 2*df + e*\sqrt{e^2 - 4*df}) + f*(2*af - b*(e + \sqrt{e^2 - 4*df})))^{3/2} - (\sqrt{2}*f^2*\sqrt{c*(e^2 - 2*df - e*\sqrt{e^2 - 4*df}) + f*(2*af + b*(-e + \sqrt{e^2 - 4*df}))}) * \text{ArcTanh}[(4*af + 2*c*(-e + \sqrt{e^2 - 4*df}))*x + b*(-e + \sqrt{e^2 - 4*df} + 2*f*x)] / (2*\sqrt{2}*\sqrt{c*(e^2 - 2*df - e*\sqrt{e^2 - 4*df}) + f*(2*af + b*(-e + \sqrt{e^2 - 4*df}))}) * \sqrt{a + x*(b + c*x)}}}{(c*(-e^2 + 2*df + e*\sqrt{e^2 - 4*df}) + f*(-2*af + b*(e - \sqrt{e^2 - 4*df})))^2} / \sqrt{e^2 - 4*df}$$

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x^2+b\*x+a)^(3/2)/(f\*x^2+e\*x+d),x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

*sage2*

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x^2+b\*x+a)^(3/2)/(f\*x^2+e\*x+d),x, algorithm="giac")

[Out] sage2

**maple** [B] time = 0.03, size = 4099, normalized size = 6.15

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c\*x^2+b\*x+a)^(3/2)/(f\*x^2+e\*x+d),x)

[Out] 
$$\frac{-2/(-4*df+e^2)^{1/2}/(2*af^2-b*ef-2*c*df+c*e^2-(-4*df+e^2)^{1/2}*bf+(-4*df+e^2)^{1/2}*c*e)*f^2/((x+1/2*(e+(-4*df+e^2)^{1/2}))/f)^2*c+(bf-c*e-(-4*df+e^2)^{1/2}*c)*(x+1/2*(e+(-4*df+e^2)^{1/2}))/f/f+1/2*(2*af^2-b*ef-2*c*df+c*e^2-(-4*df+e^2)^{1/2}*bf+(-4*df+e^2)^{1/2}*c*e)/f^2)^{1/2}-4*df/(2*af^2-b*ef-2*c*df+c*e^2-(-4*df+e^2)^{1/2}*bf+(-4*df+e^2)^{1/2}*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*df+e^2)*c^2-b^2)/((x+1/2*(e+(-4*df+e^2)^{1/2}))/f)^2*c+(bf-c*e-(-4*df+e^2)^{1/2}*c)*(x+1/2*(e+(-4*df+e^2)^{1/2}))/f/f+1/2*(2*af^2-b*ef-2*c*df+c*e^2-(-4*df+e^2)^{1/2}*bf+(-4*df+e^2)^{1/2}*c*e)/f^2)^{1/2}}{(c*(-e^2 + 2*df + e*\sqrt{e^2 - 4*df}) + f*(-2*af + b*(e - \sqrt{e^2 - 4*df})))^2} / \sqrt{e^2 - 4*df}$$



$$2)) / f) / f + 1/2 * (2 * a * f^2 - b * e * f - 2 * c * d * f + c * e^2 + (-4 * d * f + e^2)^{1/2} * b * f - (-4 * d * f + e^2)^{1/2} * c * e) / f^2)^{1/2} * x * b * c + 4 / (-4 * d * f + e^2)^{1/2} * f / (2 * a * f^2 - b * e * f - 2 * c * d * f + c * e^2 + (-4 * d * f + e^2)^{1/2} * b * f - (-4 * d * f + e^2)^{1/2} * c * e) / (4 * a * c - 4 * c^2 / f * d + c^2 / f^2 * e^2 - 1 / f^2 * (-4 * d * f + e^2) * c^2 - b^2) / ((x - 1/2 * (-e + (-4 * d * f + e^2)^{1/2})) / f)^2 * c + (b * f - c * e + (-4 * d * f + e^2)^{1/2} * c) * (x - 1/2 * (-e + (-4 * d * f + e^2)^{1/2})) / f) / f + 1/2 * (2 * a * f^2 - b * e * f - 2 * c * d * f + c * e^2 + (-4 * d * f + e^2)^{1/2} * b * f - (-4 * d * f + e^2)^{1/2} * c * e) / f^2)^{1/2} * x * c^2 * e - 2 * f / (2 * a * f^2 - b * e * f - 2 * c * d * f + c * e^2 + (-4 * d * f + e^2)^{1/2} * b * f - (-4 * d * f + e^2)^{1/2} * c * e) / (4 * a * c - 4 * c^2 / f * d + c^2 / f^2 * e^2 - 1 / f^2 * (-4 * d * f + e^2) * c^2 - b^2) / ((x - 1/2 * (-e + (-4 * d * f + e^2)^{1/2})) / f)^2 * c + (b * f - c * e + (-4 * d * f + e^2)^{1/2} * c) * (x - 1/2 * (-e + (-4 * d * f + e^2)^{1/2})) / f) / f + 1/2 * (2 * a * f^2 - b * e * f - 2 * c * d * f + c * e^2 + (-4 * d * f + e^2)^{1/2} * b * f - (-4 * d * f + e^2)^{1/2} * c * e) / f^2)^{1/2} * b * c - 2 / (-4 * d * f + e^2)^{1/2} * f^2 / (2 * a * f^2 - b * e * f - 2 * c * d * f + c * e^2 + (-4 * d * f + e^2)^{1/2} * b * f - (-4 * d * f + e^2)^{1/2} * c * e) / (4 * a * c - 4 * c^2 / f * d + c^2 / f^2 * e^2 - 1 / f^2 * (-4 * d * f + e^2) * c^2 - b^2) / ((x - 1/2 * (-e + (-4 * d * f + e^2)^{1/2})) / f)^2 * c + (b * f - c * e + (-4 * d * f + e^2)^{1/2} * c) * (x - 1/2 * (-e + (-4 * d * f + e^2)^{1/2})) / f) / f + 1/2 * (2 * a * f^2 - b * e * f - 2 * c * d * f + c * e^2 + (-4 * d * f + e^2)^{1/2} * b * f - (-4 * d * f + e^2)^{1/2} * c * e) / f^2)^{1/2} * b^2 + 2 / (-4 * d * f + e^2)^{1/2} * f / (2 * a * f^2 - b * e * f - 2 * c * d * f + c * e^2 + (-4 * d * f + e^2)^{1/2} * b * f - (-4 * d * f + e^2)^{1/2} * c * e) / (4 * a * c - 4 * c^2 / f * d + c^2 / f^2 * e^2 - 1 / f^2 * (-4 * d * f + e^2) * c^2 - b^2) / ((x - 1/2 * (-e + (-4 * d * f + e^2)^{1/2})) / f)^2 * c + (b * f - c * e + (-4 * d * f + e^2)^{1/2} * c) * (x - 1/2 * (-e + (-4 * d * f + e^2)^{1/2})) / f) / f + 1/2 * (2 * a * f^2 - b * e * f - 2 * c * d * f + c * e^2 + (-4 * d * f + e^2)^{1/2} * b * f - (-4 * d * f + e^2)^{1/2} * c * e) / f^2)^{1/2} * b * c * e - 2 / (-4 * d * f + e^2)^{1/2} / (2 * a * f^2 - b * e * f - 2 * c * d * f + c * e^2 + (-4 * d * f + e^2)^{1/2} * b * f - (-4 * d * f + e^2)^{1/2} * c * e) * f^2 * 2^{1/2} / ((2 * a * f^2 - b * e * f - 2 * c * d * f + c * e^2 + (-4 * d * f + e^2)^{1/2} * b * f - (-4 * d * f + e^2)^{1/2} * c * e) / f^2)^{1/2} * \ln(((b * f - c * e + (-4 * d * f + e^2)^{1/2} * c) * (x - 1/2 * (-e + (-4 * d * f + e^2)^{1/2})) / f) / f + (2 * a * f^2 - b * e * f - 2 * c * d * f + c * e^2 + (-4 * d * f + e^2)^{1/2} * b * f - (-4 * d * f + e^2)^{1/2} * c * e) / f^2 + 1/2 * 2^{1/2} * ((2 * a * f^2 - b * e * f - 2 * c * d * f + c * e^2 + (-4 * d * f + e^2)^{1/2} * b * f - (-4 * d * f + e^2)^{1/2} * c * e) / f^2)^{1/2} * (4 * (x - 1/2 * (-e + (-4 * d * f + e^2)^{1/2})) / f)^2 * c + 4 * (b * f - c * e + (-4 * d * f + e^2)^{1/2} * c) * (x - 1/2 * (-e + (-4 * d * f + e^2)^{1/2})) / f) / f + 2 * (2 * a * f^2 - b * e * f - 2 * c * d * f + c * e^2 + (-4 * d * f + e^2)^{1/2} * b * f - (-4 * d * f + e^2)^{1/2} * c * e) / f^2)^{1/2} / (x - 1/2 * (-e + (-4 * d * f + e^2)^{1/2})) / f))$$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x^2+b\*x+a)^(3/2)/(f\*x^2+e\*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*d\*f-e^2>0)', see `assume?` for more details)Is 4\*d\*f-e^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(cx^2 + bx + a)^{3/2} (fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2)), x)

[Out] int(1/((a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x\*\*2+b\*x+a)\*\*(3/2)/(f\*x\*\*2+e\*x+d), x)

[Out] Timed out

$$3.118 \quad \int \frac{(d+ex+fx^2)^3}{(a+bx+cx^2)^{5/2}} dx$$

Optimal. Leaf size=891

$$\frac{x\sqrt{cx^2+bx+a}f^3}{2c^3} + \frac{(12ce-11bf)\sqrt{cx^2+bx+a}f^2}{4c^4} + \frac{(24(e^2+df)c^2-20f(3be+af)c+35b^2f^2)\tanh^{-1}\left(\frac{x}{2\sqrt{c}}\right)}{8c^{9/2}}$$

[Out]  $2/3*(3*a*b^4*c*e*f^2-a*b^5*f^3+a*b^3*c*f*(5*a*f^2-3*c*(d*f+e^2))-b*c^2*(c^3*d^3+5*a^3*f^3+3*a*c^2*d*(d*f+e^2)-9*a^2*c*f*(d*f+e^2))-a*b^2*c^2*e*(12*a*f^2-c*(6*d*f+e^2))+2*a*c^3*e*(3*c^2*d^2+3*a^2*f^2-a*c*(6*d*f+e^2))-(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*(a^2*c^2*f^2-4*a*b^2*c*f^2+7*a*b*c^2*e*f-2*a*c^3*d*f-3*a*c^3*e^2+b^4*f^2-2*b^3*c*e*f+b^2*c^2*d*f+b^2*c^2*e^2-b*c^3*d*e+c^4*d^2)*x/c^5/(-4*a*c+b^2)/(c*x^2+b*x+a)^(3/2)+1/8*f*(35*b^2*f^2-20*c*f*(a*f+3*b*e)+24*c^2*(d*f+e^2))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(9/2)-2/3*(3*b^6*c*e*f^2-b^7*f^3+3*b^5*c*f*(6*a*f^2-c*(d*f+e^2))-3*b^3*c^2*(2*9*a^2*f^3+c^2*d*(d*f+e^2)-10*a*c*f*(d*f+e^2))-4*b*c^3*(2*c^3*d^3-29*a^3*f^3+3*a*c^2*d*(d*f+e^2)+24*a^2*c*f*(d*f+e^2))-24*a^2*c^4*e*(6*a*f^2-c*(6*d*f+e^2))-b^4*c^2*e*(42*a*f^2-c*(6*d*f+e^2))+6*b^2*c^3*e*(2*c^2*d^2+28*a^2*f^2-a*c*(6*d*f+e^2))-c*(16*c^6*d^3-10*b^6*f^3+3*b^4*c*f^2*(26*a*f+7*b*e)-24*c^5*d*(b*d*e-a*(d*f+e^2))-6*b^2*c^2*f*(25*a*b*e*f+27*a^2*f^2+2*b^2*(d*f+e^2))+6*c^4*(b^2*d*(d*f+e^2)-16*a^2*f*(d*f+e^2)-2*a*b*e*(6*d*f+e^2))+c^3*(240*a^2*b*e*f^2+56*a^3*f^3+84*a*b^2*f*(d*f+e^2)+b^3*(6*d*e*f+e^3)))*x/c^5/(-4*a*c+b^2)^(1/2)/(c*x^2+b*x+a)^(1/2)+1/4*f^2*(-11*b*f+12*c*e)*(c*x^2+b*x+a)^(1/2)/c^4+1/2*f^3*x*(c*x^2+b*x+a)^(1/2)/c^3$

**Rubi [A]** time = 1.77, antiderivative size = 891, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {1660, 1661, 640, 621, 206}

$$\frac{x\sqrt{cx^2+bx+a}f^3}{2c^3} + \frac{(12ce-11bf)\sqrt{cx^2+bx+a}f^2}{4c^4} + \frac{(24(e^2+df)c^2-20f(3be+af)c+35b^2f^2)\tanh^{-1}\left(\frac{x}{2\sqrt{c}}\right)}{8c^{9/2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x + f\*x^2)^3/(a + b\*x + c\*x^2)^(5/2), x]

[Out]  $(2*(3*a*b^4*c*e*f^2 - a*b^5*f^3 + a*b^3*c*f*(5*a*f^2 - 3*c*(e^2 + d*f)) - b*c^2*(c^3*d^3 + 5*a^3*f^3 + 3*a*c^2*d*(e^2 + d*f) - 9*a^2*c*f*(e^2 + d*f)) - a*b^2*c^2*e*(12*a*f^2 - c*(e^2 + 6*d*f)) + 2*a*c^3*e*(3*c^2*d^2 + 3*a^2*f$

$$\begin{aligned} &^2 - a*c*(e^2 + 6*d*f)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*(c^4*d^2 - b* \\ &c^3*d*e + b^2*c^2*e^2 - 3*a*c^3*e^2 + b^2*c^2*d*f - 2*a*c^3*d*f - 2*b^3*c*e \\ &*f + 7*a*b*c^2*e*f + b^4*f^2 - 4*a*b^2*c*f^2 + a^2*c^2*f^2)*x)/(3*c^5*(b^2 \\ &- 4*a*c)*(a + b*x + c*x^2)^{(3/2)}) - (2*(3*b^6*c*e*f^2 - b^7*f^3 + 3*b^5*c* \\ &f*(6*a*f^2 - c*(e^2 + d*f)) - 3*b^3*c^2*(29*a^2*f^3 + c^2*d*(e^2 + d*f) - 1 \\ &0*a*c*f*(e^2 + d*f)) - 4*b*c^3*(2*c^3*d^3 - 29*a^3*f^3 + 3*a*c^2*d*(e^2 + d \\ &*f) + 24*a^2*c*f*(e^2 + d*f)) - 24*a^2*c^4*e*(6*a*f^2 - c*(e^2 + 6*d*f)) - \\ &b^4*c^2*e*(42*a*f^2 - c*(e^2 + 6*d*f)) + 6*b^2*c^3*e*(2*c^2*d^2 + 28*a^2*f^ \\ &2 - a*c*(e^2 + 6*d*f)) - c*(16*c^6*d^3 - 10*b^6*f^3 + 3*b^4*c*f^2*(7*b*e + \\ &26*a*f) - 24*c^5*d*(b*d*e - a*(e^2 + d*f)) - 6*b^2*c^2*f*(25*a*b*e*f + 27*a \\ &^2*f^2 + 2*b^2*(e^2 + d*f)) + 6*c^4*(b^2*d*(e^2 + d*f) - 16*a^2*f*(e^2 + d \\ &f) - 2*a*b*e*(e^2 + 6*d*f)) + c^3*(240*a^2*b*e*f^2 + 56*a^3*f^3 + 84*a*b^2* \\ &f*(e^2 + d*f) + b^3*(e^3 + 6*d*e*f)))*x)/(3*c^5*(b^2 - 4*a*c)^2*sqrt[a + b \\ &*x + c*x^2]) + (f^2*(12*c*e - 11*b*f)*sqrt[a + b*x + c*x^2])/(4*c^4) + (f^3 \\ &*x*sqrt[a + b*x + c*x^2])/(2*c^3) + (f*(35*b^2*f^2 - 20*c*f*(3*b*e + a*f) + \\ &24*c^2*(e^2 + d*f))*ArcTanh[(b + 2*c*x)/(2*sqrt[c]*sqrt[a + b*x + c*x^2])]) \\ &)/(8*c^(9/2)) \end{aligned}$$

### Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rule 621

```
Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 640

```
Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol
] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d -
b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

### Rule 1660

```
Int[(Pq_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
```

- 4\*a\*c, 0] && LtQ[p, -1]

### Rule 1661

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(q + 2\*p + 1)), x] + Dist[1/(c\*(q + 2\*p + 1)), Int[(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*(q + 2\*p + 1)\*Pq - a\*e\*(q - 1)\*x^(q - 2) - b\*e\*(q + p)\*x^(q - 1) - c\*e\*(q + 2\*p + 1)\*x^q, x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

### Rubi steps

$$\int \frac{(d + ex + fx^2)^3}{(a + bx + cx^2)^{5/2}} dx = \frac{2(3ab^4cef^2 - ab^5f^3 + ab^3cf(5af^2 - 3c(e^2 + df)) - bc^2(c^3d^3 + 5a^3f^3 + 3ac^2d(e^2 + df))}{(a + bx + cx^2)^{5/2}}$$

$$= \frac{2(3ab^4cef^2 - ab^5f^3 + ab^3cf(5af^2 - 3c(e^2 + df)) - bc^2(c^3d^3 + 5a^3f^3 + 3ac^2d(e^2 + df))}{(a + bx + cx^2)^{5/2}}$$

$$= \frac{2(3ab^4cef^2 - ab^5f^3 + ab^3cf(5af^2 - 3c(e^2 + df)) - bc^2(c^3d^3 + 5a^3f^3 + 3ac^2d(e^2 + df))}{(a + bx + cx^2)^{5/2}}$$

$$= \frac{2(3ab^4cef^2 - ab^5f^3 + ab^3cf(5af^2 - 3c(e^2 + df)) - bc^2(c^3d^3 + 5a^3f^3 + 3ac^2d(e^2 + df))}{(a + bx + cx^2)^{5/2}}$$

$$= \frac{2(3ab^4cef^2 - ab^5f^3 + ab^3cf(5af^2 - 3c(e^2 + df)) - bc^2(c^3d^3 + 5a^3f^3 + 3ac^2d(e^2 + df))}{(a + bx + cx^2)^{5/2}}$$

**Mathematica [A]** time = 2.19, size = 872, normalized size = 0.98

$$\frac{-105f^3x^2b^7 - 10f^2x(21af + 2cx(7fx - 9e))b^6 - 3f(35a^2f^2 - 10acx(12e + 23fx))f + c^2x^2(24e^2 - 80fxe + 7f^2x^2)}{}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x + f\*x^2)^3/(a + b\*x + c\*x^2)^(5/2), x]

[Out]  $(-105b^7f^3x^2 - 10b^6f^2x(21af + 2cx(-9e + 7fx)) + 6b^4c^2f(5a^2f(6e + 53fx) - 6acx(4e^2 + 4df + 30efx - 31f^2x^2) + c^2x^3(-16e^2 - 16df + 6efx + f^2x^2)) - 3b^5f(35a^2f^2 - 10acx(12e + 23fx) + c^2x^2(24e^2 + 24df - 80efx + 7f^2x^2)) - 48b^2c^2(27a^4f^3 - 4c^4d^2x^2(d - ex) + a^2c^2(-4d^2f + 4e^3x - 64ef^2x^3 + 7f^3x^4 - 4d^2e(e - 6fx)) - 2a^3c^3(d^3 - e^3x^3 + 3d^2ex^2(e - 2fx) + 3d^2x(-e + fx)) - 2a^3c^3f(5e^2 + 39efx + f(5d - 14fx^2))) - 8b^3c^3(-95a^3f^3 + c^3(d^3 - e^3x^3 + 9d^2x(e - fx) - 3d^2ex^2(3e + 2fx)) - 3ac^2fx^2(18e^2 - 74efx + f(18d + 7fx^2)) + 3a^2c^3f(3e^2 + 105efx + f(3d + 29fx^2))) + 32c^3(4c^4d^3x^3 + 3a^4f^2(16e + 5fx) + 6ac^3d^2x(d^2 + e^2x^2 + d^2fx^2) - 2a^3c^3(2e^3 + 9e^2fx + f^2x(9d - 10fx^2) + 12ef(d - 3fx^2)) - 3a^2c^2(2d^2e + 4d^2fx^2(3e + 2fx) + x^2(2e^3 + 8e^2fx - 6ef^2x^2 - f^3x^3))) - 48b^2c^2(a^3f^2(25e + 63fx) - c^3d^2x(d^2 + e^2x^2 + d^2x(-6e + fx)) + a^2c^3f^2(-21e^2 - 12efx + 7f(-3d + 7fx^2)) + ac^2(d^2(e - 6fx) - 2d^2x(3e^2 - 3efx + 7f^2x^2) + x^2(e^3 - 14e^2fx + 6ef^2x^2 + f^3x^3))))/(12c^4(b^2 - 4ac)^2(a + x(b + cx))^(3/2)) + (f(35b^2f^2 - 20c^2f(3be + af) + 24c^2(e^2 + df))*Log[b + 2cx + 2Sqrt[c]*Sqrt[a + x(b + cx)]])/(8c^(9/2))$

**fricas [B]** time = 12.87, size = 3995, normalized size = 4.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)^3/(c\*x^2+b\*x+a)^(5/2), x, algorithm="fricas")

[Out]  $[-1/48(3((24(b^4c^4 - 8ab^2c^5 + 16a^2c^6)e^2f + 5(7b^6c^2 - 60ab^4c^3 + 144a^2b^2c^4 - 64a^3c^5))f^3 + 12(2(b^4c^4 - 8ab^2c^5 + 16a^2c^6)d - 5(b^5c^3 - 8ab^3c^4 + 16a^2b^2c^5)e))f^2)x^4 + 24(a^2b^4c^2 - 8a^3b^2c^3 + 16a^4c^4)e^2f + 5(7a^2b^6 - 60a^3b^4c + 144a^4b^2c^2 - 64a^5c^3))f^3 + 2(24(b^5c^3 - 8ab^3c^4 + 16a^2b^2c^5)e^2f + 5(7b^7c - 60ab^5c^2 + 144a^2b^3c^3 - 64a^3b^2c^4))f^3 + 12(2(b^5c^3 - 8ab^3c^4 + 16a^2b^2c^5)d - 5(b^6c^2$



$$\begin{aligned}
& 2 - 8*a*b^4*c^3 + 16*a^2*b^2*c^4)*e)*f^2)*x^3 + 12*(2*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d - 5*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*e)*f^2 + (24*(b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*e^2*f + 5*(7*b^8 - 46*a*b^6*c + 24*a^2*b^4*c^2 + 224*a^3*b^2*c^3 - 128*a^4*c^4)*f^3 + 12*(2*(b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*d - 5*(b^7*c - 6*a*b^5*c^2 + 32*a^3*b*c^4)*e)*f^2)*x^2 + 2*(24*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*e^2*f + 5*(7*a*b^7 - 60*a^2*b^5*c + 144*a^3*b^3*c^2 - 64*a^4*b*c^3)*f^3 + 12*(2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*d - 5*(a*b^6*c - 8*a^2*b^4*c^2 + 16*a^3*b^2*c^3)*e)*f^2)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(192*a^2*b*c^5*d*e^2 - 128*a^3*c^5*e^3 + 6*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*f^3*x^5 + 3*(12*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*e*f^2 - 7*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*f^3)*x^4 - 8*(b^3*c^5 - 12*a*b*c^6)*d^3 - 48*(a*b^2*c^5 + 4*a^2*c^6)*d^2*e - (105*a^2*b^5*c - 760*a^3*b^3*c^2 + 1296*a^4*b*c^3)*f^3 + 4*(32*c^8*d^3 - 48*b*c^7*d^2*e + 12*(b^2*c^6 + 4*a*c^7)*d*e^2 + 2*(b^3*c^5 - 12*a*b*c^6)*e^3 - (35*b^6*c^2 - 279*a*b^4*c^3 + 588*a^2*b^2*c^4 - 160*a^3*c^5)*f^3 - 12*(2*(b^4*c^4 - 7*a*b^2*c^5 + 8*a^2*c^6)*d - (5*b^5*c^3 - 37*a*b^3*c^4 + 64*a^2*b*c^5)*e)*f^2 + 12*((b^2*c^6 + 4*a*c^7)*d^2 + (b^3*c^5 - 12*a*b*c^6)*d*e - 2*(b^4*c^4 - 7*a*b^2*c^5 + 8*a^2*c^6)*e^2)*f)*x^3 - 12*(2*(3*a^2*b^3*c^3 - 20*a^3*b*c^4)*d - (15*a^2*b^4*c^2 - 100*a^3*b^2*c^3 + 128*a^4*c^4)*e)*f^2 + 3*(64*b*c^7*d^3 - 96*b^2*c^6*d^2*e + 24*(b^3*c^5 + 4*a*b*c^6)*d*e^2 - 16*(a*b^2*c^5 + 4*a^2*c^6)*e^3 - (35*b^7*c - 230*a*b^5*c^2 + 232*a^2*b^3*c^3 + 448*a^3*b*c^4)*f^3 - 12*(2*(b^5*c^3 - 6*a*b^3*c^4)*d - (5*b^6*c^2 - 30*a*b^4*c^3 + 16*a^2*b^2*c^4 + 64*a^3*c^5)*e)*f^2 + 24*((b^3*c^5 + 4*a*b*c^6)*d^2 - 4*(a*b^2*c^5 + 4*a^2*c^6)*d*e - (b^5*c^3 - 6*a*b^3*c^4)*e^2)*f)*x^2 + 24*(8*a^2*b*c^5*d^2 - 32*a^3*c^5*d*e - (3*a^2*b^3*c^3 - 20*a^3*b*c^4)*e^2)*f + 6*(48*a*b^2*c^5*d*e^2 - 32*a^2*b*c^5*e^3 + 8*(b^2*c^6 + 4*a*c^7)*d^3 - 12*(b^3*c^5 + 4*a*b*c^6)*d^2*e - (35*a*b^6*c - 265*a^2*b^4*c^2 + 504*a^3*b^2*c^3 - 80*a^4*c^4)*f^3 - 12*(2*(a*b^4*c^3 - 7*a^2*b^2*c^4 + 4*a^3*c^5)*d - (5*a*b^5*c^2 - 35*a^2*b^3*c^3 + 52*a^3*b*c^4)*e)*f^2 + 24*(2*a*b^2*c^5*d^2 - 8*a^2*b*c^5*d*e - (a*b^4*c^3 - 7*a^2*b^2*c^4 + 4*a^3*c^5)*e^2)*f)*x)*sqrt(c*x^2 + b*x + a))/(a^2*b^4*c^5 - 8*a^3*b^2*c^6 + 16*a^4*c^7 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*x^4 + 2*(b^5*c^6 - 8*a*b^3*c^7 + 16*a^2*b*c^8)*x^3 + (b^6*c^5 - 6*a*b^4*c^6 + 32*a^3*c^8)*x^2 + 2*(a*b^5*c^5 - 8*a^2*b^3*c^6 + 16*a^3*b*c^7)*x), -1/24*(3*((24*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*e^2*f + 5*(7*b^6*c^2 - 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*f^3 + 12*(2*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*d - 5*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*e)*f^2)*x^4 + 24*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*e^2*f + 5*(7*a^2*b^6 - 60*a^3*b^4*c + 144*a^4*b^2*c^2 - 64*a^5*c^3)*f^3 + 2*(24*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*e^2*f + 5*(7*b^7*c - 60*a*b^5*c^2 + 144*a^2*b^3*c^3 - 64*a^3*b*c^4)*f^3 + 12*(2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*d - 5*(b^6*c^2 - 8*a*b^4*c^3 + 16*a^2*b^2*c^4)*e)*f^2)*x^3 + 12*(2*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d - 5*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*e)*f^2 + (24*(b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*e^2*f + 5*(7*b^8 - 46*a*b^6*c + 24*a^2*b^4*c^2 + 224*a^3*b^2*c^3 - 128*a^4*c^4)*f^3 + 12*(
\end{aligned}$$

$$\begin{aligned}
& 2*(b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*d - 5*(b^7*c - 6*a*b^5*c^2 + 32*a^3* \\
& b*c^4)*e)*f^2)*x^2 + 2*(24*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*e^2*f \\
& + 5*(7*a*b^7 - 60*a^2*b^5*c + 144*a^3*b^3*c^2 - 64*a^4*b*c^3)*f^3 + 12*(2* \\
& (a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*d - 5*(a*b^6*c - 8*a^2*b^4*c^2 + \\
& 16*a^3*b^2*c^3)*e)*f^2)*x)*\sqrt{-c}*\arctan(1/2*\sqrt{c*x^2 + b*x + a}*(2*c* \\
& x + b)*\sqrt{-c}/(c^2*x^2 + b*c*x + a*c)) - 2*(192*a^2*b*c^5*d*e^2 - 128*a^3 \\
& *c^5*e^3 + 6*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*f^3*x^5 + 3*(12*(b^4*c^4 \\
& - 8*a*b^2*c^5 + 16*a^2*c^6)*e*f^2 - 7*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5 \\
& )*f^3)*x^4 - 8*(b^3*c^5 - 12*a*b*c^6)*d^3 - 48*(a*b^2*c^5 + 4*a^2*c^6)*d^2* \\
& e - (105*a^2*b^5*c - 760*a^3*b^3*c^2 + 1296*a^4*b*c^3)*f^3 + 4*(32*c^8*d^3 \\
& - 48*b*c^7*d^2*e + 12*(b^2*c^6 + 4*a*c^7)*d*e^2 + 2*(b^3*c^5 - 12*a*b*c^6)* \\
& e^3 - (35*b^6*c^2 - 279*a*b^4*c^3 + 588*a^2*b^2*c^4 - 160*a^3*c^5)*f^3 - 12 \\
& *(2*(b^4*c^4 - 7*a*b^2*c^5 + 8*a^2*c^6)*d - (5*b^5*c^3 - 37*a*b^3*c^4 + 64* \\
& a^2*b*c^5)*e)*f^2 + 12*((b^2*c^6 + 4*a*c^7)*d^2 + (b^3*c^5 - 12*a*b*c^6)*d* \\
& e - 2*(b^4*c^4 - 7*a*b^2*c^5 + 8*a^2*c^6)*e^2)*f)*x^3 - 12*(2*(3*a^2*b^3*c^ \\
& 3 - 20*a^3*b*c^4)*d - (15*a^2*b^4*c^2 - 100*a^3*b^2*c^3 + 128*a^4*c^4)*e)*f \\
& ^2 + 3*(64*b*c^7*d^3 - 96*b^2*c^6*d^2*e + 24*(b^3*c^5 + 4*a*b*c^6)*d*e^2 - \\
& 16*(a*b^2*c^5 + 4*a^2*c^6)*e^3 - (35*b^7*c - 230*a*b^5*c^2 + 232*a^2*b^3*c^ \\
& 3 + 448*a^3*b*c^4)*f^3 - 12*(2*(b^5*c^3 - 6*a*b^3*c^4)*d - (5*b^6*c^2 - 30* \\
& a*b^4*c^3 + 16*a^2*b^2*c^4 + 64*a^3*c^5)*e)*f^2 + 24*((b^3*c^5 + 4*a*b*c^6) \\
& *d^2 - 4*(a*b^2*c^5 + 4*a^2*c^6)*d*e - (b^5*c^3 - 6*a*b^3*c^4)*e^2)*f)*x^2 \\
& + 24*(8*a^2*b*c^5*d^2 - 32*a^3*c^5*d*e - (3*a^2*b^3*c^3 - 20*a^3*b*c^4)*e^2 \\
& )*f + 6*(48*a*b^2*c^5*d*e^2 - 32*a^2*b*c^5*e^3 + 8*(b^2*c^6 + 4*a*c^7)*d^3 \\
& - 12*(b^3*c^5 + 4*a*b*c^6)*d^2*e - (35*a*b^6*c - 265*a^2*b^4*c^2 + 504*a^3* \\
& b^2*c^3 - 80*a^4*c^4)*f^3 - 12*(2*(a*b^4*c^3 - 7*a^2*b^2*c^4 + 4*a^3*c^5)*d \\
& - (5*a*b^5*c^2 - 35*a^2*b^3*c^3 + 52*a^3*b*c^4)*e)*f^2 + 24*(2*a*b^2*c^5*d \\
& ^2 - 8*a^2*b*c^5*d*e - (a*b^4*c^3 - 7*a^2*b^2*c^4 + 4*a^3*c^5)*e^2)*f)*x)*\sqrt{c*x^2 + b*x + a} \\
& )/(a^2*b^4*c^5 - 8*a^3*b^2*c^6 + 16*a^4*c^7 + (b^4*c^7 \\
& - 8*a*b^2*c^8 + 16*a^2*c^9)*x^4 + 2*(b^5*c^6 - 8*a*b^3*c^7 + 16*a^2*b*c^8)* \\
& x^3 + (b^6*c^5 - 6*a*b^4*c^6 + 32*a^3*c^8)*x^2 + 2*(a*b^5*c^5 - 8*a^2*b^3*c \\
& ^6 + 16*a^3*b*c^7)*x]
\end{aligned}$$

**giac** [A] time = 0.45, size = 1401, normalized size = 1.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)^3/(c\*x^2+b\*x+a)^(5/2),x, algorithm="giac")

[Out] 1/12\*(((3\*(2\*(b^4\*c^3\*f^3 - 8\*a\*b^2\*c^4\*f^3 + 16\*a^2\*c^5\*f^3)\*x/(b^4\*c^4 - 8\*a\*b^2\*c^5 + 16\*a^2\*c^6) - (7\*b^5\*c^2\*f^3 - 56\*a\*b^3\*c^3\*f^3 + 112\*a^2\*b\*c^4\*f^3 - 12\*b^4\*c^3\*f^2\*e + 96\*a\*b^2\*c^4\*f^2\*e - 192\*a^2\*c^5\*f^2\*e)/(b^4\*c^4 - 8\*a\*b^2\*c^5 + 16\*a^2\*c^6))\*x + 4\*(32\*c^7\*d^3 + 12\*b^2\*c^5\*d^2\*f + 48\*a\*c^6\*d^2\*f - 24\*b^4\*c^3\*d\*f^2 + 168\*a\*b^2\*c^4\*d\*f^2 - 192\*a^2\*c^5\*d\*f^2 - 35\*b^6\*c\*f^3 + 279\*a\*b^4\*c^2\*f^3 - 588\*a^2\*b^2\*c^3\*f^3 + 160\*a^3\*c^4\*f^3 - 4

$$\begin{aligned}
& 8*b*c^6*d^2*e + 12*b^3*c^4*d*f*e - 144*a*b*c^5*d*f*e + 60*b^5*c^2*f^2*e - 4 \\
& 44*a*b^3*c^3*f^2*e + 768*a^2*b*c^4*f^2*e + 12*b^2*c^5*d*e^2 + 48*a*c^6*d*e^2 \\
& - 24*b^4*c^3*f*e^2 + 168*a*b^2*c^4*f*e^2 - 192*a^2*c^5*f*e^2 + 2*b^3*c^4* \\
& e^3 - 24*a*b*c^5*e^3)/(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6))*x + 3*(64*b*c^6 \\
& *d^3 + 24*b^3*c^4*d^2*f + 96*a*b*c^5*d^2*f - 24*b^5*c^2*d*f^2 + 144*a*b^3*c \\
& ^3*d*f^2 - 35*b^7*f^3 + 230*a*b^5*c*f^3 - 232*a^2*b^3*c^2*f^3 - 448*a^3*b*c \\
& ^3*f^3 - 96*b^2*c^5*d^2*e - 96*a*b^2*c^4*d*f*e - 384*a^2*c^5*d*f*e + 60*b^6 \\
& *c*f^2*e - 360*a*b^4*c^2*f^2*e + 192*a^2*b^2*c^3*f^2*e + 768*a^3*c^4*f^2*e \\
& + 24*b^3*c^4*d*e^2 + 96*a*b*c^5*d*e^2 - 24*b^5*c^2*f*e^2 + 144*a*b^3*c^3*f* \\
& e^2 - 16*a*b^2*c^4*e^3 - 64*a^2*c^5*e^3)/(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6) \\
& ))*x + 6*(8*b^2*c^5*d^3 + 32*a*c^6*d^3 + 48*a*b^2*c^4*d^2*f - 24*a*b^4*c^2 \\
& *d*f^2 + 168*a^2*b^2*c^3*d*f^2 - 96*a^3*c^4*d*f^2 - 35*a*b^6*f^3 + 265*a^2* \\
& b^4*c*f^3 - 504*a^3*b^2*c^2*f^3 + 80*a^4*c^3*f^3 - 12*b^3*c^4*d^2*e - 48*a* \\
& b*c^5*d^2*e - 192*a^2*b*c^4*d*f*e + 60*a*b^5*c*f^2*e - 420*a^2*b^3*c^2*f^2* \\
& e + 624*a^3*b*c^3*f^2*e + 48*a*b^2*c^4*d*e^2 - 24*a*b^4*c^2*f*e^2 + 168*a^2 \\
& *b^2*c^3*f*e^2 - 96*a^3*c^4*f*e^2 - 32*a^2*b*c^4*e^3)/(b^4*c^4 - 8*a*b^2*c^5 \\
& + 16*a^2*c^6))*x - (8*b^3*c^4*d^3 - 96*a*b*c^5*d^3 - 192*a^2*b*c^4*d^2*f \\
& + 72*a^2*b^3*c^2*d*f^2 - 480*a^3*b*c^3*d*f^2 + 105*a^2*b^5*f^3 - 760*a^3*b^ \\
& 3*c*f^3 + 1296*a^4*b*c^2*f^3 + 48*a*b^2*c^4*d^2*e + 192*a^2*c^5*d^2*e + 768 \\
& *a^3*c^4*d*f*e - 180*a^2*b^4*c*f^2*e + 1200*a^3*b^2*c^2*f^2*e - 1536*a^4*c^ \\
& 3*f^2*e - 192*a^2*b*c^4*d*e^2 + 72*a^2*b^3*c^2*f*e^2 - 480*a^3*b*c^3*f*e^2 \\
& + 128*a^3*c^4*e^3)/(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6))/(c*x^2 + b*x + a) \\
& ^{(3/2)} - 1/8*(24*c^2*d*f^2 + 35*b^2*f^3 - 20*a*c*f^3 - 60*b*c*f^2*e + 24*c^2 \\
& *f*e^2)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^{(9/2)} \\
& )
\end{aligned}$$

**maple [B]** time = 0.03, size = 4635, normalized size = 5.20

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\int (f*x^2+e*x+d)^3/(c*x^2+b*x+a)^{(5/2)}, x$

[Out]  $5/6*f^3*a/c^2*x^3/(c*x^2+b*x+a)^{(3/2)}+5/2*f^3*a/c^3*x/(c*x^2+b*x+a)^{(1/2)}-6$   
 $*b/c*a/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)}*x*d*e*f-5/2*f^3*a/c^3*b^2/(4*a*c-b^2$   
 $)/(c*x^2+b*x+a)^{(1/2)}*x-33/4*f^3*b^2/c^3*a^2/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2}$   
 $)*x-66*f^3*b^2/c^2*a^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)}*x-1/8*b^4/c^3/(4*a$   
 $*c-b^2)/(c*x^2+b*x+a)^{(3/2)}*x*e^2*f+4*b^3/c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/$   
 $2)}*x*d*e*f-3*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)}*d*e*f-48*b*a/(4*a*c-$   
 $b^2)^2/(c*x^2+b*x+a)^{(1/2)}*x*d*e*f-24*b^2/c*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{($   
 $1/2)}*d*e*f+12*b^2/c*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)}*x*d*f^2+12*b^2/c*a/$   
 $(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)}*x*e^2*f+3/2*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+$   
 $b*x+a)^{(3/2)}*x*d*f^2+3/2*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)}*x*e^2*f+$   
 $12*e*f^2*a^2/c^2*b/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)}*x+96*e*f^2*a^2/c*b/(4*a*$   
 $c-b^2)^2/(c*x^2+b*x+a)^{(1/2)}*x-19/4*e*f^2*b^3/c^3*a/(4*a*c-b^2)/(c*x^2+b*x+$

$$\begin{aligned}
& a)^{(3/2)} * x^{-38} * e * f^2 * b^3 / c^2 * a / (4 * a * c - b^2)^2 / (c * x^2 + b * x + a)^{(1/2)} * x + 1/2 * b^3 / c \\
& ^2 / (4 * a * c - b^2) / (c * x^2 + b * x + a)^{(3/2)} * x * d * e * f + 2/3 * d^3 / (4 * a * c - b^2) / (c * x^2 + b * x + a) \\
& )^{(3/2)} * b - x^2 / c / (c * x^2 + b * x + a)^{(3/2)} * e^3 + 1/24 * b^2 / c^3 / (c * x^2 + b * x + a)^{(3/2)} * e^3 - 2/3 * a / c^2 / (c * x^2 + b * x + a)^{(3/2)} * e^3 + 3/c^{(5/2)} * \ln((c * x + 1/2 * b) / c^{(1/2)} + (c * x^2 \\
& + b * x + a)^{(1/2)}) * d * f^2 + 3/c^{(5/2)} * \ln((c * x + 1/2 * b) / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}) * \\
& e^2 * f + 35/16 * f^3 * b^3 / c^5 / (c * x^2 + b * x + a)^{(1/2)} - 5/2 * f^3 * a / c^{(7/2)} * \ln((c * x + 1/2 * b) \\
& ) / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)} + 35/8 * f^3 * b^2 / c^{(9/2)} * \ln((c * x + 1/2 * b) / c^{(1/2)} + \\
& (c * x^2 + b * x + a)^{(1/2)}) + 1/2 * f^3 * x^5 / c / (c * x^2 + b * x + a)^{(3/2)} - 35/384 * f^3 * b^5 / c^6 / ( \\
& c * x^2 + b * x + a)^{(3/2)} - d^2 * e / c / (c * x^2 + b * x + a)^{(3/2)} - 33/16 * f^3 * b^2 / c^4 * a * x / (c * x^2 \\
& + b * x + a)^{(3/2)} - 33/8 * f^3 * b^3 / c^4 * a^2 / (4 * a * c - b^2) / (c * x^2 + b * x + a)^{(3/2)} + 23/16 * f^3 \\
& * b^5 / c^5 * a / (4 * a * c - b^2) / (c * x^2 + b * x + a)^{(3/2)} + 23/2 * f^3 * b^5 / c^4 * a / (4 * a * c - b^2)^2 / (c * x^2 + b * x + a)^{(1/2)} - 5/4 * f^3 * a / c^4 * b^3 / (4 * a * c - b^2) / (c * x^2 + b * x + a)^{(1/2)} - 35/ \\
& 192 * f^3 * b^6 / c^5 / (4 * a * c - b^2) / (c * x^2 + b * x + a)^{(3/2)} * x + 35/8 * f^3 * b^4 / c^4 / (4 * a * c - b \\
& ^2) / (c * x^2 + b * x + a)^{(1/2)} * x + 1/4 * b^3 / c^2 / (4 * a * c - b^2) / (c * x^2 + b * x + a)^{(3/2)} * f * d^2 \\
& + 1/4 * b^3 / c^2 / (4 * a * c - b^2) / (c * x^2 + b * x + a)^{(3/2)} * e^2 * d + 4 * b^2 / (4 * a * c - b^2)^2 / (c * x \\
& ^2 + b * x + a)^{(1/2)} * x * f * d^2 + 4 * b^2 / (4 * a * c - b^2)^2 / (c * x^2 + b * x + a)^{(1/2)} * x * e^2 * d + 2 * b \\
& ^3 / c / (4 * a * c - b^2)^2 / (c * x^2 + b * x + a)^{(1/2)} * f * d^2 + 2 * b^3 / c / (4 * a * c - b^2)^2 / (c * x^2 + b \\
& * x + a)^{(1/2)} * e^2 * d + 2 * a / (4 * a * c - b^2) / (c * x^2 + b * x + a)^{(3/2)} * x * f * d^2 - 15/4 * e * f^2 * b^ \\
& 4 / c^4 / (4 * a * c - b^2) / (c * x^2 + b * x + a)^{(1/2)} + 3/2 * b / c^2 * x^2 / (c * x^2 + b * x + a)^{(3/2)} * d * f \\
& ^2 + 3/2 * b / c^2 * x^2 / (c * x^2 + b * x + a)^{(3/2)} * e^2 * f + 3/8 * b^2 / c^3 * x / (c * x^2 + b * x + a)^{(3/2)} \\
& ) * d * f^2 + 3/8 * b^2 / c^3 * x / (c * x^2 + b * x + a)^{(3/2)} * e^2 * f - 1/16 * b^5 / c^4 / (4 * a * c - b^2) / (c \\
& * x^2 + b * x + a)^{(3/2)} * d * f^2 - 1/16 * b^5 / c^4 / (4 * a * c - b^2) / (c * x^2 + b * x + a)^{(3/2)} * e^2 * f - \\
& 1/2 * b^5 / c^3 / (4 * a * c - b^2)^2 / (c * x^2 + b * x + a)^{(1/2)} * d * f^2 - 1/2 * b^5 / c^3 / (4 * a * c - b^2) \\
& ^2 / (c * x^2 + b * x + a)^{(1/2)} * e^2 * f + b / c^3 * a / (c * x^2 + b * x + a)^{(3/2)} * d * f^2 + b / c^3 * a / (c * x \\
& ^2 + b * x + a)^{(3/2)} * e^2 * f + 3/2 / c^3 * b^3 / (4 * a * c - b^2) / (c * x^2 + b * x + a)^{(1/2)} * d * f^2 + 3/2 \\
& / c^3 * b^3 / (4 * a * c - b^2) / (c * x^2 + b * x + a)^{(1/2)} * e^2 * f + 12 * e * f^2 * a / c^2 * x^2 / (c * x^2 + b * \\
& x + a)^{(3/2)} + 5/2 * e * f^2 * b / c^2 * x^3 / (c * x^2 + b * x + a)^{(3/2)} - 15/4 * e * f^2 * b^2 / c^3 * x^2 / ( \\
& c * x^2 + b * x + a)^{(3/2)} - 15/16 * e * f^2 * b^3 / c^4 * x / (c * x^2 + b * x + a)^{(3/2)} + 5/32 * e * f^2 * b^6 \\
& / c^5 / (4 * a * c - b^2) / (c * x^2 + b * x + a)^{(3/2)} + 5/4 * e * f^2 * b^6 / c^4 / (4 * a * c - b^2)^2 / (c * x^2 \\
& + b * x + a)^{(1/2)} - 3 * e * f^2 * b^2 / c^4 * a / (c * x^2 + b * x + a)^{(3/2)} + 15/2 * e * f^2 * b / c^3 * x / (c * x \\
& ^2 + b * x + a)^{(1/2)} - 35/24 * f^3 * b^6 / c^4 / (4 * a * c - b^2)^2 / (c * x^2 + b * x + a)^{(1/2)} * x - 33/4 * \\
& f^3 * b / c^3 * a * x^2 / (c * x^2 + b * x + a)^{(3/2)} - 33 * f^3 * b^3 / c^3 * a^2 / (4 * a * c - b^2)^2 / (c * x^2 \\
& + b * x + a)^{(1/2)} + 2 * a / (4 * a * c - b^2) / (c * x^2 + b * x + a)^{(3/2)} * x * e^2 * d + 8 * a / (4 * a * c - b^2)^2 \\
& / (c * x^2 + b * x + a)^{(1/2)} * b * f * d^2 + 8 * a / (4 * a * c - b^2)^2 / (c * x^2 + b * x + a)^{(1/2)} * b * e^2 * d - \\
& 2 * d^2 * e * b / (4 * a * c - b^2) / (c * x^2 + b * x + a)^{(3/2)} * x - d^2 * e * b^2 / c / (4 * a * c - b^2) / (c * x^2 + \\
& b * x + a)^{(3/2)} - 6 * x^2 / c / (c * x^2 + b * x + a)^{(3/2)} * d * e * f + 1/4 * b^2 / c^3 / (c * x^2 + b * x + a)^{(3 \\
& / 2)} * d * e * f + 1/12 * b^3 / c^2 / (4 * a * c - b^2) / (c * x^2 + b * x + a)^{(3/2)} * x * e^3 + 2/3 * b^3 / c / (4 * a \\
& * c - b^2)^2 / (c * x^2 + b * x + a)^{(1/2)} * x * e^3 - 1/2 * b^2 / c^2 * a / (4 * a * c - b^2) / (c * x^2 + b * x + a) \\
& )^{(3/2)} * e^3 - 8 * b * a / (4 * a * c - b^2)^2 / (c * x^2 + b * x + a)^{(1/2)} * x * e^3 - 4 * b^2 / c * a / (4 * a * c - b \\
& ^2)^2 / (c * x^2 + b * x + a)^{(1/2)} * e^3 - 4 * a / c^2 / (c * x^2 + b * x + a)^{(3/2)} * d * e * f - 16 * d^2 * e * b * \\
& c / (4 * a * c - b^2)^2 / (c * x^2 + b * x + a)^{(1/2)} * x - b^4 / c^2 / (4 * a * c - b^2)^2 / (c * x^2 + b * x + a)^{( \\
& 1/2)} * x * e^2 * f + 3/4 * b^3 / c^3 * a / (4 * a * c - b^2) / (c * x^2 + b * x + a)^{(3/2)} * d * f^2 + 3/4 * b^3 / c^ \\
& 3 * a / (4 * a * c - b^2) / (c * x^2 + b * x + a)^{(3/2)} * e^2 * f + 6 * b^3 / c^2 * a / (4 * a * c - b^2)^2 / (c * x^2 + \\
& b * x + a)^{(1/2)} * d * f^2 + 6 * b^3 / c^2 * a / (4 * a * c - b^2)^2 / (c * x^2 + b * x + a)^{(1/2)} * e^2 * f + 3/c^ \\
& 2 * b^2 / (4 * a * c - b^2) / (c * x^2 + b * x + a)^{(1/2)} * x * d * f^2 - b^4 / c^2 / (4 * a * c - b^2)^2 / (c * x^2 +
\end{aligned}$$

$$\begin{aligned}
& b*x+a)^{(1/2)}*x*d*f^2-3/2*b/c^2*x/(c*x^2+b*x+a)^{(3/2)}*d*e*f+1/4*b^4/c^3/(4*a \\
& *c-b^2)/(c*x^2+b*x+a)^{(3/2)}*d*e*f+2*b^4/c^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/ \\
& 2)}*d*e*f-b/c*a/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)}*x*e^3+5/2*e*f^2*b^5/c^3/(4*a \\
& *c-b^2)^2/(c*x^2+b*x+a)^{(1/2)}*x+48*e*f^2*a^2/c^2*b^2/(4*a*c-b^2)^2/(c*x^2+b \\
& *x+a)^{(1/2)}+3*e*f^2*a/c^3*b*x/(c*x^2+b*x+a)^{(3/2)}+6*e*f^2*a^2/c^3*b^2/(4*a* \\
& c-b^2)/(c*x^2+b*x+a)^{(3/2)}-19/8*e*f^2*b^4/c^4*a/(4*a*c-b^2)/(c*x^2+b*x+a)^{( \\
& 3/2)}-19*e*f^2*b^4/c^3*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)}-15/2*e*f^2*b^3/c^ \\
& 3/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x+5/16*e*f^2*b^5/c^4/(4*a*c-b^2)/(c*x^2+b \\
& *x+a)^{(3/2)}*x+23/8*f^3*b^4/c^4*a/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)}*x+23*f^3*b \\
& ^4/c^3*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)}*x+1/2*b^2/c/(4*a*c-b^2)/(c*x^2+b \\
& *x+a)^{(3/2)}*x*e^2*d+a/c/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)}*b*f*d^2+a/c/(4*a*c- \\
& b^2)/(c*x^2+b*x+a)^{(3/2)}*b*e^2*d+16*a*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)}*x \\
& *f*d^2+16*a*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)}*x*e^2*d+3/c^2*b^2/(4*a*c-b^ \\
& 2)/(c*x^2+b*x+a)^{(1/2)}*x*e^2*f-1/8*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)}* \\
& x*d*f^2+1/2*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)}*x*f*d^2-5/4*f^3*a/c^4*b/( \\
& c*x^2+b*x+a)^{(1/2)}+173/96*f^3*b^3/c^5*a/(c*x^2+b*x+a)^{(3/2)}-35/8*f^3*b^2/c^ \\
& 4*x/(c*x^2+b*x+a)^{(1/2)}+35/16*f^3*b^5/c^5/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}-1 \\
& 1/2*f^3*b/c^4*a^2/(c*x^2+b*x+a)^{(3/2)}-7/4*f^3*b/c^2*x^4/(c*x^2+b*x+a)^{(3/2)} \\
& -35/24*f^3*b^2/c^3*x^3/(c*x^2+b*x+a)^{(3/2)}+35/16*f^3*b^3/c^4*x^2/(c*x^2+b*x \\
& +a)^{(3/2)}+35/64*f^3*b^4/c^5*x/(c*x^2+b*x+a)^{(3/2)}-35/384*f^3*b^7/c^6/(4*a*c \\
& -b^2)/(c*x^2+b*x+a)^{(3/2)}-35/48*f^3*b^7/c^5/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/ \\
& 2)}+8*e*f^2*a^2/c^3/(c*x^2+b*x+a)^{(3/2)}-15/2*e*f^2*b/c^(7/2)*ln((c*x+1/2*b)/ \\
& c^(1/2)+(c*x^2+b*x+a)^(1/2))+3*e*f^2*x^4/c/(c*x^2+b*x+a)^{(3/2)}+5/32*e*f^2*b \\
& ^4/c^5/(c*x^2+b*x+a)^{(3/2)}-15/4*e*f^2*b^2/c^4/(c*x^2+b*x+a)^{(1/2)}-x^3/c/(c* \\
& x^2+b*x+a)^{(3/2)}*d*f^2-x^3/c/(c*x^2+b*x+a)^{(3/2)}*e^2*f-1/16*b^3/c^4/(c*x^2+ \\
& b*x+a)^{(3/2)}*d*f^2-1/16*b^3/c^4/(c*x^2+b*x+a)^{(3/2)}*e^2*f-3/c^2*x/(c*x^2+b* \\
& x+a)^{(1/2)}*d*f^2-3/c^2*x/(c*x^2+b*x+a)^{(1/2)}*e^2*f+3/2/c^3*b/(c*x^2+b*x+a)^ \\
& (1/2)*d*f^2+3/2/c^3*b/(c*x^2+b*x+a)^{(1/2)}*e^2*f-1/4*b/c^2*x/(c*x^2+b*x+a)^{( \\
& 3/2)}*e^3+1/24*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)}*e^3+1/3*b^4/c^2/(4*a* \\
& c-b^2)^2/(c*x^2+b*x+a)^{(1/2)}*e^3-3/2*x/c/(c*x^2+b*x+a)^{(3/2)}*f*d^2-3/2*x/c/ \\
& (c*x^2+b*x+a)^{(3/2)}*e^2*d+1/4*b/c^2/(c*x^2+b*x+a)^{(3/2)}*f*d^2+1/4*b/c^2/(c* \\
& x^2+b*x+a)^{(3/2)}*e^2*d-8*d^2*e*b^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)}+4/3*d^ \\
& 3/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)}*x*c+32/3*d^3*c^2/(4*a*c-b^2)^2/(c*x^2+b*x \\
& +a)^{(1/2)}*x+16/3*d^3*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)}*b
\end{aligned}$$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)^3/(c\*x^2+b\*x+a)^(5/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation \*may\* h

elp (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details) Is  $4ac - b^2$  zero or nonzero?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(fx^2 + ex + d)^3}{(cx^2 + bx + a)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x + f\*x^2)^3/(a + b\*x + c\*x^2)^(5/2), x)

[Out] int((d + e\*x + f\*x^2)^3/(a + b\*x + c\*x^2)^(5/2), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x\*\*2+e\*x+d)\*\*3/(c\*x\*\*2+b\*x+a)\*\*(5/2), x)

[Out] Timed out

$$3.119 \quad \int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{5/2}} dx$$

**Optimal.** Leaf size=444

$$\frac{2(-x(c^2(2a^2f^2 + 6abef + b^2(2df + e^2)) - 2b^2cf(2af + be) - 2c^3(a(2df + e^2) + bde) + b^4f^2 + 2c^4d^2) - bc(-3c^3(b^2 - 4ac)(a + bx + cx^2)^{3/2})}{3c^3(b^2 - 4ac)(a + bx + cx^2)^{3/2}}$$

[Out] 2/3\*(2\*a\*b^2\*c\*e\*f-a\*b^3\*f^2+4\*a\*c^2\*e\*(-a\*f+c\*d)-b\*c\*(c^2\*d^2-3\*a^2\*f^2+a\*c\*(2\*d\*f+e^2))-(2\*c^4\*d^2+b^4\*f^2-2\*b^2\*c\*f\*(2\*a\*f+b\*e)-2\*c^3\*(b\*d\*e+a\*(2\*d\*f+e^2))+c^2\*(6\*a\*b\*e\*f+2\*a^2\*f^2+b^2\*(2\*d\*f+e^2)))\*x)/c^3/(-4\*a\*c+b^2)/(c\*x^2+b\*x+a)^(3/2)+f^2\*arctanh(1/2\*(2\*c\*x+b)/c^(1/2)/(c\*x^2+b\*x+a)^(1/2))/c^(5/2)-2/3\*(2\*b^4\*c\*e\*f+48\*a^2\*c^3\*e\*f-b^5\*f^2+4\*b^2\*c^2\*e\*(-3\*a\*f+2\*c\*d)+b^3\*c\*(10\*a\*f^2-c\*(2\*d\*f+e^2))-4\*b\*c^2\*(2\*c^2\*d^2+8\*a^2\*f^2+a\*c\*(2\*d\*f+e^2))-2\*c\*(8\*c^4\*d^2-2\*b^4\*f^2+b^2\*c\*f\*(14\*a\*f+b\*e)-c^3\*(8\*b\*d\*e-4\*a\*(2\*d\*f+e^2))-c^2\*(12\*a\*b\*e\*f+16\*a^2\*f^2-b^2\*(2\*d\*f+e^2)))\*x)/c^3/(-4\*a\*c+b^2)^2/(c\*x^2+b\*x+a)^(1/2)

**Rubi [A]** time = 0.45, antiderivative size = 444, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$ , Rules used = {1660, 12, 621, 206}

$$\frac{2(-2cx(-c^2(16a^2f^2 + 12abef + b^2(-(2df + e^2)))) + b^2cf(14af + be) - c^3(8bde - 4a(2df + e^2)) - 2b^4f^2 + 3c^3(b^2 - 4ac)(a + bx + cx^2)^{3/2})}{3c^3(b^2 - 4ac)(a + bx + cx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x + f\*x^2)^2/(a + b\*x + c\*x^2)^(5/2), x]

[Out] (2\*(2\*a\*b^2\*c\*e\*f - a\*b^3\*f^2 + 4\*a\*c^2\*e\*(c\*d - a\*f) - b\*c\*(c^2\*d^2 - 3\*a^2\*f^2 + a\*c\*(e^2 + 2\*d\*f)) - (2\*c^4\*d^2 + b^4\*f^2 - 2\*b^2\*c\*f\*(b\*e + 2\*a\*f) - 2\*c^3\*(b\*d\*e + a\*(e^2 + 2\*d\*f)) + c^2\*(6\*a\*b\*e\*f + 2\*a^2\*f^2 + b^2\*(e^2 + 2\*d\*f)))\*x)/(3\*c^3\*(b^2 - 4\*a\*c)\*(a + b\*x + c\*x^2)^(3/2)) - (2\*(2\*b^4\*c\*e\*f + 48\*a^2\*c^3\*e\*f - b^5\*f^2 + 4\*b^2\*c^2\*e\*(2\*c\*d - 3\*a\*f) + b^3\*c\*(10\*a\*f^2 - c\*(e^2 + 2\*d\*f)) - 4\*b\*c^2\*(2\*c^2\*d^2 + 8\*a^2\*f^2 + a\*c\*(e^2 + 2\*d\*f)) - 2\*c\*(8\*c^4\*d^2 - 2\*b^4\*f^2 + b^2\*c\*f\*(b\*e + 14\*a\*f) - c^3\*(8\*b\*d\*e - 4\*a\*(e^2 + 2\*d\*f)) - c^2\*(12\*a\*b\*e\*f + 16\*a^2\*f^2 - b^2\*(e^2 + 2\*d\*f)))\*x)/(3\*c^3\*(b^2 - 4\*a\*c)^2\*sqrt[a + b\*x + c\*x^2]) + (f^2\*ArcTanh[(b + 2\*c\*x)/(2\*sqrt[c]\*sqrt[a + b\*x + c\*x^2])])/c^(5/2)

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]
```

### Rubi steps



$$\begin{aligned}
\int \frac{(d + ex + fx^2)^2}{(a + bx + cx^2)^{5/2}} dx &= \frac{2(2ab^2cef - ab^3f^2 + 4ac^2e(cd - af) - bc(c^2d^2 - 3a^2f^2 + ac(e^2 + 2df)) - (2c^4d^2 + b^3d^2 + 2ab^2cd^2))}{3c^3(b^2 - 4ac)} \\
&= \frac{2(2ab^2cef - ab^3f^2 + 4ac^2e(cd - af) - bc(c^2d^2 - 3a^2f^2 + ac(e^2 + 2df)) - (2c^4d^2 + b^3d^2 + 2ab^2cd^2))}{3c^3(b^2 - 4ac)} \\
&= \frac{2(2ab^2cef - ab^3f^2 + 4ac^2e(cd - af) - bc(c^2d^2 - 3a^2f^2 + ac(e^2 + 2df)) - (2c^4d^2 + b^3d^2 + 2ab^2cd^2))}{3c^3(b^2 - 4ac)} \\
&= \frac{2(2ab^2cef - ab^3f^2 + 4ac^2e(cd - af) - bc(c^2d^2 - 3a^2f^2 + ac(e^2 + 2df)) - (2c^4d^2 + b^3d^2 + 2ab^2cd^2))}{3c^3(b^2 - 4ac)} \\
&= \frac{2(2ab^2cef - ab^3f^2 + 4ac^2e(cd - af) - bc(c^2d^2 - 3a^2f^2 + ac(e^2 + 2df)) - (2c^4d^2 + b^3d^2 + 2ab^2cd^2))}{3c^3(b^2 - 4ac)}
\end{aligned}$$

**Mathematica [A]** time = 1.26, size = 387, normalized size = 0.87

$$\frac{2(b^3(-3a^2f^2 + 18acf^2x^2 + c^2(-d^2 + 6dx(fx - e) + ex^2(3e + 2fx))) + 2b^2c(21a^2f^2x - 2ac(d(e - 6fx) + x(-3e + 2fx))))}{(a + bx + cx^2)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x + f\*x^2)^2/(a + b\*x + c\*x^2)^(5/2), x]

[Out] (2\*(-3\*b^5\*f^2\*x^2 - 2\*b^4\*f^2\*x\*(3\*a + 2\*c\*x^2) + 4\*b\*c\*(5\*a^3\*f^2 + 2\*c^3\*d\*x^2\*(3\*d - 2\*e\*x) + 2\*a^2\*c\*(e^2 + 2\*d\*f - 6\*e\*f\*x) + 3\*a\*c^2\*(d - e\*x)\*(d + x\*(-e + 2\*f\*x))) + b^3\*(-3\*a^2\*f^2 + 18\*a\*c\*f^2\*x^2 + c^2\*(-d^2 + 6\*d\*x\*(-e + f\*x) + e\*x^2\*(3\*e + 2\*f\*x))) + 8\*c^2\*(2\*c^3\*d^2\*x^3 - a^3\*f\*(4\*e + 3\*f\*x) + a\*c^2\*x\*(3\*d^2 + e^2\*x^2 + 2\*d\*f\*x^2) - 2\*a^2\*c\*(d\*e + f\*x^2\*(3\*e + 2\*f\*x))) + 2\*b^2\*c\*(21\*a^2\*f^2\*x + c^2\*x\*(3\*d^2 + e^2\*x^2 + 2\*d\*x\*(-6\*e + f\*x)) - 2\*a\*c\*(d\*(e - 6\*f\*x) + x\*(-3\*e^2 + 3\*e\*f\*x - 7\*f^2\*x^2))))/(3\*c^2\*(b^2 - 4\*a\*c)^2\*(a + x\*(b + c\*x))^(3/2)) + (f^2\*Log[b + 2\*c\*x + 2\*Sqrt[c]\*Sqrt[a + x\*(b + c\*x)])]/c^(5/2))

**fricas [A]** time = 6.52, size = 1581, normalized size = 3.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)^2/(c\*x^2+b\*x+a)^(5/2),x, algorithm="fricas")

[Out] [1/6\*(3\*((b^4\*c^2 - 8\*a\*b^2\*c^3 + 16\*a^2\*c^4)\*f^2\*x^4 + 2\*(b^5\*c - 8\*a\*b^3\*c^2 + 16\*a^2\*b\*c^3)\*f^2\*x^3 + (b^6 - 6\*a\*b^4\*c + 32\*a^3\*c^3)\*f^2\*x^2 + 2\*(a\*b^5 - 8\*a^2\*b^3\*c + 16\*a^3\*b\*c^2)\*f^2\*x + (a^2\*b^4 - 8\*a^3\*b^2\*c + 16\*a^4\*c^2)\*f^2)\*sqrt(c)\*log(-8\*c^2\*x^2 - 8\*b\*c\*x - b^2 - 4\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(c) - 4\*a\*c) + 4\*(8\*a^2\*b\*c^3\*e^2 + 2\*(8\*c^6\*d^2 - 8\*b\*c^5\*d\*e + (b^2\*c^4 + 4\*a\*c^5)\*e^2 - 2\*(b^4\*c^2 - 7\*a\*b^2\*c^3 + 8\*a^2\*c^4)\*f^2 + (2\*(b^2\*c^4 + 4\*a\*c^5)\*d + (b^3\*c^3 - 12\*a\*b\*c^4)\*e)\*f)\*x^3 - (b^3\*c^3 - 12\*a\*b\*c^4)\*d^2 - 4\*(a\*b^2\*c^3 + 4\*a^2\*c^4)\*d\*e - (3\*a^2\*b^3\*c - 20\*a^3\*b\*c^2)\*f^2 + 3\*(8\*b\*c^5\*d^2 - 8\*b^2\*c^4\*d\*e + (b^3\*c^3 + 4\*a\*b\*c^4)\*e^2 - (b^5\*c - 6\*a\*b^3\*c^2)\*f^2 + 2\*((b^3\*c^3 + 4\*a\*b\*c^4)\*d - 2\*(a\*b^2\*c^3 + 4\*a^2\*c^4)\*e)\*f)\*x^2 + 16\*(a^2\*b\*c^3\*d - 2\*a^3\*c^3\*e)\*f + 6\*(2\*a\*b^2\*c^3\*e^2 + (b^2\*c^4 + 4\*a\*c^5)\*d^2 - (b^3\*c^3 + 4\*a\*b\*c^4)\*d\*e - (a\*b^4\*c - 7\*a^2\*b^2\*c^2 + 4\*a^3\*c^3)\*f^2 + 4\*(a\*b^2\*c^3\*d - 2\*a^2\*b\*c^3\*e)\*f)\*x)\*sqrt(c\*x^2 + b\*x + a))/(a^2\*b^4\*c^3 - 8\*a^3\*b^2\*c^4 + 16\*a^4\*c^5 + (b^4\*c^5 - 8\*a\*b^2\*c^6 + 16\*a^2\*c^7)\*x^4 + 2\*(b^5\*c^4 - 8\*a\*b^3\*c^5 + 16\*a^2\*b\*c^6)\*x^3 + (b^6\*c^3 - 6\*a\*b^4\*c^4 + 32\*a^3\*c^6)\*x^2 + 2\*(a\*b^5\*c^3 - 8\*a^2\*b^3\*c^4 + 16\*a^3\*b\*c^5)\*x), -1/3\*(3\*((b^4\*c^2 - 8\*a\*b^2\*c^3 + 16\*a^2\*c^4)\*f^2\*x^4 + 2\*(b^5\*c - 8\*a\*b^3\*c^2 + 16\*a^2\*b\*c^3)\*f^2\*x^3 + (b^6 - 6\*a\*b^4\*c + 32\*a^3\*c^3)\*f^2\*x^2 + 2\*(a\*b^5 - 8\*a^2\*b^3\*c + 16\*a^3\*b\*c^2)\*f^2\*x + (a^2\*b^4 - 8\*a^3\*b^2\*c + 16\*a^4\*c^2)\*f^2)\*sqrt(-c)\*arctan(1/2\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(-c)/(c^2\*x^2 + b\*c\*x + a\*c)) - 2\*(8\*a^2\*b\*c^3\*e^2 + 2\*(8\*c^6\*d^2 - 8\*b\*c^5\*d\*e + (b^2\*c^4 + 4\*a\*c^5)\*e^2 - 2\*(b^4\*c^2 - 7\*a\*b^2\*c^3 + 8\*a^2\*c^4)\*f^2 + (2\*(b^2\*c^4 + 4\*a\*c^5)\*d + (b^3\*c^3 - 12\*a\*b\*c^4)\*e)\*f)\*x^3 - (b^3\*c^3 - 12\*a\*b\*c^4)\*d^2 - 4\*(a\*b^2\*c^3 + 4\*a^2\*c^4)\*d\*e - (3\*a^2\*b^3\*c - 20\*a^3\*b\*c^2)\*f^2 + 3\*(8\*b\*c^5\*d^2 - 8\*b^2\*c^4\*d\*e + (b^3\*c^3 + 4\*a\*b\*c^4)\*e^2 - (b^5\*c - 6\*a\*b^3\*c^2)\*f^2 + 2\*((b^3\*c^3 + 4\*a\*b\*c^4)\*d - 2\*(a\*b^2\*c^3 + 4\*a^2\*c^4)\*e)\*f)\*x^2 + 16\*(a^2\*b\*c^3\*d - 2\*a^3\*c^3\*e)\*f + 6\*(2\*a\*b^2\*c^3\*e^2 + (b^2\*c^4 + 4\*a\*c^5)\*d^2 - (b^3\*c^3 + 4\*a\*b\*c^4)\*d\*e - (a\*b^4\*c - 7\*a^2\*b^2\*c^2 + 4\*a^3\*c^3)\*f^2 + 4\*(a\*b^2\*c^3\*d - 2\*a^2\*b\*c^3\*e)\*f)\*x)\*sqrt(c\*x^2 + b\*x + a))/(a^2\*b^4\*c^3 - 8\*a^3\*b^2\*c^4 + 16\*a^4\*c^5 + (b^4\*c^5 - 8\*a\*b^2\*c^6 + 16\*a^2\*c^7)\*x^4 + 2\*(b^5\*c^4 - 8\*a\*b^3\*c^5 + 16\*a^2\*b\*c^6)\*x^3 + (b^6\*c^3 - 6\*a\*b^4\*c^4 + 32\*a^3\*c^6)\*x^2 + 2\*(a\*b^5\*c^3 - 8\*a^2\*b^3\*c^4 + 16\*a^3\*b\*c^5)\*x)]

**giac** [A] time = 0.34, size = 587, normalized size = 1.32

$$\frac{f^2 \log \left( \left| -2 \left( \sqrt{c} x - \sqrt{c x^2 + b x + a} \right) \sqrt{c} - b \right| \right)}{c^{\frac{5}{2}}} + \frac{2 \left( \left( \frac{2(8c^5d^2 + 2b^2c^3df + 8ac^4df - 2b^4cf^2 + 14ab^2c^2f^2 - 16a^2c^3f^2 - 8bc^4de + b^3c^2fe - b^4c^2 - 8ab^2c^3 + 16a^2c^4}{b^4c^2 - 8ab^2c^3 + 16a^2c^4} \right) \right)}{b^4c^2 - 8ab^2c^3 + 16a^2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)^2/(c\*x^2+b\*x+a)^(5/2),x, algorithm="giac")

[Out] 
$$-f^2 \log(\text{abs}(-2 \cdot (\sqrt{c})x - \sqrt{c x^2 + b x + a}) \sqrt{c} - b) / c^{5/2} + 2/3 \cdot (((2 \cdot (8 c^5 d^2 + 2 b^2 c^3 d f + 8 a c^4 d f - 2 b^4 c f^2 + 14 a b^2 c^2 f^2 - 16 a^2 c^3 f^2 - 8 b c^4 d e + b^3 c^2 f e - 12 a b c^3 f e + b^2 c^3 e^2 + 4 a c^4 e^2) x / (b^4 c^2 - 8 a b^2 c^3 + 16 a^2 c^4) + 3 \cdot (8 b c^4 d^2 + 2 b^3 c^2 d f + 8 a b c^3 d f - b^5 f^2 + 6 a b^3 c f^2 - 8 b^2 c^3 d e - 4 a b^2 c^2 f e - 16 a^2 c^3 f e + b^3 c^2 e^2 + 4 a b c^3 e^2) / (b^4 c^2 - 8 a b^2 c^3 + 16 a^2 c^4)) x + 6 \cdot (b^2 c^3 d^2 + 4 a c^4 d^2 + 4 a b^2 c^2 d f - a b^4 f^2 + 7 a^2 b^2 c f^2 - 4 a^3 c^2 f^2 - b^3 c^2 d e - 4 a b c^3 d e - 8 a^2 b c^2 f e + 2 a b^2 c^2 e^2) / (b^4 c^2 - 8 a b^2 c^3 + 16 a^2 c^4)) x - (b^3 c^2 d^2 - 12 a b c^3 d^2 - 16 a^2 b c^2 d f + 3 a^2 b^3 f^2 - 20 a^3 b c f^2 + 4 a b^2 c^2 d e + 16 a^2 c^3 d e + 32 a^3 c^2 f e - 8 a^2 b c^2 e^2) / (b^4 c^2 - 8 a b^2 c^3 + 16 a^2 c^4)) / (c x^2 + b x + a)^{3/2}$$

**maple [B]** time = 0.01, size = 1786, normalized size = 4.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x^2+e\*x+d)^2/(c\*x^2+b\*x+a)^(5/2),x)

[Out] 
$$-2/3 d e / c / (c x^2 + b x + a)^{3/2} + 2/3 d^2 / (4 a c - b^2) / (c x^2 + b x + a)^{3/2} * b - 1/2 x / c / (c x^2 + b x + a)^{3/2} * e^2 + 1/12 b / c^2 / (c x^2 + b x + a)^{3/2} * e^2 - 1/3 f^2 x^3 / c / (c x^2 + b x + a)^{3/2} - 1/48 f^2 b^3 / c^4 / (c x^2 + b x + a)^{3/2} - f^2 / c^2 x / (c x^2 + b x + a)^{1/2} + 1/2 f^2 / c^3 b / (c x^2 + b x + a)^{1/2} - 2 e f b / c a / (4 a c - b^2) / (c x^2 + b x + a)^{3/2} * x + 16/3 a / (4 a c - b^2)^2 / (c x^2 + b x + a)^{1/2} * b d f - 1/24 f^2 b^4 / c^3 / (4 a c - b^2) / (c x^2 + b x + a)^{3/2} * x - 1/3 f^2 b^4 / c^2 / (4 a c - b^2)^2 / (c x^2 + b x + a)^{1/2} * x + 1/4 f^2 b^3 / c^3 a / (4 a c - b^2) / (c x^2 + b x + a)^{3/2} + 2 f^2 b^3 / c^2 a / (4 a c - b^2)^2 / (c x^2 + b x + a)^{1/2} + f^2 / c^2 b^2 / (4 a c - b^2) / (c x^2 + b x + a)^{1/2} * x - 4/3 d e b / (4 a c - b^2) / (c x^2 + b x + a)^{3/2} * x - 2/3 d e b^2 / c / (4 a c - b^2) / (c x^2 + b x + a)^{3/2} - 1/2 e f b / c^2 x / (c x^2 + b x + a)^{3/2} + 1/12 e f b^4 / c^3 / (4 a c - b^2) / (c x^2 + b x + a)^{3/2} + 2/3 e f b^4 / c^2 / (4 a c - b^2)^2 / (c x^2 + b x + a)^{1/2} + 1/6 b^2 / c / (4 a c - b^2) / (c x^2 + b x + a)^{3/2} * x * e^2 + 1/6 b^3 / c^2 / (4 a c - b^2) / (c x^2 + b x + a)^{3/2} * d f + 8/3 b^2 / (4 a c - b^2)^2 / (c x^2 + b x + a)^{1/2} * x * d f + 4/3 b^3 / c / (4 a c - b^2)^2 / (c x^2 + b x + a)^{1/2} * d f + 4/3 a / (4 a c - b^2) / (c x^2 + b x + a)^{3/2} * x * d f + 1/3 a / c / (4 a c - b^2) / (c x^2 + b x + a)^{3/2} * b * e^2 + 1/6/3 a c / (4 a c - b^2)^2 / (c x^2 + b x + a)^{1/2} * x * e^2 + f^2 / c^{5/2} * \ln((c x + 1/2 b) / c^{1/2} + (c x^2 + b x + a)^{1/2}) - 2 e f x^2 / c / (c x^2 + b x + a)^{3/2} + 1/12 e f b^2 / c^3 / (c x^2 + b x + a)^{3/2} - 4/3 e f a / c^2 / (c x^2 + b x + a)^{3/2} + 1/2 f^2 b / c^2 x^2 / (c x^2 + b x + a)^{3/2} + 1/8 f^2 b^2 / c^3 x / (c x^2 + b x + a)^{3/2} - 1/48 f^2 b^5 / c^4 / (4 a c - b^2) / (c x^2 + b x + a)^{3/2} - 1/6 f^2 b^5 / c^3 / (4 a c - b^2)^2 / (c x^2 + b x + a)^{1/2} + 1/3 f^2 b / c^3 a / (c x^2 + b x + a)^{3/2} + 1/2 f^2 / c^3 b^3 / (4 a c - b^2) / (c x^2 + b x + a)^{1/2} + 2/3 b^3 / c / (4 a c - b^2)^2 / (c x^2 + b x + a)^{1/2} * e^2 + 2/3 a / (4 a c$$

$$\frac{c-b^2}{(c*x^2+b*x+a)^{3/2}}*x*e^2+8/3*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{1/2}*b*e^2-16/3*d*e*b^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{1/2}+4/3*d^2/(4*a*c-b^2)/(c*x^2+b*x+a)^{3/2}*x*c+32/3*d^2*c^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{1/2}*x+16/3*d^2*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{1/2}*b-x/c/(c*x^2+b*x+a)^{3/2}*d*f+1/6*b/c^2/(c*x^2+b*x+a)^{3/2}*d*f+32/3*a*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{1/2}*x*d*f-32/3*d*e*b*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{1/2}*x-16*e*f*b*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{1/2}*x-8*e*f*b^2/c*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{1/2}+1/6*e*f*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^{3/2}*x+1/2*f^2*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^{3/2}*x+4*f^2*b^2/c*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{1/2}*x+1/3*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^{3/2}*x*d*f+2/3*a/c/(4*a*c-b^2)/(c*x^2+b*x+a)^{3/2}*b*d*f+1/12*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^{3/2}*e^2+4/3*b^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{1/2}*x*e^2+4/3*e*f*b^3/c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{1/2}*x-e*f*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^{3/2}$$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)^2/(c\*x^2+b\*x+a)^(5/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details)Is 4\*a\*c-b^2 zero or nonzero?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f x^2 + e x + d)^2}{(c x^2 + b x + a)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x + f\*x^2)^2/(a + b\*x + c\*x^2)^(5/2),x)

[Out] int((d + e\*x + f\*x^2)^2/(a + b\*x + c\*x^2)^(5/2), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x\*\*2+e\*x+d)\*\*2/(c\*x\*\*2+b\*x+a)\*\*(5/2),x)

[Out] Timed out

$$3.120 \quad \int \frac{d+ex+fx^2}{(a+bx+cx^2)^{5/2}} dx$$

**Optimal.** Leaf size=131

$$\frac{2\left(c\left(2ae - b\left(\frac{af}{c} + d\right)\right) - x\left(-2acf + b^2f - bce + 2c^2d\right)\right)}{3c(b^2 - 4ac)(a + bx + cx^2)^{3/2}} + \frac{2(b + 2cx)\left(4af + \frac{b^2f}{c} - 4be + 8cd\right)}{3(b^2 - 4ac)^2 \sqrt{a + bx + cx^2}}$$

[Out]  $2/3*(c*(2*a*e - b*(d + a*f/c)) - (-2*a*c*f + b^2*f - b*c*e + 2*c^2*d)*x)/c/(-4*a*c + b^2)/((c*x^2 + b*x + a)^(3/2)) + 2/3*(8*c*d - 4*b*e + 4*a*f + b^2*f/c)*(2*c*x + b)/(-4*a*c + b^2)^2/(c*x^2 + b*x + a)^(1/2)$

**Rubi [A]** time = 0.09, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$ , Rules used = {1660, 12, 613}

$$\frac{2\left(c\left(2ae - b\left(\frac{af}{c} + d\right)\right) - x\left(-2acf + b^2f - bce + 2c^2d\right)\right)}{3c(b^2 - 4ac)(a + bx + cx^2)^{3/2}} + \frac{2(b + 2cx)\left(4af + \frac{b^2f}{c} - 4be + 8cd\right)}{3(b^2 - 4ac)^2 \sqrt{a + bx + cx^2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x + f\*x^2)/(a + b\*x + c\*x^2)^(5/2), x]

[Out]  $(2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x))/((3*c*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(3/2)) + (2*(8*c*d - 4*b*e + 4*a*f + (b^2*f)/c)*(b + 2*c*x)))/(3*(b^2 - 4*a*c)^2*sqrt[a + b*x + c*x^2])$

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 613

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-3/2), x\_Symbol] :> Simp[(-2\*(b + 2\*c\*x))/((b^2 - 4\*a\*c)\*sqrt[a + b\*x + c\*x^2]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 1660

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x +

$c*x^2, x], x, 1] \}, \text{Simp}[(b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + \text{Dist}[1/((p + 1)*(b^2 - 4*a*c)), \text{Int}[(a + b*x + c*x^2)^(p + 1)*\text{ExpandToSum}[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x]] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{LtQ}[p, -1]$

### Rubi steps

$$\begin{aligned} \int \frac{d + ex + fx^2}{(a + bx + cx^2)^{5/2}} dx &= \frac{2 \left( c \left( 2ae - b \left( d + \frac{af}{c} \right) \right) - (2c^2d - bce + b^2f - 2acf) x \right)}{3c (b^2 - 4ac) (a + bx + cx^2)^{3/2}} - \frac{2 \int \frac{8cd - 4be + 4af + \frac{b^2f}{c}}{2(a + bx + cx^2)^{3/2}} dx}{3(b^2 - 4ac)} \\ &= \frac{2 \left( c \left( 2ae - b \left( d + \frac{af}{c} \right) \right) - (2c^2d - bce + b^2f - 2acf) x \right)}{3c (b^2 - 4ac) (a + bx + cx^2)^{3/2}} - \frac{\left( 8cd - 4be + 4af + \frac{b^2f}{c} \right) \int \frac{1}{(a + bx + cx^2)^{3/2}} dx}{3(b^2 - 4ac)} \\ &= \frac{2 \left( c \left( 2ae - b \left( d + \frac{af}{c} \right) \right) - (2c^2d - bce + b^2f - 2acf) x \right)}{3c (b^2 - 4ac) (a + bx + cx^2)^{3/2}} + \frac{2 \left( 8cd - 4be + 4af + \frac{b^2f}{c} \right) (b + cx)}{3(b^2 - 4ac)^2 \sqrt{a + bx + cx^2}} \end{aligned}$$

**Mathematica [A]** time = 0.38, size = 147, normalized size = 1.12

$$\frac{8b(2a^2f + 3ac(d - ex + fx^2) - 2c^2x^2(ex - 3d)) + 16c(-a^2e + acx(3d + fx^2) + 2c^2dx^3) - 4b^2(a(e - 6fx) - cx^2)}{3(b^2 - 4ac)^2(a + x(b + cx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x + f\*x^2)/(a + b\*x + c\*x^2)^(5/2), x]

[Out] (-2\*b^3\*(d + 3\*x\*(e - f\*x)) + 16\*c\*(-(a^2\*e) + 2\*c^2\*d\*x^3 + a\*c\*x\*(3\*d + f\*x^2)) - 4\*b^2\*(a\*(e - 6\*f\*x) - c\*x\*(3\*d - 6\*e\*x + f\*x^2)) + 8\*b\*(2\*a^2\*f - 2\*c^2\*x^2\*(-3\*d + e\*x) + 3\*a\*c\*(d - e\*x + f\*x^2)))/(3\*(b^2 - 4\*a\*c)^2\*(a + x\*(b + c\*x))^(3/2))

**fricas [B]** time = 4.02, size = 286, normalized size = 2.18

$$\frac{2(8a^2bf + 2(8c^3d - 4bc^2e + (b^2c + 4ac^2)f)x^3 + 3(8bc^2d - 4b^2ce + (b^3 + 4abc)f)x^2 - (b^3 - 12abc)d - 2(ab^2c^2 - 2ac^3))}{3(a^2b^4 - 8a^3b^2c + 16a^4c^2 + (b^4c^2 - 8ab^2c^3 + 16a^2c^4)x^4 + 2(b^5c - 8ab^3c^2 + 16a^2bc^3)x^3 + (b^6 - 12abc^2 + 16a^2c^3)x^2 - (b^7 - 12abc^2 + 16a^2c^3)x - b^8 + 16abc^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(5/2),x, algorithm="fricas")

[Out]  $\frac{2}{3}*(8*a^2*b*f + 2*(8*c^3*d - 4*b*c^2*e + (b^2*c + 4*a*c^2)*f)*x^3 + 3*(8*b*c^2*d - 4*b^2*c*e + (b^3 + 4*a*b*c)*f)*x^2 - (b^3 - 12*a*b*c)*d - 2*(a*b^2 + 4*a^2*c)*e + 3*(4*a*b^2*f + 2*(b^2*c + 4*a*c^2)*d - (b^3 + 4*a*b*c)*e)*x)*\sqrt{c*x^2 + b*x + a}/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x)$

**giac** [A] time = 0.32, size = 240, normalized size = 1.83

$$\frac{2 \left( \left( \frac{2(8c^3d + b^2cf + 4ac^2f - 4bc^2e)x}{b^4 - 8ab^2c + 16a^2c^2} + \frac{3(8bc^2d + b^3f + 4abcf - 4b^2ce)}{b^4 - 8ab^2c + 16a^2c^2} \right) x + \frac{3(2b^2cd + 8ac^2d + 4ab^2f - b^3e - 4abce)}{b^4 - 8ab^2c + 16a^2c^2} \right) x - \frac{b^3d - 12abcd - 8a^2bf + 2a^3c^2}{b^4 - 8ab^2c + 16a^2c^2}}{3(cx^2 + bx + a)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(5/2),x, algorithm="giac")

[Out]  $\frac{2}{3}*((2*(8*c^3*d + b^2*c*f + 4*a*c^2*f - 4*b*c^2*e)*x/(b^4 - 8*a*b^2*c + 16*a^2*c^2) + 3*(8*b*c^2*d + b^3*f + 4*a*b*c*f - 4*b^2*c*e)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))*x + 3*(2*b^2*c*d + 8*a*c^2*d + 4*a*b^2*f - b^3*e - 4*a*b*c*e)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))*x - (b^3*d - 12*a*b*c*d - 8*a^2*b*f + 2*a*b^2*e + 8*a^2*c*e)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))/(c*x^2 + b*x + a)^{(3/2)}$

**maple** [A] time = 0.01, size = 185, normalized size = 1.41

$$\frac{\frac{16}{3}a^2c^2fx^3 + \frac{4}{3}b^2cfx^3 - \frac{16}{3}bc^2ex^3 + \frac{32}{3}c^3dx^3 + 8abcfx^2 + 2b^3fx^2 - 8b^2cex^2 + 16bc^2dx^2 + 8ab^2fx - 8abcex}{(cx^2 + bx + a)^{\frac{3}{2}}(16a^2c^2 - 8ab^2c + b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(5/2),x)

[Out]  $\frac{2}{3}/(c*x^2+b*x+a)^{(3/2)}*(8*a*c^2*f*x^3+2*b^2*c*f*x^3-8*b*c^2*e*x^3+16*c^3*d*x^3+12*a*b*c*f*x^2+3*b^3*f*x^2-12*b^2*c*e*x^2+24*b*c^2*d*x^2+12*a*b^2*f*x-12*a*b*c*e*x+24*a*c^2*d*x-3*b^3*e*x+6*b^2*c*d*x+8*a^2*b*f-8*a^2*c*e-2*a*b^2*e+12*a*b*c*d-b^3*d)/(16*a^2*c^2-8*a*b^2*c+b^4)$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 zero or nonzero?
```

**mupad [B]** time = 3.71, size = 175, normalized size = 1.34

$$\frac{2(8fa^2b - 8ea^2c + 12fab^2x - 2eab^2 + 12fabcx^2 - 12eabcx + 12dabc + 8fac^2x^3 + 24dac^2x + 3fb^3)}{3(4ac - b^2)^2(cx^2 + bx + a)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/(a + b*x + c*x^2)^(5/2),x)
```

```
[Out] (2*(16*c^3*d*x^3 - b^3*d + 3*b^3*f*x^2 - 2*a*b^2*e + 8*a^2*b*f - 8*a^2*c*e - 3*b^3*e*x + 24*a*c^2*d*x + 12*a*b^2*f*x + 6*b^2*c*d*x + 24*b*c^2*d*x^2 - 12*b^2*c*e*x^2 + 8*a*c^2*f*x^3 - 8*b*c^2*e*x^3 + 2*b^2*c*f*x^3 + 12*a*b*c*d - 12*a*b*c*e*x + 12*a*b*c*f*x^2))/(3*(4*a*c - b^2)^2*(a + b*x + c*x^2)^(3/2))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Timed out
```



$$3.121 \quad \int \frac{1}{\sqrt{-7+2x+5x^2} (8+12x+5x^2)} dx$$

**Optimal.** Leaf size=51

$$\frac{1}{10} \tan^{-1} \left( \frac{5(x+2)}{2\sqrt{5x^2+2x-7}} \right) + \frac{1}{5} \tanh^{-1} \left( \frac{5(x+1)}{\sqrt{5x^2+2x-7}} \right)$$

[Out] 1/10\*arctan(5/2\*(2+x)/(5\*x^2+2\*x-7)^(1/2))+1/5\*arctanh(5\*(1+x)/(5\*x^2+2\*x-7)^(1/2))

**Rubi [A]** time = 0.07, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$ , Rules used = {986, 1029, 203, 207}

$$\frac{1}{10} \tan^{-1} \left( \frac{5(x+2)}{2\sqrt{5x^2+2x-7}} \right) + \frac{1}{5} \tanh^{-1} \left( \frac{5(x+1)}{\sqrt{5x^2+2x-7}} \right)$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[-7 + 2\*x + 5\*x^2]\*(8 + 12\*x + 5\*x^2)),x]

[Out] ArcTan[(5\*(2 + x))/(2\*Sqrt[-7 + 2\*x + 5\*x^2])]/10 + ArcTanh[(5\*(1 + x))/Sqrt[-7 + 2\*x + 5\*x^2]]/5

### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rule 207

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTanh[(Rt[b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

### Rule 986

Int[1/(((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*Sqrt[(d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2]), x\_Symbol] := With[{q = Rt[(c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f), 2]}, Dist[1/(2\*q), Int[(c\*d - a\*f + q + (c\*e - b\*f)\*x)/((a + b\*x + c\*x^2)\*Sqrt[d + e\*x + f\*x^2]), x], x] - Dist[1/(2\*q), Int[(c\*d - a\*f - q + (c\*e - b\*f)\*x)/((a + b\*x + c\*x^2)\*Sqrt[d + e\*x + f\*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && NeQ[c\*e -

$b*f, 0] \ \&\& \ \text{NegQ}[b^2 - 4*a*c]$

### Rule 1029

$\text{Int}[(g_.) + (h_.)*(x_.)]/((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)*\text{Sqrt}[(d_.) + (e_.)*(x_.) + (f_.)*(x_.)^2], x\_Symbol] := \text{Dist}[-2*g*(g*b - 2*a*h), \text{Subst}[\text{Int}[1/\text{Simp}[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, \text{Simp}[g*b - 2*a*h - (b*h - 2*g*c)*x, x]/\text{Sqrt}[d + e*x + f*x^2]], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, h\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[e^2 - 4*d*f, 0] \ \&\& \ \text{NeQ}[b*d - a*e, 0] \ \&\& \ \text{EqQ}[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f), 0]$

### Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{-7+2x+5x^2} (8+12x+5x^2)} dx &= -\left( \frac{1}{50} \int \frac{-100-50x}{\sqrt{-7+2x+5x^2} (8+12x+5x^2)} dx \right) + \frac{1}{50} \int \frac{-50}{\sqrt{-7+2x+5x^2}} dx \\ &= 400 \text{Subst} \left( \int \frac{1}{160000+100x^2} dx, x, \frac{200+100x}{\sqrt{-7+2x+5x^2}} \right) + 1600 \text{Subst} \left( \int \frac{-1}{10000+x^2} dx, x, \frac{200+100x}{\sqrt{-7+2x+5x^2}} \right) \\ &= \frac{1}{10} \tan^{-1} \left( \frac{5(2+x)}{2\sqrt{-7+2x+5x^2}} \right) + \frac{1}{5} \tanh^{-1} \left( \frac{5(1+x)}{\sqrt{-7+2x+5x^2}} \right) \end{aligned}$$

**Mathematica [C]** time = 0.04, size = 81, normalized size = 1.59

$$\left( \frac{1}{10} - \frac{i}{20} \right) \tanh^{-1} \left( \frac{\left( \frac{1}{100} + \frac{i}{50} \right) ((100 - 40i)x + (164 - 8i))}{\sqrt{5x^2 + 2x - 7}} \right) - \left( \frac{1}{20} - \frac{i}{10} \right) \tanh^{-1} \left( \frac{\left( \frac{1}{50} + \frac{i}{100} \right) ((-100 - 40i)x - (164 - 8i))}{\sqrt{5x^2 + 2x - 7}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[-7 + 2\*x + 5\*x^2]\*(8 + 12\*x + 5\*x^2)),x]

[Out] (-1/20 + I/10)\*ArcTan[((1/50 + I/100)\*((-164 - 8\*I) - (100 + 40\*I)\*x))/Sqrt[-7 + 2\*x + 5\*x^2]] + (1/10 - I/20)\*ArcTanh[((1/100 + I/50)\*((164 - 8\*I) + (100 - 40\*I)\*x))/Sqrt[-7 + 2\*x + 5\*x^2]]

**fricas [B]** time = 0.76, size = 154, normalized size = 3.02

$$\frac{1}{20} \arctan \left( \frac{27x^2 + 20\sqrt{5x^2 + 2x - 7}(x + 2) + 36x}{31x^2 + 16x - 56} \right) + \frac{1}{20} \arctan \left( -\frac{27x^2 - 20\sqrt{5x^2 + 2x - 7}(x + 2) + 36x}{31x^2 + 16x - 56} \right) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5\*x^2+12\*x+8)/(5\*x^2+2\*x-7)^(1/2),x, algorithm="fricas")

[Out]  $\frac{1}{20} \arctan\left(\frac{27x^2 + 20\sqrt{5x^2 + 2x - 7}(x + 2) + 36x}{31x^2 + 16x - 56}\right) + \frac{1}{20} \arctan\left(-\frac{27x^2 - 20\sqrt{5x^2 + 2x - 7}(x + 2) + 36x}{31x^2 + 16x - 56}\right) + \frac{1}{20} \log\left(\frac{15x^2 + 5\sqrt{5x^2 + 2x - 7}(x + 1) + 26x + 9}{x^2}\right) - \frac{1}{20} \log\left(\frac{15x^2 - 5\sqrt{5x^2 + 2x - 7}(x + 1) + 26x + 9}{x^2}\right)$

**giac** [B] time = 0.28, size = 205, normalized size = 4.02

$$-\frac{1}{10} \arctan\left(-\frac{5\sqrt{5}x + 6\sqrt{5} - 5\sqrt{5x^2 + 2x - 7} + 5}{2(\sqrt{5} + 5)}\right) - \frac{1}{10} \arctan\left(\frac{5\sqrt{5}x + 6\sqrt{5} - 5\sqrt{5x^2 + 2x - 7} - 5}{2(\sqrt{5} - 5)}\right) + \frac{1}{10} \log\left(\frac{5\sqrt{5}x + 6\sqrt{5} - 5\sqrt{5x^2 + 2x - 7} - 5}{2(\sqrt{5} - 5)}\right) + \frac{1}{10} \log\left(\frac{5\sqrt{5}x + 6\sqrt{5} - 5\sqrt{5x^2 + 2x - 7} + 5}{2(\sqrt{5} + 5)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5\*x^2+12\*x+8)/(5\*x^2+2\*x-7)^(1/2),x, algorithm="giac")

[Out]  $-\frac{1}{10} \arctan\left(\frac{-1/2(5\sqrt{5}x + 6\sqrt{5} - 5\sqrt{5x^2 + 2x - 7} + 5)}{\sqrt{5} + 5}\right) - \frac{1}{10} \arctan\left(\frac{1/2(5\sqrt{5}x + 6\sqrt{5} - 5\sqrt{5x^2 + 2x - 7} - 5)}{\sqrt{5} - 5}\right) + \frac{1}{10} \log\left(\frac{5(\sqrt{5}x - \sqrt{5x^2 + 2x - 7})^2 + 2(\sqrt{5}x - \sqrt{5x^2 + 2x - 7})(6\sqrt{5} + 5) + 20\sqrt{5} + 65}{5(\sqrt{5}x - \sqrt{5x^2 + 2x - 7})^2 + 2(\sqrt{5}x - \sqrt{5x^2 + 2x - 7})(6\sqrt{5} - 5) - 20\sqrt{5} + 65}\right)$

**maple** [B] time = 0.02, size = 144, normalized size = 2.82

$$\frac{\sqrt{-\frac{4(x+2)^2}{(-x-1)^2} + 9} \left( 2 \operatorname{arctanh}\left(\frac{\sqrt{-\frac{4(x+2)^2}{(-x-1)^2} + 9}}{5}\right) + \operatorname{arctan}\left(\frac{5\sqrt{-\frac{4(x+2)^2}{(-x-1)^2} + 9}(x+2)}{2\left(\frac{4(x+2)^2}{(-x-1)^2} - 9\right)(-x-1)}\right) \right)}{10 \sqrt{-\frac{4(x+2)^2}{(-x-1)^2} - 9} \left(1 + \frac{x+2}{-x-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(5\*x^2+12\*x+8)/(5\*x^2+2\*x-7)^(1/2),x)

[Out]  $-\frac{1}{10} \left(-4(x+2)^2/(-1-x)^2+9\right)^{1/2} \left(2 \operatorname{arctanh}\left(\frac{1/5(-4(x+2)^2/(-1-x)^2+9)^{1/2}}{\sqrt{-4(x+2)^2/(-1-x)^2+9}}\right) + \operatorname{arctan}\left(\frac{5/2(-4(x+2)^2/(-1-x)^2+9)^{1/2}}{4(x+2)^2/(-1-x)^2-9}\right) \right) \left(\frac{(x+2)}{(-1-x)}\right) / \left(-4(x+2)^2/(-1-x)^2-9\right) / \left(1 + \frac{(x+2)}{(-1-x)}\right)^2)^{1/2} / \left(1 + \frac{(x+2)}{(-1-x)}\right)$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(5x^2 + 12x + 8)\sqrt{5x^2 + 2x - 7}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5\*x^2+12\*x+8)/(5\*x^2+2\*x-7)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((5\*x^2 + 12\*x + 8)\*sqrt(5\*x^2 + 2\*x - 7)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{5x^2 + 2x - 7} (5x^2 + 12x + 8)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((2\*x + 5\*x^2 - 7)^(1/2)\*(12\*x + 5\*x^2 + 8)),x)

[Out] int(1/((2\*x + 5\*x^2 - 7)^(1/2)\*(12\*x + 5\*x^2 + 8)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(x-1)(5x+7)} (5x^2 + 12x + 8)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5\*x\*\*2+12\*x+8)/(5\*x\*\*2+2\*x-7)\*\*(1/2),x)

[Out] Integral(1/(sqrt((x - 1)\*(5\*x + 7))\*(5\*x\*\*2 + 12\*x + 8)), x)

$$3.122 \quad \int \frac{1}{\sqrt{a+bx+cx^2} \sqrt{d+ex+fx^2}} dx$$

Optimal. Leaf size=1432

$$\sqrt[4]{db^2 + \left(\sqrt{b^2 - 4ac}d - ae\right)b - a\left(2cd + \sqrt{b^2 - 4ac}e - 2af\right)} \left(b + 2cx + \sqrt{b^2 - 4ac}\right)^{3/2} \sqrt{2a + \left(b + \sqrt{b^2 - 4ac}\right)}$$

---

[Out]  $-(\cos(2*\arctan((2*c^2*d-b*c*e+b^2*f-2*a*c*f-(-b*f+c*e))*(-4*a*c+b^2)^(1/2)))^(1/4)*(2*a+x*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(b^2*d+b*(-a*e+d*(-4*a*c+b^2)^(1/2))-a*(2*c*d-2*a*f+e*(-4*a*c+b^2)^(1/2)))^(1/4)/(b+2*c*x+(-4*a*c+b^2)^(1/2)))^(1/2))/\cos(2*\arctan((2*c^2*d-b*c*e+b^2*f-2*a*c*f-(-b*f+c*e))*(-4*a*c+b^2)^(1/2)))^(1/4)*(2*a+x*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(b^2*d+b*(-a*e+d*(-4*a*c+b^2)^(1/2))-a*(2*c*d-2*a*f+e*(-4*a*c+b^2)^(1/2)))^(1/4)/(b+2*c*x+(-4*a*c+b^2)^(1/2)))^(1/2))*\text{EllipticF}(\sin(2*\arctan((2*c^2*d-b*c*e+b^2*f-2*a*c*f-(-b*f+c*e))*(-4*a*c+b^2)^(1/2)))^(1/4)*(2*a+x*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(b^2*d+b*(-a*e+d*(-4*a*c+b^2)^(1/2))-a*(2*c*d-2*a*f+e*(-4*a*c+b^2)^(1/2)))^(1/4)/(b+2*c*x+(-4*a*c+b^2)^(1/2))),1/2*(2+(2*a*f-b*e+2*c*d)*(b+(-4*a*c+b^2)^(1/2))/(b^2*d+b*(-a*e+d*(-4*a*c+b^2)^(1/2))-a*(2*c*d-2*a*f+e*(-4*a*c+b^2)^(1/2)))^(1/2)/(2*c^2*d+b*f*(b+(-4*a*c+b^2)^(1/2))-c*(b*e+2*a*f+e*(-4*a*c+b^2)^(1/2)))^(1/2)))^(1/2)*(b+2*c*x+(-4*a*c+b^2)^(1/2))^(3/2)*(b^2*d+b*(-a*e+d*(-4*a*c+b^2)^(1/2))-a*(2*c*d-2*a*f+e*(-4*a*c+b^2)^(1/2)))^(1/4)*(2*a+x*(b+(-4*a*c+b^2)^(1/2)))^(1/2)*((f*x^2+e*x+d)*(4*a*c-(b+(-4*a*c+b^2)^(1/2))^(1/2))^2)/(b+2*c*x+(-4*a*c+b^2)^(1/2))^2/(4*a^2*f-2*a*e*(b+(-4*a*c+b^2)^(1/2))+d*(b+(-4*a*c+b^2)^(1/2))^2))^(1/2)*(1+(2*a+x*(b+(-4*a*c+b^2)^(1/2))))*(2*c^2*d-b*c*e+b^2*f-2*a*c*f-(-b*f+c*e))*(-4*a*c+b^2)^(1/2))/(b+2*c*x+(-4*a*c+b^2)^(1/2))/(b^2*d+b*(-a*e+d*(-4*a*c+b^2)^(1/2))-a*(2*c*d-2*a*f+e*(-4*a*c+b^2)^(1/2)))^(1/2))*((1+(2*a+x*(b+(-4*a*c+b^2)^(1/2))))^2*(4*c^2*d-2*c*e*(b+(-4*a*c+b^2)^(1/2))+f*(b+(-4*a*c+b^2)^(1/2))^2)/(b+2*c*x+(-4*a*c+b^2)^(1/2))^2/(4*a^2*f-2*a*e*(b+(-4*a*c+b^2)^(1/2))+d*(b+(-4*a*c+b^2)^(1/2))^2)-(2*a*f-b*e+2*c*d)*(b+(-4*a*c+b^2)^(1/2))*(2*a+x*(b+(-4*a*c+b^2)^(1/2))))/(b+2*c*x+(-4*a*c+b^2)^(1/2))/(b^2*d+b*(-a*e+d*(-4*a*c+b^2)^(1/2))-a*(2*c*d-2*a*f+e*(-4*a*c+b^2)^(1/2))))/(1+(2*a+x*(b+(-4*a*c+b^2)^(1/2))))*(2*c^2*d-b*c*e+b^2*f-2*a*c*f-(-b*f+c*e))*(-4*a*c+b^2)^(1/2))/(b+2*c*x+(-4*a*c+b^2)^(1/2))/(b^2*d+b*(-a*e+d*(-4*a*c+b^2)^(1/2))-a*(2*c*d-2*a*f+e*(-4*a*c+b^2)^(1/2)))^(1/2))^2)/(2*c^2*d-b*c*e+b^2*f-2*a*c*f-(-b*f+c*e))*(-4*a*c$

$$+b^2)^{(1/2))^{(1/4)}/(4*a*c-(b+(-4*a*c+b^2)^{(1/2))^{(1/2)})/(c*x^2+b*x+a)^{(1/2)/(f*x^2+e*x+d)^{(1/2)/(1+(2*a*x*(b+(-4*a*c+b^2)^{(1/2))^{(1/2))^{(1/2)}+f*(b+(-4*a*c+b^2)^{(1/2))^{(1/2)})^2/(4*c^2*d-2*c*e*(b+(-4*a*c+b^2)^{(1/2))^{(1/2)}+f*(b+(-4*a*c+b^2)^{(1/2))^{(1/2)})^2/(b+2*c*x+(-4*a*c+b^2)^{(1/2))^{(1/2)})^2/(4*a^2*f-2*a*e*(b+(-4*a*c+b^2)^{(1/2))^{(1/2)}+d*(b+(-4*a*c+b^2)^{(1/2))^{(1/2)})^2-(2*a*f-b*e+2*c*d)*(b+(-4*a*c+b^2)^{(1/2))^{(1/2)}*(2*a*x*(b+(-4*a*c+b^2)^{(1/2))^{(1/2)})))/(b+2*c*x+(-4*a*c+b^2)^{(1/2))^{(1/2)})/(b^2*d+b*(-a*e+d*(-4*a*c+b^2)^{(1/2))^{(1/2)}-a*(2*c*d-2*a*f+e*(-4*a*c+b^2)^{(1/2))^{(1/2)}))^{(1/2)}$$

**Rubi [A]** time = 6.22, antiderivative size = 1432, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$ , Rules used = {992, 935, 1103}

$$\sqrt[4]{db^2 + \left(\sqrt{b^2 - 4ac}d - ae\right)b - a\left(2cd + \sqrt{b^2 - 4ac}e - 2af\right)} \left(b + 2cx + \sqrt{b^2 - 4ac}\right)^{3/2} \sqrt{2a + \left(b + \sqrt{b^2 - 4ac}\right)}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + b\*x + c\*x^2]\*Sqrt[d + e\*x + f\*x^2]),x]

[Out] -(((b^2\*d + b\*(Sqrt[b^2 - 4\*a\*c]\*d - a\*e) - a\*(2\*c\*d + Sqrt[b^2 - 4\*a\*c]\*e - 2\*a\*f))^(1/4)\*(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)^(3/2)\*Sqrt[2\*a + (b + Sqrt[b^2 - 4\*a\*c])\*x]\*Sqrt[((4\*a\*c - (b + Sqrt[b^2 - 4\*a\*c])^2)^2\*(d + e\*x + f\*x^2)))/(((b + Sqrt[b^2 - 4\*a\*c])^2\*d - 2\*a\*(b + Sqrt[b^2 - 4\*a\*c])\*e + 4\*a^2\*f)\*(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)^2)]\*(1 + (Sqrt[2\*c^2\*d - b\*c\*e + b^2\*f - 2\*a\*c\*f - Sqrt[b^2 - 4\*a\*c]\*(c\*e - b\*f)]\*(2\*a + (b + Sqrt[b^2 - 4\*a\*c])\*x)))/(Sqrt[b^2\*d + b\*(Sqrt[b^2 - 4\*a\*c]\*d - a\*e) - a\*(2\*c\*d + Sqrt[b^2 - 4\*a\*c]\*e - 2\*a\*f)]\*(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x))\*Sqrt[(1 - ((b + Sqrt[b^2 - 4\*a\*c])\*(2\*c\*d - b\*e + 2\*a\*f)\*(2\*a + (b + Sqrt[b^2 - 4\*a\*c])\*x)))/((b^2\*d + b\*(Sqrt[b^2 - 4\*a\*c]\*d - a\*e) - a\*(2\*c\*d + Sqrt[b^2 - 4\*a\*c]\*e - 2\*a\*f))\*(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)) + ((4\*c^2\*d - 2\*c\*(b + Sqrt[b^2 - 4\*a\*c])\*e + (b + Sqrt[b^2 - 4\*a\*c])^2\*f)\*(2\*a + (b + Sqrt[b^2 - 4\*a\*c])\*x)^2)/(((b + Sqrt[b^2 - 4\*a\*c])^2\*d - 2\*a\*(b + Sqrt[b^2 - 4\*a\*c])\*e + 4\*a^2\*f)\*(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)^2)]/(1 + (Sqrt[2\*c^2\*d - b\*c\*e + b^2\*f - 2\*a\*c\*f - Sqrt[b^2 - 4\*a\*c]\*(c\*e - b\*f)]\*(2\*a + (b + Sqrt[b^2 - 4\*a\*c])\*x))/(Sqrt[b^2\*d + b\*(Sqrt[b^2 - 4\*a\*c]\*d - a\*e) - a\*(2\*c\*d + Sqrt[b^2 - 4\*a\*c]\*e - 2\*a\*f)]\*(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x))^2)\*EllipticF[2\*ArcTan[((2\*c^2\*d - b\*c\*e + b^2\*f - 2\*a\*c\*f - Sqrt[b^2 - 4\*a\*c]\*(c\*e - b\*f))^(1/4)\*Sqrt[2\*a + (b + Sqrt[b^2 - 4\*a\*c])\*x])]/((b^2\*d + b\*(Sqrt[b^2 - 4\*a\*c]\*d - a\*e) - a\*(2\*c\*d +

$$\text{Sqrt}[b^2 - 4ac]e - 2af)^{1/4} \text{Sqrt}[b + \text{Sqrt}[b^2 - 4ac] + 2cx]],$$

$$(2 + ((b + \text{Sqrt}[b^2 - 4ac])(2cd - be + 2af))/(\text{Sqrt}[b^2d + b(\text{Sqrt}[b^2 - 4ac]d - ae) - a(2cd + \text{Sqrt}[b^2 - 4ac]e - 2af)] \text{Sqrt}[2c^2d + b(b + \text{Sqrt}[b^2 - 4ac])f - c(be + \text{Sqrt}[b^2 - 4ac]e + 2af)]))$$

$$/4))/((4ac - (b + \text{Sqrt}[b^2 - 4ac])^2)(2c^2d - bce + b^2f - 2ac^2f - \text{Sqrt}[b^2 - 4ac](ce - bf))^{1/4} \text{Sqrt}[a + bx + cx^2] \text{Sqrt}[d + ex + fx^2] \text{Sqrt}[1 - ((b + \text{Sqrt}[b^2 - 4ac])(2cd - be + 2af)(2a + (b + \text{Sqrt}[b^2 - 4ac])x))/((b^2d + b(\text{Sqrt}[b^2 - 4ac]d - ae) - a(2cd + \text{Sqrt}[b^2 - 4ac]e - 2af))(b + \text{Sqrt}[b^2 - 4ac] + 2cx)) + ((4c^2d - 2c(b + \text{Sqrt}[b^2 - 4ac])e + (b + \text{Sqrt}[b^2 - 4ac])^2f)(2a + (b + \text{Sqrt}[b^2 - 4ac])x)^2)/(((b + \text{Sqrt}[b^2 - 4ac])^2d - 2a(b + \text{Sqrt}[b^2 - 4ac])e + 4a^2f)(b + \text{Sqrt}[b^2 - 4ac] + 2cx)^2)))]$$

### Rule 935

$$\text{Int}[1/(\text{Sqrt}[(d_.) + (e_.)x] \text{Sqrt}[(f_.) + (g_.)x] \text{Sqrt}[(a_.) + (b_.)x + (c_.)x^2]), x\_Symbol] := \text{Dist}[(-2(d + ex) \text{Sqrt}[(ef - dg)^2(a + bx + cx^2)])/((cf^2 - bfg + ag^2)(d + ex)^2)]/((ef - dg) \text{Sqrt}[a + bx + cx^2]), \text{Subst}[\text{Int}[1/\text{Sqrt}[1 - ((2cdf - bdf - bdf + 2aefg)x^2)/(cf^2 - bfg + ag^2) + ((cd^2 - bde + ae^2)x^4)/(cf^2 - bfg + ag^2)], x], x, \text{Sqrt}[f + gx]/\text{Sqrt}[d + ex]], x] /; \text{FreeQ}\{a, b, c, d, e, f, g\}, x] \&\& \text{NeQ}[ef - dg, 0] \&\& \text{NeQ}[b^2 - 4ac, 0] \&\& \text{NeQ}[cd^2 - bde + ae^2, 0]$$

### Rule 992

$$\text{Int}[1/(\text{Sqrt}[(a_.) + (b_.)x + (c_.)x^2] \text{Sqrt}[(d_.) + (e_.)x + (f_.)x^2]), x\_Symbol] := \text{With}\{r = \text{Rt}[b^2 - 4ac, 2]\}, \text{Dist}[(\text{Sqrt}[b + r + 2cx] \text{Sqrt}[2a + (b + r)x])/ \text{Sqrt}[a + bx + cx^2], \text{Int}[1/(\text{Sqrt}[b + r + 2cx] \text{Sqrt}[2a + (b + r)x] \text{Sqrt}[d + ex + fx^2]), x], x]] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b^2 - 4ac, 0] \&\& \text{NeQ}[e^2 - 4df, 0]$$

### Rule 1103

$$\text{Int}[1/\text{Sqrt}[(a_.) + (b_.)x^2 + (c_.)x^4], x\_Symbol] := \text{With}\{q = \text{Rt}[c/a, 4]\}, \text{Simp}[(1 + q^2x^2) \text{Sqrt}[(a + bx^2 + cx^4)/(a(1 + q^2x^2)^2)] * \text{EllipticF}[2 \text{ArcTan}[qx], 1/2 - (bq^2)/(4c)]/(2q \text{Sqrt}[a + bx^2 + cx^4]), x]] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4ac, 0] \&\& \text{PosQ}[c/a]$$

### Rubi steps

$$\int \frac{1}{\sqrt{a+bx+cx^2}\sqrt{d+ex+fx^2}} dx = \frac{\left(\sqrt{b+\sqrt{b^2-4ac}}+2cx\sqrt{2a+(b+\sqrt{b^2-4ac})x}\right) \int \frac{1}{\sqrt{b+\sqrt{b^2-4ac}+2cx}\sqrt{2a+(b+\sqrt{b^2-4ac})x}} dx}{\sqrt{a+bx+cx^2}}$$

$$= \frac{\left(2(b+\sqrt{b^2-4ac}+2cx)^{3/2}\sqrt{2a+(b+\sqrt{b^2-4ac})x}\sqrt{\frac{4ac}{(b+\sqrt{b^2-4ac})^2d-4ac}}\right)}{\sqrt{a+bx+cx^2}}$$

$$= \frac{\sqrt[4]{b^2d+b(\sqrt{b^2-4ac}d-ae)}-a(2cd+\sqrt{b^2-4ac}e-2af)(b+\sqrt{b^2-4ac})}{\sqrt{a+bx+cx^2}}$$

**Mathematica [A]** time = 2.34, size = 670, normalized size = 0.47

$$\frac{\left(\sqrt{b^2-4ac}-b-2cx\right)\left(-\sqrt{e^2-4df}+e+2fx\right)\sqrt{\frac{c\sqrt{b^2-4ac}\left(\sqrt{e^2-4df}+e+2fx\right)}{\left(\sqrt{b^2-4ac}-b-2cx\right)\left(f\left(\sqrt{b^2-4ac}+b\right)-c\left(\sqrt{e^2-4df}+e\right)\right)}}}{\sqrt{a+x(b+cx)}\sqrt{d+x(e+fx)}\left(f\left(\sqrt{b^2-4ac}\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(Sqrt[a + b\*x + c\*x^2]\*Sqrt[d + e\*x + f\*x^2]),x]

[Out] -(((b + Sqrt[b^2 - 4\*a\*c] - 2\*c\*x)\*(e - Sqrt[e^2 - 4\*d\*f] + 2\*f\*x)\*Sqrt[-(c\*Sqrt[b^2 - 4\*a\*c]\*(e + Sqrt[e^2 - 4\*d\*f] + 2\*f\*x))/((b + Sqrt[b^2 - 4\*a\*c])\*f - c\*(e + Sqrt[e^2 - 4\*d\*f]))\*(-b + Sqrt[b^2 - 4\*a\*c] - 2\*c\*x)))]\*Sqrt[-((c\*(4\*a\*f + Sqrt[b^2 - 4\*a\*c])\*Sqrt[e^2 - 4\*d\*f] - 2\*Sqrt[b^2 - 4\*a\*c]\*f\*x + 2\*c\*Sqrt[e^2 - 4\*d\*f]\*x - e\*(Sqrt[b^2 - 4\*a\*c] + 2\*c\*x) + b\*(-e + Sqrt[e^2 - 4\*d\*f])))]



$$\frac{[e^2 - 4*d*f] + 2*f*x)}{((b + \sqrt{b^2 - 4*a*c})*f + c*(-e + \sqrt{e^2 - 4*d*f}))} * (-b + \sqrt{b^2 - 4*a*c} - 2*c*x)) * \text{EllipticF}[\text{ArcSin}[\sqrt{((-b + \sqrt{b^2 - 4*a*c})*f + c*(e - \sqrt{e^2 - 4*d*f})) * (b + \sqrt{b^2 - 4*a*c} + 2*c*x)) / ((b + \sqrt{b^2 - 4*a*c})*f + c*(-e + \sqrt{e^2 - 4*d*f})) * (-b + \sqrt{b^2 - 4*a*c} - 2*c*x))}], (2*c*d - b*e + 2*a*f - \sqrt{b^2 - 4*a*c} * \sqrt{e^2 - 4*d*f}) / (2*c*d - b*e + 2*a*f + \sqrt{b^2 - 4*a*c} * \sqrt{e^2 - 4*d*f})] / ((-b + \sqrt{b^2 - 4*a*c})*f + c*(e - \sqrt{e^2 - 4*d*f})) * \sqrt{(c*\sqrt{b^2 - 4*a*c}*(-e + \sqrt{e^2 - 4*d*f} - 2*f*x)) / ((b + \sqrt{b^2 - 4*a*c})*f + c*(-e + \sqrt{e^2 - 4*d*f})) * (-b + \sqrt{b^2 - 4*a*c} - 2*c*x))} * \sqrt{a + x*(b + c*x)} * \sqrt{d + x*(e + f*x))}]$$

**fricas** [F] time = 1.04, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{\sqrt{cx^2 + bx + a} \sqrt{fx^2 + ex + d}}{cfx^4 + (ce + bf)x^3 + (cd + be + af)x^2 + ad + (bd + ae)x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x^2+b\*x+a)^(1/2)/(f\*x^2+e\*x+d)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(c\*x^2 + b\*x + a)\*sqrt(f\*x^2 + e\*x + d)/(c\*f\*x^4 + (c\*e + b\*f)\*x^3 + (c\*d + b\*e + a\*f)\*x^2 + a\*d + (b\*d + a\*e)\*x), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + bx + a} \sqrt{fx^2 + ex + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x^2+b\*x+a)^(1/2)/(f\*x^2+e\*x+d)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(c\*x^2 + b\*x + a)\*sqrt(f\*x^2 + e\*x + d)), x)

**maple** [A] time = 0.27, size = 928, normalized size = 0.65

$$8 \left( 2b f^2 x^2 - 2cef x^2 + 2befx - 8cdfx + 2\sqrt{-4df + e^2} cf x^2 + 2\sqrt{-4ac + b^2} f^2 x^2 - 2bdf + b e^2 + 2\sqrt{-4df + e^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c\*x^2+b\*x+a)^(1/2)/(f\*x^2+e\*x+d)^(1/2),x)

```
[Out] 8*(2*b*f^2*x^2-2*c*e*f*x^2+2*x^2*c*f*(-4*d*f+e^2)^(1/2)+2*(-4*a*c+b^2)^(1/2)
)*f^2*x^2+2*b*e*f*x+2*x*b*f*(-4*d*f+e^2)^(1/2)-8*c*d*f*x+2*x*e*f*(-4*a*c+b^
2)^(1/2)+2*x*f*(-4*d*f+e^2)^(1/2)*(-4*a*c+b^2)^(1/2)-2*b*d*f+b*e^2+b*e*(-4*
d*f+e^2)^(1/2)-2*c*d*e-2*c*d*(-4*d*f+e^2)^(1/2)-2*(-4*a*c+b^2)^(1/2)*d*f+e^
2*(-4*a*c+b^2)^(1/2)+e*(-4*d*f+e^2)^(1/2)*(-4*a*c+b^2)^(1/2))*EllipticF((((
-4*d*f+e^2)^(1/2)*c-(-4*a*c+b^2)^(1/2)*f-b*f+c*e)*(-2*f*x+(-4*d*f+e^2)^(1/2
)-e)/((-4*d*f+e^2)^(1/2)*c+(-4*a*c+b^2)^(1/2)*f+b*f-c*e)/(2*f*x+(-4*d*f+e^2
)^(1/2)+e))^(1/2), (((-4*d*f+e^2)^(1/2)*c+(-4*a*c+b^2)^(1/2)*f-b*f+c*e)*((-4
*d*f+e^2)^(1/2)*c+(-4*a*c+b^2)^(1/2)*f+b*f-c*e)/((-4*d*f+e^2)^(1/2)*c-(-4*a
*c+b^2)^(1/2)*f+b*f-c*e)/((-4*d*f+e^2)^(1/2)*c-(-4*a*c+b^2)^(1/2)*f-b*f+c*e
))^(1/2))*((-4*d*f+e^2)^(1/2)*(2*c*x+b+(-4*a*c+b^2)^(1/2))*f/((-4*d*f+e^2)^(
1/2)*c+(-4*a*c+b^2)^(1/2)*f+b*f-c*e)/(2*f*x+(-4*d*f+e^2)^(1/2)+e))^(1/2)*(-
(-4*d*f+e^2)^(1/2)*(-2*c*x-b+(-4*a*c+b^2)^(1/2))*f/((-4*d*f+e^2)^(1/2)*c-(-
4*a*c+b^2)^(1/2)*f+b*f-c*e)/(2*f*x+(-4*d*f+e^2)^(1/2)+e))^(1/2)*(((4*d*f+
e^2)^(1/2)*c-(-4*a*c+b^2)^(1/2)*f-b*f+c*e)*(-2*f*x+(-4*d*f+e^2)^(1/2)-e)/((-
4*d*f+e^2)^(1/2)*c+(-4*a*c+b^2)^(1/2)*f+b*f-c*e)/(2*f*x+(-4*d*f+e^2)^(1/2
)+e))^(1/2)*(c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)^(1/2)/(1/c/f*(-2*f*x+(-4*d*f+e
^2)^(1/2)-e)*(2*f*x+(-4*d*f+e^2)^(1/2)+e)*(-2*c*x-b+(-4*a*c+b^2)^(1/2))*
(2*c*x+b+(-4*a*c+b^2)^(1/2)))^(1/2)/(-4*d*f+e^2)^(1/2)/((-4*a*c+b^2)^(1/2)*f-(-
4*d*f+e^2)^(1/2)*c+b*f-c*e)/(c*f*x^4+b*f*x^3+c*e*x^3+a*f*x^2+b*e*x^2+c*d*x
^2+a*e*x+b*d*x+a*d)^(1/2)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + bx + a} \sqrt{fx^2 + ex + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(f*x^2 + e*x + d)), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{cx^2 + bx + a} \sqrt{fx^2 + ex + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)^(1/2)),x)
```

```
[Out] int(1/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)^(1/2)), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + bx + cx^2} \sqrt{d + ex + fx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x + c*x**2)*sqrt(d + e*x + f*x**2)), x)
```

$$3.123 \quad \int \frac{1}{\sqrt{3-x+2x^2} \sqrt{2+3x+5x^2}} dx$$

**Optimal.** Leaf size=652

$$\sqrt{\frac{23}{11}} (-4x - i\sqrt{23} + 1) \sqrt{4x + i\sqrt{23} - 1} \sqrt{6 - (1 - i\sqrt{23})x} \sqrt{\frac{(-\sqrt{23} + 11i)(5x^2 + 3x + 2)}{(\sqrt{23} + 7i)(-4x - i\sqrt{23} + 1)^2}} \left( 1 - \frac{\sqrt{-\frac{-\sqrt{23} + 3i}{\sqrt{23} + 7i}} (6 - (1 - i\sqrt{23})x)}{-4x - i\sqrt{23} + 1} \right)$$

$$(23 + i\sqrt{23})^4 \sqrt{-\frac{-\sqrt{23} + 3i}{\sqrt{23} + 7i}} \sqrt{2x^2 - x + 3} \sqrt{5x^2 + 3x + 2} \sqrt{-\frac{11}{}}$$

[Out]  $\frac{1}{11} * (\cos(2 * \arctan(\frac{(-3 * I + 23^{(1/2)})}{(7 * I + 23^{(1/2)})}))^{(1/4)} * (6 - x * (1 - I * 23^{(1/2)}))^{(1/2)} / (-1 + 4 * x + I * 23^{(1/2)})^{(1/2)})^{(1/2)} / \cos(2 * \arctan(\frac{(-3 * I + 23^{(1/2)})}{(7 * I + 23^{(1/2)})}))^{(1/4)} * (6 - x * (1 - I * 23^{(1/2)}))^{(1/2)} / (-1 + 4 * x + I * 23^{(1/2)})^{(1/2)}) * \text{EllipticF}(\sin(2 * \arctan(\frac{(-3 * I + 23^{(1/2)})}{(7 * I + 23^{(1/2)})}))^{(1/4)} * (6 - x * (1 - I * 23^{(1/2)}))^{(1/2)} / (-1 + 4 * x + I * 23^{(1/2)})^{(1/2)}), 1/22 * 11^{(1/2)} * ((66 * I - 22 * 23^{(1/2)} + 41 * (-23 * (3 * I - 23^{(1/2)}) / (7 * I + 23^{(1/2)}))^{(1/2)} + 41 * I * ((-3 * I + 23^{(1/2)}) / (7 * I + 23^{(1/2)}))^{(1/2)} / (3 * I - 23^{(1/2)}))^{(1/2)}) * 253^{(1/2)} * (1 - 4 * x - I * 23^{(1/2)}) * (6 - x * (1 - I * 23^{(1/2)}))^{(1/2)} * (-1 + 4 * x + I * 23^{(1/2)})^{(1/2)} * (1 - (6 - x * (1 - I * 23^{(1/2)})) * ((-3 * I + 23^{(1/2)}) / (7 * I + 23^{(1/2)}))^{(1/2)} / (1 - 4 * x - I * 23^{(1/2)})) * ((5 * x^2 + 3 * x + 2) * (11 * I - 23^{(1/2)}) / (1 - 4 * x - I * 23^{(1/2)})^2 / (7 * I + 23^{(1/2)}))^{(1/2)} * ((11 - 11 * (6 - x * (1 - I * 23^{(1/2)}))^{(1/2)})^2 * (3 * I - 23^{(1/2)}) / (1 - 4 * x - I * 23^{(1/2)})^2 / (7 * I + 23^{(1/2)}) - 41 * (6 - x * (1 - I * 23^{(1/2)})) * (I + 23^{(1/2)}) / (1 - 4 * x - I * 23^{(1/2)}) / (7 * I + 23^{(1/2)})) / (1 - (6 - x * (1 - I * 23^{(1/2)})) * ((-3 * I + 23^{(1/2)}) / (7 * I + 23^{(1/2)}))^{(1/2)} / (1 - 4 * x - I * 23^{(1/2)})^2)^{(1/2)} / (23 + I * 23^{(1/2)}) / ((-3 * I + 23^{(1/2)}) / (7 * I + 23^{(1/2)}))^{(1/4)} / (2 * x^2 - x + 3)^{(1/2)} / (5 * x^2 + 3 * x + 2)^{(1/2)} / (11 - 11 * (6 - x * (1 - I * 23^{(1/2)}))^{(1/2)})^2 * (3 * I - 23^{(1/2)}) / (1 - 4 * x - I * 23^{(1/2)})^2 / (7 * I + 23^{(1/2)}) - 41 * (6 - x * (1 - I * 23^{(1/2)})) * (I + 23^{(1/2)}) / (1 - 4 * x - I * 23^{(1/2)}) / (7 * I + 23^{(1/2)}))^{(1/2)}$

**Rubi [A]** time = 0.68, antiderivative size = 652, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$ , Rules used = {992, 935, 1103}

$$\sqrt{\frac{23}{11}} (-4x - i\sqrt{23} + 1) \sqrt{4x + i\sqrt{23} - 1} \sqrt{6 - (1 - i\sqrt{23})x} \sqrt{\frac{(-\sqrt{23} + 11i)(5x^2 + 3x + 2)}{(\sqrt{23} + 7i)(-4x - i\sqrt{23} + 1)^2}} \left( 1 - \frac{\sqrt{-\frac{-\sqrt{23} + 3i}{\sqrt{23} + 7i}} (6 - (1 - i\sqrt{23})x)}{-4x - i\sqrt{23} + 1} \right)$$

$$(23 + i\sqrt{23})^4 \sqrt{-\frac{-\sqrt{23} + 3i}{\sqrt{23} + 7i}} \sqrt{2x^2 - x + 3} \sqrt{5x^2 + 3x + 2} \sqrt{-\frac{11}{}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[3 - x + 2\*x^2]\*Sqrt[2 + 3\*x + 5\*x^2]),x]

[Out] (Sqrt[23/11]\*(1 - I\*Sqrt[23] - 4\*x)\*Sqrt[-1 + I\*Sqrt[23] + 4\*x]\*Sqrt[6 - (1 - I\*Sqrt[23])\*x]\*Sqrt[((11\*I - Sqrt[23])\*(2 + 3\*x + 5\*x^2))/((7\*I + Sqrt[23])\*(1 - I\*Sqrt[23] - 4\*x)^2)]\*(1 - (Sqrt[-((3\*I - Sqrt[23])/(7\*I + Sqrt[23]))])\*(6 - (1 - I\*Sqrt[23])\*x)))/(1 - I\*Sqrt[23] - 4\*x)\*Sqrt[(11 - (41\*(I + Sqrt[23]))\*(6 - (1 - I\*Sqrt[23])\*x)))/((7\*I + Sqrt[23])\*(1 - I\*Sqrt[23] - 4\*x)) - (11\*(3\*I - Sqrt[23])\*(6 - (1 - I\*Sqrt[23])\*x)^2)/((7\*I + Sqrt[23])\*(1 - I\*Sqrt[23] - 4\*x)^2)]/(1 - (Sqrt[-((3\*I - Sqrt[23])/(7\*I + Sqrt[23]))])\*(6 - (1 - I\*Sqrt[23])\*x)))/(1 - I\*Sqrt[23] - 4\*x)^2\*EllipticF[2\*ArcTan[(((3\*I - Sqrt[23])/(7\*I + Sqrt[23]))^(1/4)\*Sqrt[6 - (1 - I\*Sqrt[23])\*x])/Sqrt[-1 + I\*Sqrt[23] + 4\*x]], (44 - (41\*(I + Sqrt[23]))/Sqrt[11 + I\*Sqrt[23]])/88)/((23 + I\*Sqrt[23])\*(-(3\*I - Sqrt[23])/(7\*I + Sqrt[23]))^(1/4)\*Sqrt[3 - x + 2\*x^2]\*Sqrt[2 + 3\*x + 5\*x^2]\*Sqrt[11 - (41\*(I + Sqrt[23]))\*(6 - (1 - I\*Sqrt[23])\*x)))/((7\*I + Sqrt[23])\*(1 - I\*Sqrt[23] - 4\*x)) - (11\*(3\*I - Sqrt[23])\*(6 - (1 - I\*Sqrt[23])\*x)^2)/((7\*I + Sqrt[23])\*(1 - I\*Sqrt[23] - 4\*x)^2)]

### Rule 935

Int[1/(Sqrt[(d\_.) + (e\_.)\*(x\_.)]\*Sqrt[(f\_.) + (g\_.)\*(x\_.)]\*Sqrt[(a\_.) + (b\_.)\*(x\_.) + (c\_.)\*(x\_.)^2]), x\_Symbol] := Dist[(-2\*(d + e\*x)\*Sqrt[((e\*f - d\*g)^2\*(a + b\*x + c\*x^2))/((c\*f^2 - b\*f\*g + a\*g^2)\*(d + e\*x)^2)]/((e\*f - d\*g)\*Sqrt[a + b\*x + c\*x^2]), Subst[Int[1/Sqrt[1 - ((2\*c\*d\*f - b\*e\*f - b\*d\*g + 2\*a\*e\*g)\*x^2)/(c\*f^2 - b\*f\*g + a\*g^2) + ((c\*d^2 - b\*d\*e + a\*e^2)\*x^4)/(c\*f^2 - b\*f\*g + a\*g^2)], x], x, Sqrt[f + g\*x]/Sqrt[d + e\*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0]

### Rule 992

Int[1/(Sqrt[(a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]\*Sqrt[(d\_) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2]), x\_Symbol] := With[{r = Rt[b^2 - 4\*a\*c, 2]}, Dist[(Sqrt[b + r + 2\*c\*x]\*Sqrt[2\*a + (b + r)\*x])/Sqrt[a + b\*x + c\*x^2], Int[1/(Sqrt[b + r + 2\*c\*x]\*Sqrt[2\*a + (b + r)\*x]\*Sqrt[d + e\*x + f\*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0]

### Rule 1103

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4], x\_Symbol] := With[{q = Rt[c/a, 4]}, Simp[((1 + q^2\*x^2)\*Sqrt[(a + b\*x^2 + c\*x^4)/(a\*(1 + q^2\*x^2)^2)]\*EllipticF[2\*ArcTan[q\*x], 1/2 - (b\*q^2)/(4\*c)]/(2\*q\*Sqrt[a + b\*x^2 + c\*x^4]), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && PosQ[c/a]

Rubi steps

$$\int \frac{1}{\sqrt{3-x+2x^2}\sqrt{2+3x+5x^2}} dx = \frac{\left(\sqrt{-1+i\sqrt{23}+4x}\sqrt{6+(-1+i\sqrt{23})x}\right) \int \frac{1}{\sqrt{-1+i\sqrt{23}+4x}\sqrt{6+(-1+i\sqrt{23})x}\sqrt{2+3x+5x^2}} dx}{\sqrt{3-x+2x^2}}$$

$$= \frac{\left(2(-1+i\sqrt{23}+4x)^{3/2}\sqrt{6+(-1+i\sqrt{23})x}\sqrt{\frac{(24-(-1+i\sqrt{23}))^2(2+3x+5x^2)}{(180-18(-1+i\sqrt{23}))+2(-1+i\sqrt{23}))}}\right)}{(24-(-1+i\sqrt{23}))^2}$$

$$= \frac{\sqrt{\frac{23}{11}}(-1+i\sqrt{23}+4x)^{3/2}\sqrt{6-(1-i\sqrt{23})x}\sqrt{\frac{(11i-\sqrt{23})(2+3x+5x^2)}{(7i+\sqrt{23})(1-i\sqrt{23}-4x)^2}}}{(23+i\sqrt{23})^4\sqrt{-\frac{3i-\sqrt{23}}{7i+\sqrt{23}}}\sqrt{3}}$$

**Mathematica [A]** time = 0.61, size = 390, normalized size = 0.60

$$\frac{(-4x + i\sqrt{23} + 1)(10ix + \sqrt{31} + 3i) \sqrt{\frac{20ix - 2\sqrt{31} + 6i}{(11i + 5\sqrt{23} - 2\sqrt{31})(4ix + \sqrt{23} - i)}} \sqrt{\frac{(-22 - 10i\sqrt{23} + 4i\sqrt{31})x - \sqrt{713} - i\sqrt{31} - 3i\sqrt{23} + 63}{(11i + 5\sqrt{23} + 2\sqrt{31})(4ix + \sqrt{23} - i)}} F\left(\sin^{-1}\left(\frac{(-11i + 5\sqrt{23} - 2\sqrt{31}) \sqrt{\frac{10ix + \sqrt{31} + 3i}{(11i + 5\sqrt{23} + 2\sqrt{31})(4ix + \sqrt{23} - i)}}}{\sqrt{2x^2 + 3x + 5}}\right)\right)}{\sqrt{3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(Sqrt[3 - x + 2\*x^2]\*Sqrt[2 + 3\*x + 5\*x^2]),x]

[Out] ((1 + I\*Sqrt[23] - 4\*x)\*(3\*I + Sqrt[31] + (10\*I)\*x)\*Sqrt[(6\*I - 2\*Sqrt[31] + (20\*I)\*x)/((11\*I + 5\*Sqrt[23] - 2\*Sqrt[31])\*(-I + Sqrt[23] + (4\*I)\*x))]\*Sqrt[(63 - (3\*I)\*Sqrt[23] - I\*Sqrt[31] - Sqrt[713] + (-22 - (10\*I)\*Sqrt[23] + (4\*I)\*Sqrt[31])\*x]/((11\*I + 5\*Sqrt[23] + 2\*Sqrt[31])\*(-I + Sqrt[23] + (4\*I)\*x)))\*EllipticF[ArcSin[Sqrt[2]\*Sqrt[-((-63 + (3\*I)\*Sqrt[23] + I\*Sqrt[31] + Sqrt[713] + 2\*(11 + (5\*I)\*Sqrt[23] - (2\*I)\*Sqrt[31])\*x)/((11\*I + 5\*Sqrt[23] + 2\*Sqrt[31])\*(-I + Sqrt[23] + (4\*I)\*x))]]], (1197 + 41\*Sqrt[713])/484]

$$\frac{((-11*I + 5*\sqrt{23} - 2*\sqrt{31})*\sqrt{(3*I + \sqrt{31} + (10*I)*x)} / ((11*I + 5*\sqrt{23} + 2*\sqrt{31})*(-I + \sqrt{23} + (4*I)*x))) * \sqrt{3 - x + 2*x^2}}{\sqrt{2 + 3*x + 5*x^2}}$$

**fricas** [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{5x^2 + 3x + 2}\sqrt{2x^2 - x + 3}}{10x^4 + x^3 + 16x^2 + 7x + 6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5\*x^2+3\*x+2)^(1/2)/(2\*x^2-x+3)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(5\*x^2 + 3\*x + 2)\*sqrt(2\*x^2 - x + 3)/(10\*x^4 + x^3 + 16\*x^2 + 7\*x + 6), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{5x^2 + 3x + 2}\sqrt{2x^2 - x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5\*x^2+3\*x+2)^(1/2)/(2\*x^2-x+3)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(5\*x^2 + 3\*x + 2)\*sqrt(2\*x^2 - x + 3)), x)

**maple** [A] time = 0.47, size = 418, normalized size = 0.64

$$\frac{4i\sqrt{5x^2 + 3x + 2}\sqrt{2x^2 - x + 3}(5i\sqrt{23} + 2i\sqrt{31} - 11)\sqrt{\frac{(5i\sqrt{23} - 2i\sqrt{31} + 11)(4x - 1 + i\sqrt{23})}{(5i\sqrt{23} + 2i\sqrt{31} - 11)(-4x + i\sqrt{23} + 1)}}}{23\sqrt{10x^4 + x^3 + 16x^2 + 7x + 6}(5i\sqrt{23} - 2i\sqrt{31} - 11)}(-4x + i\sqrt{23} + 1)^2\sqrt{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(5\*x^2+3\*x+2)^(1/2)/(2\*x^2-x+3)^(1/2),x)

[Out] 
$$-4/23*I*(5*x^2+3*x+2)^{(1/2)}*(2*x^2-x+3)^{(1/2)}*(5*I*23^{(1/2)}+2*I*31^{(1/2)}-11)*((5*I*23^{(1/2)}-2*I*31^{(1/2)}+11)*(-1+4*x+I*23^{(1/2)})/(5*I*23^{(1/2)}+2*I*31^{(1/2)}-11)/(I*23^{(1/2)}-4*x+1))^{(1/2)}*(I*23^{(1/2)}-4*x+1)^2*(-I*23^{(1/2)}*(I*31^{(1/2)}+10*x+3)/(5*I*23^{(1/2)}-2*I*31^{(1/2)}-11)/(I*23^{(1/2)}-4*x+1))^{(1/2)}*(I*23^{(1/2)}*(I*31^{(1/2)}-10*x-3)/(5*I*23^{(1/2)}+2*I*31^{(1/2)}-11)/(I*23^{(1/2)}-4*x+1))^{(1/2)}*23^{(1/2)}*10^{(1/2)}*\text{EllipticF}(((5*I*23^{(1/2)}-2*I*31^{(1/2)}+11)*(-1+4*x+I*23^{(1/2)})/(5*I*23^{(1/2)}+2*I*31^{(1/2)}-11)/(I*23^{(1/2)}-4*x+1))^{(1/2)}, ((5*I*23^{(1/2)}+2*I*31^{(1/2)}+11)*(5*I*23^{(1/2)}+2*I*31^{(1/2)}-11)/(5*I*23^{(1/2)}-2*I*31^{(1/2)}-11))^{(1/2)}, (5*I*23^{(1/2)}-2*I*31^{(1/2)}+11)/(5*I*23^{(1/2)}+2*I*31^{(1/2)}-11))^{(1/2)}, (5*I*23^{(1/2)}+2*I*31^{(1/2)}-11)/(5*I*23^{(1/2)}-2*I*31^{(1/2)}-11))^{(1/2)}, (5*I*23^{(1/2)}-2*I*31^{(1/2)}+11)/(5*I*23^{(1/2)}+2*I*31^{(1/2)}-11))^{(1/2)}, (5*I*23^{(1/2)}+2*I*31^{(1/2)}-11)/(5*I*23^{(1/2)}-2*I*31^{(1/2)}-11))^{(1/2)})$$

$$\frac{2\sqrt{31}-11}{(5\sqrt{23}-2\sqrt{31}+11)^{1/2}} \cdot \frac{1}{(10x^4+x^3+16x^2+7x+6)^{1/2}} \cdot \frac{1}{(5\sqrt{23}-2\sqrt{31}+11)} \cdot \frac{1}{((-1+4x+\sqrt{23})\sqrt{23}-4x+1)\sqrt{31}+10x+3} \cdot \frac{1}{(\sqrt{31}-10x-3)^{1/2}}$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{5x^2+3x+2}\sqrt{2x^2-x+3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5\*x^2+3\*x+2)^(1/2)/(2\*x^2-x+3)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(5\*x^2 + 3\*x + 2)\*sqrt(2\*x^2 - x + 3)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{2x^2-x+3}\sqrt{5x^2+3x+2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((2\*x^2 - x + 3)^(1/2)\*(3\*x + 5\*x^2 + 2)^(1/2)),x)

[Out] int(1/((2\*x^2 - x + 3)^(1/2)\*(3\*x + 5\*x^2 + 2)^(1/2)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^2-x+3}\sqrt{5x^2+3x+2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5\*x\*\*2+3\*x+2)\*\*(1/2)/(2\*x\*\*2-x+3)\*\*(1/2),x)

[Out] Integral(1/(sqrt(2\*x\*\*2 - x + 3)\*sqrt(5\*x\*\*2 + 3\*x + 2)), x)



# Chapter 4

## Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
```

```

If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
  If[LeafCount[result]<=2*LeafCount[optimal],
    "A",
    "B"],
  "C"],
If[FreeQ[result,Integrate] && FreeQ[result,Int],
  "C",
"F"]]

```

```

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)

```

```

ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
        Max[ExpnType[expn[[1]],2]],
      Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
    If[Head[expn]===Plus || Head[expn]===Times,
      Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
    If[ElementaryFunctionQ[Head[expn]],
      Max[3,ExpnType[expn[[1]]],
    If[SpecialFunctionQ[Head[expn]],
      Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
    If[HypergeometricFunctionQ[Head[expn]],
      Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
    If[AppellFunctionQ[Head[expn]],
      Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],

```

```

If[Head[expn]===RootSum,
  Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
If[Head[expn]===Integrate || Head[expn]===Int,
  Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
9]]]]]]]]]]

ElementaryFunctionQ[func_] :=
MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]

SpecialFunctionQ[func_] :=
MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]

HypergeometricFunctionQ[func_] :=
MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]

AppellFunctionQ[func_] :=
MemberQ[{AppellF1},func]

```

## 4.0.2 Maple grading function

```

# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000

```

```

#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
# see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
  debug:=false;

  leaf_count_result:=leafcount(result);
  #do NOT call ExpnType() if leaf size is too large. Recursion problem
  if leaf_count_result > 500000 then
    return "B";
  fi;

  leaf_count_optimal:=leafcount(optimal);

  ExpnType_result:=ExpnType(result);
  ExpnType_optimal:=ExpnType(optimal);

  if debug then
    print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
ExpnType_optimal);
  fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
# is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
# antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
  return "F";
end if;

if ExpnType_result<=ExpnType_optimal then
  if debug then
    print("ExpnType_result<=ExpnType_optimal");
  fi;

```

```

if is_contains_complex(result) then
  if is_contains_complex(optimal) then
    if debug then
      print("both result and optimal complex");
    fi;
    #both result and optimal complex
    if leaf_count_result<=2*leaf_count_optimal then
      return "A";
    else
      return "B";
    end if
  else #result contains complex but optimal is not
    if debug then
      print("result contains complex but optimal is not");
    fi;
    return "C";
  end if
else # result do not contain complex
  # this assumes optimal do not as well
  if debug then
    print("result do not contain complex, this assumes optimal do not
as well");
  fi;
  if leaf_count_result<=2*leaf_count_optimal then
    if debug then
      print("leaf_count_result<=2*leaf_count_optimal");
    fi;
    return "A";
  else
    if debug then
      print("leaf_count_result>2*leaf_count_optimal");
    fi;
    return "B";
  end if
end if
else #ExpnType(result) > ExpnType(optimal)
  if debug then
    print("ExpnType(result) > ExpnType(optimal)");
  fi;
  return "C";
end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false

```

```

#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  end if
end proc:

```

```

elif HypergeometricFunctionQ(op(0,expn)) then
  max(5,apply(max,map(ExpnType,[op(expn)])))
elif AppellFunctionQ(op(0,expn)) then
  max(6,apply(max,map(ExpnType,[op(expn)])))
elif op(0,expn)='int' then
  max(8,apply(max,map(ExpnType,[op(expn)]))) else
9
end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,
    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
    MmaTranslator[Mma][LeafCount](u);
end proc:
```

### 4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
        asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
        asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
        fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
        gamma,loggamma,digamma,zeta,polylog,LambertW,
        elliptic_f,elliptic_e,elliptic_pi,exp_polar
        ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```



```

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'`^`')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)
    ))
    else:
        return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))

```

```

elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif is_appell_function(expn.func):
    m1 = max(map(expnType, list(expn.args)))
    return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif isinstance(expn,RootSum):
    m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
Apply[List,expn]],7]],
    return max(7,m1)
elif str(expn).find("Integral") != -1:
    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```

## 4.0.4 SageMath grading function

```

#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fracas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:

```

```

        print ("func ", func , " is elementary_function")
    else:
        print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in
sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:

```

```

        return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(), "base_ring") and hasattr(expn.parent(), "gens"):
        return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
    return False

except AttributeError as error:
    return False

def expnType(expn):

    if debug:
        print(">>>>Enter expnType, expn=", expn)
        print(">>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #instance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(instance(expn.args[0],
Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
    elif expn.operator() == operator.pow: #instance(expn,Pow)
        if type(expn.operands()[1])==Integer: #instance(expn.args[1],Integer)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #instance(expn.args[1],
Rational)
            if type(expn.operands()[0])==Rational: #instance(expn.args[0],
Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
instance(expn,Add) or instance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))

```

```

    return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()): #is_elementary_function(expn.
func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(4,m1) #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(5,m1) #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(6,m1) #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
#is checked before calling the grading function that is passed.
#but kept it here.
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

```

```
#main function
```

```
def grade_antiderivative(result,optimal):
```

```

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex

```

```
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
    else: #result contains complex but optimal is not
        return "C"
else: # result do not contain complex, this assumes optimal do not as
well
    if leaf_count_result <= 2*leaf_count_optimal:
        return "A"
    else:
        return "B"
else:
    return "C"
```